KTH PHYSICS April 4, 2023
SH1017 Physics for Engineering Mathematics

Computer Lab 1: Driven damped pendulum

1. Download and run the code pendulum.py. It should result in an animation on the computer screen. Implement a computation of the period time T, i.e., the time for the pendulum to make one full period of the motion. There is no closed formula for T in terms of elementary functions so a numerical evaluation is useful. T can be computed as follows. In the first half period of the motion θ decreases with time, so $T/2$ can be computed as the time when θ starts to increase. In this way determine T for a few different values of θ_0.

The time step Δt in the provided code has to be adjusted. Test a few values $\Delta t = 0.1, 0.01, 0.001$ etc. A good choice makes the final results independent of Δt within the wanted numerical accuracy. The code assumes the parameter values $m = g = L = 1$.

Compare the simulation results with some analytical results by plotting T vs θ_0. The period time can be computed by numerical integration of the equation

$$T = \sqrt{2} \int_{-\theta_0}^{\theta_0} \frac{d\theta}{\sqrt{\cos(\theta) - \cos(\theta_0)}}$$

which follows from energy conservation: $E(t) = E(t_0)$. T can also be obtained from the series expansion

$$T = \frac{2\pi}{\omega_0} \left(1 + \frac{1}{16}\theta_0^2 + \frac{11}{3072}\theta_0^4 + ... \right)$$

where $\omega_0 = \sqrt{g/L}$. In the same plot include the result for the harmonic oscillator $T = 2\pi/\omega_0$ which applies for small oscillations of the pendulum. Comment on the agreement between the different results.

2. Modify the code to simulate a damped pendulum with equation of motion $\ddot{\theta} = -\gamma\dot{\theta} - \omega_0^2 \sin\theta$. The modification is done by uncommenting an existing line in the provided code. For $\gamma = 1$ determine and plot the amplitude at the turning points vs time.

The next task is to determine the characteristic time $t = \tau$ where the amplitude has reduced to half its initial value. This has to be done by interpolation in the amplitude vs time plot. Suggestions: It is always useful to look for a way to present a plot that makes the data form straight lines. The damping part of the equation of motion gives $\ddot{\theta} = -\gamma\dot{\theta} \Rightarrow \dot{\theta}(t) = C\exp(-\gamma t)$ which suggests that the pendulum amplitude also decays as $\exp(-\gamma t)$. This suggests trying a logarithmic plot. From this plot the half time is easily computed by linear interpolation between data points.

Run a set of simulations for a sequence of values of the damping: $\gamma = 0.1, 0.2, 0.4, 0.6, 0.8, 1.0, ...$ At which threshold value of γ does the motion become overdamped such that the motion never crosses the vertical $\theta = 0$? Try to plot the absolute value of the amplitude at the first turn vs γ and also vs $1/\gamma$. Can you use one of these plots to extrapolate to the γ that would give zero amplitude?

3. Modify the code to simulate the driven damped pendulum by uncommenting an existing line. The driving force is here assumed to act on the mass point of the pendulum. The equation of motion is $\ddot{\theta} = -\gamma\dot{\theta} - A\sin\omega t - \omega_0^2\sin\theta$. Use the initial conditions $\theta_0 = \dot{\theta}_0 = 0$, damping $\gamma = 3/8$ and drive frequency $\omega = 2/3$.

For small A the motion consists of two stages: the transient motion when the amplitude grows and the steady state motion when the phase portrait has reached a limit cycle and the amplitude settles to a constant value. Compute and plot the steady state amplitude vs A for $A = 0.1, 0.2, 0.4, 0.8$.

At $A \geq 1$ the solution changes character and can become very rich and complicated with period doubling, strange attractors and chaos. We will not go into detail here but to get a flavor, set $A = 1$ and describe the resulting solution. What happens to the phase portrait? Is there a limit cycle? For details see Classical Mechanics by John Taylor.

Write report in English, please.