Homework 3

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1.

2. (a)
   (b)
   (c)

3. (a)
   (b)
   (c)

4. *Proof.* Suppose there exists, two degree n polynomials, p_1 and p_2, such that

$$p_1(x_i) = y_i = p_2(x_i)$$

for all $0 \leq i \leq n$. It suffices to show that $p_1(x) = p_2(x)$. Therefore, the polynomial,

$$f(x) = p_1(x) - p_2(x)$$

is a polynomial of degree at most n, with $n+1$ distinct roots, $x_0, x_1, \ldots, x_n$. However, by the Fundamental Theorem of Algebra, f must be the 0 polynomial.[1] Therefore,

$$f(x) = 0,$$

which means that $p_1(x) = p_2(x)$, which is the desired result. □

5. (a)
   (b)
   (c)

6.

7.

[1] Since otherwise it would be a non-zero degree n polynomial with more than n distinct roots.