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Research papers

Verifying the prevalence, properties, and congruent hydraulics of at-many-stations hydraulic geometry (AMHG) for rivers in the continental United States

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ABSTRACT

Hydraulic geometry (HG) has long enabled daily discharge estimates, flood risk monitoring, and water resource and habitat assessments, among other applications. At-many-stations HG (AMHG) is a newly discovered form of HG with an evolving understanding. AMHG holds that there are temporally and spatially invariant ('congruent') depth, width, velocity, and discharge values that are shared by all stations of a river. Furthermore, these river-wide congruent hydraulics have been shown to link at-a-station HG (AHG) in space, contrary to previous expectation of AHG as spatially unpredictable. To date, AMHG has only been thoroughly examined on six rivers, and its congruent hydraulics are not well understood. To address the limited understanding of AMHG, we calculated AMHG for 191 rivers in the United States using USGS field-measured data from over 1900 gauging stations. These rivers represent nearly all geologic and climatic settings found in the continental U.S. and allow for a robust assessment of AMHG across scales. Over 60% of rivers were found to have AMHG with strong explanatory power to predict AHG across space (defined as $r^2 > 0.6$, 118/191 rivers). We also found that derived congruent hydraulics bear little relation to their observed time-varying counterparts, and the strength of AMHG did not correlate with any available observed or congruent hydraulic parameters. We also found that AMHG is expressed at all fluvial scales in this study. Some statistically significant spatial clusters of rivers with strong and weak AMHG were identified, but further research is needed to identify why these clusters exist. Thus, this first widespread empirical investigation of AMHG leads us to conclude that AMHG is indeed a widely prevalent natural fluvial phenomenon, and we have identified linkages between known fluvial parameters and AMHG. Our work should give confidence to future researchers seeking to perform the necessary detailed hydraulic analysis of AMHG.

1. Introduction

Rivers are essential for many human and ecosystem functions, and researchers have long been interested in the form and process of rivers. In particular, fluvial geomorphologists have used the concept of hydraulic geometry to understand the relationships between discharge and channel geometry for decades (Leopold and Maddock, 1953), enabling advances in many fields that aim to monitor and better understand rivers. Hydraulic geometry augmented development of rating curves already used by agencies throughout the world to relate river stage and discharge, and these resultant gauging stations have become essential for monitoring water resources and flood risks. This further understanding of hydraulic geometry has allowed agencies such as the United States Geological Survey (USGS) to provide daily discharge data for rivers throughout the US. In addition, hydrologic modeling also uses hydraulic geometry to calculate runoff routing times after large precipitation events and parametrize subgrid channels for streamflow simulation (e.g. Paik and Kumar, 2004; Neal et al., 2012), and hydraulic geometry has also been utilized to assess paleo flow conditions (Tinkler and Pengelly, 1995; Larson and Lamb, 2016). River restoration efforts also rely on hydraulic geometry as an easy-to-observe metric which can be related to other common fisheries management indices that are difficult to observe (Lamouroux and Souchon, 2002; Mosley, 1982; Rosenfeld et al., 2007).

The study of hydraulic geometry was launched by Leopold and Maddock in 1953 when they identified three power law equations that related discharge Q through a given station of a river with the flow width w, mean depth d, and mean velocity v of that station. They coined these equations (Eqs. (1)–(3)) as at-a-station hydraulic geometry (AHG).

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$$w = aQ^b \quad (1)$$

$$d = cQ^f \quad (2)$$

$$v = kQ^m \quad (3)$$

The coefficients (a, c, and k) and exponents (b, f, and m) are empirical parameters obtained by calibrating repeated field measurements of width, depth, and velocity with discharge in log-log space. AHG is only valid for in-bank flows, and out of bank flows exhibit distinct shifts in the rating curve that AHG power laws are unable to describe (Gleason, 2015). Similarly, Leopold and Maddock proposed downstream hydraulic geometry (DHG), which relates discharge with river width, depth, and velocity for stations along a river in the downstream direction for a given flow recurrence frequency. They found the relationship between these DHG parameters to take on the same power law form as AHG and used the same nomenclature as Eqs. (1)–(3) to describe DHG, though the coefficients and exponents in Eqs. (1)–(3) represent different fluvial properties between the two forms of hydraulic geometry.

Following Leopold and Maddock's, 1953 paper, AHG became an increasingly popular research topic. Many studies sought to verify the empirical relationships across a range of physiographic settings, while others aimed to discover the theoretical basis that explained the empirical observations. Through empirical analysis, Park (1977) and Rhodes (1977, 1978, 1987) separately found that AHG exponents are not similar among rivers in similar physiographic settings, leaving the question of what physical parameters determine the exponents of AHG unanswered. Despite objections made to the use of power law equations to describe the observed AHG relationships (Richards, 1973), a definitive theoretical explanation for AHG was provided by Ferguson (1986) through the derivation of AHG for different channel geometries using flow resistance equations. Attempts have been made to derive explicit formulations for the AHG coefficients and exponents, and Dingman (2007) showed that both the coefficients and exponents can be determined given a flow resistance equation and channel geometry *a priori*. Unlike the theoretical basis for AHG, no such definitive explanation has been found for DHG, resulting in the continued use of empirical data in order to identify factors affecting DHG (Gleason, 2015).

Calibrating the coefficients and exponents of the AHG and DHG equations requires data specific to discrete river stations, limiting the applicability of AHG and DHG to river stations for which these data have been gathered. This limitation, along with Ferguson's (1986) work, led to the conclusion that AHG is fundamentally site-specific and unrelated through space (Phillips, 1990). Dingman (2007) found that coefficients and exponents are related, but his first-principles analysis holds only at a single station. Gleason and Smith (2014), however, discovered a spatial relationship between the AHG coefficients and exponents they termed at-many-stations hydraulic geometry (AMHG). This finding seemingly contradicts the assumption that these parameters are spatially independent and site specific, and states that the AHG coefficient may be determined from the AHG exponent, and vice versa. Thus, Dingman's, 2007 important at-a-station conclusions cannot predict why multiple stations should have a spatially predictable variation in coefficients and exponents. AMHG was first introduced as a log-linear relationship between a river's AHG coefficients and exponents (a and b, c and f, k and m), effectively halving the number of unknown AHG parameters from six to three if AMHG were known (Gleason and Smith, 2014). Gleason and Wang (2015) advanced the understanding of AMHG when they showed that multiple AHG curves representing different stations on the same river reliably converge to congruent hydraulic quantities when plotted on the same axes (coined congruent discharge Q_c, width w_c, depth d_c, and velocity v_c), and they argued that AMHG exists as a consequence of this convergence, further confirmed by Shen et al. (2016). Gleason and Wang argued that the values of the congruent hydraulics are represented by the spatial mode of the time mean for each of these station values and as a result concluded that the strength (r^2) of AMHG was a geomorphic index indicating the degree of convergence of AHG curves. Shen et al. (2016) built upon this work and stated that the 'similar time mean' condition was a sufficient but not necessary condition for AMHG, leaving open the question of whether or not AMHG is a mathematical construct or geomorphic phenomenon. Finally, Gleason and Wang proposed p_{int} as a topological index indicating the percentage of rating curves intersecting within the range of observed hydraulic quantities. They argued that this can indicate the strength of AMHG *a priori*, but this hypothesis has never been tested on a wide range of rivers.

AMHG's most prominent application to date has been in remote sensing of river discharge (Bonnema et al., 2016; Durand et al., 2016; Gleason and Smith, 2014; Gleason et al., 2014). Despite a lack of understanding regarding the physical meaning of AMHG, and in particular the meaning of the congruent discharge, AMHG's importance has been demonstrated through its influence on reliable discharge estimation. It has been shown that the success of the AMHG discharge estimation algorithm strongly relates to whether the congruent discharge is within the range of a river's observed discharge (Gleason and Wang, 2015), though the likelihood of this occurrence is unknown.

These latest developments push AMHG toward maturity, but large-scale testing of the ideas discussed above has not been performed. Although hydraulic congruence has been shown to exist in theory and from limited datasets, the physical hydraulic quantities represented by congruent hydraulics are not currently understood, and little research has focused on uncovering their meaning. Furthermore, to date, the full width, depth, and velocity AMHG has been robustly examined on only six rivers (Gleason and Smith, 2014). AMHG has also been demonstrated on 57 other rivers by Shen et al. (2016) using USGS gauge data, though these data were employed without a rigorous verification of the phenomenon. Additionally, fluvial geomorphic or climatic factors that influence the formation of AMHG remain poorly understood, and possible spatial patterns in AMHG have not been identified.

To these ends, we here compile the largest dataset ever used to test the full suite of width, depth, and velocity AMHG in order to address unanswered questions regarding the driving factors of AMHG. We first calculate AMHG for 191 rivers and then analyze spatial patterns and geomorphic/climatic controls in these data. Our work thus parallels that of earlier hydraulic geometry researchers seeking to understand AHG across spatial and climactic scales (e.g. Park, 1977; Richards, 1973; Rhodes, 1977, 1978, 1987). We then assess previous assumptions/conclusions about AMHG, including the relation of congruent hydraulics to observed hydraulics, and, critically, whether or not the full suite of AMHG is observed beyond the six rivers for which it has been previously demonstrated. The paper concludes with discussion of the implications of our findings on the application of AMHG to a wide range of fluvial problems.

2. Methods

We relied on field-measured quantities of width, velocity, and discharge provided by the USGS to assess AMHG. The USGS maintains numerous gauging stations across the US, and each one of these requires periodic measurements to assess shifts in the rating curve due to sediment scour/deposition or other changes to channel geometry. These data are provided free to the public by the USGS, and two forms of data are available: the stage recorded by the gauge and its corresponding computed discharge, and the raw measurements of width, depth, velocity, and discharge used to determine the rating curve. We rely on these latter data in this paper, as gauge reported discharges are typically less accurate than in-situ measured discharge and as velocity and width are typically not reported along with the stage.

The USGS provides a Data Retrieval package written for the open-source R software program for ease of interface with its databases of measured hydraulic quantities (https://owi.usgs.gov/R/). Information regarding station number, name, latitude, longitude, hydrologic unit code, and available measured quantities were retrieved from the USGS archives (https://waterservices.usgs.gov/) using a modified version of the R package. This allowed us access to measurements made at all USGS stations across the US. The next step was to determine which rivers had enough stations from which to derive an AMHG: since AMHG is a spatial phenomenon, multiple stations on the same river are needed, and we determined that a minimum of six stations were required to assess AMHG. This falls short of a traditional statistically significant number of stations on which to test AMHG but greatly increased the amount of available data. Furthermore, only stations that had at least 20 measurements of river width, depth, velocity, and discharge within a 20 year period were used in order to reduce the impacts of the natural changes to channel geometry over time and ensure that each AHG curve was built from a reasonable amount of data (Gleason and Smith, 2014; Phillips, 1990; Ran et al., 2012).

Text files containing measured depth, width, velocity, and discharge data were downloaded from the USGS website for all rivers that met these specified criteria using the R script. We then manually filtered these data to ensure station data duplicates had not been retrieved for rivers containing parts of the same name (e.g. Root River and Bitterroot River). The hydrologic unit code of each station was used in order to distinguish between different rivers with the same name, as e.g. dozens of streams named 'Mill Creek' exist across the US.

After the above search and filtering were performed, a total of 191 rivers were found to have at least 6 gauging stations with a minimum of 20 measurements of river width, depth, velocity, and discharge over a 20 year period. Analysis of this data allowed us to increase the number of rivers for which AMHG has been robustly tested by a factor of more than thirty (from 6 to 191). First, we calculated the AHG coefficients and exponents for each station using the width, velocity, and discharge data downloaded from the USGS website. We used hydraulic mean depth (hereafter 'depth', discharge divided by the product of width and velocity) rather than measured depth for this process. To derive AHG, we fit a linear least-squares regression between discharge and each of width, depth, and velocity in log-log space. Stations that had a b exponent value less than 0.02 were then removed as these "low-b" rivers have widths that vary little with discharge and have been found to exhibit weak AMHG (Gleason et al., 2014). AMHG was determined following Gleason and Smith (2014) by fitting a linear regression through a semi-log plot of AHG coefficients and exponents for each station along each river. The strength of the width, depth, and velocity AMHG was then determined as the goodness of fit (r^2) of this regression through each of the log-linear plots $\log a$-b, $\log c$-f, and $\log k$-m.

Rivers showing a strong AMHG presence were selected for further analysis. We extracted the congruent width w_c, depth d_c, and velocity v_c values from the least-squares regression of AMHG:

$$\log w_c = \frac{\text{width AMHG intercept}}{\text{width AMHG slope}} \tag{4}$$

The congruent depth and velocity were determined in a parallel manner using their respective intercepts and slopes. The congruent discharge Q_c was calculated as the inverse of the depth AMHG slope, following Gleason and Wang (2015):

$$\log Q_c = -\frac{1}{\text{depth AMHG slope}} \tag{5}$$

These mathematical definitions of congruent hydraulic quantities prevent a more mechanistic understanding of their origin. However, these definitions are convenient for the present analysis and allow us to make the tests herein without relying on unavailable data for slope and flow resistance that would be needed for deeper understanding (per Dingman, 2007). Further analysis revealed that the width and velocity AMHG, while often as strong as depth AMHG, often yield congruent hydraulic parameters (e.g., w_c and v_c) well outside the range of observed values (see Sections 3 and 4). Thus, we use the depth Q_c as representative of the congruent discharge for all three variants.

Finally, spatial analyses were conducted using ArcGIS v10.5. An Anselin Local Moran's I cluster and outlier analysis was performed for all 191 rivers in order to determine areas in the US which had statistically significant (p value <.05) clusters of rivers with high and low AMHG values. Based on the cluster analysis results, regions in the US were identified as having overall strong or weak AMHG presence, and possible explanations for the observed results are considered below. Additionally, the implications of geologic and climatic settings for AMHG are discussed. To further the understanding of the congruent parameters (discharge, width, depth, and velocity), spatial patterns for these variables were examined for rivers that exhibited strong AMHG following the above procedure. Using the Global Moran's I test, we tested for spatial autocorrelation of the congruent parameters and AMHG strength for all 191 rivers.

3. Results

The 191 rivers in this study encompass a range of geologic and climatic settings in the US, allowing for widespread analysis of AMHG and its associated congruent parameters. The average reported discharge of all rivers in this study was 415 m^3/s and the average peak discharge was 2060 m^3/s, covering a wide range of rivers in the US (Fig. 1). The mean depth, width, and velocity

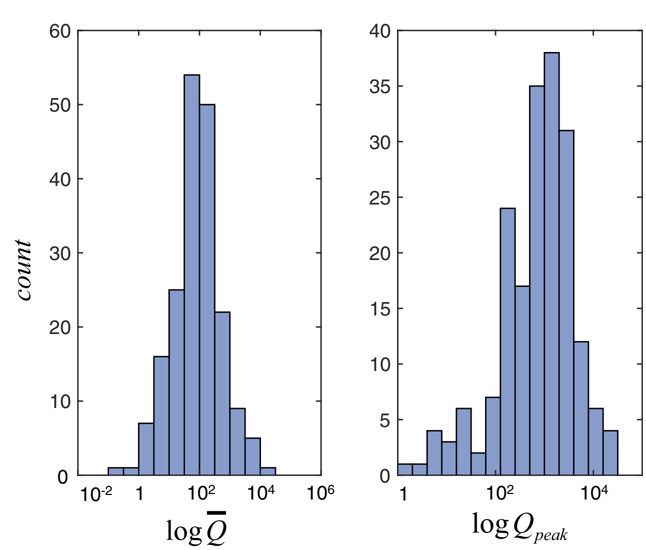

Fig. 1. Average and peak discharge are characterized for all 191 rivers in this study in the histograms above. The average discharge is 415 m^3/s and the average peak discharge is 2060 m^3/s. Rivers show a strong central tendency, but do encompass a wide range of flows, suggesting that our conclusions reflect a broad array of fluvial conditions.

of the rivers in this dataset are 2.9 m, 112 m, and 0.62 m/s respectively. AMHG was calculated for all 191 rivers and those with a depth AMHG r^2 greater than 0.6 were considered to exhibit a strong AMHG presence. Depth AMHG was chosen as representative of AMHG for several reasons: depth is an important fluvial quantity, and the depth-velocity relationship in particular was shown by Dingman (2007) to give rise to AHG when imposed on channel geometry. In addition, the depth AMHG has less extreme values across our dataset (Fig. 2), and we thus assume that these extreme values of AMHG are the result of the sensitive log-log fits of AMHG and outliers within the data (as discussed later). Finally, Shen et al. (2016) showed in their analysis of over 50 rivers that the depth AMHG is almost always the strongest, even when width and velocity AMHG are poor. Of the 191 rivers for which AMHG was calculated, 118 were found to have a depth AMHG r^2 greater than 0.6, providing evidence for AMHG as a widespread fluvial phenomenon. Additionally, these 118 rivers are distributed throughout the US, confirming its existence on rivers in various geologic and climatic environments (Fig. 3). This study represents the first time AMHG has been calculated and analyzed for such a large range of rivers, and can therefore provide new insights into the drivers of AMHG and factors that affect its strength.

Gleason and Wang (2015) showed that a congruent discharge exists for each of the width, depth, and velocity AMHG. These three congruent discharge values, they argued, correspond to congruent width, depth, and velocity values and together each pair (i.e. congruent discharge and congruent width) represents the point of intersection of rating curves that defines AMHG. However, given the effects of conservation of mass on the unity of AHG coefficients and exponents, it may be expected that these three separate congruent discharges in fact represent a single, driving Q_c. It is difficult to assess whether these three separate congruent discharges exist, or whether there is a single congruent discharge for all quantities. Fig. 2 shows that the depth AMHG is quite stable and predictable, producing congruent depths and discharges within the range of observed values that correlate with their hydraulic counterparts. However, the width and velocity AMHG produce results with many more outliers, and so assessing Q_c is difficult, as it is problematic to determine which outliers are true artifacts of the data and which faithfully represent the congruent hydraulic quantities. If we limit the comparison of Q_c to only those realistic values (assuming Q_c corresponds to a physical flow) falling between 1 m^3/s and 10,000 m^3/s, we find that there is a 41.3% difference (relative RMSE) between the velocity and depth Q_c values, and a 139.7% difference between the depth and width Q_c values. Reasons for these discrepancies are discussed below, and we have used the depth Q_c for the remaining analysis.

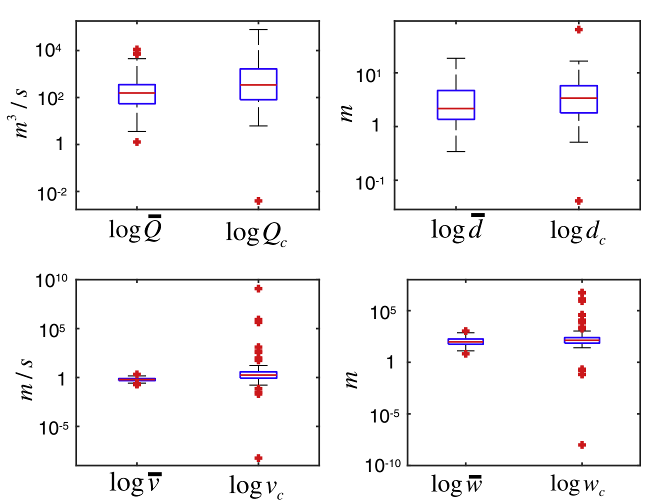

Fig. 2. The congruent hydraulics (Q_c, d_c, v_c, w_c) for all rivers with strong AMHG are shown here compared to their average observed hydraulic counterparts ($\bar{Q}, \bar{d}, \bar{v}, \bar{w}$). Q_c and d_c have only a few outliers and are therefore much more similar to their observed values than v_c and w_c which each have many outliers. This finding provides some explanation for the relationships observed between $\bar{Q}$ vs Q_c, and similarly $\bar{d}$ vs d_c, but the lack of such relationships between $\bar{d}$ vs v_c and $\bar{w}$ vs w_c (Fig. 4). Q_c is thus defined as that derived from depth for this reason.

The congruent hydraulic parameters were also analyzed for rivers with strong AMHG to determine their relationship to their observed hydraulic counterparts. It was found that the median of each of the congruent parameters (Q_c = 343 m^3/s, d_c = 3.38 m, v_c = 1.78 m/s, w_c = 135 m) was higher than the median of the observed parameters (Q = 156 m^3/s, $\bar{d}$ = 2.16 m, $\bar{v}$ = 0.62 m/s, $\bar{w}$ = 93 m), as seen in Fig. 2. Overall, v_c and w_c have many more outliers than Q_c and d_c, which results in dissimilar average values to the observed hydraulic counterparts. This also highlights the sensitivity to the semi-log fits of AMHG: small changes in a given intercept can manifest as extreme values of congruent hydraulics that would be rare or not possible for a given river.

We sought correlations between common fluvial parameters and indices to investigate potential driving factors of AMHG for rivers with a strong AMHG presence, using all 191 rivers to make this test. No correlation was found between AMHG strength and Q_c, and similarly no correlations were found to exist between the strength of AMHG and the other congruent hydraulic parameters (width, depth, velocity) or with the average values of discharge, width, depth, and velocity. We also found no correlation between the number of stations used to calculate AMHG and AMHG strength. Taken together, these results suggest that AMHG is not restricted to rivers of a particular scale or to rivers with a certain amount of available data.

The physical meaning of Q_c and the other congruent hydraulic parameters remains poorly understood. Gleason and Wang (2015) proposed that the spatial modes of time mean quantities serves as the congruent hydraulic variable in their argument that AMHG was a mathematical construct. They concluded that the congruent values *must* be the spatial mode of time mean quantities if the average discharge and average width are similar along the stations of a river. To test this hypothesis, we sought correlations between the calculated congruent hydraulics and the spatial modes of time mean quantities. The spatial modes of time mean quantities were found by first taking the average of the logged measurements for each station. For example, if a river had eight stations, then eight separate averages would be found for each *Q, w, d, v*. The mean values for each hydraulic quantity were then rounded to the nearest tenth of a unit in log space (m, m/s, or m^3/s). The mode of these rounded averages was the final value used to represent the spatial mode of time mean quantities. Our results found no correlation between the congruent values and their corresponding spatial mode of time mean quantities. In this, our results agree with Shen et al. (2016), who suggested that the similar-time-mean condition for AMHG was sufficient but not necessary. In addition, we investigated the relationships between congruent and observed hydraulics to begin to uncover more about the physical meaning of these congruent parameters. A slight positive trend (r = 0.51) is observed between the congruent discharge and the average discharge, as well as between the congruent depth and the average depth (r = 0.54). After removal of two extreme congruent width outliers, a weak positive trend (r = 0.35) is observed between the congruent width and the average width. Two extreme congruent velocity outliers were also removed in order to determine if a relationship exists between the congruent velocity and average velocity, however no trend is observed (Fig. 4). Based on the comparison between the congruent parameters and the observed parameters, the results suggest that the con-

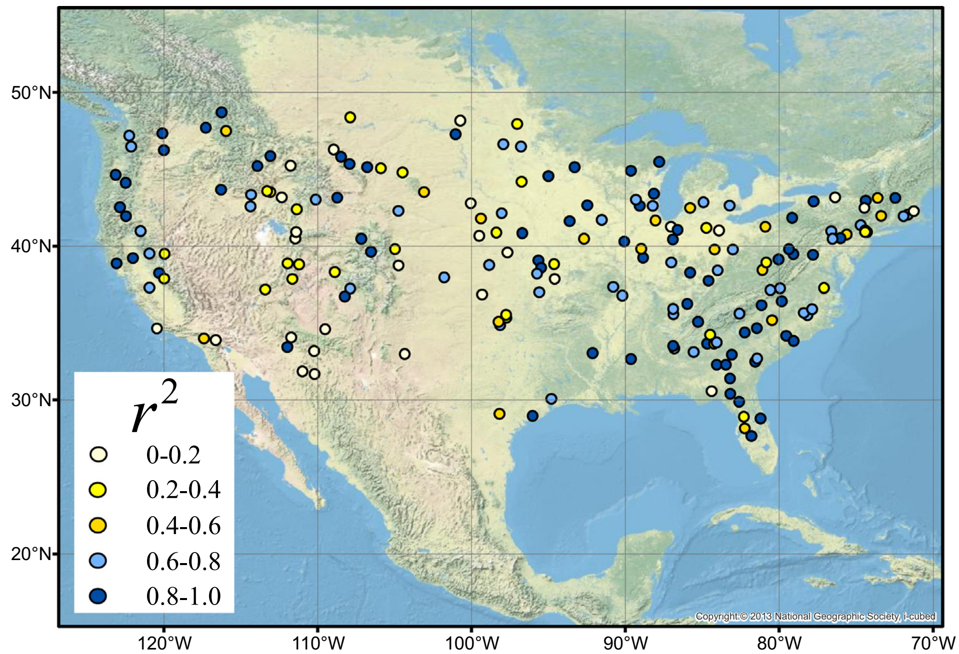

Fig. 3. This figure plots the strength of AMHG for all 191 rivers in this study. We define a strong AMHG if its r^2 is greater than 0.6, which suggests that AMHG is able to predict AHG coefficients and exponents with reasonable accuracy. Given this threshold, rivers with weak AMHG are shown in hues of yellow and strong AMHG are shown in hues of blue. Each river is represented by its mid-river station, and the number of stations used to calculate AMHG per river ranged from 6 to 43. The National Geographic Society USA Topo Map is used as a basemap in this figure and was accessed through ArcGIS. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

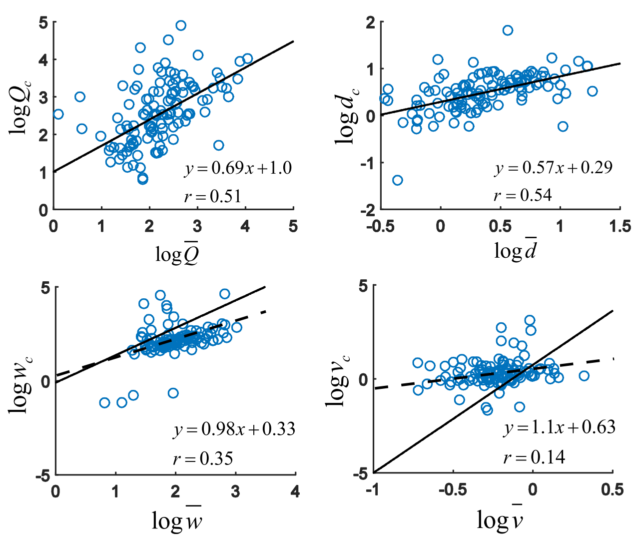

Fig. 4. A weak positive trend (r = 0.51) exists between Q_c and $\bar{Q}$. The physical meaning of Q_c is currently not well understood, however this correlation suggests that Q_c is partially driven by actual channel discharges. Similarly, a weak positive trend (r = 0.54) exists between d_c and $\bar{d}$. The width and velocity panels show two trendlines each. The solid trendlines are calculated from the entire strong AMHG dataset (118 rivers). The dashed lines are calculated after removal of two extreme y-axis outliers in each panel (not shown in the figure), and the equations and r values shown on the figure correspond to the dashed trendlines. After removal of the extreme outliers, a weak positive trend (r = 0.35) is observed between w_c and $\bar{w}$. No trend, however, was found to exist between the velocity parameters (r = 0.14). The weak positive trends identified provide evidence to support the hypothesis that the congruent parameters are related to their physical counterparts, however more research is needed for verification.

gruent discharge, depth, and width values are much more closely related to their observed counterparts than the velocity parameters.

To determine if any spatial patterns exist for the 191 rivers, a cluster analysis was performed to identify statistically significant spatial clusters of high and low AMHG r^2 values (corresponding to strong and weak AMHG respectively). The western edge of the Rocky Mountains was determined to have a statistically significant (p values <.05) cluster of fourteen rivers with weak AMHG. A total of five and eleven rivers along the Northwest and Southeast coasts, respectively, were identified as having statistically significant (p values <.05) clusters of strong AMHG (Fig. 5). This analysis suggests that climatic factors may indeed affect AMHG strength. A cluster analysis was also performed to determine if the congruent hydraulic parameters are influenced by physiographic or climatic factors. This analysis was restricted to rivers with strong AMHG and revealed no spatial patterns in w_c, d_c, v_c, or Q_c. Additionally, we tested for spatial autocorrelation of AMHG strength and the congruent parameters (w_c, d_c, v_c, Q_c) and found all to be randomly dispersed. We then normalized Q_c and d_c by $\bar{Q}$ and also found these to be randomly dispersed.

Finally, an important application of AMHG is its use to estimate river discharge from remotely sensed imagery (e.g. Durand et al., 2016). As discussed in Gleason and Wang (2015), AMHG leads to successful discharge estimation if Q_c is within the range of observed discharge. It is therefore important to understand if there are rivers with strong AMHG but with a Q_c outside the range of observed discharge and determine the frequency of this occurrence to better understand expected discharge accuracy *a priori*. Rivers with strong AMHG presence ($r^2 > 0.6$, 118/191 rivers) and weak AMHG presence ($r^2 < 0.6$, 73/191 rivers) were investigated separately. Over 77% (91/118) of rivers with strong AMHG had a value of Q_c that fell within the range of observed discharges. Conversely, only 30% (22/73 rivers) of rivers with weak AMHG had Q_c values within the range of observed discharges (Fig. 6). Gleason and Wang (2015) hypothesized that p_{int}, the percentage of rating curves intersecting within the observed discharge range, can predict the strength of AMHG, and our results thus confirm their findings.

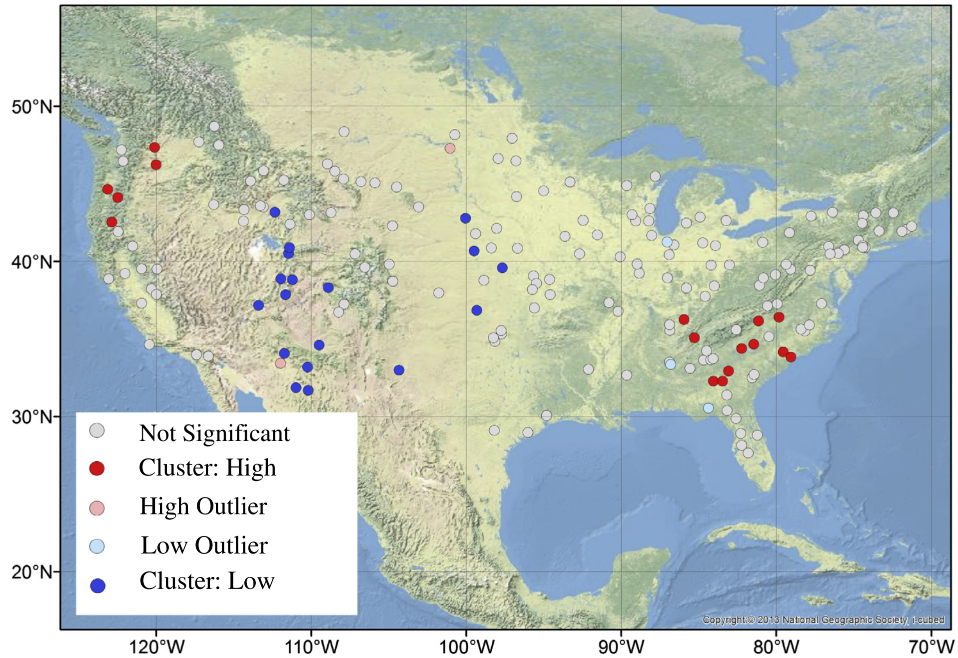

Fig. 5. Statistically significant (p < .05; Anselin Local Morans I) clusters of weak AMHG (shown in blue) are observed just west of the Rocky Mountains and in the Midwest. Clusters of strong AMHG (shown in red) are observed in the northwest and southeast coasts. Rivers represented by pink and light blue are outliers within the respective weak and strong AMHG clusters. Grey dots signify rivers that are not part of a cluster. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

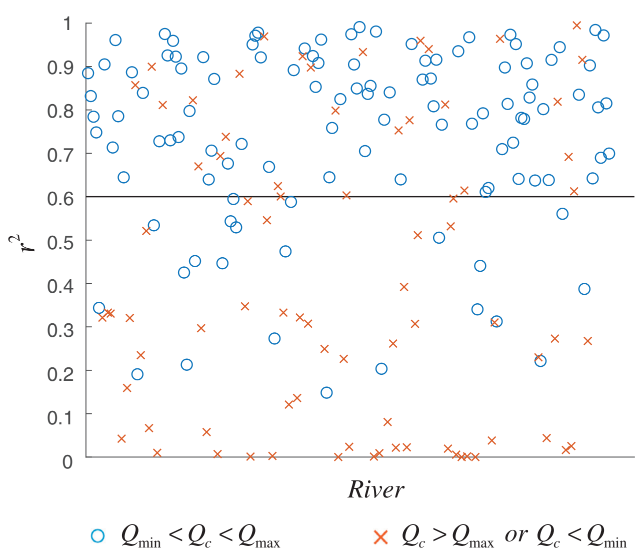

Fig. 6. This figure shows that rivers that had a congruent discharge Q_c within the range of observed discharge typically have a stronger AMHG than those that do not: 76% of rivers with strong AMHG ($r^2 > 0.6$) had a Q_c value within the range of observed discharge, while only 30% of rivers with weak AMHG ($r^2 < 0.6$) had a Q_c value within the range of observed discharge. Interestingly, the other 24% of rivers with strong AMHG but a Q_c external to observed values are all defined as 'low-*b*' with an average *b* exponent of just 0.05. Gleason and Wang (2015) demonstrated that AMHG-based discharge estimation is successful if Q_c is within the range of observed discharge, and these results augment this conclusion by demonstrating that strong AMHG is most often found on rivers for which Q_c is within the observed discharge range.

4. Discussion

AMHG is a recently discovered fluvial phenomenon that has been empirically observed but remains theoretically immature. Prior to this study, robust AMHG analysis on the full suite of hydraulic parameters had been performed on only six rivers globally (Gleason and Smith, 2014; Gleason and Wang, 2015). The newly curated dataset presented in this study has enabled the calculation of the full width, depth, and velocity AMHG for 191 rivers in the continental US, representing the first widespread, thorough verification of AMHG. Rivers in this study cover a variety of geologic and climatic regions as well as diverse hydraulic geometries and properties (discharge, width, depth, and velocity). The large geographic and hydraulic range allows us to verify the prevalence of AMHG and, for the first time, perform analysis into its driving factors. We can verify AMHG's existence as a widespread fluvial phenomenon, as over 60% (118/191) of all rivers in this study exhibit strong depth AMHG. The 118 rivers with strong AMHG span the entire US, providing evidence that AMHG is observed for a range of physiographic and climatic settings. Similar to previous work, we find the depth AMHG to be more stable than the width or velocity AMHG.

We also sought to further understand congruent hydraulics. The slight positive trend (r = 0.51) observed between $\bar{Q}$ and Q_c indicates that generally Q_c increases as $\bar{Q}$ increases, though $\bar{Q}$ cannot provide a reliable prediction of Q_c ($r^2 = 0.26$). In addition, Gleason and Wang (2015) suggested that Q_c must represent the mode of time mean discharge through space, which we do not support as necessary as no such trends were found to exist based on analysis of the 118 rivers with strong AMHG. These findings may be affected by the tendency of the USGS to take measurements during very high and very low flow events, though the extent to which this influences the results is untested. The correlation between $\bar{Q}$ and Q_c, in conjunction with the finding that the relation between Q_c and the observed discharge range greatly impacts the strength of AMHG, suggests that the physical hydraulic quantity which Q_c represents is related to the natural flow of the river. A positive trend is also observed between $\bar{d}$ and d_c (r = 0.54) as well as between $\bar{w}$ and w_c (r = 0.35), further suggesting that observed hydraulic quantities have some bearing on their congruent counterparts. Other unidentified factors, however, must also influence the interaction between these parameters. Our findings emphasize the need to uncover all factors influencing AMHG, including the important physical controls of slope and flow resistance, which were not available for this study. Thus, while congruent hydraulics are a necessary facet of AMHG, *why* rivers should exhibit these congruent hydraulics is a further open question.

We sought predictive variables and indices that can predict the strength of AMHG *a priori*, as knowing this strength prior to making measurements has a direct bearing on the application of AMHG. No direct correlation was found between Q_c and AMHG strength. This finding suggests that AMHG is not a phenomenon restricted to a particular scale of river. Similarly, no correlation was found between AMHG strength and the remaining hydraulic and congruent parameters ($\bar{w}, \bar{d}, \bar{v}$) and w_c, d_c, v_c). We can therefore conclude that AMHG is a phenomenon that has equal predictive power across fluvial scales.

To investigate the potential influence of non-hydraulic factors on AMHG, spatial analysis was performed to determine if physiographic or climatic factors affect its strength. Initial spatial analysis of the 191 rivers reveals that the western side of the Rocky Mountains and a few rivers in the Midwestern US have statistically significant spatial clusters of rivers with weak AMHG. In contrast, the Northwest and Southeast coasts have significant clusters of rivers with strong AMHG. To date, the potential correlation between AMHG strength and various climactic/hydrologic factors has not been explored, but the findings of our research indicate that these merit further investigation.

Our results do not suggest the existence of a single Q_c shared by all AMHG variants, however the sensitive logged fits of our data may have a significant effect on this conclusion. Thus, AMHG is frequently observed and is a robust phenomenon across scales, but the resultant congruent hydraulics are difficult to interpret. For instance, only the depth AMHG produces congruent hydraulics with obvious correlation to observed river flows, making it difficult to test hypotheses on the number of congruent discharges or characterize the congruent hydraulics beyond the simple summaries presented here. As an example, Q_c is calculated from the slope of the best-fit coefficient vs exponent relationship, and we can track the sensitivity of Q_c to changes in coefficients and exponents via Eq. (6) (note that in Eq. (6b) we assume that slope is negative per Gleason and Wang, 2015):

$$slope \stackrel{\text{def}}{=} r\frac{\sigma_{exp}}{\sigma_{coeff}} \tag{6a}$$

$$Q_c \stackrel{\text{def}}{=} 10^{-1}/slope \tag{6b}$$

$$Q_c = 10^{\frac{-1}{r\frac{\sigma_{exp}}{\sigma_{coeff}}}} \tag{6c}$$

Thus, a change in the slope of AMHG can lead to dramatic changes in Q_c per Eq. (6b): a slope of -0.3 produces a Qc of 2154 m^3/s while a slope of -0.2 produces a Q_c of 100,000 m^3/s. The likelihood of such a slope difference is difficult to predict given that it is a function of the ratio of the variance of coefficients and exponents and that each coefficient is geomorphologically a function of its corresponding exponent (Dingman, 2007), but it is evident that it is quite sensitive to variations in the data, particularly outliers. As can be seen from Eq. (4), the intercepts of the best-fit lines determine the congruent width, depth, and velocity. Eq. (4) also indicates that there is a direct linear relationship between the log of the congruent quantity and the intercept. The direct linear relationship in log space causes small changes in log space to result in exponential changes when brought out of log space. Thus, congruent widths, depths, and velocities are exponentially sensitive to changes in their respective AMHG intercepts. This agrees with the findings of Shen et al. (2016), who also found that the depth AMHG was more stable than the other forms of AMHG. The variations in stability among the width, depth, and velocity AMHG raises the question as to what 'strong AMHG' entails: we assume that a strong depth AMHG is sufficient for the label of 'strong AMHG,' which does not account for all three variants. When attempting to discover more about the fundamental nature of congruent hydraulics, future work must be mindful of the congruent hydraulics sensitivity to small changes in their intercepts. Going forward, the issue of sensitivity could be avoided by using a formulation for AMHG that does not first calculate slope and intercept but rather fits congruent hydraulics directly.

The physical hydraulic quantity to which Q_c corresponds has yet to be uncovered, but it is now clear that the relation between Q_c and observed discharge has a primary influence on the strength of AMHG. The results of this study have shown, through separate analysis of rivers with strong and weak AMHG, that AMHG strength strongly predicts whether or not Q_c is within a river's observed discharge range. This is as previously suspected: rivers with strong AMHG are much more likely to have a Q_c value within the range of observed discharges (e.g. these rivers have a high p_{int}). Since Q_c is defined as the discharge at which all rating curves intersect (Gleason and Wang, 2015), a Q_c outside of the observed discharge indicates that this intersection happens in a hydraulic parameter space (usually at a much higher discharge) that is never observed in the river. This marks an interesting case, and we have shown here that some (23%, 27/118) of the rivers in this study have a strong AMHG and such a Q_c, but these rivers are in the minority. This finding is important, as Gleason and Wang (2015) showed that applying AMHG to estimate discharge cannot be successful in this situation. Thus, we expect that about a quarter of US rivers with strong AMHG are unsuitable for discharge estimation.

However, upon further exploration of the significance of Q_c in relation to the range of observed discharge, we found that the 27 rivers which exhibit strong AMHG but have a Q_c value outside the range of observed discharge have an average b exponent value of 0.15 and a median b value of 0.14. These values are notably lower when compared to an average b value of 0.19 and a median b value of 0.18 for the 91 rivers with strong AMHG and Q_c within the observed discharge range. The distribution of b values can be seen in Fig. 7 for the four combinations of strong/weak AMHG and Q_c within/outside the observed Q range. Fig. 7C shows that for rivers with strong AMHG but a Q_c value outside the observed Q range, the majority of b values trend toward the low b range. Rivers in this category (strong AMHG and Q_c value outside the observed Q range) can therefore be considered 'low-b'. Eq. (6) shows that a Q_c of great magnitude is mathematically produced even when b values are highly variable but all low (, as is the case for the 27 rivers with strong AMHG but Q_c outside the observed Q range. The fact that a large Q_c results from 'low-b' rivers corroborates our observations that frequently $Q_c > Q_{max}$ when AMHG is strong but Q_c is not within observed Q limits. These results further bolster the finding that AMHG discharge estimation is not suitable for low-b rivers (Gleason et al., 2014), and also lead us to conclude that AMHG is unlikely to exist in artificially constrained rivers, as restricted width variability often indicates imposed control (but could also indicate a highly topographically-constrained morphology). Width variability with respect to discharge can be approximated *a priori*, as low-b stations will have a much lower coefficient of variation of time varying widths as compared to nearby stations (Durand et al., 2016). Therefore, the ability to estimate width variability allows for this exception to strong AMHG to be identified prior to attempted application of AMHG.

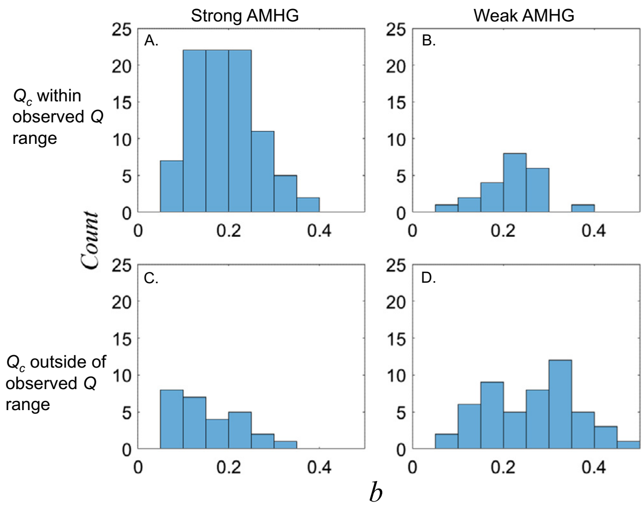

Fig. 7. The distribution of the b exponent value can be seen in histograms A-D. (A) The spread of b values for rivers with strong AMHG that have a Q_c within the observed Q range (91 rivers). (B) The spread of b values for rivers with weak AMHG that have a Q_c value within the observed Q range (22 rivers). (C) Rivers with strong AMHG and a Qc value outside the observed Q range (27 rivers). (D) Rivers with weak AMHG that have a Q_c value outside of the observed Q range (51 rivers). Notably, the average b value for (A) (rivers with strong AMHG and Q_c within the observed Q range) is 0.19, while the average b value for (C) (rivers with strong AMHG and Q_c outside the observed Q range) is 0.15.

The findings presented here are the result of data gathered solely from USGS gauging stations. This data source may introduce a bias in the data as these gauging stations are often placed at locations with large variability in depth (Rantz et al., 1982) and at stable, single-channel sites such as near permanent structures (e.g. bridges) which can affect the width of a river (Allen and Pavelsky, 2015), and our data show maximum b values less than 0.6 (Fig. 7). To address potential data bias, data sets that do not rely on gauging stations are required. Due to the large-scale nature of this study, the use of gauging stations was required to gain access to decades of data from thousands of stations in a variety of settings. Although this represents the first widespread study of AMHG, it is restricted to rivers in the continental US, and therefore a global assessment of AMHG is still needed to understand global trends and spatial factors. It has been shown that certain regions of the US generally exhibit weaker AMHG, and research in other areas of the world is needed for confirmation. Despite these limitations, AMHG has been proven to be a commonly occurring phenomenon, justifying future research into its driving factors and congruent parameters.

5. Conclusions

AMHG is a newly discovered fluvial phenomenon which remains poorly understood despite its implications on conventional AHG and its impact on discharge estimation in ungauged basins. Robust AMHG analysis has previously been limited to six rivers throughout the world, limiting the confidence in its widespread existence. This research verifies that AMHG is, in fact, a commonly occurring phenomenon and characterizes the congruent hydraulics unique to AMHG. We have also shown that AMHG is not solely defined as a mathematical construct as previously suggested, and confirmed earlier work that the depth AMHG is the most stable variant of AMHG. Future work should consider rivers outside the US in order to expand the data set for which AMHG is calculated and analyzed. As AMHG is investigated on rivers globally, potential physiographic and climatic influences on AMHG will continue to be revealed. Additionally, the importance of Q_c has been demonstrated, but much remains unknown about this congruent parameter and therefore further investigation is required to understand its influence on AMHG and its physical meaning. In addition to the finding of AMHG as a naturally occurring fluvial phenomenon and its implications on the understanding of hydraulic geometry, our results suggest that AMHG should be reliably observed and thus applied, opening up a range of potential applications for regions with ungauged rivers.

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