

Import biblioteki **TensorFlow** (<https://www.tensorflow.org/>) z której będziemy korzystali w **uczeniu maszynowym**:

```
import tensorflow as tf
import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
df = pd.read_csv('Boston.csv')
print(df)
```

	Unnamed: 0	crim	zn	indus	chas	nox	rm	age	dis	rad	\
0	1	0.00632	18.0	2.31	0	0.538	6.575	65.2	4.0900	1	
1	2	0.02731	0.0	7.07	0	0.469	6.421	78.9	4.9671	2	
2	3	0.02729	0.0	7.07	0	0.469	7.185	61.1	4.9671	2	
3	4	0.03237	0.0	2.18	0	0.458	6.998	45.8	6.0622	3	
4	5	0.06905	0.0	2.18	0	0.458	7.147	54.2	6.0622	3	
..	
501	502	0.06263	0.0	11.93	0	0.573	6.593	69.1	2.4786	1	
502	503	0.04527	0.0	11.93	0	0.573	6.120	76.7	2.2875	1	
503	504	0.06076	0.0	11.93	0	0.573	6.976	91.0	2.1675	1	
504	505	0.10959	0.0	11.93	0	0.573	6.794	89.3	2.3889	1	
505	506	0.04741	0.0	11.93	0	0.573	6.030	80.8	2.5050	1	
..	
501	296	15.3	396.90	4.98	24.0						
1	242	17.8	396.90	9.14	21.6						
2	242	17.8	392.83	4.03	34.7						
3	222	18.7	394.63	2.94	33.4						
4	222	18.7	396.90	5.33	36.2						
..						
501	273	21.0	391.99	9.67	22.4						
502	273	21.0	396.90	9.08	20.6						
503	273	21.0	396.90	5.64	23.9						
504	273	21.0	393.45	6.48	22.0						
505	273	21.0	396.90	7.88	11.9						

[506 rows x 15 columns]

```
df.head()
```

	Unnamed: 0	crim	zn	indus	chas	nox	rm	age	dis	rad	tax	ptratio
0	1	0.00632	18.0	2.31	0	0.538	6.575	65.2	4.0900	1	296	15.3
1	2	0.02731	0.0	7.07	0	0.469	6.421	78.9	4.9671	2	242	17.8
2	3	0.02729	0.0	7.07	0	0.469	7.185	61.1	4.9671	2	242	17.8
3	4	0.03237	0.0	2.18	0	0.458	6.998	45.8	6.0622	3	222	18.7

```
nox=df.iloc[:,6]
```

nox

```
0      6.575
1      6.421
2      7.185
3      6.998
4      7.147
```

...

```
501    6.593
502    6.120
503    6.976
504    6.794
505    6.030
```

Name: rm, Length: 506, dtype: float64

indus=df.iloc[:,14]

indus

```
0      24.0
1      21.6
2      34.7
3      33.4
4      36.2
```

...

```
501    22.4
502    20.6
503    23.9
504    22.0
505    11.9
```

Name: medv, Length: 506, dtype: float64

plt.figure(figsize=(20, 5))

features = ['nox']

target = df['indus']

for i, col in enumerate(features):

plt.subplot(1, len(features) , i+1)

x = df[col]

y = target

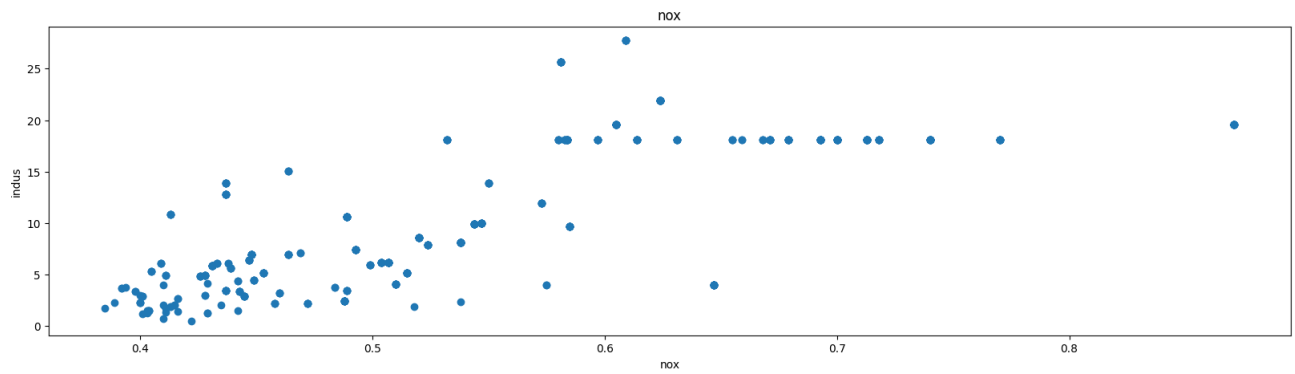
plt.scatter(x, y, marker='o')

plt.title(col)

plt.xlabel(col)

plt.ylabel('indus')

plt.show()



nox

```
0      6.575
1      6.421
2      7.185
3      6.998
4      7.147
...
501    6.593
502    6.120
503    6.976
504    6.794
505    6.030
Name: rm, Length: 506, dtype: float64
```

df.corr()

	Unnamed: 0	crim	zn	indus	chas	nox	rm	
df.corr()	Unnamed: 0	1.000000	0.407407	-0.103393	0.399439	-0.003759	0.398736	-0.079971
	crim	0.407407	1.000000	-0.200469	0.406583	-0.055892	0.420972	-0.219247
	zn	-0.103393	-0.200469	1.000000	-0.533828	-0.042697	-0.516604	0.311991
	indus	0.399439	0.406583	-0.533828	1.000000	0.062938	0.763651	-0.391676
	chas	-0.003759	-0.055892	-0.042697	0.062938	1.000000	0.091203	0.091251
	nox	0.398736	0.420972	-0.516604	0.763651	0.091203	1.000000	-0.302188
	rm	-0.079971	-0.219247	0.311991	-0.391676	0.091251	-0.302188	1.000000
	age	0.203784	0.352734	-0.569537	0.644779	0.086518	0.731470	-0.240265
	dis	-0.302211	-0.379670	0.664408	-0.708027	-0.099176	-0.769230	0.205246
	rad	0.686002	0.625505	-0.311948	0.595129	-0.007368	0.611441	-0.209847
	tax	0.666626	0.582764	-0.314563	0.720760	-0.035587	0.668023	-0.292048
	ptratio	0.291074	0.289946	-0.391679	0.383248	-0.121515	0.188933	-0.355501
	black	-0.295041	-0.385064	0.175520	-0.356977	0.048788	-0.380051	0.128069
	lstat	0.258465	0.455621	-0.412995	0.603800	-0.053929	0.590879	-0.613808

```
real_x = np.array(nox)
real_y = np.array(indus)
```

```
real_x
```

```
array([6.575, 6.421, 7.185, 6.998, 7.147, 6.43 , 6.012, 6.172, 5.631,
        6.004, 6.377, 6.009, 5.889, 5.949, 6.096, 5.834, 5.935, 5.99 ,
        5.456, 5.727, 5.57 , 5.965, 6.142, 5.813, 5.924, 5.599, 5.813,
        6.047, 6.495, 6.674, 5.713, 6.072, 5.95 , 5.701, 6.096, 5.933,
        5.841, 5.85 , 5.966, 6.595, 7.024, 6.77 , 6.169, 6.211, 6.069,
        5.682, 5.786, 6.03 , 5.399, 5.602, 5.963, 6.115, 6.511, 5.998,
        5.888, 7.249, 6.383, 6.816, 6.145, 5.927, 5.741, 5.966, 6.456,
        6.762, 7.104, 6.29 , 5.787, 5.878, 5.594, 5.885, 6.417, 5.961,
        6.065, 6.245, 6.273, 6.286, 6.279, 6.14 , 6.232, 5.874, 6.727,
        6.619, 6.302, 6.167, 6.389, 6.63 , 6.015, 6.121, 7.007, 7.079,
        6.417, 6.405, 6.442, 6.211, 6.249, 6.625, 6.163, 8.069, 7.82 ,
        7.416, 6.727, 6.781, 6.405, 6.137, 6.167, 5.851, 5.836, 6.127,
        6.474, 6.229, 6.195, 6.715, 5.913, 6.092, 6.254, 5.928, 6.176,
```

```

6.021, 5.872, 5.731, 5.87 , 6.004, 5.961, 5.856, 5.879, 5.986,
5.613, 5.693, 6.431, 5.637, 6.458, 6.326, 6.372, 5.822, 5.757,
6.335, 5.942, 6.454, 5.857, 6.151, 6.174, 5.019, 5.403, 5.468,
4.903, 6.13 , 5.628, 4.926, 5.186, 5.597, 6.122, 5.404, 5.012,
5.709, 6.129, 6.152, 5.272, 6.943, 6.066, 6.51 , 6.25 , 7.489,
7.802, 8.375, 5.854, 6.101, 7.929, 5.877, 6.319, 6.402, 5.875,
5.88 , 5.572, 6.416, 5.859, 6.546, 6.02 , 6.315, 6.86 , 6.98 ,
7.765, 6.144, 7.155, 6.563, 5.604, 6.153, 7.831, 6.782, 6.556,
7.185, 6.951, 6.739, 7.178, 6.8 , 6.604, 7.875, 7.287, 7.107,
7.274, 6.975, 7.135, 6.162, 7.61 , 7.853, 8.034, 5.891, 6.326,
5.783, 6.064, 5.344, 5.96 , 5.404, 5.807, 6.375, 5.412, 6.182,
5.888, 6.642, 5.951, 6.373, 6.951, 6.164, 6.879, 6.618, 8.266,
8.725, 8.04 , 7.163, 7.686, 6.552, 5.981, 7.412, 8.337, 8.247,
6.726, 6.086, 6.631, 7.358, 6.481, 6.606, 6.897, 6.095, 6.358,
6.393, 5.593, 5.605, 6.108, 6.226, 6.433, 6.718, 6.487, 6.438,
6.957, 8.259, 6.108, 5.876, 7.454, 8.704, 7.333, 6.842, 7.203,
7.52 , 8.398, 7.327, 7.206, 5.56 , 7.014, 8.297, 7.47 , 5.92 ,
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6.415, 6.431, 6.312, 6.083, 5.868, 6.333, 6.144, 5.706, 6.031,
6.316, 6.31 , 6.037, 5.869, 5.895, 6.059, 5.985, 5.968, 7.241,
6.54 , 6.696, 6.874, 6.014, 5.898, 6.516, 6.635, 6.939, 6.49 ,
6.579, 5.884, 6.728, 5.663, 5.936, 6.212, 6.395, 6.127, 6.112,
6.398, 6.251, 5.362, 5.803, 8.78 , 3.561, 4.963, 3.863, 4.97 ,
6.683, 7.016, 6.216, 5.875, 4.906, 4.138, 7.313, 6.649, 6.794,
6.38 , 6.223, 6.968, 6.545, 5.536, 5.52 , 4.368, 5.277, 4.652,
5. , 4.88 , 5.39 , 5.713, 6.051, 5.036, 6.193, 5.887, 6.471,
6.405, 5.747, 5.453, 5.852, 5.987, 6.343, 6.404, 5.349, 5.531,
5.683, 4.138, 5.608, 5.617, 6.852, 5.757, 6.657, 4.628, 5.155,
4.519, 6.434, 6.782, 5.304, 5.957, 6.824, 6.411, 6.006, 5.648,
6.103, 5.565, 5.896, 5.837, 6.202, 6.193, 6.38 , 6.348, 6.833,
6.425, 6.436, 6.208, 6.629, 6.461, 6.152, 5.935, 5.627, 5.818,
6.406, 6.219, 6.485, 5.854, 6.459, 6.341, 6.251, 6.185, 6.417,
6.749, 6.655, 6.297, 7.393, 6.728, 6.525, 5.976, 5.936, 6.301,
6.081, 6.701, 6.376, 6.317, 6.513, 6.209, 5.759, 5.952, 6.003,
5.926, 5.713, 6.167, 6.229, 6.437, 6.98 , 5.427, 6.162, 6.484,
5.304, 6.185, 6.229, 6.242, 6.75 , 7.061, 5.762, 5.871, 6.312,
6.114, 5.905, 5.454, 5.414, 5.093, 5.983, 5.983, 5.707, 5.926,
5.67 , 5.39 , 5.794, 6.019, 5.569, 6.027, 6.593, 6.12 , 6.976,
6.794, 6.03 ])
```

real_y

```

array([24. , 21.6, 34.7, 33.4, 36.2, 28.7, 22.9, 27.1, 16.5, 18.9, 15. ,
18.9, 21.7, 20.4, 18.2, 19.9, 23.1, 17.5, 20.2, 18.2, 13.6, 19.6,
15.2, 14.5, 15.6, 13.9, 16.6, 14.8, 18.4, 21. , 12.7, 14.5, 13.2,
13.1, 13.5, 18.9, 20. , 21. , 24.7, 30.8, 34.9, 26.6, 25.3, 24.7,
21.2, 19.3, 20. , 16.6, 14.4, 19.4, 19.7, 20.5, 25. , 23.4, 18.9,
35.4, 24.7, 31.6, 23.3, 19.6, 18.7, 16. , 22.2, 25. , 33. , 23.5,
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33.2, 27.5, 26.5, 18.6, 19.3, 20.1, 19.5, 19.5, 20.4, 19.8, 19.4,
21.7, 22.8, 18.8, 18.7, 18.5, 18.3, 21.2, 19.2, 20.4, 19.3, 22. ,
20.3, 20.5, 17.3, 18.8, 21.4, 15.7, 16.2, 18. , 14.3, 19.2, 19.6,
```

```

23. , 18.4, 15.6, 18.1, 17.4, 17.1, 13.3, 17.8, 14. , 14.4, 13.4,
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17. , 15.6, 13.1, 41.3, 24.3, 23.3, 27. , 50. , 50. , 50. , 22.7,
25. , 50. , 23.8, 23.8, 22.3, 17.4, 19.1, 23.1, 23.6, 22.6, 29.4,
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20. , 21.7, 19.3, 22.4, 28.1, 23.7, 25. , 23.3, 28.7, 21.5, 23. ,
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22.2, 23.7, 17.6, 18.5, 24.3, 20.5, 24.5, 26.2, 24.4, 24.8, 29.6,
42.8, 21.9, 20.9, 44. , 50. , 36. , 30.1, 33.8, 43.1, 48.8, 31. ,
36.5, 22.8, 30.7, 50. , 43.5, 20.7, 21.1, 25.2, 24.4, 35.2, 32.4,
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22.8, 20.3, 16.1, 22.1, 19.4, 21.6, 23.8, 16.2, 17.8, 19.8, 23.1,
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8.8, 8.4, 16.7, 14.2, 20.8, 13.4, 11.7, 8.3, 10.2, 10.9, 11. ,
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29.8, 13.8, 13.3, 16.7, 12. , 14.6, 21.4, 23. , 23.7, 25. , 21.8,
20.6, 21.2, 19.1, 20.6, 15.2, 7. , 8.1, 13.6, 20.1, 21.8, 24.5,
23.1, 19.7, 18.3, 21.2, 17.5, 16.8, 22.4, 20.6, 23.9, 22. , 11.9])

```

Batch Stochastic Gradient Descent - wykorzystujemy cały zbiór danych

Definicja błędu:

```

def loss_fn(real_y, pred_y):
    return tf.reduce_mean((real_y - pred_y)**2)

```

```
import random
```

```
Loss = []
epochs = 2000
learning_rate = 0.02

a = tf.Variable(random.random())
b = tf.Variable(random.random())

for _ in range(epochs):

    with tf.GradientTape() as tape:
        pred_y = a * real_x + b
        #print(pred_y)
        loss = loss_fn(real_y, pred_y)
        Loss.append(loss.numpy())
        grad_a, grad_b = tape.gradient(loss, (a, b))

    a.assign_sub(learning_rate*grad_a)
    b.assign_sub(learning_rate*grad_b)

np.max(Loss), np.min(Loss)

(318.53137, 45.89997)

print(a.numpy())
print(b.numpy())

6.955526
-21.015951

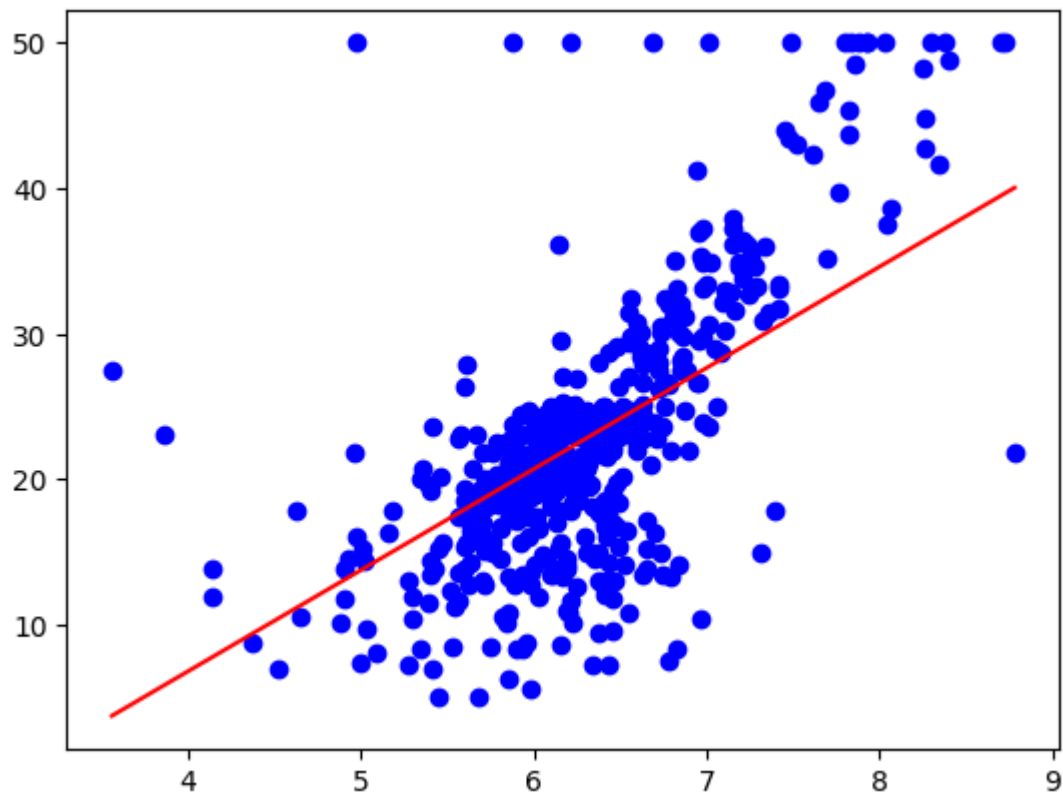
plt.scatter(np.arange(epochs), Loss)
plt.show()
```

```

max = np.max(nox)
min = np.min(nox)

X = np.linspace(min, max, num=10)
plt.plot(X,a.numpy()*X+b.numpy(),c='r')
plt.scatter(nox,indus,c="b")
plt.show()

```



Mini-batch Stochastic Gradient Descent - wykorzystujemy część zbioru danych

Definiujemy tablicę:

```

arr = np.arange(10)
arr

array([0, 1, 2, 3, 4, 5, 6, 7, 8, 9])

```

Mieszamy zawartość tablicy:

```

np.random.shuffle(arr)
arr

array([6, 8, 4, 0, 1, 2, 5, 7, 9, 3])

```

Funkcja do przetestowania:


```
def subset_dataset(x_dataset, y_dataset, subset_size):
    arr = np.arange(len(x_dataset))
    np.random.shuffle(arr)
    x_train = x_dataset[arr[0:subset_size]]
    y_train = y_dataset[arr[0:subset_size]]
    return x_train, y_train
```

Uzupełnik poniższy kod, tak aby możliwe było testowanie różnych wielkości próbki treningowej.

```
def mini_batch_stochastic_gradient_descent(batch_size):
    Loss = []
    epochs = 2000
    learning_rate = 0.02
    batch_size = batch_size      #wielkość zbioru wykorzystanego do treningu

    a = tf.Variable(random.random())
    b = tf.Variable(random.random())

    for i in range(epochs):

        real_x_batch, real_y_batch = subset_dataset(real_x, real_y, batch_size)

        with tf.GradientTape() as tape:
            pred_y = a * real_x_batch + b
            loss = loss_fn(real_y_batch, pred_y)
            Loss.append(loss.numpy())

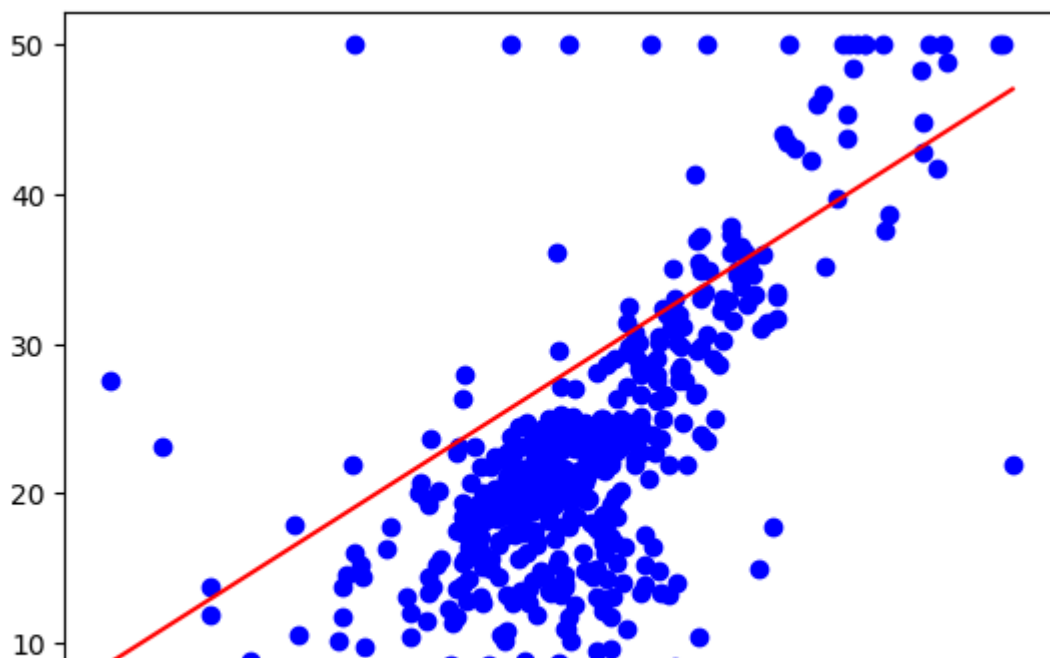
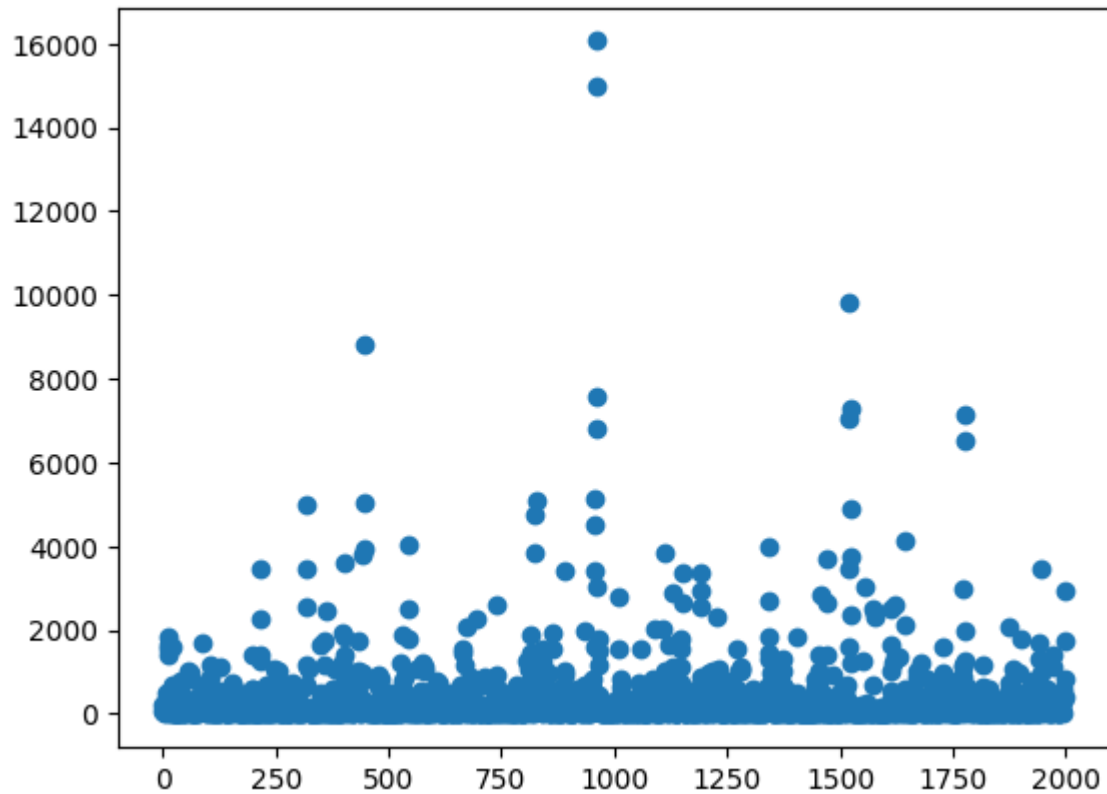
        dloss_da, dloss_db = tape.gradient(loss, (a, b))

        a.assign_sub(learning_rate*dloss_da)  #a = a - alpha*dloss_da
        b.assign_sub(learning_rate*dloss_db)  #b = b - alpha*dloss_db
        print("last one loss", str(loss))

    plt.scatter(np.arange(epochs), Loss)
    plt.show()
    max = np.max(nox)
    min = np.min(nox)
    X = np.linspace(min, max, num=10)
    plt.plot(X, a.numpy()*X+b.numpy(), c='r')
    plt.scatter(nox, indus, c="b")
    plt.show()

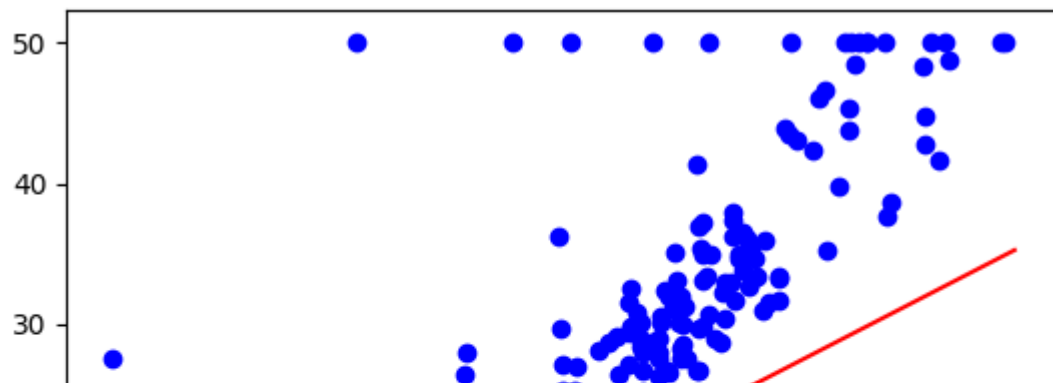
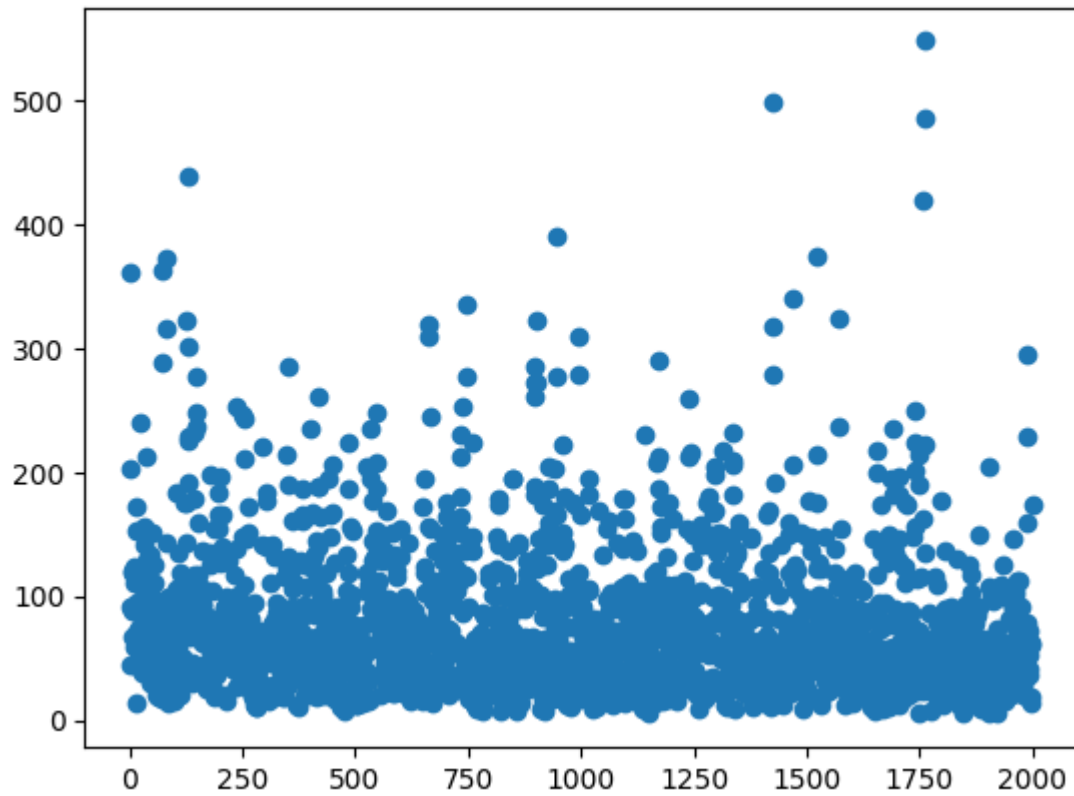
mini_batch_stochastic_gradient_descent(1)
```

```
last one loss tf.Tensor(428.42133, shape=(), dtype=float32)
```



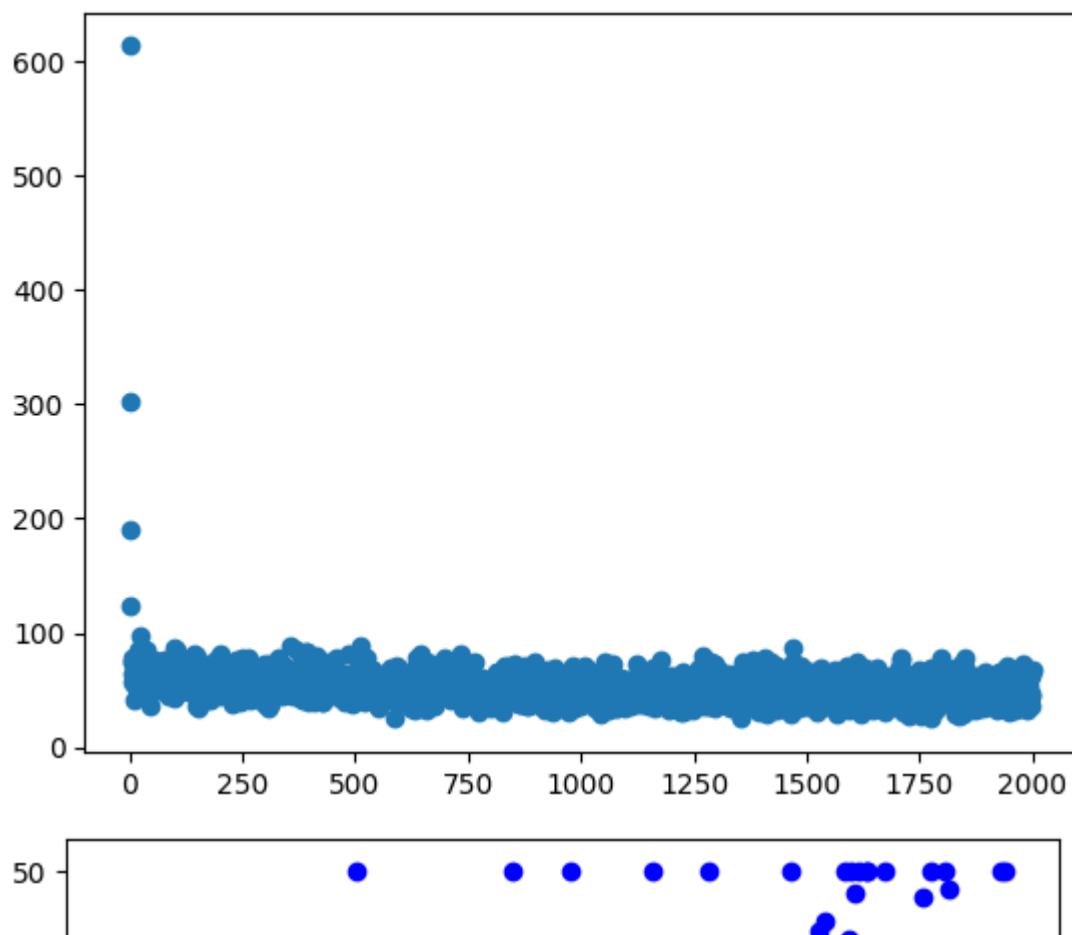
```
mini_batch_stochastic_gradient_descent(10)
```

```
last one loss tf.Tensor(173.78526, shape=(), dtype=float32)
```



```
mini_batch_stochastic_gradient_descent(100)
```

last one loss tf.Tensor(66.96526, shape=(), dtype=float32)



Wykres zmian błędu:

#do uzupełnienia

```
def subset_dataset(x_dataset, y_dataset, subset_size):
    arr = np.arange(len(x_dataset))
    np.random.shuffle(arr)
    x_train = x_dataset[arr[0:subset_size]]
    y_train = y_dataset[arr[0:subset_size]]
    return x_train, y_train
```

```
def loss_fn(real_y, pred_y):
    return tf.reduce_mean((real_y - pred_y)**2)
```

```

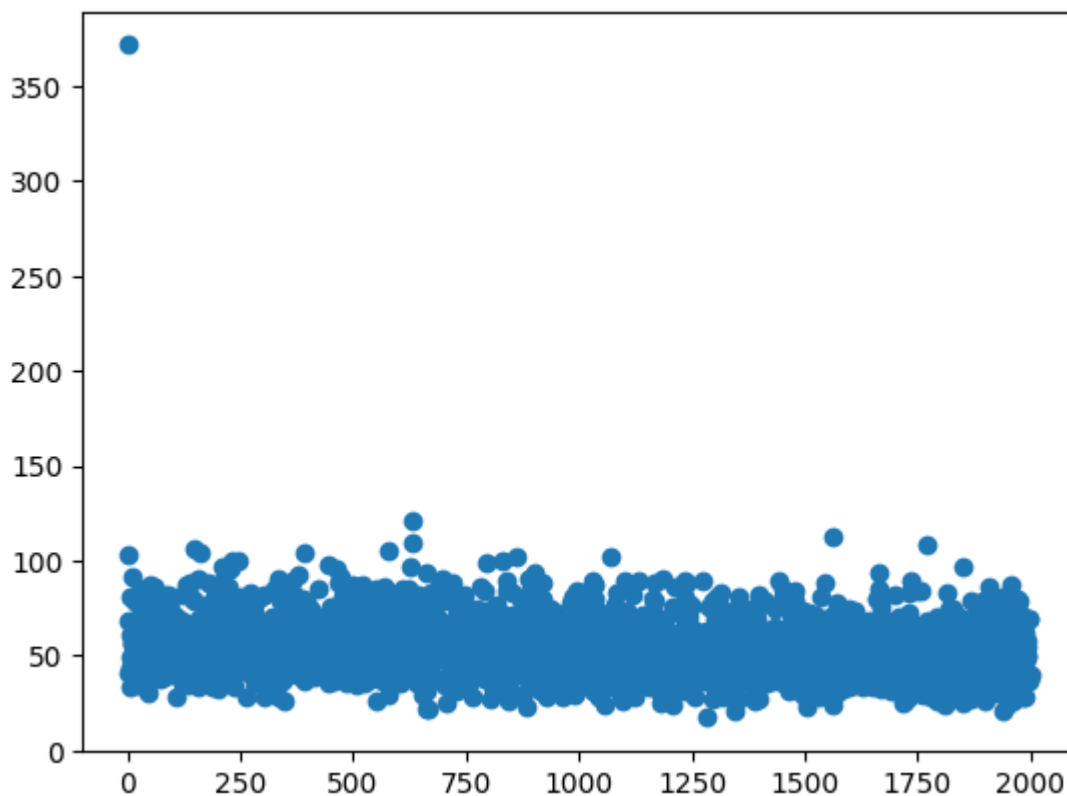
Loss = []
epochs = 2000
learning_rate = 0.01
batch_size = 50
a = tf.Variable(random.random())
b = tf.Variable(random.random())
for _ in range(epochs):
    real_nox_batch, real_medv_batch = subset_dataset(real_x, real_y, batch_size)
    with tf.GradientTape() as tape:
        pred_medv = a * real_nox_batch + b
        loss = loss_fn(real_medv_batch, pred_medv)
        Loss.append(loss.numpy())

    dloss_da, dloss_db = tape.gradient(loss, (a, b))

    a.assign_sub(learning_rate*dloss_da) #a = a - alpha*dloss_da
    b.assign_sub(learning_rate*dloss_db) #b = b - alpha*dloss_db

plt.scatter(np.arange(epochs), Loss)
plt.show()

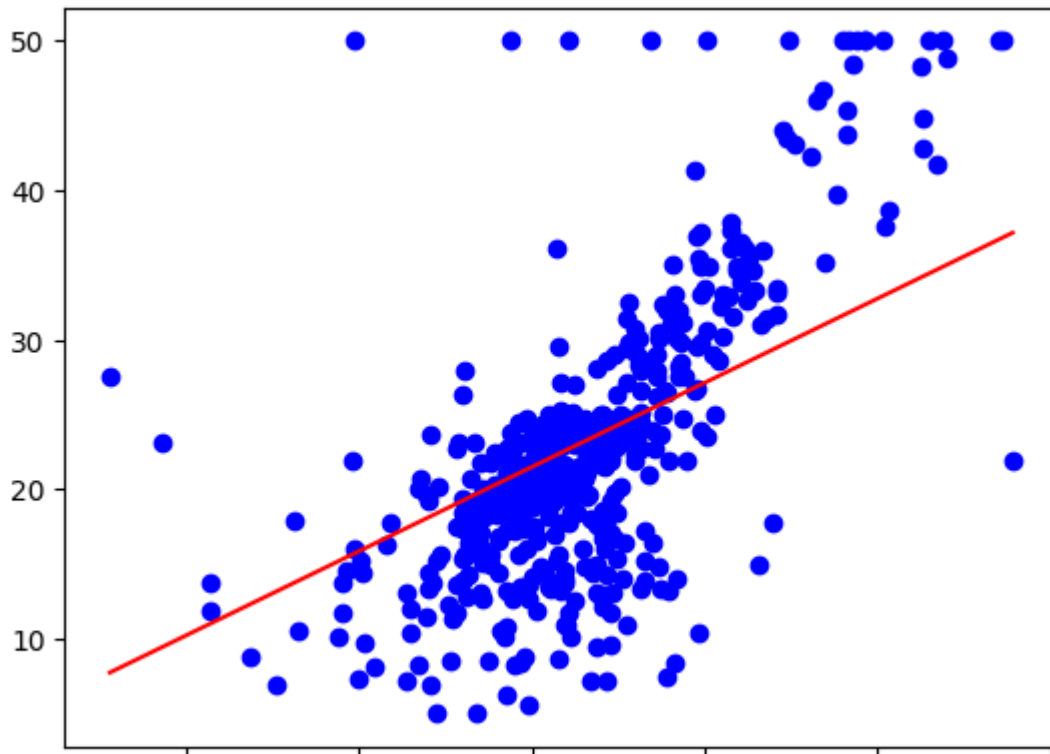
```



```

max = np.max(nox)
min = np.min(nox)
X = np.linspace(min, max, num=10)
plt.plot(X, a.numpy()*X+b.numpy(), c='r')
plt.scatter(nox, indus, c="b")
plt.show()

```



▼ Podsumowanie

Na uczenie modelu ma największy wpływ użycie batcha (bez batcha jest podawany cały zbiór uczący), małe batche mogą przyspieszyć proces uczenia, ponieważ aktualizacje wag modelu są wykonywane częściej. Dzięki temu wprowadza to pewną losowość w procesie uczenia, pomaga uniknąć utknięcia w minimach lokalnych. Model uczony z minibatchem osiąga lepsze rezultaty jeżeli chodzi o wyniki uczenia (lepiej znaleziona prosta) oraz mniejszy błąd. Model lepiej i szybciej się uczy gdy mini-batch jest większy niż gdy jest on mniejszy. Ponadto na proces uczenia modelu ma wpływ ilość epok. Za mała ilość epok skutkuje niedouczeniem modelu (model nie nauczył się wystarczająco dobrze dostosowywać się do danych treningowych), zaś gdy ilość epok jest zbyt duża następuje przeuczenie modelu (model nieugulnia zgromadzonej wiedzy tylko "uczy się na pamięć" zbioru treningowego co sprawia, że jest nieskuteczny lub mało skuteczny dla nowych danych). Ostatnim sprawdzonym przeze mnie parametrem jest współczynnik uczenia. Jego zbyt duża wartość rowadzi do skakania wokół minimum globalnego przy czym model go nie osiągnie. W przypadku zastosowania zbyt małej wartości współczynnika uczenia proces uczenia jest bardzo wolny, a model "utyka" w minimach lokalnych.

Nie można połączyć się z usługą reCAPTCHA. Sprawdź połączenie z internetem i załaduj ponownie zadanie reCAPTCHA.