Chapter 3: Discrete Random Variables and Probability Distributions

Course Name: PROBABILITY & STATISTICS

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Content

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1. Discrete Random Variables

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Example 1

A voice communication system for a business contains 48 external lines. At a particular time, the system is observed, and some of the lines are being used. Let the random variable X denote the number of lines in use. Then, X can assume any of the integer values 0 through 48. When the system is observed, if 10 lines are in use, $x = 10$.

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Example 2

Let X be the number of heads obtained when two coins are tossed. X is a random variable that can take on values of $0; 1; 2$.

1. Discrete Random Variables

Exercise

Determine the range (possible values) of the random variable:
1) The random variable is the number of nonconforming solder connections on a printed circuit board with 1000 connections.

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Determine the range (possible values) of the random variable:

1) The random variable is the number of nonconforming solder connections on a printed circuit board with 1000 connections.

2) In a voice communication system with 50 lines, the random variable is the number of lines in use at a particular time.

3) A batch of 500 machined parts contains 10 that do not conform to customer requirements. The random variable is the number of parts in a sample of five parts that do not conform to customer requirements.

1. Discrete Random Variables

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Determine the range (possible values) of the random variable:

1) The random variable is the number of nonconforming solder connections on a printed circuit board with 1000 connections.

2) In a voice communication system with 50 lines, the random variable is the number of lines in use at a particular time.

3) A batch of 500 machined parts contains 10 that do not conform to customer requirements. The random variable is the number of parts in a sample of five parts that do not conform to customer requirements.

4) The random variable is the moisture content of a lot of raw material, measured to the nearest percentage point.

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4) The random variable is the moisture content of a lot of raw material, measured to the nearest percentage point.
5) The random variable is the number of surface flaws in a large coil of galvanized steel.

Content

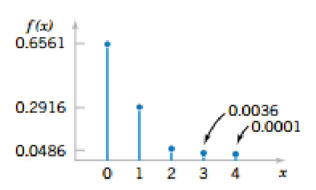
2. Probability Distributions and Probability Mass Functions

The **probability distribution** of a random variable X is a description of the probabilities associated with the possible values of X. For a discrete random variable, the distribution is often specified by just a list of the possible values along with the probability of each.

Probability Mass Function

For a discrete random variable X with possible values $x_1, x_2, ..., x_n$, a **probability mass function** is a function such that

1) $f(x_i) = P(X = x_i)$
2) $f(x_i) \geq 0$
3) $\sum_{i=1}^{n} f(x_i) = 1$



2. Probability Distributions and Probability Mass Functions

Example 3

Let the random variable X denote the number of heads that appear when 2 coins are tossed. Assume that the probability of a heads appearing is 0.5. Determine the probability distribution of X.

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We have $X = \{0; 1; 2\}$
$f(0) = P(X = 0) = 0.5 \times 0.5 = 0.25;$
$f(1) = P(X = 1) = 0.5 \times 0.5 + 0.5 \times 0.5 = 0.5;$
$f(2) = P(X = 2) = 0.5 \times 0.5 = 0.25.$

2. Probability Distributions and Probability Mass Functions

Exercise 1

The sample space of a random experiment is $\{a, b, c, d, e, f\}$ and each outcome is equally likely. A random variable is defined as follows:

outcome	a	b	c	d	e	f
x	0	0	1.5	1.5	2	3

Determine the probability mass function of X. Use the probability mass function to determine the following probabilities:

a) $P(X = 1.5)$

b) $P(0.5 < X < 2.7)$

c) $P(X > 3)$

d) $P(0 \leq X < 2)$

e) $P(X = 0 \quad \text{or} \quad \text{X} = 2)$

2. Probability Distributions and Probability Mass Functions

Exercise 2

x	-2	-1	0	1	2
$f(x)$	1/8	2/8	2/8	2/8	1/8

Determine

a) $P(X \leq 2)$

b) $P(X > -2)$

c) $P(-1 \leq X \leq 1)$

d) $P(X \leq -1 \quad \text{or} \quad \text{X} = 2)$

2. Probability Distributions and Probability Mass Functions

Exercise 3

$f(x) = (8/7)(1/2)^x, \ x = 1, 2, 3$

Determine

a) $P(X \leq 1)$

b) $P(X > 1)$

c) $P(2 < X < 6)$

d) $P(X \leq 1 \quad \text{or} \quad X > 1)$

2. Probability Distributions and Probability Mass Functions

Exercise 3

$f(x) = (8/7)(1/2)^x, \; x = 1, 2, 3$

Determine

a) $P(X \leq 1)$

b) $P(X > 1)$

c) $P(2 < X < 6)$

d) $P(X \leq 1 \quad \text{or} \quad X > 1)$

Exercise 4

A disk drive manufacturer sells storage devices with capacities of one terabyte, 500 gigabytes, and 100 gigabytes with probabilities 0.5, 0.3, and 0.2, respectively. The revenues associated with the sales in that year are estimated to be \$50 million, \$25 million, and \$10 million, respectively. Let X denote the revenue of storage devices during that year. Determine the probability mass function of X.

2. Probability Distributions and Probability Mass Functions

Exercise 5

An optical inspection system is to distinguish among different part types. The probability of a correct classification of any part is 0.98. Suppose that three parts are inspected and that the classifications are independent. Let the random variable X denote the number of parts that are correctly classified. Determine the probability mass function of X.

2. Probability Distributions and Probability Mass Functions

Exercise 5

An optical inspection system is to distinguish among different part types. The probability of a correct classification of any part is 0.98. Suppose that three parts are inspected and that the classifications are independent. Let the random variable X denote the number of parts that are correctly classified. Determine the probability mass function of X.

Exercise 6

An assembly consists of two mechanical components. Suppose that the probabilities that the first and second components meet specifications are 0.95 and 0.98. Assume that the components are independent. Determine the probability mass function of the number of components in the assembly that meet specifications.

2. Probability Distributions and Probability Mass Functions

Exercise 7

The data from 200 endothermic reactions involving sodium bicarbonate are summarized as follows:

Final Temperature Conditions	Number of Reactions
266 K	48
271 K	60
274 K	92

Calculate the probability mass function of final temperature.

Exercise 8

The distributor of a machine for cytogenics has developed a new model. The company estimates that when it is introduced into the market, it will be very successful with a probability 0.6, moderately successful with a probability 0.3, and not successful with probability 0.1. The estimated yearly profit associated with the model being very successful is \$15 million and with it being moderately successful is \$5 million; not successful would result in a loss of \$500,000. Let X be the yearly profit of the new model. Determine the probability mass function of X.

Content

3. Cumulative Distribution Function

Cumulative Distribution Function

The **cumulative distribution function** of a discrete random variable X, denoted as $F(x)$, is

$$F(x) = P(X \leq x) = \sum_{x_i \leq x} f(x_i).$$

For a discrete random variable X, satisfies the following properties:
1) $F(x) = P(X \leq x) = \sum_{x_i \leq x} f(x_i)$
2) $0 \leq F(x) \leq 1$
3) If $x \leq y$, then $F(x) \leq F(y)$

Example 4

Suppose that a day's production of 850 manufactured parts contains 50 parts that do not conform to customer requirements. Two parts are selected at random, without replacement, from the batch. Let the random variable X equal the number of nonconforming parts in the sample. What is the cumulative distribution function of X?

3. Cumulative Distribution Function

Example 4

Suppose that a day's production of 850 manufactured parts contains 50 parts that do not conform to customer requirements. Two parts are selected at random, without replacement, from the batch. Let the random variable X equal the number of nonconforming parts in the sample. What is the cumulative distribution function of X?

We first find the probability mass function of X:

$$f(0) = P(X = 0) = \frac{800}{850}.\frac{799}{849} = 0.886$$

$$f(1) = P(X = 1) = 2.\frac{800}{850}.\frac{50}{849} = 0.111$$

$$f(2) = P(X = 2) = \frac{50}{850}.\frac{49}{849} = 0.003$$

3. Cumulative Distribution Function

Example 4

Therefore,

$$F(0) = P(X \leq 0) = 0.886$$
$$F(1) = P(X \leq 1) = 0.886 + 0.111 = 0.997$$
$$F(2) = P(X \leq 2) = 1$$

The cumulative distribution function for this example is graphed in Fig. 3-4. Note that $F(x)$ is defined for all x from $-\infty < x < +\infty$ and not only for 0, 1, and 2.

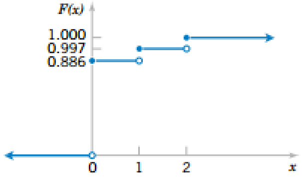


Figure 3-4 Cumulative distribution function for Example 3-8.

Example 4

Therefore,

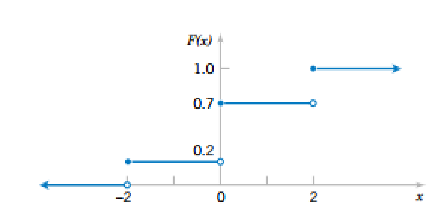
$$F(x) = \begin{cases} 0 & x < 0 \\ 0.886 & 0 \leq x < 1 \\ 0.997 & 1 \leq x < 2 \\ 1 & 2 \leq x \end{cases}$$

Example 5

Determine the probability mass function of X from the following cumulative distribution function:

$$F(x) = \begin{cases} 0 & x < -2 \\ 0.2 & -2 \leq x < 0 \\ 0.7 & 0 \leq x < 2 \\ 1 & 2 \leq x \end{cases}$$

Example 5



$$f(-2) = 0.2 - 0 = 0.2$$
$$f(0) = 0.7 - 0.2 = 0.5$$
$$f(2) = 1.0 - 0.7 = 0.3$$

Exercise 1

The sample space of a random experiment is $\{a, b, c, d, e, f\}$ and each outcome is equally likely. A random variable is defined as follows:

outcome	a	b	c	d	e	f
x	0	0	1.5	1.5	2	3

Determine the cumulative distribution function of X.

Exercise 2

x	-2	-1	0	1	2
$f(x)$	1/8	2/8	2/8	2/8	1/8

Determine the cumulative distribution function for X; also determine the following probabilities:
a) $P(X \leq 1.25)$
b) $P(X \leq 2.2)$
c) $P(-1.1 < X \leq 1)$
d) $P(X > 0)$

Exercise 3

The thickness of wood paneling (in inches) that a customer orders is a random variable with the following cumulative distribution function:

$$F(x) = \begin{cases} 0 & x < 1/8 \\ 0.2 & 1/8 \le x < 1/4 \\ 0.9 & 1/4 \le x < 3/8 \\ 1 & 3/8 \le x \end{cases}$$

Determine the following probabilities:

a) $P(X \le 1/18)$
b) $P(X \le 1/4)$
c) $P(X \le 5/16)$
d) $P(X > 1/4)$
e) $P(X \le 1/2)$

Exercise 4

Errors in an experimental transmission channel are found when the transmission is checked by a certifier that detects missing pulses. The number of errors found in an eightbit byte is a random variable with the following distribution:

$$F(x) = \begin{cases} 0 & x < 1 \\ 0.7 & 1 \leq x < 4 \\ 0.9 & 4 \leq x < 7 \\ 1 & 7 \leq x \end{cases}$$

Determine the following probabilities:

a) $P(X \leq 4)$
b) $P(X > 7)$
c) $P(X \leq 5)$
d) $P(X > 4)$
e) $P(X \leq 2)$

Question 1

According to police sources a car with a certain protection system will be recovered 78% of the time. Find the probability that 3 of 8 stolen cars will be recovered.

Select one:

- a. 0.5711
- b. 0.8754
- c. 0.0440
- d. 0.0137

Question 2

The probability that an individual is left-handed is 0.15. In a class of 30 students, what is the probability of finding five left-handers?

Select one:

- a. 0.153
- b. 0.002
- c. 0.186
- d. 0.054

Question 3

An airline reports that it has been experiencing a 12% rate of no-shows on advanced reservations. Among 100 advanced reservations, find the probability that there will be fewer than 15 no-shows.

Select one:

- a. 0.7549
- b. 0.7840
- c. 0.251
- d. 0.3187

Question 4

To calculate the probability of obtaining three aces in eight draws of a card with replacement from an ordinary deck, we would use the

Select one:

- a. binomial distribution.
- b. multinomial distribution.
- c. Poisson distribution.
- d. hypergeometric distribution.

Question 5

Find the probability that in 20 tosses of a fair six-sided die, a five will be obtained at least 5 times.

Select one:

- a. 0.2313
- b. 0.3875
- c. 0.1223
- d. 0.0871

Question 6

Find the probability that in 40 tosses of a fair six-sided die, a five will be obtained at most 11 times.

Select one:

- a. 0.9739
- b. 0.9106
- c. 0.8810
- d. 0.0853

Question 7

A test consists of 10 true/false questions. To pass the test a student must answer at least 4 questions correctly. If a student guesses on each question, what is the probability that the student will pass the test?

Select one:

- a. 0.117
- b. 0.172
- c. 0.8281
- d. 0.945

Question 8

Suppose that 14% of people are left handed. If 5 people are selected at random, what is the probability that exactly 2 of them are left handed?

Select one:

- a. 0.1139
- b. 0.1247
- c. 0.2278
- d. 0.0121

Question 9

A basketball player has made 95% of his foul shots during the season. If he shoots 3 foul shots in tonight's game, what is the probability that he makes all of the shots?

Select one:

- a. 0.09
- b. 0.21
- c. 0.857
- d. 0.343

Question 10

Police estimate that 22% of drivers drive without their seat belts. If they stop 4 drivers at random, find the probability that all of them are wearing their seat belts.

Select one:

- a. 0.1781
- b. 0.4582
- c. 0.3701
- d. 0.1536

Question 11

Police estimate that 22% of drivers drive without their seat belts. If they stop 4 drivers at random, find the probability that all of them are wearing their seat belts.

Select one:

- a. 0.1781
- b. 0.4582
- c. 0.3701
- d. 0.1536

Question 12

The on-line access computer service industry is growing at an extraordinary rate. Current estimates suggest that 10% of people with home-based computers have access to on-line services. Suppose that 8 people with home-based computers were randomly and independently sampled. What is the probability that at least 1 of those sampled have access to on-line services at home?

Select one:

- a. 0.0352
- b. 0.5695
- c. 0.8329
- d. 0.9648

Question 13

Samples of 10 parts from a metal punching process are selected every hour. Let X denote the number of parts in the sample of 10 that require rework. If the percentage of parts that require rework at 3%, what is the probability that X exceeds 2?

Select one:

- a. 0.0028
- b. 0.3152
- c. 0.0159
- d. 0.4114

Content

4. Mean and Variance of a Discrete Random Variable

Mean of a Discrete Random Variable

The **mean** or **expected value** of the discrete random variable X, denoted as μ or $E(X)$ is

$$\mu = E(X) = \sum_{x} x f(x)$$

4. Mean and Variance of a Discrete Random Variable

The expected or mean value of a random variable X is of special importance in statistics because it reflects the central value of the random variable's probability distribution. However, the mean does not give a complete description of the shape of the distribution.

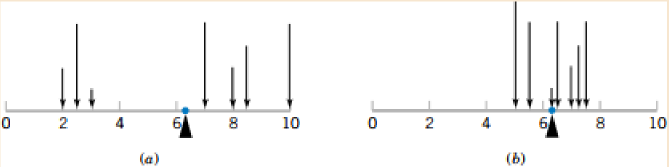


Figure 3-5 A probability distribution can be viewed as a loading with the mean equal to the balance point. Parts (a) and (b) illustrate equal means, but Part (a) illustrates a larger variance.

4. Mean and Variance of a Discrete Random Variable

Variance of a Discrete Random Variable

The **variance** of X, denoted as σ^2 or $V(X)$, is

$$\sigma^2 = V(X) = E(X-\mu)^2 = \sum_x (x-\mu)^2 f(x) = \sum_x x^2 f(x) - \mu^2$$

The variance of a random variable X is a measure of dispersion or scatter in the possible values for X.

The **standard deviation** of X is $\sigma = \sqrt{\sigma^2}$.

4. Mean and Variance of a Discrete Random Variable

Example 5

The number of messages sent per hour over a computer network has the following distribution:

x = number of messages	10	11	12	13	14	15
$f(x)$	0.08	0.15	0.30	0.20	0.20	0.07

Determine the mean and standard deviation of the number of messages sent per hour.

4. Mean and Variance of a Discrete Random Variable

Example 5

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x = number of messages	10	11	12	13	14	15
$f(x)$	0.08	0.15	0.30	0.20	0.20	0.07

Determine the mean and standard deviation of the number of messages sent per hour.

$$E(X) = 10(0.08) + 11(0.15) + ... + 15(0.07) = 12.5$$

4. Mean and Variance of a Discrete Random Variable

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x = number of messages	10	11	12	13	14	15
$f(x)$	0.08	0.15	0.30	0.20	0.20	0.07

Determine the mean and standard deviation of the number of messages sent per hour.

$$E(X) = 10(0.08) + 11(0.15) + ... + 15(0.07) = 12.5$$
$$V(X) = 10^2(0.08) + 11^2(0.15) + ... + 15^2(0.07) - 12.5^2 = 1.85$$
$$\sigma = \sqrt{\sigma^2} = \sqrt{1.85} = 1.36$$

4. Mean and Variance of a Discrete Random Variable

Now, we will extend the concept of the mean of a random variable X to a random variable related to X, the random variable h(X).

Expected Value of a Function of a Discrete Random Variable

If X is a discrete random variable with probability mass function $f(x)$

$$E[h(X)] = \sum_{x} h(x) f(x)$$

In the special case that $h(X) = aX + b$ for any constants a and b, $E(h(X)) = aE(X) + b$.

4. Mean and Variance of a Discrete Random Variable

Exercise 1

If the range of X is the set $\{0, 1, 2, 3, 4\}$ and $P(X = x) = 0.2$, determine the mean and variance of the random variable.

4. Mean and Variance of a Discrete Random Variable

Exercise 1

If the range of X is the set $\{0, 1, 2, 3, 4\}$ and $P(X = x) = 0.2$, determine the mean and variance of the random variable.

Exercise 2

The range of the random variable X is $\{0; 1; 2; 3; x\}$ where x is unknown. If each value is equally likely and the mean of X is 6, determine x.

4. Mean and Variance of a Discrete Random Variable

Exercise 3

The sample space of a random experiment is $\{a, b, c, d, e, f\}$ and each outcome is equally likely. A random variable is defined as follows:

outcome	a	b	c	d	e	f
x	0	0	1.5	1.5	2	3

Determine the mean and variance of the random variable.

4. Mean and Variance of a Discrete Random Variable

Exercise 3

The sample space of a random experiment is $\{a, b, c, d, e, f\}$ and each outcome is equally likely. A random variable is defined as follows:

outcome	a	b	c	d	e	f
x	0	0	1.5	1.5	2	3

Determine the mean and variance of the random variable.

Exercise 4

x	-2	-1	0	1	2
$f(x)$	1/8	2/8	2/8	2/8	1/8

Determine the mean and variance of the random variable.

Question 1

Find the standard deviation for the probability distribution.

x	0	1	2	3	4
$P(x)$	0.1296	0.3456	0.3456	0.1536	0.0256

Select one:

- a. 0.98
- b. 1.88
- c. 1.12
- d. 0.96

Question 2

Find the standard deviation for the given probability distribution.

x	0	1	2	3	4
$P(x)$	0.37	0.05	0.13	0.25	0.20

Select one:

- a. 2.45
- b. 2.56
- c. 1.71
- d. 1.60

Question 3

Find the variance for the given probability distribution.

x	0	1	2	3	4
$P(x)$	0.17	0.28	0.05	0.15	0.35

Select one:

- a. 2.69
- b. 2.63
- c. 7.43
- d. 2.46

Question 4

The following table is the probability distribution of the number of golf balls ordered by customers

x	3	6	9	12	15
P(x)	0.11	0.14	0.36	0.29	0.10

Find the mean of the this probability distribution.

Select one:

- a. 9.39
- b. 8.22
- c. 6.63
- d. 9.3

Question 5

Find the mean of the following probability distribution.

x	0	1	2	3	4
$P(x)$	0.19	0.37	0.16	0.26	0.02

Select one:

- a. 1.55
- b. 1.74
- c. 1.64
- d. 1.45

Question 6

The accompanying table shows the probability distribution for x, the number that shows up when a loaded die is rolled. Find the variance for the probability distribution.

x	1	2	3	4	5	6
P(x)	0.16	0.19	0.22	0.21	0.12	0.10

Select one:

- a. 2.41
- b. 9.62
- c. 2.36
- d. 2.03

Question 7

The probabilities that a batch of 4 computers will contain 0, 1, 2, 3, and 4 defective computers are 0.4521, 0.3970, 0.1307, 0.0191, and 0.0010, respectively. Find the variance for the probability distribution.

Select one:

- a. 0.59
- b. 1.11
- c. 0.51
- d. 0.69

Question 8

The random variable ***X*** represents the number of tests that a patient entering a hospital will have along with the corresponding probabilities. Find the mean and standard deviation for the random variable ***X***.

x	0	1	2	3	4
P(x)	5/17	3/17	6/17	2/17	1/17

Select one:

- a. mean: 1.59; standard deviation: 1.09
- b. mean: 1.47; standard deviation: 1.19
- c. mean: 1.59; standard deviation: 3.72
- d. mean: 1.47; standard deviation: 1.42

Question 9

The random variable **X** represents the number of credit cards that adults have along with the corresponding probabilities. Find the mean and standard deviation.

x	0	1	2	3	4
P(x)	0.05	0.49	0.32	0.07	0.07

Select one:

- a. mean: 1.18; standard deviation: 1.30
- b. mean: 1.18; standard deviation: 0.64
- c. mean: 1.62; standard deviation: 1.50
- d. mean: 1.62; standard deviation: 0.95

Question 10

In a pizza takeout restaurant, the following probability distribution was obtained. The random variable **X** represents the number of toppings for a large pizza. Find the mean and standard deviation for the random variable **X**.

x	0	1	2	3	4
P(x)	0.40	0.30	0.20	0.06	0.04

Select one:

- a. mean: 1.04; standard deviation: 1.09
- b. mean: 1.30; standard deviation: 2.38
- c. mean: 1.04; standard deviation: 0.49
- d. mean: 1.30; standard deviation: 1.54

Question 11

A basketball player is asked to shot free throws in sets of four. The player shoots 100 sets of 4 free throws. The probability distribution for making a particular number of free throws id given below. Determine the standard deviation for this discrete probability distribution.

x	0	1	2	3	4
$P(x)$	0.42	0.07	0.22	0.27	0.02

Select one:

- a. 1.05
- b. 1.21
- c. 1.32
- d. 1.10

Question 12

Find the variance of the following probability distribution.

x	1	2	3	4	5	6	7	8	9	10
$P(x)$	0.05	0.19	0.20	0.25	0.12	0.10	0	0.08	0	0.01

Select one:

- a. 1.95
- b. 0.56
- c. 3.57
- d. 3.97

Question 13

What is the standard deviation of the following probability distribution?

x	0	1	2	3	4	5	6
P(x)	0.30	0.25	0.20	0.12	0.07	0.04	0.02

Select one:

- a. 2.23
- b. 1.82
- c. 1.16
- d. 1.54

Content

5. Discrete Uniform Distribution

Discrete Uniform Distribution

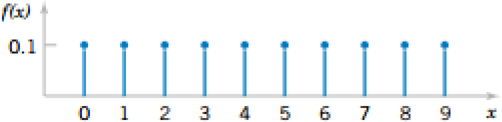
A random variable X has a discrete uniform distribution if each of the n values in its range, say, $x_1, x_2, ..., x_n$ has equal probability. Then,

$$f(x_i) = \frac{1}{n}$$

Example 6

The first digit of a part's serial number is equally likely to be any one of the digits 0 through 9. If one part is selected from a large batch and X is the first digit of the serial number, X has a discrete uniform distribution with probability 0.1 for each value in $S = \{0, 1, 2, ..., 9\}$. That is, $f(x) = 0.1$ for each value in S. The probability mass function of X is shown in Fig. 3-7.

Figure 3-7 Probability mass function for a discrete uniform random variable.



5. Discrete Uniform Distribution

Mean and Variance

Suppose X is a discrete uniform random variable on the consecutive integers $a, a+1, a+2, ..., b$, for $a \leq b$. The mean of X is

$$\mu = E(X) = \frac{b+a}{2}$$

The variance of X is

$$\sigma^2 = \frac{(b-a+1)^2 - 1}{12}$$

Example 7

Let the random variable X denote the number of the 48 voice lines that are in use at a particular time. Assume that X is a discrete uniform random variable with a range of 0 to 48. Then

$$E(X) = (48 + 0)/2 = 24$$

and

$$\sigma = [(48 - 0 + 1)^2 - 1]/12^{1/2} = 14.14$$

5. Discrete Uniform Distribution

Exercise 1

Let the random variable X have a discrete uniform distribution on the integers $0 \leq x \leq 99$. Determine the mean and variance of X.

5. Discrete Uniform Distribution

Exercise 1

Let the random variable X have a discrete uniform distribution on the integers $0 \leq x \leq 99$. Determine the mean and variance of X.

Exercise 2

Product codes of two, three, four, or five letters are equally likely. What is the mean and standard deviation of the number of letters in the codes?

5. Discrete Uniform Distribution

Exercise 1

Let the random variable X have a discrete uniform distribution on the integers $0 \leq x \leq 99$. Determine the mean and variance of X.

Exercise 2

Product codes of two, three, four, or five letters are equally likely. What is the mean and standard deviation of the number of letters in the codes?

Exercise 3

The probability of an operator entering alphanumeric data incorrectly into a field in a database is equally likely. The random variable X is the number of fields on a data entry form with an error. The data entry form has 28 fields. Is X a discrete uniform random variable? Why or why not?

Exercise 4

Suppose that X has a discrete uniform distribution on the integers 0 through 9. Determine the mean, variance, and standard deviation of the random variable $Y = 5X$ and compare to the corresponding results for X.

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Suppose that X has a discrete uniform distribution on the integers 0 through 9. Determine the mean, variance, and standard deviation of the random variable $Y = 5X$ and compare to the corresponding results for X.

Exercise 5

Thickness measurements of a coating process are made to the nearest hundredth of a millimeter. The thickness measurements are uniformly distributed with values 0.15, 0.16, 0.17, 0.18, and 0.19. Determine the mean and variance of the coating thickness for this process?

Question 1

Suppose that X has a discrete uniform distribution on the integers 0 through 5. Determine the mean of the random variable Y = 4X

Select one:

- a. 2.5
- b. 6
- c. 5
- d. 10

Question 2

The range of the random variable X is {1, 2, 3, 6, u}, where u is unknown. If each value is equally likely and the mean of X is 10, determine the value of u.

Select one:

- a. 12
- b. 24
- c. 19
- d. 38

Question 3

Let the random variable X have a discrete uniform distribution on the integers $1 \leq X \leq 10$. Determine P(X < 6).

Select one:

- a. 0.7
- b. 0.4
- c. 0.6
- d. 0.5

Question 4

Product codes of 3, 4 or 5 letters are equally likely. What is the mean of the number of letters in 20 codes?

Select one:

- a. 80
- b. 4
- c. 40
- d. 8

Question 5

Let the random variable X have a discrete uniform distribution on the integers 12, 13, ..., 19. Find the value of P(X > 17).

Select one:

- a. 3/8
- b. 0.6
- c. 1/3
- d. 0.25

Question 6

Product codes of 1, 2 or 3 letters are equally likely. What is the mean of the number of letters in 50 codes?

Select one:

- a. 2
- b. 20
- c. 80
- d. 100

Question 7

Suppose that X has a discrete uniform distribution on the integers 2 to 5. Find V(4X).

Select one:

- a. 12.3
- b. 20
- c. 10
- d. 4.47
- e. None of the other choices is correct

Question 8

Product codes of 6, 7, 8 or 9 letters are equally likely. Which of the following statements are true?

(i) Standard deviation of the number of letters in one code is 1.25.

(ii) The probability of the event that the code has at least 7 letters is 0.5

Select one:

- a. (ii) only
- b. None of the other choices is correct
- c. (i) only
- d. Both (i) and (ii)

Question 9

The thickness measurements of a coating process are uniform distributed with values 0.1, 0.14, 0.18, 0.16. Determine the standard deviation of the coating thickness for this process.

Select one:

- a. 0.02
- b. 0.0009
- c. 0.03
- d. None of the other choices is correct
- e. 0.01

Question 10

Suppose that X has a discrete uniform distribution on the integers 20 to 79. Which of the followings are true?

(i) $P(X > 41) = 13/20$

(ii) $E(10X) = 495$

Select one:

- a. (i) only
- b. None of the other choices is correct
- c. (ii) only
- d. Both (i) and (ii)

Question 11

Suppose that X has a discrete uniform distribution on the integers 2 to 8. Which of the following are true?

(i) E(4X) = 20

(ii) σ(X) = 4

Select one:

- a. Both (i) and (ii)
- b. (ii) only
- c. (i) only
- d. None of the other choices is correct

Content

6. Binomial Distribution

Bernoulli trial

A trial with only two possible outcomes is used so frequently as a building block of a random experiment that it is called a **Bernoulli trial**.
The outcome from each trial either meets the criterion that X counts or it does not; consequently, each trial can be summarized as resulting in either a *success* or a *failure*. Then, $X = \{0; 1\}$

6. Binomial Distribution

Bernoulli trial

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The outcome from each trial either meets the criterion that X counts or it does not; consequently, each trial can be summarized as resulting in either a *success* or a *failure*. Then, $X = \{0; 1\}$

For example:

- Flip a coin. Let X = number of heads obtained.

6. Binomial Distribution

Bernoulli trial

A trial with only two possible outcomes is used so frequently as a building block of a random experiment that it is called a **Bernoulli trial**.

The outcome from each trial either meets the criterion that X counts or it does not; consequently, each trial can be summarized as resulting in either a *success* or a *failure*. Then, $X = \{0; 1\}$

For example:

- Flip a coin. Let X = number of heads obtained.
- In the multiple-choice experiment, for each question, only the choice that is correct is considered a success. Choosing any one of the three incorrect choices results in the trial being summarized as a failure.

6. Binomial Distribution

Binomial Distribution

A random experiment consists of n Bernoulli trials such that
1) The trials are independent.
2) Each trial results in only two possible outcomes, labeled as "success" and "failure".
3) The probability of a success in each trial, denoted as p, remains constant.

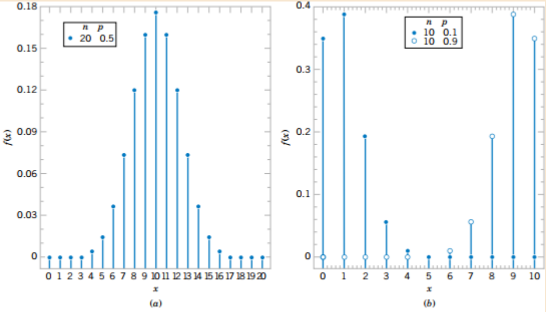
The random variable X that equals the number of trials that result in a success has a **binomial random variable** with parameters $0 < p < 1$ and $n = 1, 2, ...$. The probability mass function of X is

$$f(x) = C_x^n p^x (1-p)^{n-x}, \qquad x = 0, 1, ..., n$$

6. Binomial Distribution

For example:

- Flip a coin 10 times. Let X = number of heads obtained.
- A multiple-choice test contains 10 questions, each with four choices, and you guess at each question. Let X = the number of questions answered correctly.
- In the next 20 births at a hospital. Let X = the number of female births.



6. Binomial Distribution

Example 8

Each sample of water has a 10% chance of containing a particular organic pollutant. Assume that the samples are independent with regard to the presence of the pollutant.

a) Find the probability that in the next 18 samples, exactly 2 contain the pollutant.

b) Determine the probability that at least four samples contain the pollutant.

6. Binomial Distribution

Example 8

Each sample of water has a 10% chance of containing a particular organic pollutant. Assume that the samples are independent with regard to the presence of the pollutant.

a) Find the probability that in the next 18 samples, exactly 2 contain the pollutant.

b) Determine the probability that at least four samples contain the pollutant.

Let X = the number of samples that contain the pollutant in the next 18 samples analyzed. Then X is a binomial random variable with $p = 0.1$ and $n = 18$. Therefore,

$$P(X = 2) = C_2^{18}(0.1)^2(0.9)^{16} = 153(0.1)^2(0.9)^{16} = 0.284$$

$$P(X \geq 4) = 1 - P(X < 4) = 1 - \sum_{x=0}^{3} C_x^{18}(0.1)^x(0.9)^{18-x} = 0.098$$

6. Binomial Distribution

Mean and Variance

If X is a binomial random variable with parameters p and n,

$$\mu = E(X) = np$$

and

$$\sigma^2 = V(X) = np(1-p)$$

Exercise 1

Let X be a binomial random variable with $p = 0.2$ and $n = 20$. Use the binomial table in Appendix A to determine the following probabilities:

a) $P(X \leq 3)$ b) $P(X > 10)$

c) $P(X = 6)$ d) $P(6 \leq X \leq 11)$

6. Binomial Distribution

Exercise 1

Let X be a binomial random variable with $p = 0.2$ and $n = 20$. Use the binomial table in Appendix A to determine the following probabilities:

a) $P(X \leq 3)$ b) $P(X > 10)$

c) $P(X = 6)$ d) $P(6 \leq X \leq 11)$

Exercise 2

The random variable X has a binomial distribution with $n = 10$ and $p = 0.5$. Determine the following probabilities:

a) $P(X = 5)$ b) $P(X \leq 2)$

c) $P(X \geq 9)$ d) $P(3 \leq X < 5)$

6. Binomial Distribution

Exercise 3

Determine the cumulative distribution function of a binomial random variable with $n = 3$ and $p = 1/2$.

6. Binomial Distribution

Exercise 3

Determine the cumulative distribution function of a binomial random variable with $n = 3$ and $p = 1/2$.

Exercise 4

A multiple-choice test contains 25 questions, each with four answers. Assume a student just guesses on each question.
a) What is the probability that the student answers more than 20 questions correctly?
b) What is the probability the student answers less than five questions correctly?

6. Binomial Distribution

Exercise 3

Determine the cumulative distribution function of a binomial random variable with $n = 3$ and $p = 1/2$.

Exercise 4

A multiple-choice test contains 25 questions, each with four answers. Assume a student just guesses on each question.
a) What is the probability that the student answers more than 20 questions correctly?
b) What is the probability the student answers less than five questions correctly?

Exercise 5

An electronic product contains 40 integrated circuits. The probability that any integrated circuit is defective is 0.01, and the integrated circuits are independent. The product operates only if there are no defective integrated circuits. What is the probability that the product operates?

Question 1

Find the mean for the binomial distribution which has the values of n = 33 and p = 0.2. Round answer to the nearest tenth.

Select one:

- a. 6.9
- b. 7.3
- c. 6.1
- d. 6.6

Question 2

Find the mean for the binomial distribution which has the stated values of n = 20 and p = 3/5. Round answer to the nearest tenth.

Select one:

- a. 12.3
- b. 11.5
- c. 12.0
- d. 12.7

Question 3

Find the standard deviation for the binomial distribution which has the stated values of $n = 2661$ and $p = 0.63$. Round your answer to the nearest hundredth.

Select one:

- a. 28.18
- b. 29.03
- c. 24.91
- d. 22.50

Question 4

The probability that a house in an urban area will be burglarized is 15%. If 30 houses are randomly selected, what is the mean of the number of houses burglarized?

Select one:

- a. 2
- b. 1.5
- c. 4.5
- d. 1

Question 5

According to a college survey, 12% of all students work full time. Find the mean for the number of students who work full time in samples of size 54.

Select one:

- a. 6.48
- b. 4.00
- c. 0.22
- d. 3.52

Question 6

According to a college survey, 15% of all students work full time. Find the mean for the random variable X, the number of students who work full time in samples of size 42.

Select one:

- a. 4.00
- b. 2.75
- c. 3.52
- d. 6.30

Question 7

A die is rolled 22 times and the number of times that two shows on the upper face is counted. If this experiment is repeated many times, find the mean for the number of twos.

Select one:

- a. 1.67
- b. 8.33
- c. 3.67
- d. 2.98

Question 8

On a 50-question multiple choice test , each question has four possible answers, one of which is correct. For students who guess at all answers, find the mean for the random variable X, the number of correct answers.

Select one:

- a. 5
- b. 12.5
- c. 22.5
- d. 2.5

Question 9

The probability that a person has immunity to a particular disease is 0.06. Find the mean for the random variable X, the number who have immunity in samples of size 106.

Select one:

- a. 6.36
- b. 15.6
- c. 10.4
- d. 6.84

Question 10

The probability is 0.85 that a person shopping at a certain store will spend less than $20. For random samples of 82 customers, find the mean number of shoppers who spend less than $20.

Select one:

- a. 69.7
- b. 44.0
- c. 62.0
- d. 19.6

Question 11

According to a CNN poll taken in February of 2008, 67% of respondents disapproved of the overall job that President Bush was doing. Based on this poll, for samples of size 140, what is the mean number of American adults who disapprove of the overall job that President Bush is doing?

Select one:

- a. 93.8
- b. 67
- c. 44.22
- d. 134

Question 12

According to a college survey, 18% of all students work full time. Find the standard deviation for the random variable X, the number of students who work full time in samples of size 35.

Select one:

- a. 3.52
- b. 1.66
- c. 1.88
- d. 2.27

Question 13

A die is rolled 80 times and the number of twos that come up is tallied. If this experiment is repeated many times, find the standard deviation for the random variable X, the number of twos.

Select one:

- a. 3.33
- b. 1.58
- c. 1.62
- d. 1.73

Question 14

On a multiple choice test with 12 questions, each question has four possible answers, one of which is correct. For students who guess at all answers, find the standard deviation for the random variable X, the number of correct answers.

Select one:

- a. 1.500
- b. 1.746
- c. 1.732
- d. 1.600

Question 15

The probability that a radish seed will germinate is 0.26. A gardener plants seeds in batches of 52. Find the standard deviation for the random variable X, the number of seeds germinating in each batch.

Select one:

- a. 3.25
- b. 1.77
- c. 3.16
- d. 1.52

Question 16

The probability of winning a certain lottery is 1/9999. For people who play 246 times, find the standard deviation for the random variable X, the number of wins.

Select one:

- a. 0.1038
- b. 0.1223
- c. 0.1568
- d. 0.0108

Question 17

A salesperson knows that 20% of her presentations result in sales. Find the probabilities that in the next 60 presentations at least 9 result in sales.

Select one:

- a. 0.1241
- b. 0.8732
- c. 0.8189
- d. 0.6421

Question 18

In a binomial distribution with 10 trials, which of the following is true?

Select one:

- a. $P(x < 6) = 1 - P(x \geq 7)$
- b. $P(x > 7) = P(x \geq 8)$
- c. $P(3 \leq x \leq 5) = P(3 < x < 5)$
- d. $P(x < 4) = P(x \geq 5) - P(x \geq 4)$

Question 19

If the probability of a newborn child being female is 0.5, find the probability that in 50 births, 35 or more will be female.

Select one:

- a. 0.0033
- b. 0.0606
- c. 0.1841
- d. 0.8059

Question 20

Assume that a procedure yields a binomial distribution with a trial repeated 4 times. Use the binomial probability formula to find the probability of 3 successes given the probability 1/6 of success on a single trial.

Select one:

- a. 0.0231
- b. 0.0154
- c. 0.0116
- d. 0.0039

Question 21

Assume that a procedure yields a binomial distribution with a trial repeated 12 times. Use the binomial probability formula to find the probability of 5 successes given the probability 0.25 of success on a single trial.

Select one:

- a. 0.091
- b. 0.082
- c. 0.027
- d. 0.103

Question 22

Assume that a procedure yields a binomial distribution with a trial repeated 64 times. Use the binomial probability formula to find the probability of 3 successes given the probability 0.04 of success on a single trial.

Select one:

- a. 0.221
- b. 0.139
- c. 0.091
- d. 0.375

Question 23

The random variable X represents the number of girls in a family of three children. Assuming that boys and girls are equally likely, find the probability that the number of girls is two or more.

Select one:

- a. 0.75
- b. 0.50
- c. 0.40
- d. 0.25

Question 24

In a recent survey, 80% of the community favored building a police substation in their neighborhood. If 15 citizens are chosen, what is the probability that the number favoring the substation is more than 12?

Select one:

- a. 0.398
- b. 0.1208
- c. 0.3518
- d. 0.6019

Question 25

In a recent survey, 95% of the community favored building a police substation in their neighborhood. If 50 citizens are chosen, what is the probability that the number favoring the substation is exactly 42?

Select one:

- a. 0.6218
- b. 0.0046
- c. 0.0024
- d. 0.5501

Question 26

In a recent survey, 85% of the community favored building a police substation in their neighborhood. If 20 citizens are chosen, what is the probability that the number favoring the substation is exactly 12?

Select one:

- a. 0.0046
- b. 0.0059
- c. 0.5501
- d. 0.6218

Question 27

A telemarketer found that there was a 1.5% chance of a sale from his phone solicitations. Find the probability of getting 28 or more sales for 1000 telephone calls.

Select one:

- a. 0.0401
- b. 0.8810
- c. 0.0016

Question 28

A card game is played in which the player wins if a face card is drawn (king, queen, jack) from a deck of 52 cards. If the player plays 10 times, what is the probability that the number of wins for the player is 5?

Select one:

- a. 0.0444
- b. 0.0132
- c. 0.9868
- d. 0.5821

Question 29

According to the 2003 National Survey on Drug Use and Health, 55.3% of males have never used marijuana. Based on this percentage, what is the probability that more than 50 males who have used marijuana for samples of size 120?

Select one:

- a. 0.717
- b. 0.0010
- c. 0.0018
- d. 0.8921

Question 30

An archer is able to hit the bull's-eye 57% of the time. If she shoots 15 arrows, what is the probability that she gets exactly 6 bull's-eyes? Assume each shot is independent of the others.

Select one:

- a. 0.0038
- b. 0.2627
- c. 0.0863
- d. 0.1719

Question 31

A tennis player makes a successful first serve 53% of the time. If she serves 6 times, what is the probability that she gets exactly 3 first serves in? Assume that each serve is independent of the others.

Select one:

- a. 0.3091
- b. 0.4062
- c. 0.7069
- d. 0.2031

Question 32

A multiple choice test has 22 questions each of which has 4 possible answers, only one of which is correct. If Judy, who forgot to study for the test, guesses on all questions, what is the probability that she will answer exactly 8 questions correctly?

Select one:

- a. 0.0021
- b. 0.2503
- c. 0.0869
- d. 0.5006

Question 33

In one city, the probability that a person will pass his or her driving test on the first attempt is 0.59. 23 people are selected at random from among those taking their driving test for the first time. What is the probability that among these 23 people, the number passing the test is between 15 and 18 inclusive?

Select one:

- a. 0.0345
- b. 0.0308
- c. 0.3362
- d. 0.0299

Content

Geometric Distribution

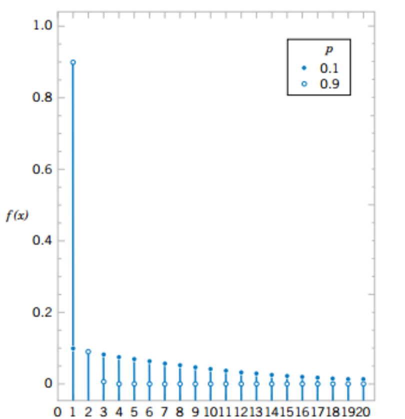
Geometric Distribution

In a series of Bernoulli trials (independent trials with constant probability p of a success), let the random variable X denote the number of trials until the first success. Then X is a geometric random variable with parameter $0 < p < 1$ and

$$f(x) = (1-p)^{x-1}p, \quad x = 1, 2, \ldots$$

Geometric Distribution

Examples of the probability mass functions for geometric random variables are shown in Fig. 3-9. Note that the height of the line at x is $(1-p)$ times the height of the line at $x-1$. That is, the probabilities decrease in a geometric progression. The distribution acquires its name from this result.



Geometric Distribution

Example 9

The probability that a wafer contains a large particle of contamination is 0.01. If it is assumed that the wafers are independent, what is the probability that exactly 125 wafers need to be analyzed before a large particle is detected?

Geometric Distribution

Example 9

The probability that a wafer contains a large particle of contamination is 0.01. If it is assumed that the wafers are independent, what is the probability that exactly 125 wafers need to be analyzed before a large particle is detected?

Let X denote the number of samples analyzed until a large particle is detected. Then X is a geometric random variable with $p = 0.01$. The requested probability is

$$P(X = 125) = (0.99)^{124} 0.01 = 0.0029$$

Geometric Distribution

Mean and Variance

If X is a geometric random variable with parameter p

$$\mu = E(X) = 1/p$$

and

$$\sigma^2 = V(X) = (1-p)/p^2$$

Negative Binomial Distribution

Negative Binomial Distribution

In a series of Bernoulli trials (independent trials with constant probability pof a success), let the random variable X denote the number of trials until r successes occur. Then X is a **negative binomial random variable** with parameters $0 < p < 1$ and $r = 1, 2, ...$, and

$$f(x) = C_{r-1}^{x-1}(1-p)^{x-r}p^r, \quad x = r, r+1, r+2, ...$$

In the special case that $r = 1$, a negative binomial random variable is a geometric random variable.

Negative Binomial Distribution

Mean and Variance

If X is a negative binomial random variable with parameters p and r,

$$\mu = E(X) = r/p$$

and

$$\sigma^2 = V(X) = r(1-p)/p^2$$

Negative Binomial Distribution

Example 10

The probability that a bit transmitted through a digital transmission channel is received in error is 0.1. Assume the transmissions are independent events, and let the random variable X denote the number of bits transmitted until the fourth error.

Then, X has a negative binomial distribution with $r = 4$. Probabilities involving Xcan be found as follows. $P(X) = 10$ is the probability that exactly three errors occur in the first nine trials and then trial 10 results in the fourth error. The probability that exactly three errors occur in the first nine trials is determined from the binomial distribution to be

$$C_3^9(0.1)^3(0.9)^6$$

Because the trials are independent, the probability that exactly three errors occur in the first 9 trials and trial 10 results in the fourth error is the product of the probabilities of these two events, namely

$$C_3^9(0.1)^3(0.9)^6(0.1) = C_3^9(0.1)^4(0.9)^6$$

7. Geometric and Negative Binomial Distributions

Exercise 1

Suppose the random variable X has a geometric distribution with $p = 0.5$. Determine the following probabilities:

a) $P(X = 1)$ b) $P(X = 4)$

c) $P(X = 8)$ d) $P(X \leq 2)$ e) $P(X > 2)$

7. Geometric and Negative Binomial Distributions

Exercise 1

Suppose the random variable X has a geometric distribution with $p = 0.5$. Determine the following probabilities:

a) $P(X = 1)$ b) $P(X = 4)$

c) $P(X = 8)$ d) $P(X \leq 2)$ e) $P(X > 2)$

Exercise 2

Suppose the random variable X has a geometric distribution with a mean of 2.5. Determine the following probabilities:

a) $P(X = 1)$ b) $P(X = 4)$

c) $P(X = 5)$ d) $P(X \leq 3)$ e) $P(X > 3)$

7. Geometric and Negative Binomial Distributions

Exercise 3

Suppose that X is a negative binomial random variable with $p = 0.2$ and $r = 4$. Determine the following:

a) $E(X)$ b) $P(X = 20)$

c) $P(X = 19)$ d) $P(X = 21)$

7. Geometric and Negative Binomial Distributions

Exercise 3

Suppose that X is a negative binomial random variable with $p = 0.2$ and $r = 4$. Determine the following:

a) $E(X)$ b) $P(X = 20)$

c) $P(X = 19)$ d) $P(X = 21)$

Exercise 4

The probability of a successful optical alignment in the assembly of an optical data storage product is 0.8. Assume the trials are independent.

a) What is the probability that the first successful alignment requires exactly four trials?

b) What is the probability that the first successful alignment requires at most four trials?

c) What is the probability that the first successful alignment requires at least four trials?

7. Geometric and Negative Binomial Distributions

Exercise 5

In a clinical study, volunteers are tested for a gene that has been found to increase the risk for a disease. The probability that a person carries the gene is 0.1.

a) What is the probability four or more people will have to be tested before two with the gene are detected?

b) How many people are expected to be tested before two with the gene are detected?

7. Geometric and Negative Binomial Distributions

Exercise 5

In a clinical study, volunteers are tested for a gene that has been found to increase the risk for a disease. The probability that a person carries the gene is 0.1.
a) What is the probability four or more people will have to be tested before two with the gene are detected?
b) How many people are expected to be tested before two with the gene are detected?

Exercise 6

Assume that each of your calls to a popular radio station has a probability of 0.02 of connecting, that is, of not obtaining a busy signal. Assume that your calls are independent.
a) What is the probability that your first call that connects is your tenth call?
b) What is the probability that it requires more than five calls for you to connect?
c) What is the mean number of calls needed to connect?

7. Geometric and Negative Binomial Distributions

Exercise 7

A player of a video game is confronted with a series of opponents and has an 80% probability of defeating each one. Success with any opponent is independent of previous encounters. The player continues to contest opponents until defeated.

a) What is the probability mass function of the number of opponents contested in a game?

b) What is the probability that a player defeats at least two opponents in a game?

c) What is the expected number of opponents contested in a game?

d) What is the probability that a player contests four or more opponents in a game?

e) What is the expected number of game plays until a player contests four or more opponents?

Question 1

The probability of a successful optical alignment in the assembly of an optical data storage product is 0.7. Assume the trials are independent. What is the probability that the first successful alignment requires exactly 4 trials?

Select one:

- a. 0.072
- b. 0.019
- c. 0.103
- d. 0.006

Question 2

The probability of a successful optical alignment in the assembly of an optical data storage product is 0.7. Assume the trials are independent. What is the probability that the first **two** successful alignments require exactly 4 trials?

Select one:

- a. 0.132
- b. 0.017
- c. 0.402
- d. 0.005

Content

8. Hypergeometric Distribution

Hypergeometric Distribution

A set of N objects contains
K objects classified as successes
$N-K$ objects classified as failures
A sample of size n objects is selected randomly (without replacement) from the N objects, where $K \leq N$ and $n \leq N$.
Let the random variable X denote the number of successes in the sample. Then X is a **hypergeometric random variable** and

$$f(x) = \frac{C_x^K . C_{n-x}^{N-K}}{C_n^N}, \quad x = \max\{0, n+K-N\} \text{ to } \min\{\mathrm{K}, \mathrm{n}\}$$

Mean and Variance

If X is a hypergeometric random variable with parameters N, K, n, then

$$\mu = E(X) = np$$

and

$$\sigma^2 = V(X) = np(1-p)(\frac{N-n}{N-1})$$

where $p = K/N$.

8. Hypergeometric Distribution

Finite Population Correction Factor

The term in the variance of a hypergeometric random variable

$$\frac{N-n}{N-1}$$

is called the **finite population correction factor**.

If sampling were done with replacement, X would be a binomial random variable and its variance would be $np(1-p)$. Consequently, the finite population correction represents the correction to the binomial variance that results because the sampling is without replacement from the finite set of size N. If n is small relative to N, the correction is small and the hypergeometric distribution is similar to the binomial.

Example 11

A batch of parts contains 100 parts from a local supplier of tubing and 200 parts from a supplier of tubing in the next state. If four parts are selected randomly and without replacement, what is the probability they are all from the local supplier?

8. Hypergeometric Distribution

Example 11

A batch of parts contains 100 parts from a local supplier of tubing and 200 parts from a supplier of tubing in the next state. If four parts are selected randomly and without replacement, what is the probability they are all from the local supplier?

Let X equal the number of parts in the sample from the local supplier. Then, X has a hypergeometric distribution and the requested probability is $P(X = 4)$. Consequently,

$$P(X = 4) = \frac{C_4^{100}.C_0^{200}}{C_4^{300}} = 0.0119$$

What is the probability that two or more parts in the sample are from the local supplier?

$$P(X \geq 2) = P(X = 2) + P(X = 3) + P(X = 4) = 0.298 + 0.098 + 0.0119 = 0.408$$

Exercise 1

Suppose X has a hypergeometric distribution with $N = 100, n = 4$ and $K = 20$. Determine the following:

a) $P(X = 1)$ b) $P(X = 6)$ c) $P(X = 4)$

d) Determine the mean and variance of X.

8. Hypergeometric Distribution

Exercise 1

Suppose X has a hypergeometric distribution with $N = 100, n = 4$ and $K = 20$. Determine the following:

a) $P(X = 1)$ b) $P(X = 6)$ c) $P(X = 4)$

d) Determine the mean and variance of X.

Exercise 2

Suppose X has a hypergeometric distribution with $N = 10, n = 3$ and $K = 4$. Sketch the probability mass function of X. Determine the cumulative distribution function for X.

Exercise 3

A batch contains 36 bacteria cells and 12 of the cells are not capable of cellular replication. Suppose you examine three bacteria cells selected at random, without replacement.

a) What is the probability mass function of the number of cells in the sample that can replicate?

b) What are the mean and variance of the number of cells in the sample that can replicate?

c) What is the probability that at least one of the selected cells cannot replicate?

Exercise 4

A company employs 800 men under the age of 55. Suppose that 30% carry a marker on the male chromosome that indicates an increased risk for high blood pressure.
a) If 10 men in the company are tested for the marker in this chromosome, what is the probability that exactly one man has the marker?
b) If 10 men in the company are tested for the marker in this chromosome, what is the probability that more than one has the marker?

8. Hypergeometric Distribution

Exercise 5

A state runs a lottery in which six numbers are randomly selected from 40, without replacement. A player chooses six numbers before the state's sample is selected.

a) What is the probability that the six numbers chosen by a player match all six numbers in the state's sample?

b) What is the probability that five of the six numbers chosen by a player appear in the state's sample?

c) What is the probability that four of the six numbers chosen by a player appear in the state's sample?

d) If a player enters one lottery each week, what is the expected number of weeks until a player matches all six numbers in the state's sample?

Question 1

A batch contains 36 bacteria cells, in which 12 are not capable of cellular replication. Suppose you examine 7 bacteria cells selected at random, without replacement. What is the probability that exactly 3 of them are not capable of cellular replication?

Select one:

- a. 0.83
- b. 0.28
- c. 0.72
- d. 0.17

Question 2

A batch contains 36 bacteria cells, in which 12 are not capable of cellular replication. Suppose you examine 7 bacteria cells selected at random, without replacement. What is the probability that exactly 3 of them are not capable of cellular replication?

Select one:

- a. 0.83
- b. 0.28
- c. 0.72
- d. 0.17

Content

9. Poisson Distribution

Poisson Process

Consider an interval T of real numbers partitioned into subintervals of small length Δt and assume that as Δt tends to zero,

- the probability of more than one event in a subinterval tends to zero,
- the probability of one event in a subinterval tends to $\lambda \Delta t / T$,
- the event in each subinterval is independent of other subintervals.

A random experiment with these properties is called a **Poisson process**.

9. Poisson Distribution

Poisson Distribution

The random variable X that equals the number of events in a Poisson process is a **Poisson random variable** with parameter $0 < \lambda$ and the probability mass function of X is

$$f(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}, \qquad x = 0, 1, 2, \ldots$$

9. Poisson Distribution

Mean and Variance

If X is a Poisson random variable with parameter λ, then

$$\mu = E(X) = \lambda \text{ and } \sigma^2 = V(X) = \lambda$$

9. Poisson Distribution

Example 12

For the case of the thin copper wire, suppose that the number of flaws follows a Poisson distribution with a mean of 2.3 flaws per millimeter. Determine the probability of exactly two flaws in 1 millimeter of wire.

Example 12

For the case of the thin copper wire, suppose that the number of flaws follows a Poisson distribution with a mean of 2.3 flaws per millimeter. Determine the probability of exactly two flaws in 1 millimeter of wire.

Let X denote the number of flaws in 1 millimeter of wire. Then, $E(X) = 2.3$ flaws and

$$P(X = 2) = \frac{e^{-2.3} 2.3^2}{2!} = 0.265$$

9. Poisson Distribution

Example 12

For the case of the thin copper wire, suppose that the number of flaws follows a Poisson distribution with a mean of 2.3 flaws per millimeter. Determine the probability of exactly two flaws in 1 millimeter of wire.

Let X denote the number of flaws in 1 millimeter of wire. Then, $E(X) = 2.3$ flaws and

$$P(X = 2) = \frac{e^{-2.3} 2.3^2}{2!} = 0.265$$

Determine the probability of 10 flaws in 5 millimeters of wire.
Let X denote the number of flaws in 5 millimeters of wire. Then, X has a Poisson distribution with

$$E(X) = 5\text{mm} \times 2.3\text{flaws/mm} = 11.5\text{flaws}$$

Therefore,

$$P(X = 10) = \frac{e^{-11.5} 11.5^10}{10!} = 0.113$$

9. Poisson Distribution

Example 12

Determine the probability of at least one flaw in 2 millimeters of wire. Let X denote the number of flaws in 2 millimeters of wire. Then, X has a Poisson distribution with

$$E(X) = 2\text{mm} \times 2.3\text{flaws/mm} = 4.6\text{flaws}$$

Therefore,

$$P(X \geq 1) = 1 - P(X = 0) = 1 - e^{-4.6} = 0.9899$$

9. Poisson Distribution

Exercise 1

Suppose X has a Poisson distribution with a mean of 4. Determine the following probabilities:

a) $P(X = 0)$ b) $P(X \leq 2)$

c) $P(X = 4)$ d) $P(X = 8)$

9. Poisson Distribution

Exercise 1

Suppose X has a Poisson distribution with a mean of 4. Determine the following probabilities:

a) $P(X = 0)$ b) $P(X \leq 2)$

c) $P(X = 4)$ d) $P(X = 8)$

Exercise 2

Suppose that the number of customers who enter a bank in an hour is a Poisson random variable, and suppose that $P(X = 0) = 0.05$. Determine the mean and variance of X.

Exercise 3

The number of telephone calls that arrive at a phone exchange is often modeled as a Poisson random variable. Assume that on the average there are 10 calls per hour.

a) What is the probability that there are exactly five calls in one hour?

b) What is the probability that there are three or fewer calls in one hour?

c) What is the probability that there are exactly 15 calls in two hours?

d) What is the probability that there are exactly five calls in 30 minutes?

Exercise 4

The number of flaws in bolts of cloth in textile manufacturing is assumed to be Poisson distributed with a mean of 0.1 flaw per square meter.

a) What is the probability that there are two flaws in 1 square meter of cloth?

b) What is the probability that there is one flaw in 10 square meters of cloth?

c) What is the probability that there are no flaws in 20 square meters of cloth?

d) What is the probability that there are at least two flaws in 10 square meters of cloth?

9. Poisson Distribution

Exercise 5

When a computer disk manufacturer tests a disk, it writes to the disk and then tests it using a certifier. The certifier counts the number of missing pulses or errors. The number of errors on a test area on a disk has a Poisson distribution with $\lambda = 0.2$

a) What is the expected number of errors per test area?

b) What percentage of test areas have two or fewer errors?

Exercise 5

When a computer disk manufacturer tests a disk, it writes to the disk and then tests it using a certifier. The certifier counts the number of missing pulses or errors. The number of errors on a test area on a disk has a Poisson distribution with $\lambda = 0.2$

a) What is the expected number of errors per test area?

b) What percentage of test areas have two or fewer errors?

Exercise 6

The number of content changes to a Web site follows a Poisson distribution with a mean of 0.25 per day.

a) What is the probability of two or more changes in a day?

b) What is the probability of no content changes in five days?

c) What is the probability of two or fewer changes in five days?

Question 1

The number of weeds that remain living after a specific chemical has been applied averages 1.21 per square yard and follows a Poisson distribution. Based on this, what is the probability that a 1 square yard section will contain less than 5 weeds?

Select one:

- a. 0.0998
- b. 0.6324
- c. 0.5000
- d. 0.9920

Question 2

The manager of a movie theater has determined that the distribution of customers arriving at the concession stand is Poisson distributed with a standard deviation equal to 2 people per 10 minutes. If the servers can accommodate 3 customers in a 10-minute period, what is the probability that the servers will be idle for an entire ten minute period?

Select one:

- a. 0.1353
- b. 0.9807
- c. 0.2135
- d. 0.0183

Question 3

If the standard deviation for a Poisson distribution is known to be 3, the expected value of that Poison distribution is:

Select one:

- a. Can't be determined without more information.
- b. 9.
- c. about 1.73.
- d. 3.

Question 4

The number of calls to an Internet service provider during the hour between 6:00 and 7:00 p.m. is described by a Poisson distribution with mean equal to 15. Given this information, what is the expected number of calls in the first 30 minutes?

Select one:

- a. 15
- b. 7.5
- c. 3.87
- d. 225

Question 5

The number of customers that arrive at a fast-food business during a one-hour period is known to be Poisson distributed with a mean equal to 8.60. What is the probability that exactly 8 customers will arrive in a one-hour period?

Select one:

- a. 0.7832
- b. 0.0065
- c. 0.2073
- d. 0.1366

Question 6

Assume that ***x*** has a Poisson probability distribution. Find ***P***(***x*** = 6) when μ = 1.0.

Select one:

- a. .9999
- b. .0005
- c. .0031
- d. 1

Question 7

Suppose X has a Poisson probability distribution with $\lambda = 9.0$. Find μ and σ.

Select one:

- a. $\mu = 3.0, \sigma = 3.0$
- b. $\mu = 9.0, \sigma = 81.0$
- c. $\mu = 9.0, \sigma = 9.0$
- d. $\mu = 9.0, \sigma = 3.0$

Question 8

The number of visible defects on a product container is thought to be Poisson distributed with a mean equal to 4.3. Based on this, the probability that 2 containers will contain less than 2 defects is:

Select one:

- a. 0.1359
- b. 0.1850
- c. 0.0073
- d. 0.0018

Question 9

The number of visible defects on a product container is thought to be Poisson distributed with a mean equal to 2.1. Based on this, how many defects should be expected if 2 containers are inspected?

Select one:

- a. 10.5
- b. 2.1
- c. 4.2
- d. Between 4 and 7

Question 10

An automobile service center can take care of 12 cars per hour. If cars arrive at the center randomly and independently at a rate of 8 per hour on average, what is the probability of the service center being totally empty in a given hour?

Select one:

- a. 0.0003
- b. 0.1755
- c. 0.2011
- d. 0.0067

Question 11

The Columbia Power Company experiences power failures with a mean of 0.120 per day. Use the Poisson Distribution to find the probability that there are exactly two power failures in a particular day.

Select one:

- a. 0.018
- b. 0.006
- c. 0.085
- d. 0.027