맥스웰 방정식 → 이중슬릿 → 나비에-스토크스 치환

1. 나비에-스토크스 방정식 (5항)

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u}\right) = -\nabla p + \mu\nabla^2\mathbf{u} + \mathbf{f}$$

- $\mathbf{u} = (u, v, w)$: 유체 속도 - ρ: 밀도, μ: 점성계수 - $\mathbf{f}$: 외력 (전자기 포함)

2. 맥스웰 방정식과 이중슬릿 치환

맥스웰 4식

$$\nabla\cdot\mathbf{E} = \frac{\rho_e}{\varepsilon_0}, \quad \nabla\cdot\mathbf{B} = 0$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}, \quad \nabla\times\mathbf{B} = \mu_0\mathbf{J} + \mu_0\varepsilon_0\frac{\partial\mathbf{E}}{\partial t}$$

이중슬릿 전계

$$\mathbf{E}_{\text{slit}}(\mathbf{r}, t) = \Re\left\{A(\theta)(e^{i\phi_1} + e^{i\phi_2})e^{-i\omega t}\hat{\mathbf{e}}\right\}$$

$$\mathbf{B}_{\text{slit}} \approx \frac{1}{c}\hat{\mathbf{k}}\times\mathbf{E}_{\text{slit}}$$

여기서 $A(\theta) = \text{sinc}(ak\sin\theta)$, $\Delta\phi = kd\sin\theta$.

3. N-S 각 항과 맥스웰 치환

N-S 항	대응 맥스웰 항	치환 수식
시간항 $\rho\partial_t\mathbf{u}$	패러데이 법칙 $\nabla\times\mathbf{E} = -\partial_t\mathbf{B}$	$\partial_t\mathbf{u} \to \varepsilon_0\partial_t\mathbf{E}_{\text{slit}}$
대류항 $\rho(\mathbf{u}\cdot\nabla)\mathbf{u}$	앰페어-맥스웰 법칙 $\nabla\times\mathbf{B} = \mu_0\mathbf{J} + \mu_0\varepsilon_0\partial_t\mathbf{E}$	$(\mathbf{u}\cdot\nabla)\mathbf{u} \to \frac{1}{\mu_0}(\nabla\times\mathbf{B}_{\text{slit}})\times\mathbf{B}_{\text{slit}}$
압력항 $-\nabla p$	가우스 전기 $\nabla\cdot\mathbf{E} = \rho_e/\varepsilon_0$	$-\nabla p \to -\nabla(\varepsilon_0(\nabla\cdot\mathbf{E}_{\text{slit}}))$
점성항 $\mu\nabla^2\mathbf{u}$	가우스 자기 $\nabla\cdot\mathbf{B} = 0$	$\mu\nabla^2\mathbf{u} \to \mu\nabla^2\mathbf{B}_{\text{slit}}$
외력항 $\mathbf{f}$	로렌츠 힘 / 이중슬릿	$\mathbf{f} \to \varepsilon_0(\nabla\cdot\mathbf{E}_{\text{slit}})\mathbf{E}_{\text{slit}} + [\frac{1}{\mu_0}(\nabla\times\mathbf{B}_{\text{slit}}) - \varepsilon_0\partial_t\mathbf{E}_{\text{slit}}]\times\mathbf{B}_{\text{slit}}$

4. 편미분 예시 (x방향)

$$\partial_x(\rho\partial_t u) = \rho\frac{\partial^2 u}{\partial x\partial t} \approx \rho\varepsilon_0\frac{\partial^2 E_x}{\partial x\partial t}$$

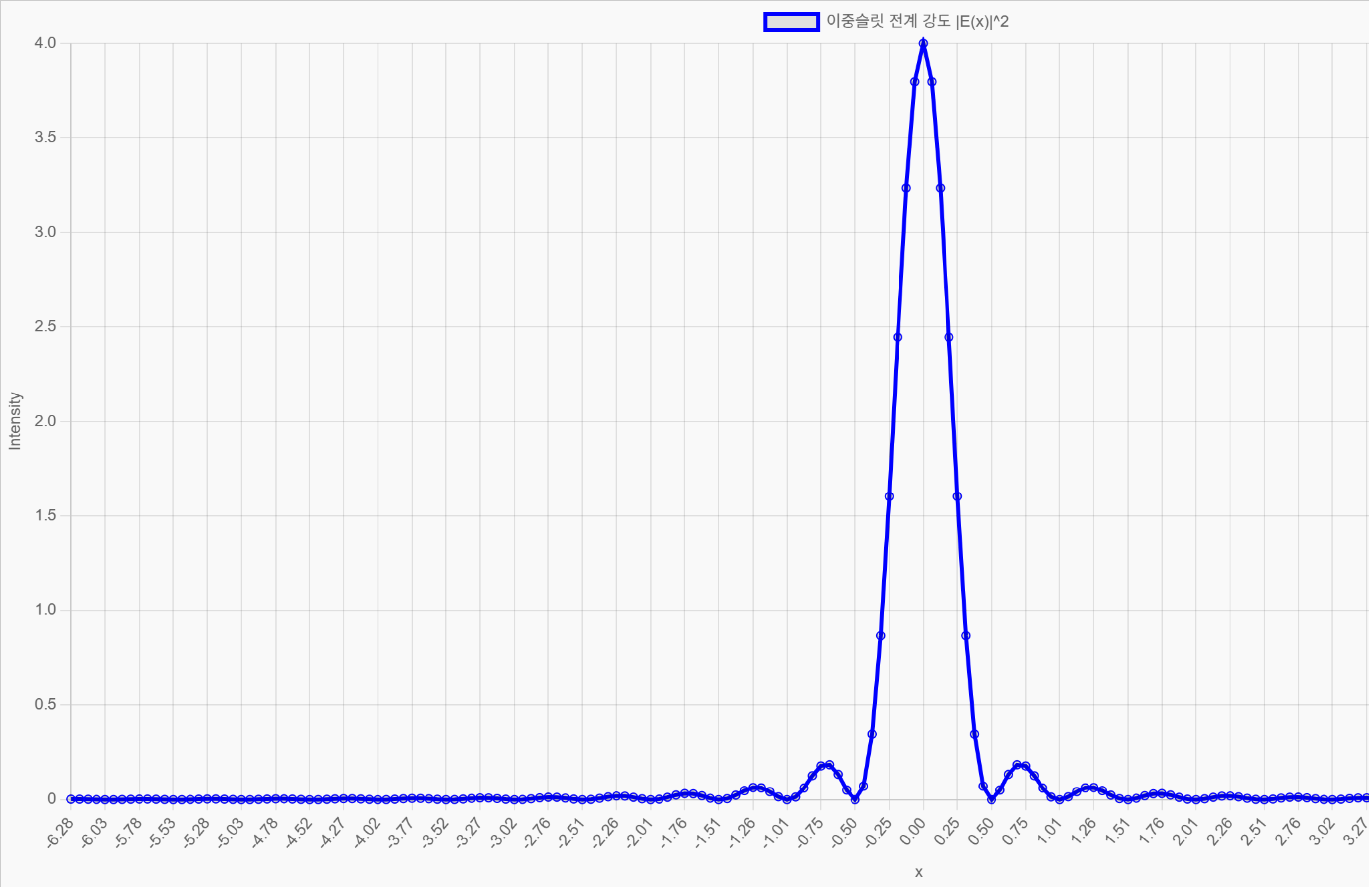
$$\partial_x(\rho(u\partial_x u + v\partial_y u + w\partial_z u)) \approx \rho\partial_x\left[\frac{1}{\mu_0}(\nabla\times\mathbf{B}_{\text{slit}})\times\mathbf{B}_{\text{slit}}\right]$$

$$\partial_x(-\nabla p) \approx -\partial_x\nabla(\varepsilon_0(\nabla\cdot\mathbf{E}_{\text{slit}}))$$

$$\partial_x(\mu\nabla^2 u) \approx \mu\nabla^2(\partial_x B_x)$$

$$\partial_x\mathbf{f}_x \approx \partial_x\left[\varepsilon_0(\nabla\cdot\mathbf{E}_{\text{slit}})E_x + \left(\frac{1}{\mu_0}(\nabla\times\mathbf{B}_{\text{slit}}) - \varepsilon_0\partial_t\mathbf{E}_{\text{slit}}\right)\times\mathbf{B}_{\text{slit}}\right]_x$$

5. 간단한 이중슬릿 시뮬레이션


이중슬릿 전계 강도 |E(x)|^2
Intensity
x
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0
-6.28
-6.03
-5.78
-5.53
-5.28
-5.03
-4.78
-4.52
-4.27
-4.02
-3.77
-3.52
-3.27
-3.02
-2.76
-2.51
-2.26
-2.01
-1.76
-1.51
-1.26
-1.01
-0.75
-0.50
-0.25
0.00
0.25
0.50
0.75
1.01
1.26
1.51
1.76
2.01
2.26
2.51
2.76
3.02
3.27