1. a) a → (¬b ∧ c)

b ↔ a

c → (¬a ∧ ¬b) ∨ (¬a ∧ b) ∨ (a ∧ ¬b)

b)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| a | b | c | A’s Report | B’s Report | C’s Report |
| T | T | T | F | T | F |
| T | T | F | F | T | T |
| T | F | T | T | F | T |
| T | F | F | F | F | T |
| F | T | T | T | F | T |
| F | T | F | T | F | T |
| F | F | T | T | T | T |
| F | F | F | T | T | T |

c) Processor c is working.

d) A’s report and C’s report are false.

e) Processor A is working, processor B is not working, processor C is working.

2. a)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| p | q | r | p ↔ q | (p ↔ q) → r |
| T | T | T | T | T |
| T | T | F | T | F |
| T | F | T | F | T |
| T | F | F | F | T |
| F | T | T | F | T |
| F | T | F | F | T |
| F | F | T | T | T |
| F | F | F | T | F |

b) ¬((p ∧ q) ∨ (¬p ∧ ¬q)) ∨ r

c) (p ∧ ¬q) ∨ (¬p ∧ q) ∨ r

3. a) “If n is not even, then so is n2”, where n is an integer.

b) Case 1: where x is an even number

Because x is even, let x be 2a, where n is an integer

x2 =(2a)2

= 4a2

Let k = a2

Therefore x2 = 4k

Case 2: where x is an odd number

Because x is an odd number, let x = 2a + 1, where m is an integer

x2 = (2a + 1)2

= 4a2 + 4a + 1

= 4(a2+a) + 1

Let (a2+a) be k

Therefore x2 = 4k + 1

c) Let c = 2x because c is assumed to be an even number.

(2x)2 = 4x2

= 2(2x2)

Since 2x2 results in any even number, then 2(2x2) is also any even number.

Therefore, if c is an even number, then so is c2.

If c2 is even, then a2 + b2 is also even.

If a is even and bis odd, or vice versa, then a2 + b2 will be odd, since c2 has to be even, a and b have to both be even, or odd.

Let a = 2y and b = 2z + 1

(2y)2 + (2z + 1)2

= 4y2 + 4z2 + 4z + 1

= 2(2y2 + 2z2 + 2z) + 1

Let m represent 2y2 + 2z2 + 2z

= 2m + 1

Therefore a and b has to be both even or both odd for c2 to equal to an even number

However, a and b cannot be both odd numbers.

Let a = 2y + 1 and b = 2z + 1

(2y + 1)2 + (2z + 1)2

= 4y2 + 4y + 1 + 4z2 + 4z + 1

= 4(y2 + y + z2 + z) + 2

Let k represent y2 + y + z2 + z

= 4k + 2

Since c2 is a perfect square and we proved in 3 b) that a perfect square can only be in the form of 4k or 4k + 1, a and b cannot both be odd.

If a is an even number:

a2 = (2k)2

a2 = 4k2

Therefore, if a and b are even a2 and b2 are also even. So, c2 = a2 + b2 if a, b, and c are all even.

Therefore if c is even then so are a and b.

4. Prove 23m-3 mod 7 = 1 → 23m mod 7 = 1

Prove 23k-3 mod 7 using m = 1

20 mod 7 = 1

Assume that 23m-3 mod 7 = 1 is true, when m = k

Prove m = k + 1 is true:

23(k+1)-3 mod 7 = (23k-3) mod 7 \* (23) mod 7

= (1)(1)

= 1

Prove 23k mod 7 using m = 1:

23 mod 7 = 1

Assume that 23m mod 7 is true, when m = k

Prove m = k + 1 is true:

23k+3 mod 7 = (23k-3) mod 7 \* 26 mod 7

= (1)(1) = 1

Therefore 23m-3 mod 7 = 1 → 23m mod 7 = 1