

*Water Resources Research*

Supporting Information for

**Constraining Remote River Discharge Estimation Using Reach-Scale Geomorphology**

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**Introduction**

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Text S1: Extracting river widths from Landsat imagery

Multi-temporal widths were extracted for every reach in the basin primarily following the process outlined in Feng et al. (2019). There were two main deviations in how this workflow was carried out. First, centerlines were defined differently, and second, intervals between cross-sections were not constant and instead were a function of the river’s mean observed width.

MERIT hydrography (Lin et al. 2019), vectorized from the MERIT Hydro digital elevation model (Yamazaki et al. 2019) was used to define river centerlines. We first generated cross sections at varying intervals along those centerlines (Table S1). Then, orthogonal lines were constructed for every cross-section. Orthogonal line lengths were defined as 2 \* mean river width for each cross section, where mean width was extracted from the MERIT Hydro (Yamazaki et al., 2019) and GRWL (Allen & Pavelsky 2018) datasets. Finally, these orthogonal lines were used as inputs to RivWidthCloud (Yang et al. 2019). RivWidthCloud classifies a cloud-free pixel as water using equation S1, where *Hw* is the half-length of each orthogonal, is the ratio of overlapped water mask and buffer area, and *W* is wetted width. Consult Yang, et al. (2019) and Feng, et al. (2019) for more information.

S1

RivWidthCloud extracts its water mask from the imagery using the following approach, with a reported accuracy of 97% (Zou et al. 2018): (mNDWI > EVI or mNDWI > NDVI) and (EVI < 0.1). These indexes are defined below in equations S2-S4, where bandgreen, bandred, bandblue, bandswir1, and bandnir are cell values for the green, red, blue, shortwave infrared, and near infrared bands for Landsat images, respectively. After filtering for clear-sky images (cloud clover < 25% as defined by Landsat surface reflectance tier 1), the classified water mask was intersected with the orthogonal lines to estimate wetted width at each cross-section.

S2

S3

S4

**References**

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Text S2: Choosing parameters for DBSCAN algorithm

While the DBSCAN unsupervised learning algorithm can determine its own number of clusters, the user must set a pre-assigned ‘maximum cluster radius’ and ‘minimum number of points’ for clusters. The results of the DBSCAN algorithm are necessarily sensitive to these parameters. The user’s needs and the unique specifications of the dataset being used generally require manual tweaking of parameters. However, for this study we wanted to test as unsupervised a workflow as possible. Thus, we implemented a heuristic method for finding an ‘optimal’ maximum cluster radius conceptually outlined by Ester et al. (1996) and implemented programmatically by Rahmah & Sukaesih (2016) to cluster peatland hotspots in Sumatra. This approach is similar to the aptly-named ‘elbow-based approach’ to determining the number of clusters for KMeans or agglomerative clustering.

We calculated the Euclidean distance to n nearest points in the feature space for every cross-section (n = 2). We then sorted these from smallest to largest and plotted the results as a line (Figure S2). The ‘optimal cluster radius’ is the point of maximum curvature, i.e. the distance where the change between points is most pronounced. We determined this optimal value to be approximately 0.5.

Unlike the maximum cluster radius, choosing a minimum number of points for a cluster is largely a subjective choice given one’s needs and the specifics of their dataset. We opted for a minimum of 5 cross-sections, which was deemed a fair compromise between within-cluster variation, number of resulting clusters, and number of ‘noisy’ cross-sections. Using both algorithm parameters, DBSCAN produced 7 clusters from 95% of the cross-sections, with the remaining 5% classified as ‘noise’.

Text S3: Updates to AMHG

Before introducing our new prior river knowledge into BAM, we updated the AMHG flow law within BAM to reflect recent work on AMHG theory (Brinkerhoff et al. 2019). This amounted to three interventions: a new flow law for AMHG, a physically-based ‘AMHG switch’, and a space-varying channel roughness prior.

Brinkerhoff et al. (2019) analyzed 155 rivers in the continental United States, finding that AMHG strength is a direct mathematical consequence of a river longitudinal profile’s strength of fit to any slope-roughness model. Further, they showed that empirically-derived *Wc* and *Qcw* coexist with Dingman (2007)’s hydraulic geometry model and thus are a valid hydraulic tuple for the given river cross-section. Consult that paper for a more thorough treatment of AMHG, but overall their analysis yielded a new AMHG expression (equation S5) defined by *p* (the generalized velocity-depth relation exponent), *K* (the generalized roughness coefficient), *Wb* (bankfull width), *Db* (bankfull depth), and *r* (a channel shape term). We take this and derive a novel flow law (equation S7) by first substituting equation S5 into the original AMHG flow law (equation S6- Gleason & Wang, 2015), as well as substituting Manning’s constants for the generalized terms. Note that *K* is equal to 1/*n* when adhering to Manning’s relation.

S5

S6

S7

Consult Dingman (2007) for an explanation of his generalizations of Chezy’s and Manning’s expressions for HG relations. It must be stressed that values for *p* and *K* have been shown to vary widely and not adhere to Manning’s (or Chezy’s) constants due to different river morphologies (e.g. Bjerklie et al. 2005; Dingman & Afshari, 2018; Dingman, 2007; Dingman & Sharma, 1997; Ferguson, 2010; Knighton, 1975). With that said, this new formulation of the AMHG flow law is flexible and in theory allows for future generalized implementations.

Finally, we derived a Bayesian likelihood function to be introduced to BAM. This was done by log-transforming Equation S7 and solving for observed width, resulting in Equation S8 (where *i* denotes space-varying quantities and *t* denotes time-varying ones).

S8

Note that observed slope is still on the right-hand side of the equation out of algebraic necessity (and that AMHG is now defined by both observed widths and slopes). Also note that through our redefinition of AMHG we have removed the *Qc* prior while maintaining the *Wc* prior, even though algebraically simplifying equation S7 to its fullest extent would remove *Wc* from the expression. This was done deliberately to preserve the AMHG hydraulic tuple (*Wc*, *Qc*)in the physical model. The mathematical definition for *Wc* is robust (Gleason & Wang 2015) and Brinkerhoff et al. (2019) showed that if AMHG is strong in a river, this tuple is but one of many valid hydraulic conditions. Thus, it was kept here to force the physical model to reflect *Qc* and no other dischargepossibly experienced in the river. Equation S8 is thus substituted into BAM in place of the AMHG likelihood function developed by Hagemann et al. (2017).

Next, we incorporated a physically-based AMHG switch into BAM. As previously stated, BAM allows for a switch to be defined, ‘turning on’ AMHG when it is suitably strong. The definition of ‘suitably strong’ used in Hagemann et al. (2017) was a standard deviation of log-transformed river widths > 0.1 and a concurrent ‘percentage of rating curve intersections’ *pint* > 0.15. We redefined ‘suitably strong’ AMHG following Brinkerhoff et al. (2019)’s finding that when observed slopes strongly fit a river-wide slope model, AMHG is strong. Here, we use the regime theory model defined by Henderson (1966) and reprinted as Equation S9. Degree-of-fit is defined by the coefficient of determination (r2), and strong fit was defined as an r2 > 0.90.

S9

*De* is the minimum grain size for entrainment, calculated here as 11­*DbS* (Henderson, 1966), and *Qb* is bankfull discharge. In order to keep this methodology viable using just remote sensing observations for running BAM, *Qb* and *Db* were predicted using regression models with river width as the sole predictor (following the method used by Hagemann et al. 2017- see Figure S6 for similar models).

Finally, BAM uses a global-scope channel roughness prior (*n*), the use of which is known to be both physically inaccurate in many scenarios and poorly reflective of the variation in roughness experienced in space and time (Tuozollo et al. 2019). In order to quantify improvements in RSQ prediction with a more physically-realistic *n* prior, we tested both space-varying and space-and-time-varying *n* variants of Equation S8 within BAM on the SWOT-simulated rivers- consult Figure S5 for the results of this test. While sensitive to the inclusion of a time-varying *n* prior, there was no discernable pattern for poorly performing rivers (NSE < 0), with some getting better and some worse. Further, there was no meaningful difference between NSE scores for rivers with NSE > 0 (Figure S5). This intervention also significantly slowed BAM, and thus we deemed the little change in performance not worth the significant computational burden it imposed on BAM. Thus, we implemented only space-varying *n* prior into the version of BAM used in this study.

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Text S4: Calculating hydraulic variables using training data

BAM requires priors on six hydraulic terms, as well as discharge and error terms. These are bankfull width *Wb*, bankfull depth *Db*, median cross-sectional area *A0*, the scaling exponent in the width AHG expression *b*, channel shape parameter *r*, and Manning’s roughness term *n*. *Wb* and *Db* were calculated at each cross-section using a return period of 2 years, acknowledging that *Qb* has been associated with a range of return periods given local geomorphology (Petit & Pauquet, 1997; Williams, 1978). However, a 1.5-2-year return period is a standard statistical definition for bankfull flow and was used here to align with existing literature. *A0* is simply the median observed cross-sectional channel area. *b* was defined empirically by fitting, at each cross-section, the equation . *r* was defined empirically following Dingman (2007) as , where *f* is the exponent calculated when fitting, at each cross-section, . Manning’s *n* was calculated using observed flow velocities and depths: , where *d* is mean channel depth, *v* is flow velocity, and *S* is bed slope.

We also calculated a suite of hydraulics variables for our geomorphic classification (Table 1). Froude number was calculated as . Shear stress was calculated as . Unit power, which is stream power normalized by channel width, was calculated as: . We opted to normalize stream power to remove the effect of river size from the variable. Finally, ‘minimum grain size entrained’ *De* was calculated using an equation from Henderson (1966) which assumes a parabolic channel shape: .

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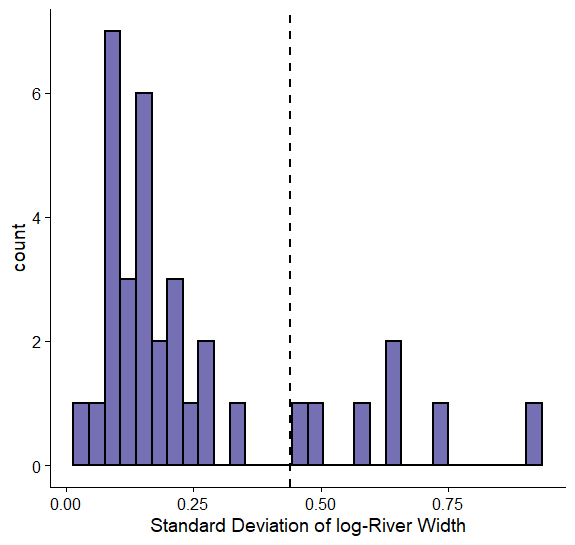
Figure S1 Map of locations of hydraulic measurements in training dataset



Figure S2 Elbow plot for DBSCAN parameter choice, where ‘index’ refers to the sorted river cross-sections. A value of 0.50 was chosen for this study (Text S2).



Figure S3 Boxplots of training data river width, segregated by river type (x-axis) using the expert classification framework.



**Figure S4** Histogram of SWOT-rivers’ standard deviation of log-transformed river width. Dashed line is the geoBAM-Expert threshold for ‘highly width variable’ rivers. There is a clear normal distribution of standard deviations across rivers, where a threshold of 1.57 meters was set for rivers deviating from this normal distribution.

A screenshot of a cell phone

Description automatically generated

Figure S5 Empirical cumulative density function of BAM NSE results on the 19 SWOT-simulated rivers, using a space-varying Manning’s *n* prior and using a space-and-time varying n prior. NSE of 0 is marked on plot by dashed line. The space-and-time-varying implementation improves NSE in some poorly performing rivers and not in others, but performance differences are negligible in rivers with an NSE approximately > 0.

A close up of a map

Description automatically generated

Figure S6 Training of global prior estimation models using just remotely-sensible width or slope from our training data. This approach is identical to that used to estimate priors in Hagemann et al. (2017).

A close up of a map

Description automatically generated

**Figure S7** Stream order vs. NSE improvement from control BAM to geoBAM-Expert in the Mackenzie River basin. No clear patterns emerge, though the variability in improvement does decrease slightly with increasing order.

Table S1 Varying interval distances between the cross-sections that we extracted multi-temporal river widths at. Interval lengths are a function of mean observed river width, retrieved from the MERIT Hydro digital elevation model (Yamazaki et al. 2019).

|  |  |
| --- | --- |
| **Mean River Width [m]** | **Cross-section Interval Length [m]** |
| 120-1000 | 500 |
| 1000-2000 | 1000 |
| 2000-3000 | 2000 |
| 3000-4000 | 3000 |
| 4000-5000 | 4000 |
| 5000-10000 | 5000 |

Table S2 Descriptions of 19 simulated rivers used to test our interventions on SWOT-like data. Models and references are reprinted from Durand et al. (2016).

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **River** | **Median Q [m3/s]** | **# Days Simulated** | **# Reaches Simulated** | **Location** | **Model** | **Reference** |
| Connecticut | 542 | 209 | 3 | USA | HEC-RAS |  |
| Cumberland | 932 | 162 | 4 | USA | HEC-RAS | Adams et al. (2010) |
| Ganges | 4,601 | 365 | 6 | Bangladesh | HEC-RAS | Siddique-E-Akbor et al. (2011); Maswood & Hossain (2016) |
| Garonne Upstream | 480 | 365 | 8 | France | HEC-RAS | Besnard and Goutal (2008); Larnier (2010) |
| Garonne Downstream | 128 | 365 | 16 | France | MASCARET |  |
| Kanawha | 510 | 162 | 4 | USA | HEC-RAS | Adams et al. (2010) |
| Mississippi Downstream | 14,199 | 162 | 6 | USA | HEC-RAS | Adams et al. (2010) |
| Mississippi Upstream | 4,869 | 162 | 3 | USA | HEC-RAS | Adams et al. (2010) |
| Ohio | 3,444 | 220 | 5 | USA | HEC-RAS | Adams et al. (2010) |
| Platte | 114 | 22 | 14 | USA | BreZo (2D) | Schubert et al. (2015) |
| Po | 1,009 | 367 | 16 | Italy | HEC-RAS | Di Baldassarre et al. (2009) |
| Sacramento Downstream | 213 | 154 | 9 | USA | HEC-RAS | Rogers (2014) |
| Sacramento Upstream | 181 | 305 | 7 | USA | HEC-RAS | Rogers (2014) |
| Seine | 200 | 365 | 4 | France | ProSe | Vilmin et al. (2015) |
| Severn | 62 | 88 | 4 | UK | LISFLOOD-FP | Neal et al. (2015) |
| St. Lawrence Downstream | 9,037 | 139 | 4 | Canada | H2D2 (2D) | Heniche et al. (2000) |
| St. Lawrence Upstream | 9,037 | 139 | 4 | Canada | H2D2 (2D) | Heniche et al. (2000) |
| Tanana | 1,450 | 100 | 9 | USA | LISFLOOD-FP | Humphries et al. (2014) |
| Wabash | 842 | 162 | 4 | USA | HEC-RAS | Adams et al. (2010) |

Table S3 Results from PCA ran on training data: PC loading vectors for the PCs cumulatively responsible for 54% of variance in the dataset. Variable definitions are in Table 1.

|  |  |  |  |
| --- | --- | --- | --- |
| **Symbol** | **PC1** | **PC2** | **PC3** |
| **W** | 0.020612 | 0.449775 | -0.060491 |
| **V** | 0.008647 | 0.272547 | 0.266181 |
| **D** | 0.068257 | 0.423103 | -0.124790 |
| **Var(W)** | -0.000500 | 0.015312 | 0.011062 |
| **Var(V)** | -0.001584 | 0.004030 | 0.620828 |
| **Var(D)** | 0.004299 | 0.130895 | 0.030030 |
| **S** | 0.326711 | -0.050378 | 0.021752 |
| **SO** | 0.034240 | 0.406142 | 0.034349 |
| **Dd** | 0.046203 | 0.435189 | 0.035897 |
| **Sn** | -0.017695 | -0.124973 | -0.072751 |
| **HRT** | 0.000624 | -0.067599 | 0.104257 |
| **WB** | -0.073919 | 0.117427 | -0.054102 |
| **r** | -0.001385 | -0.008047 | 0.005691 |
| **DA** | 0.031245 | 0.342989 | 0.053671 |
| **Fr** | -0.024476 | -0.112151 | 0.329431 |
|  | 0.349559 | -0.030114 | 0.010459 |
|  | 0.347851 | -0.027668 | 0.014022 |
| **De** | 0.349559 | -0.030114 | 0.010459 |
| **n** | 0.283657 | -0.013854 | -0.057696 |
| **Var(Fr)** | -0.001660 | -0.004760 | 0.620033 |
| **Var()** | 0.348013 | -0.030471 | 0.013733 |
| **Var()** | 0.346018 | -0.034192 | 0.012901 |
| **Var(De)** | 0.348013 | -0.030471 | 0.013733 |
| **Var(*n*)** | 0.263850 | 0.007837 | -0.035598 |

Table S4 Prior upper and lower bounds for expert classification’s ‘big’ rivers. These were manually increased to account for the notable magnitude differences in river size. Lower bounds were set to the 25th percentile of all training data, while the upper bounds were set ad-hoc (in natural log space and converted here).

|  |  |
| --- | --- |
| **Parameter** | **Value [m or m2]** |
| A0 Lower | 0.720 |
| A0 Upper | 1,000,000 |
| Wb Lower | 22 |
| Wb Upper | 22,026 |
| Db Lower | 0.386 |
| Db Upper | 148 |

Table S5 P-values of two One-Way ANOVA tests ran on the medians of the distributions of six hydraulic parameters, grouped by river types.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | A0 | Wb | Db | r | b | n |
| Expert | 0.001 | 0.001 | 0.004 | 0.442 | 0.641 | 0.031 |
| Unsupervised | 0.207 | 0.184 | 0.222 | 0.070 | 0.090 | 0.744 |