CHAPTER 13

Valuing Stock Options: The Black–Scholes–Merton Model

In the early 1970s, Fischer Black, Myron Scholes, and Robert Merton achieved a major breakthrough in the pricing of European stock options.[1] This involved the development of what has become known as the Black–Scholes or Black–Scholes–Merton model. This model has had a huge influence on the way in which traders price and hedge options. It has also been pivotal to the growth and success of financial engineering. An acknowledgment of the importance of the model came in 1997 when Myron Scholes and Robert Merton were awarded the Nobel prize for economics. Fischer Black died in 1995; otherwise, he would undoubtedly also have been one of the recipients of this prize.

How did Black, Scholes, and Merton make their breakthrough? Previous researchers had made similar assumptions to theirs and had correctly calculated the expected payoff from a European option. However, as explained in Section 12.2, it is difficult to know the correct discount rate to use for this payoff. Black and Scholes used the capital asset pricing model (see the appendix to Chapter 3) to determine the relationship between the market's required return on the option and the required return on the stock. This was not easy because the relationship depends on both the stock price and time. Merton's approach was different from that of Black and Scholes. His pricing model involved setting up a riskless portfolio consisting of the option and the underlying stock and arguing that the return on the portfolio over a short period of time must be the risk-free return. This is similar to what we did in Section 12.1—but more complicated because the portfolio changes continuously through time. Merton's approach is more general than that of Black and Scholes because it does not rely on the assumptions of the capital asset pricing model.

This chapter presents the Black–Scholes–Merton model for valuing European call and put options on a non-dividend-paying stock and discusses the assumptions on which it is based. It also considers more fully than in previous chapters the meaning of volatility and shows how volatility can be either estimated from historical data or

[1] See F. Black and M. Scholes, "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy*, 81 (May/June 1973): 637–59; and R. C. Merton, "Theory of Rational Option Pricing," *Bell Journal of Economics and Management Science* 4 (Spring 1973): 141–83.

implied from option prices. Toward the end of the chapter we explain how the Black–Scholes–Merton results can be extended to deal with European call and put options on dividend-paying stocks.

13.1 ASSUMPTIONS ABOUT HOW STOCK PRICES EVOLVE

A stock option pricing model must make some assumptions about how stock prices evolve over time. If a stock price is \$100 today, what is the probability distribution for the price in one day or in one week or in one year?

The Black–Scholes–Merton model considers a non-dividend-paying stock and assumes that the return on the stock in a short period of time is normally distributed. The returns in two different nonoverlapping periods are assumed to be independent. Define:

μ: Expected return on the stock

σ: Volatility of the stock price

The mean of the return in time Δt is $\mu\,\Delta t$. The standard deviation of the return is $\sigma\sqrt{\Delta t}$. The assumption underlying Black–Scholes–Merton is therefore that

$$\frac{\Delta S}{S} \sim \phi(\mu\,\Delta t, \sigma^2 \Delta t) \tag{13.1}$$

where ΔS is the change in the stock price S in time Δt, and $\phi(m, v)$ denotes a normal distribution with mean m and variance v. Note that it is the variance of the return, not its standard deviation, that is proportional to Δt.

When Δt is small, equation (13.1) shows that the percentage change in the stock price in time Δt is normal with standard deviation $\sigma\sqrt{\Delta t}$. Suppose that $\sigma = 0.3$, or 30% per annum, and the current stock price is \$50. The standard deviation of the percentage change in the stock price in one week ($= 1/52$ years) is, to a good approximation,

$$30\% \times \sqrt{\frac{1}{52}} = 4.16\%$$

A one-standard-deviation move in the stock price in one week is therefore $\$50 \times 0.0416$, or \$2.08.

During a short period of time, the standard deviation of the percentage change in a stock price is usually much greater than its expected percentage change. (For instance, in our example, if the expected return on the stock is, say, 15% per annum, the expected change in one week is 15%/52, or 0.29%, and the 4.16% standard deviation is over 14 times as great as this.) When calculating confidence levels for the future stock price, a commonly used short cut is to assume that the expected return is zero. There is a 95% probability that a normally distributed variable has a value within 1.96 standard deviations of the mean. In our example, it is therefore approximately true that there is a 95% chance that the change over one week is less that $1.96 \times 4.16\% = 8.2\%$ (i.e., between -8.2% and $+8.2\%$). Similarly, it is approximately true that there is a 95% chance that the dollar change will be less than $1.96 \times \$2.08 = \$4.08$.

The Lognormal Distribution

When longer time periods are considered, it is necessary to be more precise about the future stock price distribution. The assumption in equation (13.1) implies that the stock price at any future time has a *lognormal* distribution. The general shape of a lognormal distribution is shown in Figure 13.1. It can be contrasted with the more familiar normal distribution in Figure 13.2. Whereas a variable with a normal distribution can take any positive or negative value, a lognormally distributed variable is restricted to being positive. A normal distribution is symmetrical; a lognormal distribution is skewed with the mean, median, and mode all different.

A variable with a lognormal distribution has the property that its natural logarithm is normally distributed. The Black–Scholes–Merton assumption for stock prices therefore implies that $\ln S_T$ is normal, where S_T is the stock price at a future time T. The mean and standard deviation of $\ln S_T$ can be shown to be

$$\ln S_0 + \left(\mu - \frac{\sigma^2}{2}\right)T \quad \text{and} \quad \sigma\sqrt{T}$$

where S_0 is the current stock price. We can write this result as

$$\ln S_T \sim \phi\left[\ln S_0 + \left(\mu - \frac{\sigma^2}{2}\right)T,\ \sigma^2 T\right] \tag{13.2}$$

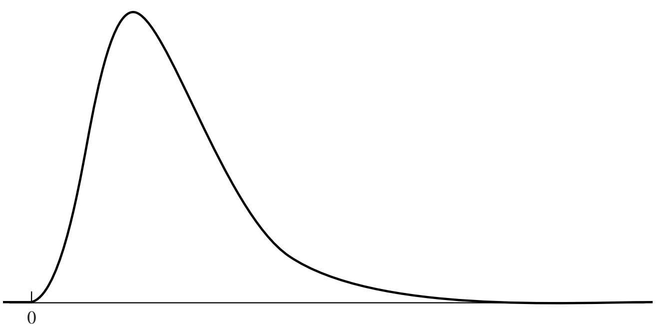

Figure 13.1 A lognormal distribution

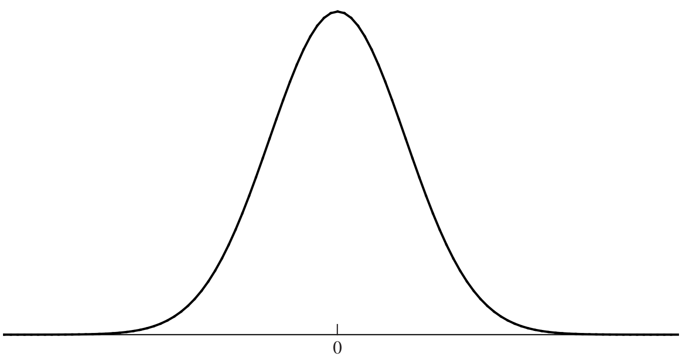

Figure 13.2 A normal distribution

The expected value (or mean) of S_T is

$$E(S_T) = S_0 e^{\mu T} \tag{13.3}$$

and the variance of S_T is

$$\operatorname{var}(S_T) = S_0^2 e^{2\mu T}(e^{\sigma^2 T} - 1)$$

Example 13.1 provides an application of these equations.

From equation (13.2) and the properties of the normal distribution, we have

$$\ln S_T - \ln S_0 \sim \phi\left[\left(\mu - \frac{\sigma^2}{2}\right)T,\ \sigma^2 T\right]$$

or

$$\ln \frac{S_T}{S_0} \sim \phi\left[\left(\mu - \frac{\sigma^2}{2}\right)T,\ \sigma^2 T\right] \tag{13.4}$$

When $T = 1$, the expression $\ln(S_T/S_0)$ is the continuously compounded return provided by the stock in one year.[2] The mean and standard deviation of the continuously compounded return in one year are therefore $\mu - \sigma^2/2$ and σ, respectively. Example 13.2 shows how confidence limits for the return can be calculated.

We now consider in more detail the nature of the expected return and volatility parameter in the lognormal stock price model.

Example 13.1 Confidence limits, mean, and variance for a future stock price

Consider a stock with an initial price of \$40, an expected return of 16% per annum, and a volatility of 20% per annum. From equation (13.2), the probability distribution of the stock price, S_T, in six months is given by

$$\ln S_T \sim \phi\left[\ln 40 + \left(0.16 - \frac{0.2^2}{2}\right)0.5,\ 0.2^2 \times 0.5\right] \quad \text{or} \quad \ln S_T \sim \phi(3.759, 0.02)$$

There is a 95% probability that a normally distributed variable has a value within 1.96 standard deviations of its mean. In this case, the standard deviation is $\sqrt{0.02} = 0.141$. Hence, with 95% confidence, we have

$$3.759 - 1.96 \times 0.141 < \ln S_T < 3.759 + 1.96 \times 0.141$$

This implies

$$e^{3.759 - 1.96\times 0.141} < S_T < e^{3.759+1.96\times 0.141} \quad \text{or} \quad 32.55 < S_T < 56.56$$

Thus, there is a 95% probability that the stock price in six months will lie between 32.55 and 56.56. The mean and variance of S_T are

$$40e^{0.16\times 0.5} = 43.33$$

and

$$40^2 e^{2\times 0.16\times 0.5}(e^{0.2\times 0.2\times 0.5} - 1) = 37.93$$

[2] We can distinguish between the continuously compounded return and the return with annual compounding. The former is $\ln(S_T/S_0)$; the latter is $(S_T - S_0)/S_0$.

Example 13.2 Confidence limits for stock price return

Consider a stock with an expected return of 17% per annum and a volatility of 20% per annum. The probability distribution for the rate of return (continuously compounded) realized over one year is normal, with mean

$$0.17 - \frac{0.2^2}{2} = 0.15$$

or 15% and standard deviation 20%. Because there is a 95% chance that a normally distributed variable will lie within 1.96 standard deviations of its mean, we can be 95% confident that the return realized over one year will be between $15 - 1.96 \times 20 = -24.2\%$ and $15 + 1.96 \times 20 = +54.2\%$.

13.2 EXPECTED RETURN

The expected return, μ, required by investors from a stock depends on the riskiness of the stock. The higher the risk, the higher the expected return. It also depends on the level of interest rates in the economy. The higher the level of interest rates, the higher the expected return required on any given stock. Fortunately, we do not have to concern ourselves with the determinants of μ in any detail. It turns out that the value of a stock option, when expressed in terms of the value of the underlying stock, does not depend on μ at all. Nevertheless, there is one aspect of the expected return from a stock that frequently causes confusion and needs to be explained.

Equation (13.1) shows that $\mu\,\Delta t$ is the expected percentage change in the stock price in a very short period of time, Δt. It is natural to assume from this that μ is the expected continuously compounded return on the stock over a longer period of time. However, this is not the case. Denote by R the continuously compounded return actually realized over a period of time of length T years. This satisfies

$$S_T = S_0 e^{RT}$$

so that

$$R = \frac{1}{T} \ln \frac{S_T}{S_0}$$

Equation (13.4) shows that the expected value, $E(R)$, of R is $\mu - \sigma^2/2$.

The reason why the expected continuously compounded return is different from μ is subtle, but important. Suppose we consider a very large number of very short periods of time of length Δt. Define S_i as the stock price at the end of the ith interval and ΔS_i as $S_{i+1} - S_i$. Under the assumptions we are making for stock price behavior (see equation (13.1)), the average of the returns on the stock in each interval is close to μ. In other words, $\mu\,\Delta t$ is close to the arithmetic mean of the $\Delta S_i/S_i$. However, the expected return over the whole period covered by the data, expressed with a compounding period of Δt, is close to the geometric mean of the $\Delta S_i/S_i$. This is $\mu - \sigma^2/2$, not μ.[3] Business Snapshot 13.1 provides a numerical example related to the mutual fund industry to illustrate this.

[3] The arguments in this section show that the term "expected return" is ambiguous. It can refer either to μ or to $\mu - \sigma^2/2$. Unless otherwise stated, it will be used to refer to μ throughout this book.

Business Snapshot 13.1 Mutual fund returns can be misleading

The difference between μ and $\mu - \sigma^2/2$ is closely related to an issue in the reporting of mutual fund returns. Suppose that the following is a sequence of returns per annum reported by a mutual fund manager over the last five years (measured using annual compounding): 15%, 20%, 30%, −20%, 25%.

The arithmetic mean of the returns, calculated by taking the sum of the returns and dividing by 5, is 14%. However, an investor would actually earn less than 14% per annum by leaving the money invested in the fund for five years. The dollar value of \$100 at the end of the five years would be

$$100 \times 1.15 \times 1.20 \times 1.30 \times 0.80 \times 1.25 = \$179.40$$

By contrast, a 14% return with annual compounding would give

$$100 \times 1.14^5 = \$192.54$$

The return that gives \$179.40 at the end of five years is 12.4%. This is because

$$100 \times (1.124)^5 = 179.40$$

What average return should the fund manager report? It is tempting for the manager to make a statement such as: "The average of the returns per year that we have realized in the last five years is 14%." Although true, this is misleading. It is much less misleading to say "The average return realized by someone who invested with us for the last five years is 12.4% per year." In some jurisdictions regulatory standards require fund managers to report returns the second way.

This phenomenon is an example of a well-known result. The geometric mean of a set of numbers (not all the same) is always less than the arithmetic mean. In our example, the return multipliers each year are 1.15, 1.20, 1.30, 0.80, and 1.25. The arithmetic mean of these numbers is 1.140, but the geometric mean is only 1.124.

For a mathematical explanation of what is going on, we start with equation (13.3):

$$E(S_T) = S_0 e^{\mu T}$$

Taking logarithms, we get

$$\ln[E(S_T)] = \ln(S_0) + \mu T$$

It is now tempting to set $\ln[E(S_T)] = E[\ln(S_T)]$, so that $E[\ln(S_T)] - \ln(S_0) = \mu T$, or $E[\ln(S_T/S_0)] = \mu T$, which leads to $E(R) = \mu$. However, we cannot do this because ln is a nonlinear function. In fact, $\ln[E(S_T)] > E[\ln(S_T)]$, so that $E[\ln(S_T/S_0)] < \mu T$, which leads to $E(R) < \mu$. (As pointed out above, $E(R) = \mu - \sigma^2/2$.)

13.3 VOLATILITY

The volatility of a stock, σ, is a measure of our uncertainty about the returns provided by the stock. Stocks typically have volatilities between 15% and 50%.

From equation (13.4), the volatility of a stock price can be defined as the standard deviation of the return provided by the stock in one year when the return is expressed using continuous compounding.

It is approximately true that our uncertainty about a future stock price, as measured by its standard deviation, increases with the square root of how far ahead we are looking. For example, the standard deviation of the stock price in four weeks is approximately twice the standard deviation in one week.

13.4 ESTIMATING VOLATILITY FROM HISTORICAL DATA

A record of stock price movements can be used to estimate volatility. The stock price is usually observed at fixed intervals of time (e.g., every day, week, or month). We define:

$n+1$: Number of observations
S_i: Stock price at end of ith interval, where $i = 0, 1, \dots, n$
τ: Length of time interval in years

and let

$$u_i = \ln\left(\frac{S_i}{S_{i-1}}\right)$$

An estimate, s, of the standard deviation of the u_i is given by

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(u_i - \bar{u})^2}$$

or

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}u_i^2 - \frac{1}{n(n-1)}\left(\sum_{i=1}^{n}u_i\right)^2}$$

where $\bar{u}$ is the mean of the u_i.[4] (See the appendix to Chapter 3.)

From equation (13.4), the standard deviation of the u_i is $\sigma\sqrt{\tau}$. The variable s is therefore an estimate of $\sigma\sqrt{\tau}$. It follows that σ itself can be estimated as $\hat{\sigma}$, where

$$\hat{\sigma} = \frac{s}{\sqrt{\tau}}$$

The standard error of this estimate can be shown to be approximately $\hat{\sigma}/\sqrt{2n}$. Example 13.3 illustrates the application of these formulas.

Choosing an appropriate value for n is not easy. More data generally lead to more accuracy, but σ does change over time and data that are too old may not be relevant for predicting the future volatility. A compromise that seems to work reasonably well is to use closing prices from daily data over the most recent 90 to 180 days. Alternatively, as a rule of thumb, we can set n equal to the number of days to which the volatility is to be applied. Thus, if the volatility estimate is to be used to value a two-year option, it is calculated from daily data over the last two years.

The foregoing analysis assumes that the stock pays no dividends. It can be adapted to accommodate dividend-paying stocks. The return, u_i, during a time interval that

[4] The value of $\bar{u}$ is often assumed to be zero when estimates of historical volatilities are made.

Example 13.3 Calculation of volatility from historical data

Table 13.1 shows a possible sequence of stock prices during 21 consecutive trading days. In this case, $n = 20$ and

$$\sum_{i=1}^{20} u_i = 0.09531 \quad \text{and} \quad \sum_{i=1}^{20} u_i^2 = 0.00326$$

and the estimate of the standard deviation of the daily return is

$$\sqrt{\frac{0.00326}{19} - \frac{0.09531^2}{20 \times 19}} = 0.01216$$

or 1.216%. Assuming that there are 252 trading days per year, $\tau = 1/252$ and the data give an estimate for the volatility per annum of $0.01216\sqrt{252} = 0.193$ or 19.3%. The standard error of this estimate is

$$\frac{0.193}{\sqrt{2 \times 20}} = 0.031$$

or 3.1% per annum.

includes an ex-dividend day is given by

$$u_i = \ln \frac{S_i + D}{S_{i-1}}$$

where D is the amount of the dividend. The return in other time intervals is still

$$u_i = \ln \frac{S_i}{S_{i-1}}$$

However, because tax factors play a part in determining returns around an ex-dividend date, it is probably best to discard altogether data for intervals that include an ex-dividend date when daily or weekly data is used.

Trading Days vs. Calendar Days

There is an important issue concerned with whether time should be measured in calendar days or trading days when volatility parameters are being estimated and used. As shown in Business Snapshot 13.2, research shows that volatility is much higher when the exchange is open for trading than when it is closed. As a result, practitioners tend to ignore days when the exchange is closed when estimating volatility from historical data and when calculating the life of an option. The volatility per annum is calculated from the volatility per trading day using the formula

$$\text{Volatility per annum} = \text{Volatility per trading day} \times \sqrt{\begin{array}{c}\text{Number of trading}\\ \text{days per annum}\end{array}}$$

This is what we did when calculating volatility from the data in Table 13.1. The number of trading days in a year is usually assumed to be 252 for stocks.

Business Snapshot 13.2 What causes volatility?

It is natural to assume that the volatility of a stock is caused by new information reaching the market. This new information causes people to revise their opinions about the value of the stock. The price of the stock changes and volatility results. This view of what causes volatility is not supported by research.

With several years of daily stock price data, researchers can calculate:

1. The variance of stock price returns between the close of trading on one day and the close of trading on the next day when there are no intervening nontrading days.
2. The variance of the stock price returns between the close of trading on Friday and the close of trading on Monday.

The second variance is the variance of returns over a three-day period. The first is a variance over a one-day period. We might reasonably expect the second variance to be three times as great as the first variance. Fama (1965), French (1980), and French and Roll (1986) show that this is not the case. These three research studies estimate the second variance to be, respectively, 22%, 19%, and 10.7% higher than the first variance.

At this stage one might be tempted to argue that these results are explained by more news reaching the market when the market is open for trading. But research by Roll (1984) does not support this explanation. Roll looked at the prices of orange juice futures. By far the most important news for orange juice futures prices is news about the weather and news about the weather is equally likely to arrive at any time. When Roll did a similar analysis to that just described for stocks he found that the second (Friday-to-Monday) variance for orange juice futures is only 1.54 times the first variance.

The only reasonable conclusion from all this is that volatility is to a large extent caused by trading itself. (Traders usually have no difficulty accepting this conclusion!)

The life of an option is also usually measured using trading days rather than calendar days. It is calculated as T years, where

$$T = \frac{\text{Number of trading days until option maturity}}{252}$$

13.5 ASSUMPTIONS UNDERLYING BLACK–SCHOLES–MERTON

The assumptions made by Black, Scholes, and Merton when they derived their option pricing formula were as follows:

1. Stock price behavior corresponds to the lognormal model (developed earlier in this chapter) with μ and σ constant.
2. There are no transactions costs or taxes. All securities are perfectly divisible.
3. There are no dividends on the stock during the life of the option.
4. There are no riskless arbitrage opportunities.

Table 13.1 Computation of volatility

Day	*Closing stock price (\$),* S_i	*Price relative,* S_i/S_{i-1}	*Daily return,* $u_i = \ln(S_i/S_{i-1})$
0	20.00		
1	20.10	1.00500	0.00499
2	19.90	0.99005	−0.01000
3	20.00	1.00503	0.00501
4	20.50	1.02500	0.02469
5	20.25	0.98780	−0.01227
6	20.90	1.03210	0.03159
7	20.90	1.00000	0.00000
8	20.90	1.00000	0.00000
9	20.75	0.99282	−0.00720
10	20.75	1.00000	0.00000
11	21.00	1.01205	0.01198
12	21.10	1.00476	0.00475
13	20.90	0.99052	−0.00952
14	20.90	1.00000	0.00000
15	21.25	1.01675	0.01661
16	21.40	1.00706	0.00703
17	21.40	1.00000	0.00000
18	21.25	0.99299	−0.00703
19	21.75	1.02353	0.02326
20	22.00	1.01149	0.01143

5. Security trading is continuous.
6. Investors can borrow or lend at the same risk-free rate of interest.
7. The short-term risk-free rate of interest, r, is constant.

Some of these assumptions have been relaxed by other researchers. For example, variations on the Black–Scholes–Merton formula can be used when r and σ are functions of time and, as we shall see later in this chapter, the formula can be adjusted to take dividends into account.

13.6 THE KEY NO-ARBITRAGE ARGUMENT

The arguments that can be used to price options are analogous to the no-arbitrage arguments used in Chapter 12 when stock price changes were assumed to be binomial. A riskless portfolio consisting of a position in the option and a position in the underlying stock is set up. In the absence of arbitrage opportunities, the return from the portfolio must be the risk-free interest rate, r. This results in a differential equation that must be satisfied by the option.

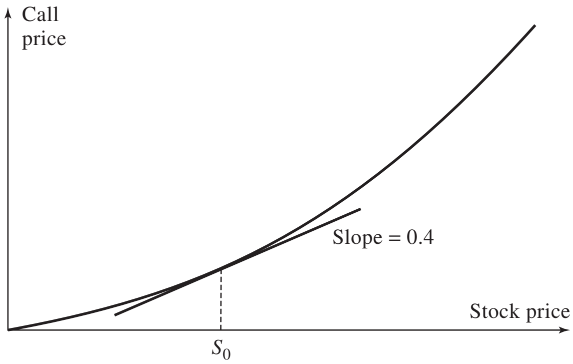

Figure 13.3 Relationship between call price and stock price. Current stock price is S_0

The reason a riskless portfolio can be set up is that the stock price and the option price are both affected by the same underlying source of uncertainty: stock price movements. In any short period of time, the price of a call option is perfectly positively correlated with the price of the underlying stock; the price of a put option is perfectly negatively correlated with the price of the underlying stock. In both cases, when an appropriate portfolio of the stock and the option is set up, the gain or loss from the stock position always offsets the gain or loss from the option position so that the overall value of the portfolio at the end of the short period of time is known with certainty.

Suppose, for example, that at some point in time the relationship between a small change in the stock price, ΔS, and the resultant small change in the price of a European call option, Δc, is given by

$$\Delta c = 0.4\Delta S$$

This means that the slope of the line representing the relationship between Δc and ΔS is 0.4, as indicated in Figure 13.3. A riskless portfolio would consist of:

1. A long position in 40 shares
2. A short position in 100 call options

Suppose that the stock price increases by 10 cents. The option price will increase by 4 cents and the $40 \times 0.10 = \$4$ gain on the shares is equal to the $100 \times 0.04 = \$4$ loss on the short option position.

There is one important difference between the analysis here and the analysis using a binomial model in Chapter 12. Here, the position that is set up is riskless for only a very short period of time. (Theoretically, it remains riskless only for an instantaneously short period of time.) To remain riskless, it must be frequently adjusted or *rebalanced*.[5] For example, the relationship between Δc and ΔS might change from $\Delta c = 0.4\,\Delta S$ today to $\Delta c = 0.5\,\Delta S$ tomorrow. (If so, an extra 0.1 shares must be purchased for each call option sold to maintain a riskless portfolio.) It is nevertheless true that the return from the riskless portfolio in any short period of time must be the risk-free interest rate. This can be used in conjunction with some stochastic calculus to produce the Black, Scholes, and Merton pricing formulas.

[5] We will examine the rebalancing of portfolios in more detail in Chapter 17.

13.7 THE BLACK–SCHOLES–MERTON PRICING FORMULAS

The Black–Scholes–Merton formulas for the prices of European calls and puts on non-dividend-paying stocks are[6]

$$c = S_0 N(d_1) - Ke^{-rT} N(d_2) \tag{13.5}$$

$$p = Ke^{-rT} N(-d_2) - S_0 N(-d_1) \tag{13.6}$$

where

$$d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

The function $N(x)$ is the cumulative probability function for a standardized normal variable. In other words, it is the probability that a variable with a standard normal distribution will be less than x. It is illustrated in Figure 13.4. The remaining notation in equations (13.5) and (13.6) should be familiar. The variables c and p are the European call and put prices, S_0 is the stock price, K is the strike price, r is the risk-free interest rate (expressed with continuous compounding), T is the time to expiration, and σ is the volatility of the stock price. Because the American call price, C, equals the European call price, c, for a non-dividend-paying stock, equation (13.5) also gives the price of an American call. There is no exact analytic formula to value an American put, but binomial trees such as those introduced in Chapter 12 can be used.

In theory, the Black–Scholes formula is correct only if the short-term interest rate, r, is constant. In practice, the formula is usually used with the interest rate, r, being set equal to the risk-free interest rate on an investment that lasts for time T.

Properties of the Black–Scholes–Merton Formulas

A full proof of the Black–Scholes–Merton formulas is beyond the scope of this book. At this stage we show that the formulas have the right general properties by considering what happens when some of the parameters take extreme values.

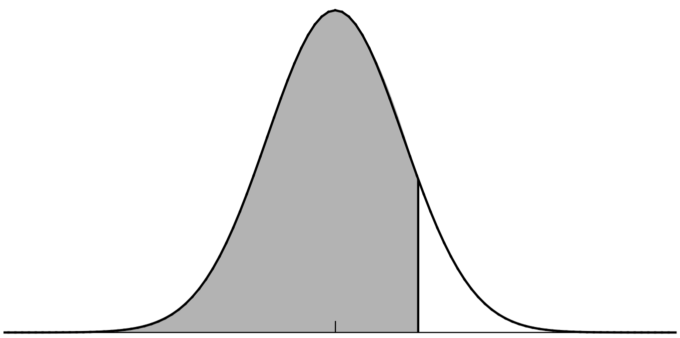

Figure 13.4 Shaded area represents $N(x)$

[6] The software, DerivaGem, that accompanies this book can be used to carry out Black–Scholes–Merton calculations for options on stocks, currencies, indices, and futures contracts.

When the stock price, S_0, becomes very large, a call option is almost certain to be exercised. It then becomes very similar to a forward contract with delivery price K. Therefore, from equation (5.5), we expect the call price to be

$$S_0 - Ke^{-rT}$$

This is, in fact, the call price given by equation (13.5) because, when S_0 becomes very large, both d_1 and d_2 become very large, and consequently $N(d_1)$ and $N(d_2)$ are both close to 1.0.

When the stock price becomes very large, the price of a European put option, p, approaches zero. This result is consistent with equation (13.6) because $N(-d_1)$ and $N(-d_2)$ are both close to zero when S_0 is large.

When the stock price becomes very small, both d_1 and d_2 become very large and negative. This means that $N(d_1)$ and $N(d_2)$ are then both very close to zero, and equation (13.5) gives a price close to zero for the call option. This is as expected. Also, $N(-d_1)$ and $N(-d_2)$ become close to 1, so that the price of the put option given by equation (13.6) is close to $Ke^{-rT} - S_0$. This is also as expected.

Example 13.4 illustrates the application of equations (13.5) and (13.6). The only difficulty is the computation of the cumulative normal distribution function, N. Tables for N are provided at the end of this book. It can also be evaluated using the NORMSDIST function in Excel.

Example 13.4 Using the Black–Scholes–Merton formulas

The stock price six months from the expiration of an option is \$42, the exercise price of the option is \$40, the risk-free interest rate is 10% per annum, and the volatility is 20% per annum. This means that $S_0 = 42$, $K = 40$, $r = 0.1$, $\sigma = 0.2$, $T = 0.5$,

$$d_1 = \frac{\ln(42/40) + (0.1 + 0.2^2/2) \times 0.5}{0.2\sqrt{0.5}} = 0.7693$$

$$d_2 = \frac{\ln(42/40) + (0.1 - 0.2^2/2) \times 0.5}{0.2\sqrt{0.5}} = 0.6278$$

$$c = 42N(0.7693) - 38.049N(0.6278)$$

$$p = 38.049N(-0.6278) - 42N(-0.7693)$$

Using Excel's NORMDIST function or the tables at the end of the book, we get

$$N(0.7693) = 0.7791, \quad N(-0.7693) = 0.2209$$

$$N(0.6278) = 0.7349, \quad N(-0.6278) = 0.2651$$

so that

$$c = 4.76 \quad \text{and} \quad p = 0.81$$

Ignoring the time value of money, the stock price has to rise by \$2.76 for the purchaser of the call to break even. Similarly, the stock price has to fall by \$2.81 for the purchaser of the put to break even.

Understanding $N(d_1)$ and $N(d_2)$

The term $N(d_2)$ in equation (13.5) has a fairly simple interpretation. It is the probability that a call option will be exercised in a risk-neutral world. The $N(d_1)$ term is not quite so easy to interpret. The expression $S_0 N(d_1)e^{rT}$ is the expected stock price at time T in a risk-neutral world when stock prices less than the strike price are counted as zero. The strike price is only paid if the stock price is greater than K and as just mentioned this has a probability of $N(d_2)$. The expected payoff in a risk-neutral world is therefore

$$S_0 N(d_1)e^{rT} - KN(d_2)$$

Present-valuing this from time T to time zero gives the Black–Scholes–Merton equation for a European call option:

$$c = S_0 N(d_1) - Ke^{-rT} N(d_2)$$

For another way of understanding this, note that the Black–Scholes–Merton equation for the value of a European call option can be written as:

$$c = e^{-rT} N(d_2)[S_0 e^{rT} N(d_1)/N(d_2) - K]$$

The terms here have the following interpretation:

e^{-rT}: Present value factor

$N(d_2)$: Probability of exercise

$S_0 e^{rT} N(d_1)/N(d_2)$: Expected stock price in risk-neutral world if option is exercised

K: Strike price paid if option is exercised.

13.8 RISK-NEUTRAL VALUATION

A very important result in the pricing of derivatives is known as risk-neutral valuation. The principle was introduced in Chapter 12 and can be stated as follows:

> Any security dependent on other traded securities can be valued on the assumption that investors are risk neutral.

Note that risk-neutral valuation does not state that investors are risk neutral. What it does state is that derivatives such as options can be valued on the assumption that investors are risk neutral. It means that investors' risk preferences have no effect on the value of a stock option when it is expressed as a function of the price of the underlying stock. It explains why equations (13.5) and (13.6) do not involve the stock's expected return, μ. Risk-neutral valuation is a very powerful tool because in a risk-neutral world two particularly simple results hold:

1. The expected return from all investment assets is the risk-free interest rate.
2. The risk-free interest rate is the appropriate discount rate to apply to any expected future cash flow.

Options and other derivatives can be valued using risk-neutral valuation. The procedure is as follows:

1. Assume that the expected return from the underlying asset is the risk-free interest rate r (i.e., assume $\mu = r$).
2. Calculate the expected payoff.
3. Discount the expected payoff at the risk-free interest rate.

Application to Forward Contracts

This procedure can be used to derive the Black–Scholes–Merton formulas, but the mathematics is fairly complicated and will not be presented here. Instead, as an illustration, we will show how the procedure can be used to value a forward contract on a non-dividend-paying stock. (This contract has already been valued in Chapter 5 using a different approach.) We will make the assumption that interest rates are constant and equal to r.

Consider a long forward contract that matures at time T with delivery price K. The value of the contract at maturity is

$$S_T - K$$

The expected value of S_T was shown earlier in this chapter to be $S_0 e^{\mu T}$. In a risk-neutral world, it becomes $S_0 e^{rT}$. The expected payoff from the contract at maturity in a risk-neutral world is therefore

$$S_0 e^{rT} - K$$

Discounting at the risk-free rate r for time T gives the value, f, of the forward contract today as

$$f = e^{-rT}(S_0 e^{rT} - K) = S_0 - Ke^{-rT}$$

This is in agreement with the result in equation (5.5).

13.9 IMPLIED VOLATILITIES

The one parameter in the Black–Scholes–Merton pricing formulas that cannot be observed directly is the volatility of the stock price. Earlier in this chapter we saw how volatility can be estimated from a history of the stock price. We now show how to calculate what is known as an *implied volatility*. This is the volatility implied by an option price observed in the market.[7]

To illustrate the basic idea, suppose that the value of a European call option on a non-dividend-paying stock is 1.90 when $S_0 = 21$, $K = 20$, $r = 0.1$, and $T = 0.25$. The implied volatility is the value of σ that, when substituted into equation (13.5), gives $c = 1.90$. It is not possible to invert equation (13.5) so that σ is expressed as a function of S_0, K, r, T, and c, but an iterative search procedure can be used to find the implied σ. We could start by trying $\sigma = 0.20$. This gives a value of c equal to 1.76, which is too low. Because c is an increasing function of σ, a higher value of σ is required. We could next try a value of 0.30 for σ. This gives a value of c equal to 2.10, which is too high,

[7] Implied volatilities for European and American options on stocks, stock indices, foreign currencies, and futures can be calculated using the DerivaGem software supplied with this book.

and means that σ must lie between 0.20 and 0.30. Next, we try a value of 0.25 for σ. This also proves to be too high, showing that σ lies between 0.20 and 0.25. Proceeding in this way, we can halve the range for σ at each iteration and thereby calculate the correct value of σ to any required accuracy.[8] In this example, the implied volatility is 0.242, or 24.2% per annum.

Implied volatilities can be used to monitor the market's opinion about the volatility of a particular stock. Whereas historical volatilities (see Section 13.4) are "backward looking," implied volatilities are "forward looking." It is therefore not surprising that predictions of a stock's future volatility based on implied volatilities tend to be slightly better than those based on historical volatilities.

Traders often quote the implied volatility of an option rather than its price. This is convenient because the implied volatility tends to be less variable than the option price. The implied volatility of an option does depend on its strike price and time to maturity. As will be explained in Chapter 19, the implied volatilities of actively traded options are used by traders to estimate appropriate implied volatilities for other options.

The VIX Index

The CBOE publishes indices of implied volatility. The most popular index, the SPX VIX, is an index of the implied volatility of 30-day options on the S&P 500 calculated from a wide range of calls and puts.[9] It is sometimes referred to as the "fear factor." An index value of 15 indicates that the calculated implied volatility of 30-day options on the S&P 500 is about 15%. Trading in futures on the VIX started in 2004 and trading in options on the VIX started in 2006. One contract is on 1,000 times the index. Example 13.5 indicates how a futures trade on the VIX works.

Trading futures on the VIX index gives a different type of bet from trading options on the S&P 500. The future value of options on the S&P 500 depend on both the level of the S&P 500 and its volatility. By contrast, a futures contract on the VIX is a bet only on volatility. Figure 13.5 shows the VIX index between January 2004 and August 2012. Between the 2004 and mid-2007, it tended to stay between 10 and 20. It reached 30 during the second half of 2007 and a record 80 in October and November 2008 after Lehman's bankruptcy. By early 2010, it had declined to more normal levels, but it

Example 13.5 Trading VIX Futures

A trader buys an April futures contract on the VIX when the futures price is 18.5 (corresponding to a 30-day S&P 500 volatility of 18.5%) and closes out the contract when when the futures price is 19.3 (corresponding to a 30-day S&P 500 volatility of 19.3%). The trader makes a gain of

$$(19.3 - 18.5) \times 1,000 = 800$$

or \$800. As is usual with a futures contract, this gain is the cumulative result of daily gains and losses on the contract during the period it is held.

[8] This method is presented for illustration. Other, more powerful, procedures are usually used in practice.

[9] For a description of how the index is calculated see J. Hull, *Options, Futures, and Other Derivatives*, 8th edn. Upper Saddle River, NJ: Pearson, 2012, Chap. 24.

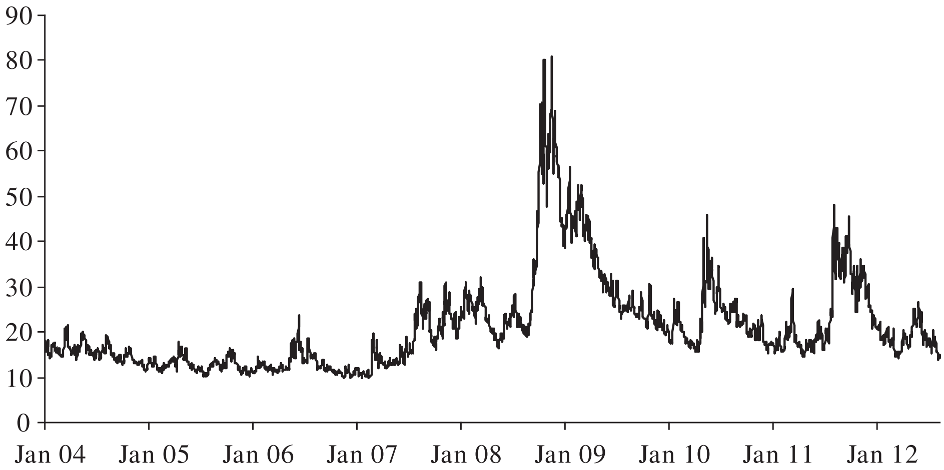

Figure 13.5 The VIX index: January 2004 to September 2012

spiked again in May 2010 and the second half of 2011 because of concerns about the European sovereign debt crisis.

13.10 DIVIDENDS

Up to now we have assumed that the stock on which the option is written pays no dividends. In practice, of course, many stocks do pay dividends. We now extend our results by assuming that the dividends paid on the stock during the life of an option can be predicted with certainty. When options last for relatively short periods of time (less than one year), the assumption is not too unreasonable. When options last for long periods of time, it is usual to assume that the dividend yield rather than the dollar amount of the dividend is known. Options can then be valued as described in Chapter 15.

The date on which the dividend is paid should be assumed to be the ex-dividend date. On this date the stock price declines by the amount of the dividend.[10] The effect is to reduce the value of calls and increase the value of puts.

European Options

European options can be analyzed by assuming that the stock price is the sum of two components: a riskless component that will be used to pay the known dividends during the life of the option and a risky component. The riskless component at any given time is the present value of all the dividends during the life of the option discounted from the ex-dividend dates to the present at the risk-free rate. The Black–Scholes–Merton formula is then correct if S_0 is set equal to the risky component. Operationally this

[10] For tax reasons the stock price may go down by somewhat less than the cash amount of the dividend. To take account of this phenomenon, we need to interpret the word *dividend* in the context of option pricing as the reduction in the stock price on the ex-dividend date caused by the dividend. Thus, if a dividend of \$1 per share is anticipated and the share price normally goes down by 80% of the dividend on the ex-dividend date, the dividend should be assumed to be \$0.80 for the purposes of the analysis.

Example 13.6 Using Black–Scholes–Merton when there are dividends

Consider a European call option on a stock with ex-dividend dates in two months and five months. The dividend on each ex-dividend date is expected to be \$0.50. The current share price is \$40, the exercise price is \$40, the stock price volatility is 30% per annum, the risk-free rate of interest is 9% per annum, and the time to maturity is six months. The present value of the dividends is

$$0.5e^{-0.09\times 2/12} + 0.5e^{-0.09\times 5/12} = 0.9741$$

The option price can therefore be calculated from the Black–Scholes–Merton formula with $S_0 = 40 - 0.9741 = 39.0259$, $K = 40$, $r = 0.09$, $\sigma = 0.3$, and $T = 0.5$:

$$d_1 = \frac{\ln(39.0259/40) + (0.09 + 0.3^2/2)\times 0.5}{0.3\sqrt{0.5}} = 0.2020$$

$$d_2 = \frac{\ln(39.0259/40) + (0.09 - 0.3^2/2)\times 0.5}{0.3\sqrt{0.5}} = -0.01012$$

Using the NORMDIST function in Excel gives

$$N(d_1) = 0.5800 \quad \text{and} \quad N(d_2) = 0.4959$$

and from equation (13.5) the call price is

$$39.0259 \times 0.5800 - 40e^{-0.09\times 0.5} \times 0.4959 = 3.67$$

or \$3.67.

means that the Black–Scholes–Merton formula can be used provided the stock price is reduced by the present value of all the dividends during the life of the option, the discounting being done from the ex-dividend dates at the risk-free rate. As already mentioned, a dividend is included in the calculations only if its ex-dividend date occurs during the life of the option. Example 13.6 illustrates the calculations.

With this procedure, σ in the Black–Scholes–Merton formula should be the volatility of the risky component of the stock price—not the volatility of the stock price itself. In practice, the two are usually assumed to be the same. In theory, the volatility of the risky component is approximately $S_0/(S_0 - D)$ times the volatility of the stock price, where D is the present value of the remaining dividends and S_0 is the stock price.

American Call Options

In Chapter 10, we saw that American call options should never be exercised early when the underlying stock pays no dividends. When dividends are paid, it is sometimes optimal to exercise at a time immediately before the stock goes ex-dividend. The reason is easy to understand. The dividend will make both the stock and the call option less valuable. If the dividend is sufficiently large and the call option is sufficiently in the money, it may be worth forgoing the remaining time value of the option in order to avoid the adverse effects of the dividend on the stock price.

Example 13.7 Using Black's approximation for an American call

Suppose that the option in Example 13.6 is American rather than European. The present value of the first dividend is given by

$$0.5e^{-0.09\times 2/12} = 0.4926$$

The value of the option on the assumption that it expires just before the final ex-dividend date can be calculated using the Black–Scholes–Merton formula, with $S_0 = 40 - 0.4926 = 39.5074$, $K = 40$, $r = 0.09$, $\sigma = 0.30$, and $T = 0.4167$. It is \$3.52. Black's approximation involves taking the greater of this value and the value of the option when it can be exercised only at the end of six months. From the previous example, we know that the latter is \$3.67. Black's approximation therefore gives the value of the American call as \$3.67.

In practice, call options are most likely to be exercised early immediately before the final ex-dividend date. The analysis in the appendix at the end of this chapter indicates why this is so and derives the conditions under which early exercise can be optimal. We now mention an approximate procedure suggested by Fischer Black for valuing American calls on dividend-paying stocks.

Black's Approximation

Black's approximation involves calculating the prices of two European options:

1. A European option that matures at the same time as the American option
2. A European option maturing just before the latest ex-dividend date that occurs during the life of the option

The strike price, initial stock price, risk-free interest rate, and volatility are the same as for the option under consideration. The American option price is set equal to the higher of these two European option prices. Example 13.7 illustrates the approach.

SUMMARY

The usual assumption in stock option pricing is that the price of a stock at some future time given its price today is lognormal. This in turn implies that the continuously compounded return from the stock in a period of time is normally distributed. Our uncertainty about future stock prices increases as we look further ahead. As a rough approximation, we can say that the standard deviation of the stock price is proportional to the square root of how far ahead we are looking.

To estimate the volatility, σ, of a stock price empirically, we need to observe the stock price at fixed intervals of time (e.g., every day, every week, or every month). For each time period, the natural logarithm of the ratio of the stock price at the end of the time period to the stock price at the beginning of the time period is calculated. The volatility is estimated as the standard deviation of these numbers divided by the square root of the length of the time period in years. Usually days when the exchanges are closed are ignored in measuring time for the purposes of volatility calculations.

Stock option valuation involves setting up a riskless portfolio of the option and the stock. Because the stock price and the option price both depend on the same underlying source of uncertainty, such a portfolio can always be created. The portfolio remains riskless for only a very short period of time. However, the return on a riskless portfolio must always be the risk-free interest rate if there are to be no arbitrage opportunities. It is this fact that enables the option price to be valued in terms of the stock price. The original Black–Scholes–Merton result gives the value of a European call or put option on a non-dividend-paying stock in terms of five variables: the stock price, the strike price, the risk-free interest rate, the volatility, and the time to expiration.

Surprisingly the expected return on the stock does not enter into the Black–Scholes–Merton equation. There is a general principle known as risk-neutral valuation, which states that any security dependent on other traded securities can be valued on the assumption that the world is risk neutral. The result proves to be very useful in practice. In a risk-neutral world the expected return from all securities is the risk-free interest rate, and the correct discount rate for expected cash flows is also the risk-free interest rate.

An implied volatility is the volatility that, when substituted into the Black–Scholes–Merton equation or its extensions, gives the market price of the option. Traders monitor implied volatilities. They often quote the implied volatility of an option rather than its price. They have developed procedures for using the volatilities implied by the prices of actively traded options to estimate the volatilities appropriate for other options.

The Black–Scholes–Merton results can be extended to cover European call and put options on dividend-paying stocks. One procedure is to use the Black–Scholes–Merton formula with the stock price reduced by the present value of the dividends anticipated during the life of the option and the volatility equal to the volatility of the stock price net of the present value of these dividends. Fischer Black has suggested an approximate way of valuing American call options on a dividend-paying stock. His approach involves setting the price equal to the greater of two European option prices. The first European option expires at the same time as the American option; the second expires immediately prior to the final ex-dividend date.

FURTHER READING

On the Black–Scholes–Merton model and its extensions

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Roll, R. "Orange Juice and Weather," *American Economic Review*, 74, 5 (December 1984): 861–80.

Quiz (Answers at End of Book)

13.1. What does the Black–Scholes–Merton stock option pricing model assume about the probability distribution of the stock price in one year? What does it assume about the probability distribution of the continuously compounded rate of return on the stock during the year?

13.2. The volatility of a stock price is 30% per annum. What is the standard deviation of the percentage price change in one trading day?

13.3. Explain how risk-neutral valuation could be used to derive the Black–Scholes–Merton formulas.

13.4. Calculate the price of a three-month European put option on a non-dividend-paying stock with a strike price of \$50 when the current stock price is \$50, the risk-free interest rate is 10% per annum, and the volatility is 30% per annum.

13.5. What difference does it make to your calculations in the previous question if a dividend of \$1.50 is expected in two months?

13.6. What is meant by implied volatility? How would you calculate the volatility implied by a European put option price?

13.7. What is Black's approximation for valuing an American call option on a dividend-paying stock?

Practice Questions

13.8. A stock price is currently \$40. Assume that the expected return from the stock is 15% and its volatility is 25%. What is the probability distribution for the rate of return (with continuous compounding) earned over a one-year period?

13.9. A stock price has an expected return of 16% and a volatility of 35%. The current price is \$38.

(a) What is the probability that a European call option on the stock with an exercise price of \$40 and a maturity date in six months will be exercised?

(b) What is the probability that a European put option on the stock with the same exercise price and maturity will be exercised?

13.10. Prove that, with the notation in the chapter, a 95% confidence interval for S_T is between

$$S_0e^{(\mu-\sigma^2/2)T-1.96\sigma\sqrt{T}} \quad \text{and} \quad S_0e^{(\mu-\sigma^2/2)T+1.96\sigma\sqrt{T}}$$

13.11. A portfolio manager announces that the average of the returns realized in each of the last 10 years is 20% per annum. In what respect is this statement misleading?

13.12. Assume that a non-dividend-paying stock has an expected return of μ and a volatility of σ. An innovative financial institution has just announced that it will trade a derivative that pays off a dollar amount equal to

$$\frac{1}{T}\ln\left(\frac{S_T}{S_0}\right)$$

at time T. The variables S_0 and S_T denote the values of the stock price at time zero and time T.

(a) Describe the payoff from this derivative.

(b) Use risk-neutral valuation to calculate the price of the derivative at time zero.

13.13. What is the price of a European call option on a non-dividend-paying stock when the stock price is \$52, the strike price is \$50, the risk-free interest rate is 12% per annum, the volatility is 30% per annum, and the time to maturity is three months?

13.14. What is the price of a European put option on a non-dividend-paying stock when the stock price is \$69, the strike price is \$70, the risk-free interest rate is 5% per annum, the volatility is 35% per annum, and the time to maturity is six months?

13.15. A call option on a non-dividend-paying stock has a market price of \$2.50. The stock price is \$15, the exercise price is \$13, the time to maturity is three months, and the risk-free interest rate is 5% per annum. What is the implied volatility?

13.16. Show that the Black–Scholes–Merton formula for a call option gives a price that tends to $\max(S_0 - K, 0)$ as $T \to 0$.

13.17. Explain carefully why Black's approach to evaluating an American call option on a dividend-paying stock may give an approximate answer even when only one dividend is anticipated. Does the answer given by Black's approach understate or overstate the true option value? Explain your answer.

13.18. Consider an American call option on a stock. The stock price is \$70, the time to maturity is eight months, the risk-free rate of interest is 10% per annum, the exercise price is \$65, and the volatility is 32%. A dividend of \$1 is expected after three months and again after six months. Use the results in the appendix to show that it can never be optimal to exercise the option on either of the two dividend dates. Use DerivaGem to calculate the price of the option.

13.19. A stock price is currently \$50 and the risk-free interest rate is 5%. Use the DerivaGem software to translate the following table of European call options on the stock into a table of implied volatilities, assuming no dividends. Are the option prices consistent with the assumptions underlying Black–Scholes–Merton?

	Maturity (months)		
Strike price (\$)	*3*	*6*	*12*
45	7.00	8.30	10.50
50	3.50	5.20	7.50
55	1.60	2.90	5.10

13.20. Show that the Black–Scholes–Merton formulas for call and put options satisfy put–call parity.

13.21. Show that the probability that a European call option will be exercised in a risk-neutral world is, with the notation introduced in this chapter, $N(d_2)$. What is an expression for the value of a derivative that pays off \$100 if the price of a stock at time T is greater than K?

Further Questions

13.22. If the volatility of a stock is 18% per annum, estimate the standard deviation of the percentage price change in (a) one day, (b) one week, and (c) one month.

13.23. A stock price is currently \$50. Assume that the expected return from the stock is 18% per annum and its volatility is 30% per annum. What is the probability distribution for the stock price in two years? Calculate the mean and standard deviation of the distribution. Determine the 95% confidence interval.

13.24. Suppose that observations on a stock price (in dollars) at the end of each of 15 consecutive weeks are as follows:

30.2, 32.0, 31.1, 30.1, 30.2, 30.3, 30.6, 33.0, 32.9, 33.0, 33.5, 33.5, 33.7, 33.5, 33.2

Estimate the stock price volatility. What is the standard error of your estimate?

13.25. A financial institution plans to offer a derivative that pays off a dollar amount equal to S_T^2 at time T, where S_T is the stock price at time T. Assume no dividends. Defining other variables as necessary use risk-neutral valuation to calculate the price of the derivative at time zero. (*Hint*: The expected value of S_T^2 can be calculated from the mean and variance of S_T given in Section 13.1.)

13.26. Consider an option on a non-dividend-paying stock when the stock price is \$30, the exercise price is \$29, the risk-free interest rate is 5% per annum, the volatility is 25% per annum, and the time to maturity is four months.
(a) What is the price of the option if it is a European call?
(b) What is the price of the option if it is an American call?
(c) What is the price of the option if it is a European put?
(d) Verify that put–call parity holds.

13.27. Assume that the stock in Problem 13.26 is due to go ex-dividend in 1.5 months. The expected dividend is 50 cents.
(a) What is the price of the option if it is a European call?
(b) What is the price of the option if it is a European put?
(c) Use the results in the appendix to this chapter to determine whether there are any circumstances under which the option is exercised early.

13.28. Consider an American call option when the stock price is \$18, the exercise price is \$20, the time to maturity is six months, the volatility is 30% per annum, and the risk-free interest rate is 10% per annum. Two equal dividends of 40 cents are expected during the life of the option, with ex-dividend dates at the end of two months and five months. Use Black's approximation and the DerivaGem software to value the option. Suppose now that the dividend is D on each ex-dividend date. Use the results in the Appendix to determine how high D can be without the American option being exercised early.

APPENDIX

The Early Exercise of American Call Options on Dividend-Paying Stocks

In Chapter 10, we saw that it is never optimal to exercise an American call option on a non-dividend-paying stock before the expiration date. A similar argument shows that the only times when a call option on a dividend-paying stock should be exercised are immediately before an ex-dividend date and on the expiration date. We assume that n ex-dividend dates are anticipated and that they are at times $t_1, t_2, \dots, t_n$, with $t_1 < t_2 < \cdots < t_n$. The dividends will be denoted by $D_1, D_2, \dots, D_n$, respectively.

We start by considering the possibility of early exercise immediately prior to the final ex-dividend date (i.e., at time t_n). If the option is exercised at time t_n, the investor receives

$$S(t_n) - K$$

where $S(t)$ denotes the stock price at time t.

If the option is not exercised, the stock price drops to $S(t_n) - D_n$. As shown in Chapter 10, a lower bound for the price of the option is then

$$S(t_n) - D_n - Ke^{-r(T-t_n)}$$

It follows that if

$$S(t_n) - D_n - Ke^{-r(T-t_n)} \geqslant S(t_n) - K$$

that is,

$$D_n \leqslant K(1 - e^{-r(T-t_n)}) \tag{13A.1}$$

it cannot be optimal to exercise at time t_n. On the other hand, if

$$D_n > K(1 - e^{-r(T-t_n)}) \tag{13A.2}$$

it can be shown that it is always optimal to exercise at time t_n for a sufficiently high value of $S(t_n)$. Inequality (13A.2) is most likely to be satisfied when the final ex-dividend date is fairly close to the maturity of the option (i.e., when $T - t_n$ is small) and the dividend is large.

Consider next time t_{n-1}, the penultimate ex-dividend date. If the option is exercised immediately prior to time t_{n-1}, the investor receives

$$S(t_{n-1}) - K$$

If the option is not exercised at time t_{n-1}, the stock price drops to $S(t_{n-1}) - D_{n-1}$ and the earliest subsequent time at which exercise could take place is t_n. A lower bound to the option price if it is not exercised at time t_{n-1} is

$$S(t_{n-1}) - D_{n-1} - Ke^{-r(t_n - t_{n-1})}$$

It follows that if

$$S(t_{n-1}) - D_{n-1} - Ke^{-r(t_n - t_{n-1})} \geqslant S(t_{n-1}) - K$$

or

$$D_{n-1} \leqslant K(1 - e^{-r(t_n - t_{n-1})})$$

Example 13A.1 Test of whether a call option should ever be exercised early

Example 13.6 considers an American call option where $S_0 = 40$, $K = 40$, $r = 0.09$, $\sigma = 0.30$, $t_1 = 0.1667$, $t_2 = 0.4167$, $T = 0.5$, $D_1 = D_2 = 0.5$, so that

$$K(1 - e^{-r(t_2 - t_1)}) = 40(1 - e^{-0.09 \times 0.25}) = 0.89$$

Because this is greater than 0.5, it follows from inequality (13A.3) that the option should never be exercised on the first ex-dividend date. Also,

$$K(1 - e^{-r(T - t_2)}) = 40(1 - e^{-0.09 \times 0.08333}) = 0.30$$

Because this is less than 0.5, it follows from inequality (13A.1) that when the option is sufficiently deep in the money it should be exercised on the second ex-dividend date.

it is not optimal to exercise at time t_{n-1}. Similarly, for any $i < n$, if

$$D_i \leqslant K(1 - e^{-r(t_{i+1} - t_i)}) \qquad \textbf{(13A.3)}$$

it is not optimal to exercise immediately prior to time t_i. Example 13A.1 illustrates the use of these results.

The inequality (13A.3) is approximately equivalent to

$$D_i \leqslant Kr(t_{i+1} - t_i)$$

Assuming that K is fairly close to the current stock price, the dividend yield on the stock has to be either close to or above the risk-free rate of interest for the inequality not to be satisfied.

We can conclude from this analysis that, in many circumstances, the most likely time for the early exercise of an American call is the final ex-dividend date, t_n. Furthermore, if the inequality (13A.3) holds for $i = 1, 2, \ldots, n-1$ and the inequality (13A.1) also holds, then we can be certain that early exercise is never optimal.

CHAPTER 14

Employee Stock Options

Employee stock options are call options on a company's stock granted by the company to its employees. The options give the employees a stake in the fortunes of the company. If the company does well so that the company's stock price moves above the strike price, employees gain by exercising the options and then selling at the market price the stock they buy at the strike price.

Employee stock options have become very popular in the last 20 years. Many companies, particularly technology companies, feel that the only way they can attract and keep the best employees is to offer them very attractive stock option packages. Some companies grant options only to senior management; others grant them to people at all levels in the organization. Microsoft was one of the first companies to use employee stock options. All Microsoft employees were granted options and, as the company's stock price rose, it is estimated that over 10,000 of them became millionaires. In 2003, Microsoft announced that it would discontinue the use of options and award shares of Microsoft to employees instead. But many other companies throughout the world continue to be enthusiastic users of employee stock options.

Employee stock options are popular with start-up companies. Often these companies do not have the resources to pay key employees as much as they could earn with an established company and they solve this problem by supplementing the salaries of the employees with stock options. If the company does well and shares are sold to the public in an IPO, the options are likely to prove to be very valuable. Some newly formed companies have even granted options to students who worked for just a few months during their summer break—and in some cases this has led to windfalls of hundreds of thousands of dollars for the students!

This chapter explains how stock option plans work and how their popularity has been influenced by their accounting treatment. It discusses whether employee stock options help to align the interests of shareholders with those of top executives running a company. It also describes how these options are valued and looks at backdating scandals.

14.1 CONTRACTUAL ARRANGEMENTS

Employee stock options often last as long as 10 to 15 years. Very often the strike price is set equal to the stock price on the grant date so that the option is initially at the money.

The following are usually features of employee stock option plans:

1. There is a vesting period during which the options cannot be exercised. This vesting period can be as long as four years.
2. When employees leave their jobs (voluntarily or involuntarily) during the vesting period, they forfeit their options.
3. When employees leave (voluntarily or involuntarily) after the vesting period, they forfeit options that are out of the money and they have to exercise vested options that are in the money almost immediately.
4. Employees are not permitted to sell the options.
5. When an employee exercises options, the company issues new shares and sells them to the employee for the strike price.

The Early Exercise Decision

The fourth feature of employee stock option plans just mentioned has important implications. If employees, for whatever reason, want to realize a cash benefit from options that have vested, they must exercise the options and sell the underlying shares. They cannot sell the options to someone else. This leads to a tendency for employee stock options to be exercised earlier than similar regular exchange-traded or over-the-counter call options.

Consider a call option on a stock paying no dividends. In Section 10.5 we showed that, if it is a regular call option, it should never be exercised early. The holder of the option will always do better by selling the option rather than exercising it before the end of its life. However, the arguments we used in Section 10.5 are not applicable to employee stock options because they cannot be sold. The only way employees can realize a cash benefit from the options (or diversify their holdings) is by exercising the options and selling the stock. It is therefore not unusual for an employee stock option to be exercised well before it would be optimal to exercise the option if it were a regular exchange-traded or over-the-counter option.

Should an employee ever exercise his or her options before maturity and then keep the stock rather than selling it? Assume that the option's strike price is constant during the life of the option and the option can be exercised at any time. To answer the question we consider two options: the employee stock option and an otherwise identical regular option that can be sold in the market. We refer to the first option as option A and the second as option B. If the stock pays no dividends, we know that option B should never be exercised early. It follows that it is not optimal to exercise option A and keep the stock. If the employee wants to maintain a stake in his or her company, a better strategy is to keep the option. This delays paying the strike price and maintains the insurance value of the option, as described in Section 10.5. Only when it is optimal to exercise option B can it be a rational strategy for an employee to exercise option A before maturity and keep the stock.[1] As discussed in the appendix to Chapter 13, it is optimal to exercise option B only when a relatively high dividend is imminent.

In practice the early exercise behavior of employees varies widely from company to company. In some companies, there is a culture of not exercising early; in others,

[1] The only exception to this could be when an executive wants to own the stock for its voting rights.

employees tend to exercise options and sell the stock soon after the end of the vesting period, even if the options are only slightly in the money.

14.2 DO OPTIONS ALIGN THE INTERESTS OF SHAREHOLDERS AND MANAGERS?

For investors to have confidence in capital markets, it is important that the interests of shareholders and managers are reasonably well aligned. This means that managers should be motivated to make decisions that are in the best interests of shareholders. Managers are the agents of the shareholders and, as discussed in Chapter 8, economists use the term *agency costs* to describe the losses shareholders experience because managers do not act in their best interests. The prison sentences that are being served in the United States by some executives who chose to ignore the interests of their shareholders can be viewed as an attempt by the United States to signal to investors that, despite Enron and other scandals, it is determined to keep agency costs low.

Do employee stock options help align the interests of employees and shareholders? The answer to this question is not straightforward. There can be little doubt that they serve a useful purpose for a start-up company. The options are an excellent way for the main shareholders, who are usually also senior executives, to motivate employees to work long hours. If the company is successful and there is an IPO, the employees will do very well; but if the company is unsuccessful, the options will be worthless.

It is the options granted to the senior executives of publicly traded companies that are most controversial. It has been estimated that employee stock options account for about 50% of the remuneration of top executives in the United States. Executive stock options are sometimes referred to as an executive's "pay for performance." If the company's stock price goes up, so that shareholders make gains, the executive is rewarded. However, this overlooks the asymmetric payoffs of options. If the company does badly then the shareholders lose money, but all that happens to the executives is that they fail to make a gain. Unlike the shareholders, they do not experience a loss.[2] A better type of pay for performance involves the simpler strategy of giving stock to executives. The gains and losses of the executives then mirror those of other shareholders.

What temptations do stock options create for a senior executive? Suppose an executive plans to exercise a large number of stock options in three months and sell the stock. He or she might be tempted to time announcements of good news—or even move earnings from one quarter to another—so that the stock price increases just before the options are exercised. Alternatively, if at-the-money options are due to be granted to the executive in three months, the executive might be tempted to take actions that reduce the stock price just before the grant date. The type of behavior we are talking about here is of course totally unacceptable—and may well be illegal. But the backdating scandals, which are discussed later in this chapter, show that the way some executives have handled issues related to stock options leaves much to be desired.

Even when there is no impropriety of the type we have just mentioned, executive stock options are liable to have the effect of motivating executives to focus on short-term

[2] When options have moved out of the money, companies have sometimes replaced them with new at-the-money options. This practice known as "repricing" leads to the executive's gains and losses being even less closely tied to those of the shareholders.

profits at the expense of longer-term performance. In some cases they might even take risks they would not otherwise take (and risks that are not in the interests of the shareholders) because of the asymmetric payoffs of options. Managers of large funds worry that, because stock options are such a huge component of an executive's compensation, they are liable to be a big source of distraction. Senior management may spend too much time thinking about all the different aspects of their compensation and not enough time running the company!

A manager's inside knowledge and ability to affect outcomes and announcements is always liable to interact with his or her trading in a way that is to the disadvantage of other shareholders. One radical suggestion for mitigating this problem is to require executives to give notice to the market—perhaps one week's notice—of an intention to buy or sell their company's stock.[3] (Once the notice of an intention to trade had been given, it would be binding on the executive.) This allows the market to form its own conclusions about why the executive is trading. As a result, the price may increase before the executive buys and decrease before the executive sells.

14.3 ACCOUNTING ISSUES

An employee stock option represents a cost to the company and a benefit to the employee just like any other form of compensation. This point, which for many is self-evident, is actually quite controversial. Many corporate executives appear to believe that an option has no value unless it is in the money. As a result, they argue that an at-the-money option issued by the company is not a cost to the company. The reality is that, if options are valuable to employees, they must represent a cost to the company's shareholders—and therefore to the company. There is no free lunch. The cost to the company of the options arises from the fact that the company has agreed that, if its stock does well, it will sell shares to employees at a price less than that which would apply in the open market.

Prior to 1995 the cost charged to the income statement of a company when it issued stock options was the intrinsic value. Most options were at the money when they were first issued, so that this cost was zero. In 1995, accounting standard FAS 123 was issued. Many people expected it to require the expensing of options at their fair value. However, as a result of intense lobbying, the 1995 version of FAS 123 only encouraged companies to expense the fair value of the options they granted on the income statement. It did not require them to do so. If fair value was not expensed on the income statement, it had to be reported in a footnote to the company's accounts.

Accounting standards have now changed to require the expensing of stock options at their fair value on the income statement. In February 2004 the International Accounting Standards Board issued IAS 2 requiring companies to start expensing stock options in 2005. In December 2004 FAS 123 was revised to require the expensing of employee stock options in the United States starting in 2005.

The effect of the new accounting standards is to require options to be valued on the grant date and the valuation amount to be expensed on the income statement. Valuation at a later time than the grant date is not required. It can be argued that

[3] This would apply to the exercise of options because, if an executive wants to exercise options and sell the stock that is acquired, then he or she would have to give notice of intention to sell.

options should be revalued at financial year ends (or every quarter) until they are exercised or reach the end of their lives.[4] This would treat them in the same way as other derivative transactions entered into by the company. If the option became more valuable from one year to the next, there would then be an additional amount to be expensed. However, if it declined in value, there would be a positive impact on income. This approach would have a number of advantages. The cumulative charge to the company would reflect the actual cost of the options (either zero if the options are not exercised or the option payoff if they are exercised). Although the charge in any year would depend on the option pricing model used, the cumulative charge over the life of the option would not.[5] Arguably there would be much less incentive for the company to engage in the backdating practices described later in the chapter. The disadvantage usually cited for accounting in this way is that it is undesirable because it introduces volatility into the income statement.[6]

Nontraditional Option Plans

It is easy to understand why pre-2005 employee stock options tended to be at the money on the grant date and have strike prices that did not change during the life of the option. Any departure from this standard arrangement was likely to require the options to be expensed. Now that accounting rules have changed so that all options are expensed at fair value, many companies are considering alternatives to the standard arrangement.

One argument against the standard arrangement is that employees do well when the stock market goes up, even if their own company's stock price does less well than the market. One way of overcoming this problem is to tie the strike price of the options to the performance of the S&P 500. Suppose that on the option grant date the stock price is $30 and the S&P 500 is 1,500. The strike price would initially be set at $30. If the S&P 500 increased by 10% to 1,650, then the strike price would also increase by 10% to $33. If the S&P 500 moved down by 15% to 1,275, then the strike price would also move down by 15% to $25.50. The effect of this is that the company's stock price performance has to beat the performance of the S&P 500 to become in the money. As an alternative to using the S&P 500 as the reference index, the company could use an index of the prices of stocks in the same industrial sector as the company.

In another variation on the standard arrangement, the strike price increases through time in a predetermined way such that the shares of the stock have to provide a certain minimum return per year for the options to be in the money. In some cases profit targets are specified and the options vest only if the profit targets are met.[7]

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[4] See J. Hull and A. White, "Accounting for Employee Stock Options: A Practical Approach to Handling the Valuation Issues," *Journal of Derivatives Accounting*, 1, 1 (2004): 3–9.

[5] Interestingly, if an option is settled in cash rather than by the company issuing new shares, it is subject to the accounting treatment proposed here. (However, there is no economic difference between an option that is settled in cash and one that is settled by selling new shares to the employee.)

[6] In fact the income statement is likely be less volatile if stock options are revalued. When the company does well, income is reduced by revaluing the executive stock options. When the company does badly, it is increased.

[7] This type of option is difficult to value because the payoff depends on reported accounting numbers as well as the stock price. Usually valuations assume that the profit targets will be achieved.

14.4 VALUATION

Accounting standards give companies some latitude in choosing how to value employee stock options. A frequently used simple approach is based on the option's *expected life*. This is the average time for which employees hold the option before it is exercised or expires. The expected life can be approximately estimated from historical data on the early exercise behavior of employees and reflects the vesting period, the impact of employees leaving the company, and the tendency mentioned above for employee stock options to be exercised earlier than regular options. The Black–Scholes–Merton model is used with the life of the option, T, set equal to the expected life. The volatility is usually estimated from several years of historical data as described in Section 13.4.

It should be emphasized that using the Black–Scholes–Merton formula in this way has no theoretical validity. There is no reason why the value of a European stock option with the time to maturity, T, set equal to the expected life should be approximately the same as the value of the American-style employee stock option in which we are interested. However, the results given by the model are not totally unreasonable. Companies, when reporting their employee stock option expense, will frequently mention the volatility and expected life used in their Black–Scholes–Merton computations. Example 14.1 describes how to value an employee stock option using this approach.

More sophisticated approaches, where the probability of exercise is estimated as a function of the stock price and time to maturity, are sometimes used. A binomial tree similar to the one in Chapter 12 is created, but with the calculations at each node being adjusted to reflect (a) whether the option has vested, (b) the probability of the employee leaving the company, and (c) the probability of the employee choosing to exercise.[8] Hull and White propose a simple rule where exercise takes place when the ratio of the stock price to the strike price reaches some multiple.[9] This requires only one parameter relating to early exercise (the multiple) to be estimated.

Example 14.1 A popular approach for valuing employee stock options

A company grants 1,000,000 options to its executives on November 1, 2013. The stock price on that date is \$30 and the strike price of the options is also \$30. The options last for 10 years and vest after 3 years. The company has issued similar at-the-money options for the last 10 years. The average time to exercise or expiry of these options is 4.5 years. The company therefore decides to use an "expected life" of 4.5 years. It estimates the long-term volatility of the stock price, using 5 years of historical data, to be 25%. The present value of dividends during the next 4.5 years is estimated to be \$4. The 4.5-year zero-coupon risk-free interest rate is 5%. The option is therefore valued using the Black–Scholes–Merton model (adjusted for dividends as described in Section 13.10) with $S_0 = 30 - 4 = 26$, $K = 30$, $r = 5\%$, $\sigma = 25\%$, and $T = 4.5$. The Black–Scholes–Merton formula gives the value of one option as \$6.31. So the income statement expense is $1{,}000{,}000 \times 6.31$, or \$6,310,000.

[8] For more details and an example, see J. Hull *Options, Futures, and Other Derivatives*, 8th edn. Pearson, 2012.

[9] See J. Hull and A. White, "How to Value Employee Stock Options," *Financial Analysts Journal*, 60, 1 (2004): 3–9. Software for implementing this approach is available at: www.rotman.utoronto.ca/~hull.

Business Snapshot 14.1 Employee stock options and dilution

Consider a company with 100,000 shares each worth \$50. It surprises the market with an announcement that it is granting 100,000 stock options to its employees with a strike price of \$50. If the market sees little benefit to the shareholders from the employee stock options in the form of reduced salaries and more highly motivated managers, the stock price will decline immediately after the announcement of the employee stock options. If the stock price declines to \$45, the dilution cost to the current shareholders is \$5 per share or \$500,000 in total.

Suppose that the company does well so that by the end of three years the share price is \$100. Suppose further that all the options are exercised at this point. The payoff to the employees is \$50 per option. It is tempting to argue that there will be further dilution in that 100,000 shares worth \$100 per share are now merged with 100,000 shares for which only \$50 is paid, so that (a) the share price reduces to \$75 and (b) the payoff to the option holders is only \$25 per option. However, this argument is flawed. The exercise of the options is anticipated by the market and already reflected in the share price. The payoff from each option exercised is \$50.

This example illustrates the general point that when markets are efficient the impact of dilution from employee stock options is reflected in the stock price as soon as they are announced and does not need to be taken into account again when the options are valued.

Dilution

The fact that a company issues new stock when an employee stock option is exercised leads to some dilution for existing stock holders because new shares are being sold to employees at below the current stock price. It is natural to assume that this dilution takes place at the time the option is exercised. However, this is not the case. Stock prices are diluted when the market first hears about a stock option grant. The possible exercise of options is anticipated and immediately reflected in the stock price. This point is emphasized by the example in Business Snapshot 14.1.

The stock price immediately after a grant is announced to the public reflects any dilution. Provided that this stock price is used in the valuation of the option, it is not necessary to adjust the option price for dilution. In many instances the market expects a company to make regular stock option grants and so the market price of the stock anticipates dilution even before the announcement is made.

14.5 BACKDATING SCANDALS

No discussion of employee stock options would be complete without mentioning backdating scandals. Backdating is the practice of marking a document with a date that precedes the current date.

Suppose that a company decides to grant at-the-money options to its employees on April 30 when the stock price is \$50. If the stock price was \$42 on April 3, it is tempting to behave as if the options were granted on April 3 and use a strike price of \$42. This is legal provided that the company reports the options as \$8 in the money on the date when the decision to grant the options is made, April 30. But it is illegal for the company to report

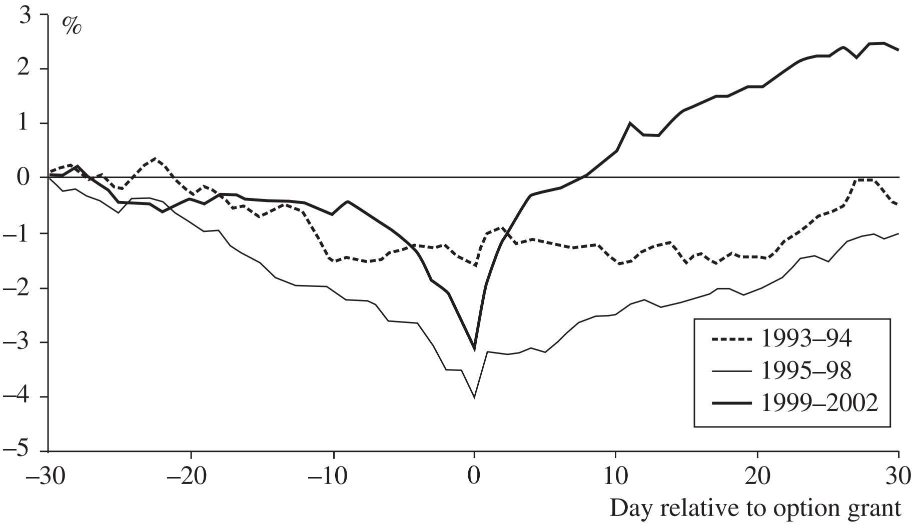

Figure 14.1 Erik Lie's results providing evidence of backdating (reproduced, with permission, from `www.biz.uiowa.edu/faculty/elie/backdating.htm`)

the options as at-the-money and granted on April 3. The value on April 3 of an option with a strike price of $42 is much less than its value on April 30. Shareholders are misled about the true cost of the decision to grant options if the company reports the options as granted on April 3.

How prevalent is backdating? To answer this question, researchers have investigated whether a company's stock price has, on average, a tendency to be low at the time of the grant date that the company reports. Early research by Yermack shows that stock prices tend to increase after reported grant dates.[10] Lie extended Yermack's work, showing that stock prices also tended to decrease before reported grant dates.[11] Furthermore he showed that the pre- and post-grant stock price patterns had become more pronounced over time. His results are summarized in Figure 14.1, which shows average abnormal returns around the grant date for the 1993–94, 1995–98, and 1999–2002 periods. (Abnormal returns are the returns after adjustments for returns on the market portfolio and the beta of the stock.) Standard statistical tests show that it is almost impossible for the patterns shown in Figure 14.1 to be observed by chance. This led both academics and regulators to conclude in 2002 that backdating had become a common practice. In August 2002 the SEC required option grants by public companies to be reported within two business days. Heron and Lie showed that this led to a dramatic reduction in the abnormal returns around the grant dates—particularly for those companies that complied with this requirement.[12] It might be argued that the patterns in Figure 14.1 are explained by managers simply choosing grant dates after bad news or before good news, but the Heron and Lie study provides compelling evidence that this is not the case.

[10] See D. Yermack, "Good timing: CEO stock option awards and company news announcements," *Journal of Finance*, 52 (1997), 449–476.

[11] See E. Lie, "On the timing of CEO stock option awards," *Management Science*, 51, 5 (May 2005), 802–12.

[12] See R. Heron and E. Lie, "Does backdating explain the stock price pattern around executive stock option grants," *Journal of Financial Economics*, 83, 2 (February 2007), 271–95.

Estimates of the number of companies that illegally backdated stock option grants in the United States vary widely. Tens and maybe hundreds of companies seem to have engaged in the practice. Many companies seem to have adopted the view that it was acceptable to backdate up to one month. Some CEOs resigned when their backdating practices came to light. In August 2007, Gregory Reyes of Brocade Communications Systems, Inc., became the first CEO to be tried for backdating stock option grants. Allegedly, Mr. Reyes said to a human resources employee: "It is not illegal if you do not get caught." In June 2010, he was sentenced to 18 months in prison and fined \$15 million. This was later reversed on appeal.

Companies involved in backdating have had to restate past financial statements and have been defendants in class action suits brought by shareholders who claim to have lost money as a result of backdating. For example, McAfee announced in December 2007 that it would restate earnings between 1995 and 2005 by \$137.4 million. In 2006, it set aside \$13.8 million to cover lawsuits.

SUMMARY

Executive compensation has increased very fast in the last 20 years and much of the increase has come from the exercise of stock options granted to the executives. Until 2005, at-the-money stock option grants were a very attractive form of compensation. They had no impact on the income statement and were very valuable to employees. Accounting standards now require options to be expensed.

There are a number of different approaches to valuing employee stock options. A common approach is to use the Black–Scholes–Merton model with the life of the option set equal to the expected time the option will remain unexercised.

Academic research has shown beyond doubt that many companies have engaged in the illegal practice of backdating stock option grants in order to reduce the strike price, while still contending that the options were at the money. The first prosecutions for this illegal practice were in 2007.

FURTHER READING

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Quiz (Answers at End of Book)

14.1. Why was it attractive for companies to grant at-the-money stock options prior to 2005? What changed in 2005?

14.2. What are the main differences between a typical employee stock option and an American call option traded on an exchange or in the over-the-counter market?

14.3. Explain why employee stock options on a non-dividend-paying stock are frequently exercised before the end of their lives, whereas an exchange-traded call option on such a stock is never exercised early.

14.4. "Stock option grants are good because they motivate executives to act in the best interests of shareholders." Discuss this viewpoint.

14.5. "Granting stock options to executives is like allowing a professional footballer to bet on the outcome of games." Discuss this viewpoint.

14.6. Why did some companies backdate stock option grants in the US prior to 2002? What changed in 2002?

14.7. In what way would the benefits of backdating be reduced if a stock option grant had to be revalued at the end of each quarter?

Practice Questions

14.8. Explain how you would do the analysis to produce a chart such as the one in Figure 14.1.

14.9. On May 31 a company's stock price is $70. One million shares are outstanding. An executive exercises 100,000 stock options with a strike price of $50. What is the impact of this on the stock price?

14.10. The notes accompanying a company's financial statements say: "Our executive stock options last 10 years and vest after 4 years. We valued the options granted this year using the Black–Scholes–Merton model with an expected life of 5 years and a volatility of 20%." What does this mean? Discuss the modeling approach used by the company.

14.11. A company has granted 500,000 options to its executives. The stock price and strike price are both $40. The options last for 12 years and vest after 4 years. The company decides to value the options using an expected life of 5 years and a volatility of 30% per annum. The company pays no dividends and the risk-free rate is 4%. What will the company report as an expense for the options on its income statement?

14.12. A company's CFO says: "The accounting treatment of stock options is crazy. We granted 10,000,000 at-the-money stock options to our employees last year when the stock price was $30. We estimated the value of each option on the grant date to be $5. At our year-end the stock price had fallen to $4, but we were still stuck with a $50 million charge to the P&L." Discuss.