Sets and Logic Cheat Sheet

Symbol	Name	Description	Example
$\{\}$	set	used to define a set	$S = \{1, 2, 3, 4, \ldots\}$
$\in$	in, element of	used to denote that an element is part of a set	$1 \in 1, 2, 3$
$\notin$	not in, not an element of	used to denote than an element is not part of a set	$4 \notin 1, 2, 3$
$\|S\|$	cardinality	used to describe the size of a set (refers to the number of unique elements if the set is finite)	$S = \{1, 2, 2, 2, 3, 4, 5, 5\}$ $\|S\| = 5$
:, |	such that	used to denote a condition, usually in set-builder notation or in a mathematical definition	$\{x^2 : x + 3 \text{ is prime}\}$
$\subseteq$	subset	set A is a subset of set B when each element in A is also an element in B	$A = \{1, 2\}$ $B = \{2, 1, 4, 3, 5\}$ $A \subseteq B$
$\subset$	proper subset	set A is a proper subset of set B when each element in A is also an element in B **and** $A \neq B$	$A = \{1, 2, 3, 4, 5\}$ $B = \{2, 1, 4, 3, 5\}$ $A \subseteq B$ is true but $A \subset B$ is not true
$\supseteq$	superset	set A is a superset of set B when B is a subset of A	$A = \{2, 4, 6, 7, 8\}$ $B = \{2, 4, 8\}$ $A \supseteq B$
$\cup$	union	a set with the elements in set A **or** in set B	$A = \{1, 2\}$ $B = \{2, 3, 5\}$ $A \cup B = \{1, 2, 3, 5\}$
$\cap$	intersection	a set with the elements in set A **and** in set B	$A = \{1, 2\}$ $B = \{2, 3, 5\}$ $A \cap B = \{2\}$
$\emptyset$	the empty set	the set with no elements	$\{1, 2, 3\} \cap \{4, 5, 6\} = \emptyset$
$-, \setminus$	set difference	elements in set A that are not in B	$A = \{1, 2, 3, 4\}$ $B = \{2, 3, 5, 8\}$ $A - B = \{1, 4\}$ $B - A = \{5, 8\}$
$\times$	Cartesian product	a set containing all possible combinations of one element from A and one element from B	$A = \{1, 2\}$ $B = \{3, 4\}$ $A \times B = \{(1, 3), (2, 3), (1, 4), (2, 4)\}$ $B \times A = \{(3, 1), (3, 2), (4, 1), (4, 2)\}$
A^c	complement	a set containing the elements of the universe U that are not in set A	$U = \{1, 2, 3, 4, 5\}, A = \{2, 4\} \implies A^c = \{1, 3, 5\}$

Symbol	Name	Description	Example
$f : A \to B$	function	the function f maps elements of the set A to elements of the set B; A is the domain and B is the codomain	$f(x) = x^2 + 5$ is an example of a function $f : \mathbb{R} \to \mathbb{R}$
$f : x \mapsto x^3$	mapping/function	the function maps any x to x^3; this notation refers to elements of sets rather than sets themselves	$f(x) = x^2 + 5$ can be written as $f : x \mapsto x^2 + 5$
$\mathbb{N}$	the set of natural numbers	the set of naturals numbers starting at 1	$\mathbb{N} = \{1, 2, 3, \ldots\}$
$\mathbb{N}_0$	the set of whole numbers	the set of whole numbers starting at 0	$\mathbb{N}_0 = \{0, 1, 2, 3, \ldots\}$
$\mathbb{Z}$	the set of integers	the union of the whole numbers with their negatives	$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
$\mathbb{Q}$	the set of rational numbers	the set of all possible combinations of one integer divided by another, with the latter integer being non-zero, i.e., $\mathbb{Q} = \{\frac{p}{q} : p, q \in \mathbb{Z}, q \neq 0\}$	$\{\frac{1}{2}, \frac{5}{14}, \frac{-17}{3}\} \subset \mathbb{Q}$
$\wedge$	conjunction/and	$P \wedge Q$ is true if both P **and** Q are true	if $P = (2 \text{ is prime}), Q = (8 \text{ is a perfect cube})$ then $P \wedge Q$ is true
$\vee$	disjunction/or	$P \vee Q$ is true if either P **or** Q is true	if $P = (2 \text{ is prime}), Q = (4 \text{ is a perfect square})$ then $P \vee Q$ is true
$\neg$	negation	$\neg P$ is true if P is false and vice versa	if $P = (35 \text{ is prime})$ then $\neg P$ is true
$\implies$	implication	$P \implies Q$ means that Q is true whenever P is true (but it does **not** say anything about what happens when P is false)	if $P = (x \text{ is divisible by } 4)$, $Q = (x \text{ is even})$ then $P \implies Q$ (but note that $P \nleftarrow Q$)
$\iff$	if and only if (iff)	$P \implies Q$ **and** $Q \implies P$	if $P = (\text{it is new year})$ and $Q = (\text{it is January } 1)$ then $P \iff Q$
$\forall$	for all	refers to all the elements in a set	if $A = \{2, 4, 10\}$ then $x \in \mathbb{N} \; \forall x \in A$
$\exists$	there exists	refers to the existence of at least one of something	$\exists x \in \mathbb{N}_0 : x = -x$
$\oplus$	XOR	either P is true or Q is true but not both	if $P = (\text{Donald Trump is a Democrat})$ and $Q = (\text{Hillary Clinton is a Democrat})$ then $P \oplus Q$ is true, but if $P = (\text{Donald Trump is a Republican})$ then $P \oplus Q$ is false