Chapter 2: Probability

Course Name: PROBABILITY & STATISTICS

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1. Sample Spaces and Events
   - Random Experiments
   - Sample Spaces
   - Events
   - Counting Techniques
2. Interpretations and Axioms of Probability
3. Addition Rules
4. Conditional probability
5. Multiplication and Total Probability Rules
6. Independence
7. Bayes' Theorem
8. Random Variables

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1. **Sample Spaces and Events**
   - Random Experiments
   - Sample Spaces
   - Events
   - Counting Techniques
2. **Interpretations and Axioms of Probability**
3. **Addition Rules**
4. **Conditional probability**
5. **Multiplication and Total Probability Rules**
6. **Independence**
7. **Bayes' Theorem**
8. **Random Variables**

1.1 Random Experiments

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Examples of random experiment:

- Tossing a coin and watching the face appear;
- Measuring rainfall in Hanoi city in January;
- Randomly selecting 10 people and measuring their height.

1.2 Sample Spaces

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- A sample space is **discrete** if it consists of a finite or countable infinite set of outcomes.
- A sample space is **continuous** if it contains an interval (either finite or infinite) of real numbers.

Example 1

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$$S = \{yes, no\},$$
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- If the objective of the analysis is to consider only whether or not the parts conform to the manufacturing specifications, the sample space can be represented by the four outcomes
$$S = \{yy, yn, ny, nn\}.$$

Tree Diagrams

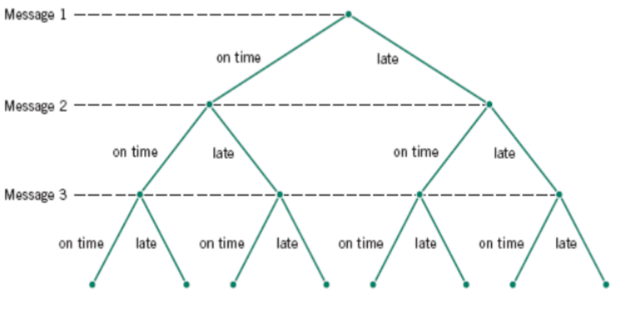
Sample spaces can also be described graphically with **tree diagrams**.

- When a sample space can be constructed in several steps or stages, we can represent each of the n_1 ways of completing the first step as a branch of a tree.
- Each of the ways of completing the second step can be represented as n_2 branches starting from the ends of the original branches, and so forth.

Example

Each message in a digital communication system is classified as to whether it is received within the time specified by the system design. If three messages are classified, use a tree diagram to represent the sample space of possible outcomes.

Each message can be received either on time or late. The possible results for three messages can be displayed by eight branches in the tree diagram.



1.3 Events

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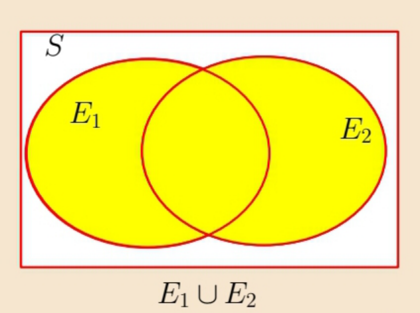
Examples of event:
- The event of coming up the head when tossing a coin $A = \{h\}$;
- Let B event that two connectors which at least one part conform to the manufacturing specifications

$$B = \{yn, ny, yy\}$$

1.3 Events

Set operations with events:

- The **union** of two events is the event that consists of all outcomes that are contained in either of the two events. We denote the union as $E_1 \cup E_2$.



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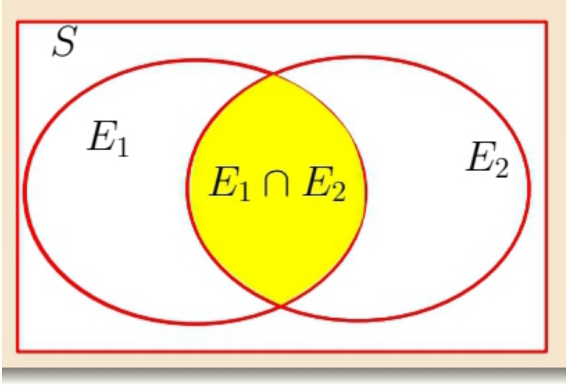
- The **intersection** of two events is the event that consists of all outcomes that are contained in both of the two events. We denote the intersection as $E_1 \cap E_2$.



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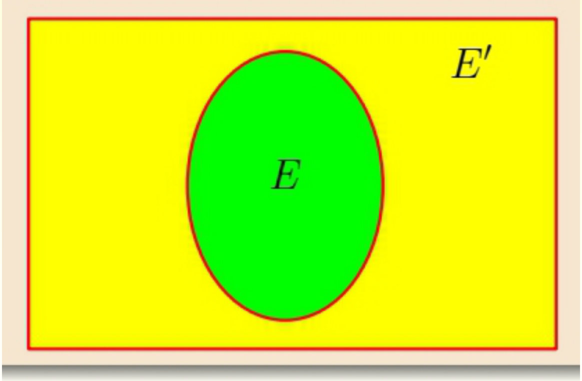
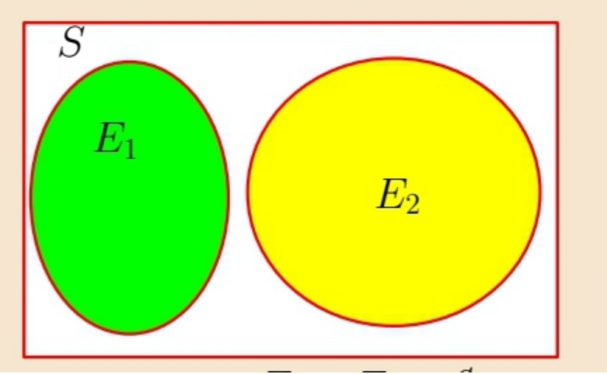
- The **complement** of an event in a sample space is the set of outcomes in the sample space that are not in the event. We denote the complement of the event E as E'.



1.3 Events

- Two events, denoted as E_1 and E_2, such that $E_1 \cap E_2 = \varnothing$ are said to be **mutually exclusive**.



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Consider the sample space $S = \{yy, yn, ny, nn\}$ in Example 2. Suppose that the subset of outcomes for which at least one part conforms is denoted as E_1.

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If $E_3 = \{yn, ny, nn\}$,

$$E_1 \cup E_3 = S, \;\; E_1 \cap E_3 = \{yn, ny\}, \;\; E_1' = \{nn\} = E_2.$$

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If $E_3 = \{yn, ny, nn\}$,

$$E_1 \cup E_3 = S,\ \ E_1 \cap E_3 = \{yn, ny\},\ \ E_1' = \{nn\} = E_2.$$

Note

- $(E')' = E$.
- $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$ and $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$.
- DeMorgan's laws imply that $(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$.

1.4 Counting Techniques

Multiplication Rule (for counting techniques)

Assume an operation can be described as a sequence of k steps, and the number of ways of completing **step 1** is n_1, and

the number of ways of completing **step 2** is n_2 for each way of completing step 1, and

the number of ways of completing **step 3** is n_3 for each way of completing step 2, and

so forth.

The total number of ways of completing the operation is

$$n_1 \times n_2 \times \ldots \times n_k.$$

Example 4

In the design of a casing for a gear housing, we can use four different types of fasteners, three different bolt lengths, and three different bolt locations. From the multiplication rule $4 \times 3 \times 3 = 36$ different designs are possible.

Permutations

Consider a set of elements, such as $S = \{a, b, c\}$. A **permutation** of the elements is an ordered sequence of the elements. For example, abc, acb, bac, bca, cab, and cba are all of the permutations of the elements of S.

The number of **permutations** of n different elements is $n!$ where

$$n! = n \times (n-1) \times ... \times 2 \times 1.$$

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Permutations of Subsets

The number of permutations of subsets of r elements selected from a set of n different elements is

$$P_r^n = n \times (n-1) \times ... \times (n-r+1) = \frac{n!}{(n-r)!}.$$

Combinations

Another counting problem of interest is the number of subsets of r elements that can be selected from a set of n elements. Here, order is not important. These are called **combinations**.

The number of combinations is denoted as $C_r^n = \frac{n!}{r!(n-r)!}$.

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  Therefore, $P_4^8 = 8 \times 7 \times 6 \times 5 = 1680$ different designs are possible.
- If 5 identical components are to be placed on the board, the number of possible designs is $C_5^8 = \frac{8!}{5!3!} = 56$.

Sample Spaces and Events

Exercise 1

Provide a reasonable description of the sample space for each of the random experiments:

a) Each of three machined parts is classified as either above or below the target specification for the part.

b) The number of hits (views) is recorded at a high-volume Web site in a day.

c) Each of 24 Web sites is classified as containing or not containing banner ads

d) The time of a chemical reaction is recorded to the nearest millisecond.

e) Calls are repeatedly placed to a busy phone line until a connection is achieved.

Exercise 2

A digital scale is used that provides weights to the nearest gram.

a) What is the sample space for this experiment?

Let A denote the event that a weight exceeds 11 grams, let B denote the event that a weight is less than or equal to 15 grams, and let C denote the event that a weight is greater than or equal to 8 grams and less than 12 grams.

Describe the following events.

b) $A \cup B$ c) $A \cap B$

d) A' e) $A \cup B \cup C$

f) $(A \cup C)'$ g) $B' \cap C$

Sample Spaces and Events

Exercise 3

A sample of two items is selected without replacement from a batch. Describe the (ordered) sample space for each of the following batches:
a) The batch contains the items $\{a, b, c, d\}$.
b) The batch contains the items $\{a, b, c, d, e, f, g\}$.
c) The batch contains 4 defective items and 20 good items.
d) The batch contains 1 defective item and 20 good items.

Sample Spaces and Events

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d) The batch contains 1 defective item and 20 good items.

Exercise 4

A batch of 140 semiconductor chips is inspected by choosing a sample of five chips. Assume 10 of the chips do not conform to customer requirements.
a) How many different samples are possible?
b) How many samples of five contain exactly one nonconforming chip?
c) How many samples of five contain at least one nonconforming chip?

Question 1

Which of the following assignments of probabilities to the sample points A, B, C and D is valid if A, B, C, and D are the only sample points in the experiment?

Select one:

- a. $P(A) = 1/9$, $P(B) = 1/4$, $P(C) = 1/2$, $P(D) = 0$
- b. $P(A) = 0$, $P(B) = 1/14$, $P(C) = 13/14$, $P(D) = 0$
- c. $P(A) = -1/4$, $P(B) = 1/2$, $P(C) = 3/4$, $P(D) = 1$
- d. $P(A) = 1/5$, $P(B) = 1/5$, $P(C) = 1/5$, $P(D) = -1$

Question 2

An experiment consists of randomly choosing a number between 1 and 10. Let *E* be the event that the number chosen is odd. List the sample points in *E*.

Select one:

- a. {2, 4, 6, 8, 10}
- b. {5}
- c. {1, 3, 5, 7, 9}
- d. {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

Question 3

If sample points *A*, *B*, *C*, and *D* are the only possible outcomes of an experiment, find the probability of *D* using the table below.

Sample Point	*A*	*B*	*C*	*D*
Probability	1/5	1/5	1/5	

Select one:

- a. $\frac{1}{5}$
- b. $\frac{2}{5}$
- c. $\frac{1}{4}$
- d. $\frac{3}{5}$

Question 4

The outcome of an experiment is the number of resulting heads when a nickel and a dime are flipped simultaneously. What is the sample space for this experiment?

Select one:

- a. {HH, HT, TT}
- b. {0, 1, 2}
- c. {HH, HT, TH, TT}
- d. {nickel, dime}

Question 5

A bag of colored candies contains 20 red, 25 yellow, 15 blue and 20 orange candies. An experiment consists of randomly choosing one candy from the bag and recording its color. What is the sample space for this experiment?

Select one:

- a. {20, 25, 15, 20}
- b. {80}
- c. {red, yellow, blue, orange}
- d. {red, yellow, orange}
- e. {1/4, 5/16, 7/16}

Question 6

Flip a coin twice, create the sample space of possible outcomes (H: Head, T: Tail).

Select one:

- a. HT TH
- b. HH HT TT
- c. HH HT TH TT
- d. HH TT HT HT

Question 7

Flip a coin three times, create the sample space of possible outcomes (H: Head, T: Tail).

Select one:

- a. HHH HHT HTH HTT THH THT TTH TTT
- b. HTT THT HTH HHH TTH TTT
- c. HHH HTT HTH TTT HTT THH HHT THT
- d. HHH TTT THT HTH HHT TTH HTH

Question 8

Hahn is having his sixth litter. The prior litters have either been three normal pups or two normal pups and a runt. Assume the probability of either outcome is 50%.

Create the sample space of possible outcomes (Normal: N, Runt: R).

Select one:

- a. NR NNR NNR
- b. NNR NNN
- c. NNN RNN NR
- d. N NN NR NNN NRN

Question 9

Both Nualart and Tom have a bag of candy containing a lollipop (LP), a cherry drop (CD), and a lemon drop (LD). Each takes out a piece and eats it. What are the possible pairs of candies eaten?

Create the sample space of possible outcomes.

Select one:

- a. CD-LD LD-LP LP-CD LP-LP LD-LD
- b. LD-LD CD-LD LP-LP LD-LP CD-CD LD-LP LP-CD CD-LP LP-LD
- c. LD-LD CD-LD LP-LP LD-CD CD-CD LD-LP LP-CD CD-LP LP-LD
- d. LD-CD LD-CD LD-CD LD-LP LD-LP LD-LP CD-LP CD-LP CD-LP

Question 10

Two white sheep mate. The male has both a white and a black fur-color gene. The female has only white fur-color genes. The fur color of the offspring depends on the pairs of fur-color genes that they receive. Assume that neither the white nor the black gene dominates. List the possible outcomes. W = white and B = black.

Select one:

- a. WW, WW
- b. WW, BW
- c. WB, BW
- d. WW, BB

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2 **Interpretations and Axioms of Probability**

3 Addition Rules

4 Conditional probability

5 Multiplication and Total Probability Rules

6 Independence

7 Bayes' Theorem

8 Random Variables

2. Interpretations and Axioms of Probability

- **Probability** is used to quantify the likelihood, or chance, that an outcome of a random experiment will occur.
- The likelihood of an outcome is quantified by assigning a number from the interval $[0, 1]$ to the outcome.
  Higher numbers indicate that the outcome is more likely than lower numbers.
  A probability of 0 indicates an outcome will not occur.
  A probability of 1 indicates an outcome will occur with certainty.

2. Interpretations and Axioms of Probability

Equally Likely Outcomes

Whenever a sample space consists of N possible outcomes that are equally likely, the probability of each outcome is $\frac{1}{N}$.

Probability of an Event

- For a discrete sample space, the probability of an event E, denoted as $P(E)$, equals the sum of the probabilities of the outcomes in E.
- When the model of equally likely outcomes is assumed, then

$$P(E) = \frac{k}{N},$$

where k, N are the numbers of elements of E, S, respectively.

2. Interpretations and Axioms of Probability

Properties of Probability

If S is the sample space and E is any event in a random experiment.

- $P(S) = 1$.
- $P(\varnothing) = 0$.
- $P(E^{'}) = 1 - P(E)$.
- $0 \leq P(E) \leq 1$.
- For two events E_1 and E_2 with $E_1 \cap E_2 = \varnothing$:

$$P(E_1 \cup E_2) = P(E_1) + P(E_2).$$

Example 6

A random experiment can result in one of the outcomes $\{a, b, c, d, e\}$ with probabilities 0.1; 0.1; 0.2; 0.4, and 0.2, respectively. Let A denote the event $\{a, b, c\}$, and let B denote the event $\{c, d, e\}$. Determine the following:

a) $P(A), P(B), P(A')$;

b) $P(A \cup B), P(A \cap B)$.

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a) $P(A), P(B), P(A^{'})$;

b) $P(A \cup B), P(A \cap B)$.

Answer: $P(A) = 0.1 + 0.1 + 0.2 = 0.4$, $P(B) = 0.2 + 0.4 + 0.2 = 0.8$, $P(A^{'}) = 0.6$, $P(A \cup B) = P(S) = 1$, $P(A \cap B) = 0.2$.

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Example 7

If the last digit of a weight measurement is equally likely to be any of the digits 0 through 9,
a) What is the probability that the last digit is 0? b) What is the probability that the last digit is greater than or equal to 5?

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A random experiment can result in one of the outcomes $\{a, b, c, d, e\}$ with probabilities 0.1; 0.1; 0.2; 0.4, and 0.2, respectively. Let A denote the event $\{a, b, c\}$, and let B denote the event $\{c, d, e\}$. Determine the following:
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a) What is the probability that the last digit is 0? b) What is the probability that the last digit is greater than or equal to 5?
Answer: a) $P(A) = 0.1$, b) $P(B) = 0.5$.

Exercise 1

Suppose your vehicle is licensed in a state that issues license plates that consist of three digits (between 0 and 9) followed by three letters (between A and Z). If a license number is selected randomly, what is the probability that yours is the one selected?

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Exercise 2

A part selected for testing is equally likely to have been produced on any one of six cutting tools.

a) What is the sample space?

b) What is the probability that the part is from tool 1?

c) What is the probability that the part is from tool 3 or tool 5?

d) What is the probability that the part is not from tool 4?

Exercise 3

Orders for a computer are summarized by the optional features that are requested as follows:

	proportion of orders
no optional features	0.3
one optional feature	0.5
more than one optional feature	0.2

a) What is the probability that an order requests at least one optional feature?
b) What is the probability that an order does not request more than one optional feature?

Exercise 4

Samples of emissions from three suppliers are classified for conformance to air-quality specifications. The results from 100 samples are summarized as follows:

		conforms	
		yes	no
	1	22	8
supplier	2	25	5
	3	30	10

Let A denote the event that a sample is from supplier 1, and let B denote the event that a sample conforms to specifications. If a sample is selected at random, determine the following probabilities:

(a) $P(A)$ (b) $P(B)$
(c) $P(A')$ (d) $P(A \cap B)$
(e) $P(A \cup B)$ (f) $P(A' \cup B)$

Question 1

The probability that a house in an urban area will be burglarized is 3%. If 30 houses are randomly selected, what is the probability that none of the houses will be burglarized?

Select one:

- a. 0.557
- b. 0.020
- c. 0.4010
- d. 0.001

Question 2

Sixty-five percent of men consider themselves knowledgeable football fans. If 15 men are randomly selected, find the probability that exactly five of them will consider themselves knowledgeable fans.

Select one:

- a. 0.0096
- b. 0.3853
- c. 0.6541
- d. 0.0341

Question 3

Assume that male and female births are equally likely and that the birth of any child does not affect the probability of the gender of any other children. Find the probability of at most two boys in five births.

Select one:

- a. 0.172
- b. 0.500
- c. 0.300
- d. 0.333

Question 4

A random number generator is set top generate integer random numbers between 0 and 9 inclusive following a uniform distribution. What is the probability of the random number generator generating a 6?

Select one:

- a. 1/9
- b. 0.07
- c. 1/2
- d. 1/10

Question 5

Sixty percent of the people that get mail-order catalogs order something. Find the probability that only three of 8 people getting these catalogs will order something.

Select one:

- a. 0.300
- b. 0.124
- c. 0.001
- d. 0.117

Question 6

The probability that a tennis set will go to a tie-breaker is 15%. What is the probability that two of three sets will go to tie-breakers?

Select one:

- a. 0.0289
- b. 0.072
- c. 0.057
- d. 0.351

Question 7

For two events A and B, P(A) = 0.4, P(B) = 0.3, and P(A and B) = 0. It follows that A and B are

Select one:

- a. neither disjoint nor independent.
- b. complementary.
- c. disjoint but not independent.
- d. both disjoint and independent.

Question 8

For two events A and B, P(A) = 0.8, P(B) = 0.2, and P(A and B) = 0.16. It follows that A and B are

Select one:

- a. neither disjoint nor independent.
- b. disjoint but not independent.
- c. both disjoint and independent.
- d. independent but not disjoint.

Question 9

At a Ohio college, 25% of students speak Spanish, 5% speak French, and 3% speak both languages. What is the probability that a student chosen at random from the college speaks Spanish but not French?

Select one:

- a. 0.22
- b. 0.24
- c. 0.19
- d. 0.17

Question 10

If two balanced die are rolled, the possible outcomes can be represented as follows.

(1, 1) (2, 1) (3, 1) (4, 1) (5, 1) (6, 1)
(1, 2) (2, 2) (3, 2) (4, 2) (5, 2) (6, 2)
(1, 3) (2, 3) (3, 3) (4, 3) (5, 3) (6, 3)
(1, 4) (2, 4) (3, 4) (4, 4) (5, 4) (6, 4)
(1, 5) (2, 5) (3, 5) (4, 5) (5, 5) (6, 5)
(1, 6) (2, 6) (3, 6) (4, 6) (5, 6) (6, 6)

Determine the probability that the sum of the dice is 7.

Select one:

- a. 3/12
- b. 1/6
- c. 5/36
- d. 2/9

Question 11

A committee of three people is to be formed. The three people will be selected from a list of six possible committee members. A simple random sample of three people is taken, without replacement, from the group of six people. Using the letters A, B, C, D, E, F to represent the six people, list the possible samples of size three and use your list to determine the probability that B is included in the sample.

(*Hint*: There are 20 possible samples.)

Select one:

- a. 3/5
- b. 1/5
- c. 2/5
- d. 7/10
- e. 1/2

Question 12

Brandon and Samantha each carry a bag containing a banana, a chocolate bar, and a licorice stick. Simultaneously, they take out a single food item and consume it. The possible pairs of food items that Brandon and Samantha consumed are as follows.

chocolate bar - chocolate bar
licorice stick - chocolate bar
banana - banana
chocolate bar - licorice stick
licorice stick - licorice stick
chocolate bar - banana
banana - licorice stick
licorice stick - banana
banana - chocolate bar

Find the probability that exactly one chocolate bar was eaten.

Select one:

- a. 5/9
- b. 4/9
- c. 1/3
- d. 1/2

Question 13

Suppose that on a particular multiple choice question, 96% of the students answered correctly. What is the probability that a randomly selected student answered the question incorrectly?

Select one:

- a. 0.48
- b. 0.04
- c. 0.96
- d. 0.14

Question 14

The distribution of B.A. degrees conferred by a local college is listed below, by major.

Major	Frequency
Engineering	868
English	2073
Mathematics	2164
Chemistry	318
Physics	856
Liberal Arts	1358
Business	1676

What is the probability that a randomly selected degree is not in Chemistry?

Select one:

- a. 0.966
- b. 0.232
- c. 0.697
- d. 0.768

Question 15

Forty percent of babies born in the U.S. in 2004 were still being breastfed at 6 months of age. If 4 children who were born in the U.S. in 2004 are randomly selected, what is the probability that none of them were breastfed for at least 6 months?

Select one:

- a. 0.4173
- b. 0.1171
- c. 0.5851
- d. 0.1296

Question 16

Which of the following is always true?

Select one:

- a. If P(A and B) = P(A or B), then A and B are independent.
- b. If A and B are disjoint, then they cannot be independent.
- c. If A and B are disjoint, P(A) + P(B) = 1
- d. If P(A and B) = 0, then A and B are independent.

Content

1. Sample Spaces and Events
   - Random Experiments
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3. **Addition Rules**
4. Conditional probability
5. Multiplication and Total Probability Rules
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3. Addition Rules

Addition Rules

- Any two events $A; B$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

- If A and B are mutually exclusive events,

$$P(A \cup B) = P(A) + P(B).$$

- A collection of events $E_1, E_2, ..., E_k$ is said to be mutually exclusive if for all pairs

$$E_i \cap E_j \neq \varnothing,$$

For a collection of mutually exclusive events,

$$P(E_1 \cup E_2 \cup ... \cup E_k) = P(E_1) + P(E_2) + ... + P(E_k).$$

- Any three events A, B, C

$$P(A \cup B \cup C) =$$
$$P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(A\cap C)+P(A\cap B\cap C).$$

3. Addition Rules

Example 8

If A, B, and C are mutually exclusive events with $P(A) = 0.2, P(B) = 0.3$, and $P(C) = 0.4$, determine the following probabilities:

a) $P(A \cup B \cup C)$
b) $P(A \cap B \cap C)$
c) $P(A \cap B)$
d) $P[(A \cup B) \cap C]$
e) $P(A' \cap B' \cap C')$

3. Addition Rules

Exercise 1

If $P(A) = 0.3, P(B) = 0.2$ and $P(A \cap B) = 0.1$ determine the following probabilities:

a) $P(A^{'})$ b) $P(A \cup B)$ c) $P(A^{'} \cap B)$

d) $P(A \cap B^{'})$ e) $P[(A \cup B)^{'}]$ f) $P(A^{'} \cup B)$

Exercise 2

Disks of polycarbonate plastic from a supplier are analyzed for scratch and shock resistance. The results from 100 disks are summarized as follows:

		shock resistance	
		high	low
scratch	high	70	9
resistance	low	16	5

a) If a disk is selected at random, what is the probability that its scratch resistance is high and its shock resistance is high?
b) If a disk is selected at random, what is the probability that its scratch resistance is high or its shock resistance is high?
c) Consider the event that a disk has high scratch resistance and the event that a disk has high shock resistance. Are these two events mutually exclusive?

3. Addition Rules

Exercise 3

A computer system uses passwords that are six characters and each character is one of the 26 letters (a–z) or 10 integers (0–9). Uppercase letters are not used. Let A denote the event that a password begins with a vowel (either a, e, i, o, or u) and let Bdenote the event that a password ends with an even number (either 0, 2, 4, 6, or 8). Suppose a hacker selects a password at random. Determine the following probabilities:

a) $P(A)$ b) $P(B)$

c) $P(A \cap B)$ d) $P(A \cup B)$

Exercise 4

Strands of copper wire from a manufacturer are analyzed for strength and conductivity. The results from 100 strands are as follows:

	strength	
	high	low
high conductivity	74	8
low conductivity	15	3

a) If a strand is randomly selected, what is the probability that its conductivity is high and its strength is high?
b) If a strand is randomly selected, what is the probability that its conductivity is low or the strength is low?
c) Consider the event that a strand has low conductivity and the event that the strand has a low strength. Are these two events mutually exclusive?

Question 1

Assume that a researcher randomly selects 14 newborn babies and counts the number of girls selected, X. The probabilities corresponding to the 14 possible values of X are summarized in the given table. Answer the question using the following table.

X(girls)	0	1	2	3	4	5	6	7	8	9	10	11	12
P(X)	0.000	0.001	0.006	0.022	0.061	0.122	0.183	0.209	0.183	0.122	0.061	0.022	0.006

Find the probability of selecting 11 or more girls.

Select one:

- a. 0.122
- b. 0.212
- c. 0.001
- d. 0.029

Question 2

According to a 2007 report published by the Columbia University, 69% of teens have family dinners five or more times a week, 11% of teens have used marijuana and the proportion of teens who have family dinners 5 or more times a week or use marijuana is 0.65. What is the probability that a teen has family dinners five or more times a week and uses marijuana?

Hint. Use the addition rules.

Select one:

- a. 0.64
- b. 0.15
- c. 0.18
- d. 0.08

Question 3

For two events A and B, P(A) = 0.4, P(B) = 0.5. Then P(A or B) equals

Select one:

- a. 0.9, if A and B are independent.
- b. 0.2, if A and B are independent.
- c. 0.7, if A and B are disjoint..
- d. 0.7, if A and B are independent.

Question 4

In 2006, the General Social Survey asked 4,491 respondents how often they attended religious services. The responses were as follows:

Frequency	Number of respondents
Never	1020
Less than once a year	302
Once a year	571
Several times a year	502
Once a month	308
Two-three times a month	380
Nearly every week	240
Every week	839
More than once a week	329

What is the probability that a randomly selected respondent attended religious services more than once a month?

Select one:

- a. 0.580
- b. 0.398
- c. 0.717
- d. 0.424

Question 5

According to the U.S. census, in 2005 25% of homicide victims were known to be female, 8.7% were known to be under the age of 18 and 2.7% were known to be females under the age of 18. What is the probability that a murder victim was known to be female or under the age of 18 based on these 2005 estimates?

Select one:

- a. 0.02
- b. 0.9
- c. 0.310
- d. 0.279

Question 6

According to a survey result, 79.6% of respondents favored the gun law, 77.8% favored the death penalty for those convicted of murder and 62.7% were in favor of both. What is the probability that a randomly selected respondent was in favor of either the gun law or the death penalty for persons convicted of murder?

Hint. Use the addition rules.

Select one:

- a. 0.796
- b. 0.947
- c. 0.847
- d. 0.527

Question 7

A survey of senior citizens at a doctor's office shows that 65% take blood pressure-lowering medication, 38% take cholesterol-lowering medication, and 7% take both medications. What is the probability that a senior citizen takes either blood pressure-lowering or cholesterol-lowering medication?

Select one:

- a. 0.96
- b. 0.85
- c. 0.14
- d. 0.90

Question 8

The probability that a student at a certain college is male is 0.55. The probability that a student at that college has a job off campus is 0.67. The probability that a student at the college is male and has a job off campus is 0.35. If a student is chosen at random from the college, what is the probability that the student is male or has an off campus job?

Select one:

- a. 0.37
- b. 0.93
- c. 0.87
- d. 0.63

Question 9

If you flip a coin three times, the possible outcomes are HHH HHT HTH HTT THH THT TTH TTT. What is the probability of getting at most one head?

Select one:

- a. 6/7
- b. 5/6
- c. 1/2
- d. 7/8

Question 10

If two balanced die are rolled, the possible outcomes can be represented as follows.

(1, 1) (2, 1) (3, 1) (4, 1) (5, 1) (6, 1)
(1, 2) (2, 2) (3, 2) (4, 2) (5, 2) (6, 2)
(1, 3) (2, 3) (3, 3) (4, 3) (5, 3) (6, 3)
(1, 4) (2, 4) (3, 4) (4, 4) (5, 4) (6, 4)
(1, 5) (2, 5) (3, 5) (4, 5) (5, 5) (6, 5)
(1, 6) (2, 6) (3, 6) (4, 6) (5, 6) (6, 6)

Determine the probability that the sum of the dice is 4 or 12.

Select one:

- a. 7/36
- b. 1/9
- c. 5/9
- d. 1/2

Question 11

The age distribution of students at a community college is given below.

Age (years)	Number of students
Under 21	410
21-24	404
25-28	276
29-32	155
33-36	97
37-40	63
Over 40	86

A student from the community college is selected at random. Find the probability that the student is 25 years or over. Give your answer as a decimal rounded to three decimal places.

Select one:

- a. 0.271
- b. 0.454
- c. 0.726
- d. 0.729

Question 12

The age distribution of students at a community college is given below.

Age (years)	Number of students
Under 21	416
21-24	419
25-28	263
29-32	151
33-36	93
37-40	59
Over 40	85

A student from the community college is selected at random. Find the probability that the student is under 37 years old. Give your answer as a decimal rounded to three decimal places.

Select one:

- a. 0.903
- b. 0.040
- c. 0.960
- d. 0.097

Question 13

A percentage distribution is given below for the size of families in one U.S. city.

Size	Percentage
2	42.8
3	21.1
4	19.2
5	11.6
6	3.3
7+	2.0

A family is selected at random. Find the probability that the size of the family is more than 4. Round your result to three decimal places.

Select one:

- a. 0.192
- b. 0.169
- c. 0.808
- d. 0.361

Question 14

A percentage distribution is given below for the size of families in one U.S. city.

Size	Percentage
2	45.1
3	22.2
4	19.7
5	8.0
6	3.1
7+	1.9

A family is selected at random. Find the probability that the size of the family is less than 6. Round your result to three decimal places.

Select one:

- a. 0.031
- b. 0.950
- c. 0.019
- d. 0.050

Question 15

Suppose that the probability that a particular brand of light bulb fails before 1000 hours of use is 0.3. If you purchase 3 of these bulbs, what is the probability that at least one of them lasts 1000 hours or more?

Select one:

- a. 0.512
- b. 0.992
- c. 0.973
- d. 0.008

Question 16

A greenhouse is offering a sale on tulip bulbs because they have inadvertently mixed pink bulbs with red bulbs. If 35% of the bulbs are pink and 65% are red, what is the probability that at least one of the bulbs will be pink if 5 bulbs are purchased?

Select one:

- a. 0.8704
- b. 0.8840
- c. 0.9744
- d. 0.2082

Question 17

According to the Center for Disease Control, in 2004, 67.5% of all adults between the ages of 18 and 44 were considered current drinkers. Based on this estimate, if three randomly selected adults between the ages of 18 and 44 are selected, what is the probability that at least one is a current drinker?

Select one:

- a. 0.97
- b. 0.43
- c. 1
- d. 0.88

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4. Conditional probability

Conditional probability

The probability of an event B under the knowledge that the outcome will be in event A is denoted as $P(B \mid A)$ and this is called the **conditional probability** of B given A, and is calculated by

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}, \; P(A) > 0.$$

4. Conditional probability

Example 9

Table 2-3 Parts Classified

		Surface Flaws		
		Yes (event F)	No	Total
Defective	Yes (event D)	10	18	28
	No	30	342	372
	Total	40	360	400

Table 2-3 provides an example of 400 parts classified by surface flaws and as (functionally) defective. For example, of the parts with surface flaws (40 parts) the number of defective ones is 10. Therefore,
$P(D \mid F) = 10/40 = 0.25$,
and of the parts without surface flaws (360 parts) the number of defective ones is 18. Therefore, $P(D \mid F^{'}) = 18/360 = 0.05$.

To select randomly implies that at each step of the sample, the items that remain in the batch are equally likely to be selected.

Example 10

A day's production of 850 manufactured parts contains 50 parts that do not meet customer requirements. Two parts are selected randomly without replacement from the batch. What is the probability that the second part is defective given that the first part is defective?

4. Conditional probability

To select randomly implies that at each step of the sample, the items that remain in the batch are equally likely to be selected.

Example 10

A day's production of 850 manufactured parts contains 50 parts that do not meet customer requirements. Two parts are selected randomly without replacement from the batch. What is the probability that the second part is defective given that the first part is defective?
Let A denote the event that the first part selected is defective, and let B denote the event that the second part selected is defective. If the first part is defective, prior to selecting the second part, the batch contains 849 parts, of which 49 are defective; therefore, $P(B \mid A) = 49/849$.

4. Conditional probability

Exercise 1

Samples of skin experiencing desquamation are analyzed for both moisture and melanin content. The results from 100 skin samples are as follows:

		melanin content	
		high	low
moisture	high	13	7
content	low	48	32

Let A denote the event that a sample has low melanin content, and let B denote the event that a sample has high moisture content. Determine the following probabilities:

a) $P(A)$ b) $P(B)$

c) $P(A|B)$ d) $P(B|A)$

Exercise 2

The analysis of results from a leaf transmutation experiment (turning a leaf into a petal) is summarized by type of transformation completed:

		total textural transformation	
		yes	no
total color transformation	yes	243	26
	no	13	18

a) If a leaf completes the color transformation, what is the probability that it will complete the textural transformation?
b) If a leaf does not complete the textural transformation, what is the probability it will complete the color transformation?

Exercise 3

The following table summarizes the analysis of samples of galvanized steel for coating weight and surface roughness:

		coating weight	
		high	low
surface	high	12	16
roughness	low	88	34

a) If the coating weight of a sample is high, what is the probability that the surface roughness is high?
b) If the surface roughness of a sample is high, what is the probability that the coating weight is high?
(c) If the surface roughness of a sample is low, what is the probability that the coating weight is low?

4. Conditional probability

Exercise 4

A lot of 100 semiconductor chips contains 20 that are defective. Two are selected randomly, without replacement, from the lot.
a) What is the probability that the first one selected is defective?
b) What is the probability that the second one selected is defective given that the first one was defective?
c) What is the probability that both are defective?
d) How does the answer to part (b) change if chips selected were replaced prior to the next selection?

Question 1

According to a survey of American households, the probability that the residents own 2 cars if annual household income is over $30,000 is 70%. Of the households surveyed, 50% had incomes over $30,000 and 70% had 2 cars. The probability that the residents of a household own 2 cars and have an income over $30,000 a year is:

Select one:

- a. 0.22
- b. 0.18
- c. 0.35
- d. 0.48

Question 2

The conditional probability of event G, given the knowledge that event H has occurred, would be written as ____.

Select one:

- a. $P(G \mid H)$
- b. $P(H \mid G)$
- c. $P(H)$
- d. $P(G)$

Question 3

After completing an inventory of three warehouses, a golf club shaft manufacturer described its stock of 14,542 shafts with the percentages given in the table. Suppose a shaft is selected at random from the 14,542 currently in stock, and the warehouse number and type of shaft are observed.

Type of Shaft

		Regular	Stiff	Extra Stiff
	1	19%	8%	3%
Warehouse	2	14%	11%	7%
	3	20%	18%	0%

Given that the shaft is produced in warehouse 2, find the probability it has an stiff shaft.

Select one:

- a. 0.721
- b. 0.344
- c. 0.356
- d. 0.219

Question 4

A research group asked the students if they carry a credit card. The responses are listed in the table.

Class	Credit Card Carrier	Not a Credit Card Carrier	Total
Freshman	50	10	60
Sophomore	30	10	40
Total	80	20	100

If a student is randomly selected, find the probability that he or she owns a credit card given that the student is a freshman. Round your answer to three decimal places.

Select one:

- a. 0.500
- b. 0.167
- c. 0.833
- d. 0.625

Question 5

The breakdown of workers in a particular state according to their political affiliation and type of job held is shown here. Suppose a worker is selected at random within the state and the worker's political affiliation and type of job are noted.

Political Affiliation

		Republican	Democrat	Independent
Type of job	White collar	9%	20%	19%
	Blue Collar	15%	18%	19%

Given the worker is a Democrat, what is the probability that the worker is in a white collar job.

Select one:

- a. 0.576
- b. 0.417
- c. 0.526
- d. 0.303

Question 6

According to a survey of American households, the probability that the residents own 2 cars if annual household income is over $35,000 is 70%. Of the households surveyed, 50% had incomes over $35,000 and 80% had 2 cars. The probability that the residents of a household do not own 2 cars and have an income over $35,000 a year is:

Select one:

- a. 0.45
- b. 0.18
- c. 0.15
- d. 0.48

Question 7

According to a survey of American households, the probability that the residents own 2 cars if annual household income is over \$20,000 is 90%. Of the households surveyed, 60% had incomes over \$20,000 and 60% had 2 cars. The probability that the residents of a household own 2 cars and have an income less than or equal to \$20,000 a year is:

Select one:

- a. 0.06
- b. 0.18
- c. 0.48
- d. 0.22

Question 8

Suppose that P(A|B) = 0.3 and P(B) = 0.4. Determine P(A' and B).

Select one:

- a. 0.12
- b. 0.75
- c. 0.35
- d. 0.28

Question 9

A group of volunteers for a clinical trial consists of 88 women and 77 men. 28 of the women and 39 of the men have high blood pressure. If one of the volunteers is selected at random find the probability that the person has high blood pressure given that it is a woman.

Select one:

- a. 0.114
- b. 0.486
- c. 0.318
- d. 0.222

Question 10

A group of volunteers for a clinical trial consists of 123 women and 178 men. 54 of the women and 46 of the men have high blood pressure. If one of the volunteers is selected at random find the probability that the person is a man given that they have high blood pressure.

Select one:

- a. 0.512
- b. 0.256
- c. 0.460
- d. 0.488

Question 11

The following table shows the political affiliation of voters in one city and their positions on stronger gun control laws.

	Favor	Oppose
Republican	0.11	0.17
Democrat	0.35	0.16
Other	0.15	0.06

What is the probability that a Democrat opposes stronger gun control laws?

Select one:

- a. 0.390
- b. 0.160
- c. 0.314
- d. 0.490

Question 12

The following table shows the political affiliation of voters in one city and their positions on stronger gun control laws.

	Favor	Oppose
Republican	0.12	0.26
Democrat	0.32	0.2
Other	0.13	0.12

What is the probability that a voter who favors stronger gun control laws is a Republican?

Select one:

- a. 0.420
- b. 0.214
- c. 0.211
- d. 0.257

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5. Multiplication and Total Probability Rules

Multiplication and Total Probability Rules

- Multiplication rule for any two events A and B

$$P(A \cap B) = P(B \mid A)P(A) = P(A \mid B)P(B).$$

- Total probability rule for any two events A and B

$$P(B) = P(B \cap A) + P(B \cap A') = P(B \mid A)P(A) + P(B \mid A')P(A').$$

- Assume $E_1, E2, ..., E_k$ are k mutually exclusive and exhaustive sets. Then

$$P(B) = P(B \cap E_1) + P(B \cap E_2) + ... + P(B \cap E_k) = P(B \mid E_1)P(E_1) + P(B \mid E_2)P(E_2) + ... + P(B \mid E_k)P(E_k).$$

A collection of sets $E_1, E2, ..., E_k$ such that $E_1 \cup E_2 \cup ... \cup E_k = S$ is said to be **exhaustive**.

5. Multiplication and Total Probability Rules

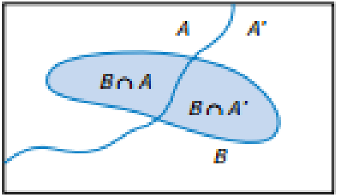
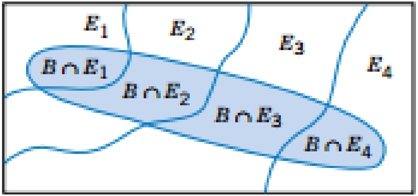


Figure 2-15 Partitioning an event into two mutually exclusive subsets.



$$B = (B \cap E_1) \cup (B \cap E_2) \cup (B \cap E_3) \cup (B \cap E_4)$$

Figure 2-16 Partitioning an event into several mutually exclusive subsets.

5. Multiplication and Total Probability Rules

Exercise 1

Suppose that $P(A|B) = 0.4$ and $P(B) = 0.5$. Determine the following:

$$a)P(A \cap B); \qquad b)P(A^{'} \cap B)$$

5. Multiplication and Total Probability Rules

Exercise 1

Suppose that $P(A|B) = 0.4$ and $P(B) = 0.5$. Determine the following:

$$a) P(A \cap B); \qquad b) P(A' \cap B)$$

Exercise 2

The probability is 1% that an electrical connector that is kept dry fails during the warranty period of a portable computer. If the connector is ever wet, the probability of a failure during the warranty period is 5%. If 90% of the connectors are kept dry and 10% are wet, what proportion of connectors fail during the warranty period?

Exercise 3

A lot of 100 semiconductor chips contains 20 that are defective.
a) Two are selected, at random, without replacement, from the lot. Determine the probability that the second chip selected is defective.
b) Three are selected, at random, without replacement, from the lot. Determine the probability that all are defective.

Exercise 4

A batch of 25 injection-molded parts contains five that have suffered excessive shrinkage.
a) If two parts are selected at random, and without replacement, what is the probability that the second part selected is one with excessive shrinkage?
b) If three parts are selected at random, and without replacement, what is the probability that the third part selected is one with excessive shrinkage?

Question 1

A company has 2 machines that produce widgets. An older machine produces 23% defective widgets, while the new machine produces only 8% defective widgets. In addition, the new machine produces 3 times as many widgets as the older machine does. Given that a widget was produced by the new machine, what is the probability it is not defective?

Select one:

- a. 0.50
- b. 0.06
- c. 0.92
- d. 0.94

Question 2

A company has 2 machines that produce widgets. An older machine produces 23% defective widgets, while the new machine produces only 8% defective widgets. In addition, the new machine produces 3 times as many widgets as the older machine does. Given a randomly chosen widget was tested and found to be defective, what is the probability it was produced by the new machine?

Select one:

- a. 0.511
- b. 0.15
- c. 0.489
- d. 0.08

Question 3

The probability is 2% that an electrical connector that is kept dry fails during the warranty period of a portable computer. If the connector is ever wet, the probability of a failure during the warranty period is 10%. If 80% of the connectors are kept dry and 20% are wet, what proportion of connectors fail during the warranty period?

Select one:

- a. 0.036
- b. 0.08
- c. 0.014
- d. 0.6

Question 4

Ms. Anne figures that there is a 40% chance that her company will set up a branch office in Ohio. If it does, she is 70% certain that she will be made manager of this new operation. What is the probability that Anne will be a Ohio branch office manager?

Select one:

- a. 0.20
- b. 0.18
- c. 0.28
- d. 0.55

Question 5

The probability is 5% that an electrical connector that is kept dry fails during the warranty period of a portable computer. If the connector is ever wet, the probability of a failure during the warranty period is 20%. If 90% of the connectors are kept dry and 10% are wet, what proportion of connectors fail during the warranty period?

Select one:

- a. 0.625
- b. 0.036
- c. 0.086
- d. 0.065

Question 6

A bin contains 15 defective (that immediately fail when put in use), 20 partially defective (that fail after a couple of hours of use), and 30 acceptable transistors. A transistor is chosen at random from the bin and put into use. If it does not immediately fail, what is the probability it is acceptable?

Select one:

- a. 0.44
- b. 0.71
- c. 0.35
- d. 0.60

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6. Independence

Independence

- Two events are independent if any one of the following equivalent statements is true:
  1) $P(A|B) = P(A)$
  2) $P(B|A) = P(B)$
  3) $P(A \cap B) = P(A)P(B)$
- The events $E_1, E_2, ..., E_n$ are independent if and only if for any subset of these events $E_{i_1}, E_{i_2}, ..., E_{i_k}$

$$P(E_{i_1} \cap E_{i_2} \cap ... \cap E_{i_k}) = P(E_{i_1}) \times P(E_{i_2}) \times ... \times P(E_{i_k})$$

Exercise 1

If $P(A|B) = 0.3, P(B) = 0.8$, and $P(A) = 0.3$ are the events B and the complement of A independent?

6. Independence

Exercise 1

If $P(A|B) = 0.3, P(B) = 0.8$, and $P(A) = 0.3$ are the events B and the complement of A independent?

Exercise 2

If $P(A) = 0.2, P(B) = 0.2$, and A and B are mutually exclusive, are they independent?

6. Independence

Exercise 1

If $P(A|B) = 0.3, P(B) = 0.8$, and $P(A) = 0.3$ are the events B and the complement of A independent?

Exercise 2

If $P(A) = 0.2, P(B) = 0.2$, and A and B are mutually exclusive, are they independent?

Exercise 3

A batch of 500 containers for frozen orange juice contains five that are defective. Two are selected, at random, without replacement, from the batch. Let A and B denote the events that the first and second containers selected are defective, respectively.

a) Are A and B independent events?

b) If the sampling were done with replacement, would A and B be independent?

Exercise 4

The probability that a lab specimen contains high levels of contamination is 0.10. Five samples are checked, and the samples are independent.

a) What is the probability that none contains high levels of contamination?

b) What is the probability that exactly one contains high levels of contamination?

c) What is the probability that at least one contains high levels of contamination?

Question 1

Two events A and B are said to be _______ if $P(A|B) = P(A)$ or if $P(B|A) = P(B)$.

Select one:

- a. simple events
- b. complementary
- c. mutually exclusive
- d. independent

Question 2

Assume that P(*A*) = 0.7 and P(*B*) = 0.2. If *A* and *B* are independent, find P(*A* and *B*).

Select one:

- a. 1.00
- b. 0.76
- c. 0.90
- d. 0.14

Question 3

If $P(A) = 0.45$, $P(B) = 0.25$, and $P(B|A) = 0.45$, are A and B independent?

Select one:

- a. no
- b. cannot determine
- c. yes

Question 4

If $P(A) = 0.72$, $P(B) = 0.11$, and A and B are independent, find $P(A|B)$.

Select one:

- a. 0.0792
- b. 0.72
- c. 0.11
- d. 0.83

Question 5

Assume that P(E) = 0.15 and P(F) = 0.48. If E and F are independent, find P(E and F).

Select one:

- a. 0.15
- b. 0.558
- c. 0.630
- d. 0.072

Question 6

If two events A and B are ________, then P(A and B) = P(A)P(B).

Select one:

- a. independent
- b. mutually exclusive
- c. simple events
- d. complements

Question 7

Assume that P(*C*) = 0.5 and P(*D*) = 0.3. If *C* and *D* are independent, find P(*C* and *D*).

Select one:

- a. 0.15
- b. 0.5
- c. 0.3
- d. 1.5

Question 8

Given that events A and B are mutually exclusive and P(A) = 0.2 and P(B) =0.7, are A and B independent?

Select one:

- a. cannot be determined
- b. no
- c. yes

Question 9

Given that events *C* and *D* are independent, P(*C*) = 0.3, and P(*D*) = 0.6, are *C* and *D* mutually exclusive?

Select one:

- a. cannot be determined
- b. no
- c. yes

Question 10

Given events A and B with probabilities $P(A) = 0.5, P(B) = 0.4$, and $P(A \text{ and } B) = 0.2$, are A and B independent?

Select one:

- a. no
- b. yes
- c. cannot be determined

Question 11

Given events C and D with probabilities $P(C) = 0.3$, $P(D) = 0.2$, and $P(C \text{ and } D) = 0.1$, are C and D independent?

Select one:

- a. cannot be determined
- b. yes
- c. no

Question 12

Given events A and B with probabilities $P(A) = 0.75$ and $P(B) = 0.15$, are A and B mutually exclusive?

Select one:

- a. no
- b. yes
- c. cannot be determined

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8. Random Variables

7. Bayes' Theorem

If A and B are any two events

$$P(A \mid B) = \frac{P(B|A)P(A)}{P(B)}, \text{ for } P(B) > 0.$$

7. Bayes' Theorem

If A and B are any two events

$$P(A \mid B) = \frac{P(B|A)P(A)}{P(B)}, \text{ for } P(B) > 0.$$

Bayes' Theorem

If $E_1, E2, ..., E_k$ are k mutually exclusive and exhaustive events and B is any event,

$$P(E_1 \mid B) = \frac{P(B|E_1)P(E_1)}{P(B|E_1)P(E_1)+P(B|E_2)P(E_2)+...+P(B|E_k)P(E_k)},$$

for $P(B) > 0$.

Example 11

A new medical procedure has been shown to be effective in the early detection of an illness, a medical screening of the population is proposed. The probability that the test correctly identifies someone with the illness as positive is 0.99, and the probability that the test correctly identifies someone without the illness as negative is 0.95. The incidence of the illness in the general population is 0.0001. You take the test, and the result is positive. What is the probability that you have the illness?

Example 11

A new medical procedure has been shown to be effective in the early detection of an illness, a medical screening of the population is proposed. The probability that the test correctly identifies someone with the illness as positive is 0.99, and the probability that the test correctly identifies someone without the illness as negative is 0.95. The incidence of the illness in the general population is 0.0001. You take the test, and the result is positive. What is the probability that you have the illness?
Let D denote the event that you have the illness, and let S denote the event that the test signals positive. The probability requested can be denoted as $P(D \mid S)$. The probability that the test correctly signals someone without the illness as negative is 0.95. Consequently, the probability of a positive test without the illness is $P(S \mid D^{'}) = 0.05$.
From Bayes' Theorem,

$$P(D \mid S) = \frac{P(S|D)P(D)}{P(S|D)P(D)+P(S|D')P(D')} = \frac{0.99\times 0.0001}{0.99\times 0.0001+0.05\times(1-0.0001)} = 0.002$$

Exercise 1

Suppose that $P(A|B) = 0.7, P(A) = 0.5$ and $P(B) = 0.2$. Determine $P(B|A)$.

7. Bayes' Theorem

Exercise 1

Suppose that $P(A|B) = 0.7, P(A) = 0.5$ and $P(B) = 0.2$. Determine $P(B|A)$.

Exercise 2

Suppose that $P(A|B) = 0.4, P(A|B^{'}) = 0.2$, and $P(B) = 0.8$. Determine $P(B|A)$.

7. Bayes' Theorem

Exercise 1

Suppose that $P(A|B) = 0.7, P(A) = 0.5$ and $P(B) = 0.2$. Determine $P(B|A)$.

Exercise 2

Suppose that $P(A|B) = 0.4, P(A|B^{'}) = 0.2$, and $P(B) = 0.8$. Determine $P(B|A)$.

Exercise 3

An inspector working for a manufacturing company has a 99% chance of correctly identifying defective items and a 0.5% chance of incorrectly classifying a good item as defective. The company has evidence that its line produces 0.9% of nonconforming items.

a) What is the probability that an item selected for inspection is classified as defective?

b) If an item selected at random is classified as nondefective, what is the probability that it is indeed good?

7. Bayes' Theorem

Exercise 4

A new analytical method to detect pollutants in water is being tested. This new method of chemical analysis is important because, if adopted, it could be used to detect three different contaminants -organic pollutants, volatile solvents, and chlorinated compounds - instead of having to use a single test for each pollutant. The makers of the test claim that it can detect high levels of organic pollutants with 99.7% accuracy, volatile solvents with 99.95% accuracy, and chlorinated compounds with 89.7% accuracy. If a pollutant is not present, the test does not signal. Samples are prepared for the calibration of the test and 60% of them are contaminated with organic pollutants, 27% with volatile solvents, and 13% with traces of chlorinated compounds. A test sample is selected randomly.

a) What is the probability that the test will signal?

b) If the test signals, what is the probability that chlorinated compounds are present?

7. Bayes' Theorem

Exercise 5

In the 2004 presidential election, exit polls from the critical state of Ohio provided the following results:

	Bush	Kerry
no college degree (62%)	50%	50%
college graduate (38%)	53%	46%

If a randomly selected respondent voted for Bush, what is the probability that the person has a college degree?

Question 1

According to a survey of American households, the probability that the residents own 3 cars if annual household income is over $25,500 is 83%. Of the households surveyed, 62% had incomes over $25,500 and 84% had 3 cars. The probability that annual household income is over $25,500 if the residents of a household own 3 cars is:

Select one:

- a. 0.69
- b. 0.50
- c. 0.61
- d. 0.42

Question 2

According to a survey of American households, the probability that the residents own 3 cars if annual household income is over $25,500 is 63%. Of the households surveyed, 62% had incomes over $25,500 and 44% had 3 cars. The probability that annual household income is over $25,500 if the residents of a household own 3 cars is:

Select one:

- a. 0.69
- b. 0.89
- c. 0.50
- d. 0.42

Question 3

Suppose that $P(A|B) = 0.6$, $P(A) = 0.5$ and $P(B) = 0.1$. Find the value of $P(B|A)$.

Select one:

- a. 0.06
- b. 0.20
- c. 0.12
- d. 0.30

Question 4

It was found that 60% of the workers were white, 30% were black and 10% are other races. Given that a worker was white, the probability that the worker had claimed bias was 30%. Given that a worker was black, the probability that the worker had claimed bias was 40%. Given that a worker was other race, the probability that the worker had claimed bias was 0%.

If a randomly selected worker had claimed bias, what is the probability that the worker is white?

Select one:

- a. 0.6
- b. 0.4
- c. 0.7
- d. 0.3

Question 5

It was found that 60% of the workers were white, 30% were black and 10% are other races. Given that a worker was white, the probability that the worker had claimed bias was 30%. Given that a worker was black, the probability that the worker had claimed bias was 40%. Given that a worker was other race, the probability that the worker had claimed bias was 0%.

If a randomly selected worker had claimed bias, what is the probability that the worker is black?

Select one:

- ○ a. 0.6
- ○ b. 0.3
- ○ c. 0.4
- ○ d. 0.7

Question 6

At a Texas college, 60% of the students are from the southern part of the state, 30% are from the northern part of the state, and the remaining 10% are from out-of-state. All students must take and pass an Entry Level Math (ELM) test. 60% of the southerners have passed the ELM, 70% of the northerners have passed the ELM, and 90% of the out-of-state have passed the ELM.

If a randomly selected student has passed the ELM, the probability the student is from out-of-state is ______.

Select one:

- a. 0.267
- b. 0.875
- c. 0.182
- d. 0.136

Question 7

In Orange County, 51% of the adults are males. One adult is randomly selected for a survey involving credit card usage. It is later learned that the selected survey subject was smoking a cigar. Also, 7.5% of males smoke cigars, whereas 1.9% of females smoke cigars. Use this additional information to find the probability that the selected subject is a male.

Select one:

- a. 0.804
- b. 0.901
- c. 0.203
- d. None of the other choices is true

Question 8

In a study of pleas and prison sentences, it is found that 35% of the subjects studied were sent to prison. Among those sent to prison, 30% chose to plead guilty. Among those not sent to prison, 50% chose to plead guilty.

If a study subject is randomly selected and it is then found that the subject entered a guilty plea, find the probability that this person was not sent to prison.

Select one:

- a. 0.863
- b. 0.756
- c. 0.347
- d. None of the other choices is true

Question 9

The New York State Health Department reports a 12% rate of the HIV virus for the "at-risk" population. Under certain conditions, a preliminary screening test for the HIV virus is correct 99% of the time. If someone is randomly selected from the at-risk population, what is the probability that they have the HIV virus if it is known that they have tested positive in the initial screening?

Select one:

- a. 0.392
- b. 0.456
- c. 0.235
- d. 0.931
- e. None of the other choices is correct

Question 10

An aircraft emergency locator transmitter (ELT) is a device designed to transmit a signal in the case of a crash. The Altigauge Manufacturing Company makes 85% of the ELTs, the Bryant Company makes 10% of them, and the Chartair Company makes the other 5%. The ELTs made by Altigauge have a 3% rate of defects, the Bryant ELTs have a 5% rate of defects, and the Chartair ELTs have a 10% rate of defects.

If a randomly selected ELT is then tested and is found to be defective, find the probability that it was made by the Altigauge Manufacturing Company.

Select one:

- a. 0.718
- b. 0.900
- c. None of the other choices is true
- d. 0.603

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8. Random Variables

Random Variable

- A **random variable** is a function that assigns a real number to each outcome in the sample space of a random experiment.

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Random Variable

- A **random variable** is a function that assigns a real number to each outcome in the sample space of a random experiment. *Notation:* A random variable is denoted by an uppercase letter such as X. After an experiment is conducted, the measured value of the random variable is denoted by a lowercase letter such as $x = 70$ milliamperes.

8. Random Variables

Random Variable

- A **random variable** is a function that assigns a real number to each outcome in the sample space of a random experiment.
  Notation: A random variable is denoted by an uppercase letter such as X. After an experiment is conducted, the measured value of the random variable is denoted by a lowercase letter such as $x = 70$ milliamperes.
- A **discrete random variable** is a random variable with a finite (or countably infinite) range. For example: number of girls in a class, number of scratches on a surface, proportion of defective parts among 1000 tested.

8. Random Variables

Random Variable

- A **random variable** is a function that assigns a real number to each outcome in the sample space of a random experiment.
  Notation: A random variable is denoted by an uppercase letter such as X. After an experiment is conducted, the measured value of the random variable is denoted by a lowercase letter such as $x = 70$ milliamperes.
- A **discrete random variable** is a random variable with a finite (or countably infinite) range. For example: number of girls in a class, number of scratches on a surface, proportion of defective parts among 1000 tested.
- A **continuous random variable** is a random variable with an interval (either finite or infinite) of real numbers for its range. For example: electrical current, length, pressure, temperature, time, voltage, weight.

8. Random Variables

Exercise 1

Decide whether a discrete or continuous random variable is the best model for each of the following variables:

a) The time until a projectile returns to earth.

b) The number of times a transistor in a computer memory changes state in one operation.

c) The volume of gasoline that is lost to evaporation during the filling of a gas tank.

d) The outside diameter of a machined shaft.

e) The number of cracks exceeding one-half inch in 10 miles of an interstate highway.

f) The weight of an injection-molded plastic part.

g) The number of molecules in a sample of gas.

h) The concentration of output from a reactor.

i) The current in an electronic circuit.

Supplemental Exercises

Exercise 1

A sample of three calculators is selected from a manufacturing line, and each calculator is classified as either defective or acceptable. Let A, B, and C denote the events that the first, second, and third calculators, respectively, are defective.

a) Describe the sample space for this experiment with a tree diagram.

Use the tree diagram to describe each of the following events:

b) A c) B

d) $A \cap B$ e) $B \cup C$

Exercise 2

A researcher receives 100 containers of oxygen. Of those containers, 10 have oxygen that is not ionized, and the rest are ionized. Two samples are randomly selected, without replacement, from the lot.

a) What is the probability that the first one selected is not ionized?

b) What is the probability that the second one selected is not ionized given that the first one was ionized?

c) What is the probability that both are ionized?

d) How does the answer in part (b) change if samples selected were replaced prior to the next selection?

Exercise 3

The analysis of shafts for a compressor is summarized by conformance to specifications:

		roundness conforms	
		yes	no
surface finish	yes	345	5
conforms	no	12	8

a) If we know that a shaft conforms to roundness requirements, what is the probability that it conforms to surface finish requirements?
b) If we know that a shaft does not conform to roundness requirements, what is the probability that it conforms to surface finish requirements?

Supplemental Exercises

Exercise 4

A lot of 50 spacing washers contains 30 washers that are thicker than the target dimension. Suppose that three washers are selected at random, without replacement, from the lot.
a) What is the probability that all three washers are thicker than the target?
b) What is the probability that the third washer selected is thicker than the target if the first two washers selected are thinner than the target?
c) What is the probability that the third washer selected is thicker than the target?

Supplemental Exercises

Exercise 5

The following table lists the history of 940 orders for features in an entry-level computer product.

		extra memory	
		no	yes
optional high-	no	514	68
speed processor	yes	112	246

Let A be the event that an order requests the optional highspeed processor, and let B be the event that an order requests extra memory. Determine the following probabilities:

a) $P(A \cup B)$ b) $P(A \cap B)$

c) $P(A' \cup B)$ d) $P(A' \cap B')$

e) What is the probability that an order requests an optional high-speed processor given that the order requests extra memory?

f) What is the probability that an order requests extra memory given that the order requests an optional high-speed processor?