

Homework 3

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Martin Coyne, Flora Dedvukaj, Jiahao Gao, Anton Karabushin, Zhihan Lin, Joshua Morales

- 1.
2. (a)
(b)
(c)
3. (a)
(b)
(c)
4. *Proof.* Suppose there exists, two degree n polynomials, p_1 and p_2 , such that

$$p_1(x_i) = y_i = p_2(x_i)$$

for all $0 \leq i \leq n$. It suffices to show that $p_1(x) = p_2(x)$. Therefore, the polynomial,

$$f(x) = p_1(x) - p_2(x)$$

is a polynomial of degree at most n , with $n + 1$ distinct roots, x_0, x_1, \dots, x_n . However, by the Fundamental Theorem of Algebra, f must be the 0 polynomial.¹ Therefore,

$$f(x) = 0,$$

which means that $p_1(x) = p_2(x)$, which is the desired result. □

5. (a) We can take $(0, 7)$, $(2, 11)$, $(3, 28)$, and $(4, 63)$ as (x_0, y_0) , (x_1, y_1) , \dots , (x_n, y_n) respectively. We find the following:

$$L_0(x) = \frac{(x - x_1)(x - x_2)(x - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)}$$

$$L_1(x) = \frac{(x - x_0)(x - x_2)(x - x_3)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)}$$

$$L_2(x) = \frac{(x - x_0)(x - x_1)(x - x_3)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)}$$

$$L_3(x) = \frac{(x - x_0)(x - x_1)(x - x_2)}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)}$$

Plugging in the points from above, we get:

$$L_0(x) = \frac{(x - 2)(x - 3)(x - 4)}{(-2)(-3)(-4)} = \frac{-1}{24}(x - 2)(x - 3)(x - 4)$$

¹Since otherwise it would be a non-zero degree n polynomial with more than n distinct roots.

$$L_1(x) = \frac{(x-0)(x-3)(x-4)}{(-2)(-1)(-2)} = \frac{1}{4}(x)(x-3)(x-4)$$

$$L_2(x) = \frac{(x-0)(x-2)(x-4)}{(3)(1)(-1)} = \frac{-1}{3}(x)(x-2)(x-4)$$

$$L_3(x) = \frac{(x-0)(x-2)(x-3)}{(4)(2)(1)} = \frac{1}{8}(x)(x-2)(x-3)$$

Using the y-values from above, we get the following interpolation:

$$P(x) = \frac{-7}{24}(x-2)(x-3)(x-4) + \frac{11}{4}(x)(x-3)(x-4) - \frac{28}{3}(x)(x-2)(x-4) + \frac{63}{8}(x)(x-2)(x-3)$$

Which can be simplified to become

$$P(x) = x^3 - 2x + 7$$

(b) To find the approximation of $f(1)$, we just plug in 1 for x in $P(x)$:

$$P(1) = 1^3 - 2(1) + 7 = 1 - 2 + 7 = 6$$

(c)

$$\int_0^4 x^3 - 2x + 7 = 76$$

6.

7.