Homework 1
NUMA01: Computational Programming with Python
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This assignment has 5 tasks.

Before you start working on this homework, sign up in Canvas for a homework group (max 2 person/ meeting).

All the functions must be *properly documented* and also tested. We recommend that you produce a report of your work with Jupyter Notebook (see lecture). You should work and present in groups by two.

Upload your solution via the homework page as a *.py file or an *.ipynb file. When uploaded book an appointment via the calendar function of the course webpage.

# Iterations

## Theory

In this homework we will approximate the log-function by an iteration method. Every iteration improves the result. The iteration is described in B. C. Carlsson: An Algorithm for Computing Logarithms nd Arctangents, MathComp. 26 (118), 1972 pp. 543-549.

It is based of computing the arithmetic and geometric mean of two values $a_i, g_i$:

- For a given value $x > 0$, initialize $a_0 = \frac{(1+x)}{2}$, $g_0 = \sqrt{x}$,
- Iterate $a_{i+1} = \frac{a_i+g_i}{2}$ and $g_{i+1} = \sqrt{a_{i+1}g_i}$,
- Consider $\frac{x-1}{a_i}$ as an approximation to $\ln(x)$.

## Task 1

Write a function `approx_ln(x, n)` that approximates the logarithm by $n$ steps of the above algorithm.

## Task 2

Plot both functions,ln and `approx_ln`, in one plot and the difference of both functions in another plot. Do this for different values of $n$.

## Task 3

Consider $x = 1.41$. Plot the absolut value of the error versus $n$.

## Task 4

In the above article a method is suggested to accelerate the convergence:

- $d_{0,i} = a_i, \quad i = 0, \ldots, n$
- $d_{k,i} = \frac{d_{k-1,i} - 4^{-k} d_{k-1,i-1}}{1-4^{-k}}, \quad k = 1, \ldots, n \quad i = 0, \ldots, n$

As approximation to ln the value $\frac{x-1}{d_{n,n}}$ is taken. Write a function `fast_approx_ln (x, n)` in which this approach is implemented.

## Task 5

Make a plot, which is similar to the plot given below.

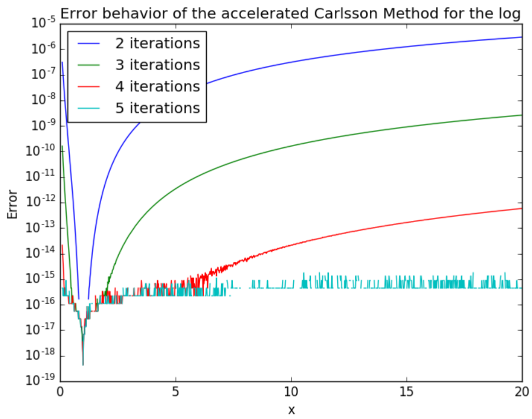


Figure 1: A plot to illustrate the speed of convergence of the approximation to ln.

Good luck!