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Hydraulic geometry of natural rivers: A review and future directions

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Abstract
The geomorphic relationships known as hydraulic geometry (HG) were first introduced by Leopold and Maddock in 1953, and their application remains critically important for assessing water resources the world over. The practical utility of HG for discharge monitoring, habitat studies, and understanding geomorphic change over time is unquestioned, but its elevation beyond empirically observed relationship to physical principle is not complete, despite universal acceptance of its existence. This review summarizes six decades of HG research while surveying rational, extremal, and empirical attempts to derive HG's underlying physical principles. In addition, so called non-Leopoldian forms of HG are discussed, expanding HG beyond its original construction to examine a range of research that invokes HG in novel ways. The recent discovery of at-many-stations hydraulic geometry (AMHG) is also discussed in the context of previous HG literature. Finally, some common themes linking disparate HG research communities are described in conjunction with suggestions for possible new directions for HG research in the future.

Keywords
hydraulic geometry, geomorphology, hydrology, fluvial geomorphology, at-many-stations hydraulic geometry, rivers, review

I Introduction

Rivers are an important component of the global hydrologic cycle that convey liquid water from the continents to the oceans, recharge groundwater storage, provide fresh water for human and agricultural consumption, and enable transportation throughout the globe. Each of these functions is critical to the water circulation of the planet and to human health and well-being. Beyond these global concerns, *how* exactly rivers respond to throughput of water is also of keen interest to fluvial geomorphologists. The field of geomorphology was founded to study landscape change through time, and the subfield of hydraulic geometry (HG) focuses specifically on the evolution of river form and how surrounding terrain and channel form influence this evolution.

When Luna Leopold and Thomas Maddock Jr published their findings on the relationships between river width, depth, velocity, and discharge (terming these relationships as HG) in 1953, they launched a field of study that is still active today. Their largely unprecedented ideas

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ran contrary to some prevailing theories of the time. Through repeated field measurements at numerous rivers in the western United States, Leopold and Maddock observed that strong log-log trends (straight lines in log-log space) emerged when time-varying measurements of instantaneous river flow width, mean depth, and mean velocity at a cross section were plotted against corresponding discharge through that cross section. They termed this finding "at-a-station hydraulic geometry," (AHG) and also coined the term "downstream hydraulic geometry" (DHG) to describe similar log-log trends between mean annual discharge and width, depth, and velocity for cross sections downstream along a river (Figure 1). Plotted in linear space, these AHG and DHG trends yield power laws, giving the now classic equations

$$w = aQ^b \tag{1}$$
$$d = cQ^f \tag{2}$$
$$v = kQ^m \tag{3}$$

Confusingly, Leopold and Maddock used the same variables and naming conventions for both AHG and DHG, and these naming conventions are still in use today. Leopold and Maddock were also quick to notice that these equations are unit sum constrained, that is to say that $b + f + m$ must equal 1, and a x c x k must also equal 1, as $Q=wdv$ by definition. As will be shown in this review, the choices that Leopold and Maddock made in presenting their data, especially their decisions to present mean values of AHG exponents across different rivers, to ignore discussion of DHG and AHG coefficients, and to use a power law to represent observed river behavior would leave lasting effects on the field that continues to the present day.

While nothing like AHG had been described before, DHG mirrored the earlier Regime Theory from engineering that also demonstrated a stable power law relationship between width and discharge along canals (Lacey, 1930). However, there are noted differences between the two ideas, and Leopold and Maddock themselves explicitly distanced HG from Regime Theory. A principal difference between these two geomorphic relationships is that the reference discharge in HG and Regime Theory are different: mean annual discharge for HG and a stable, regime discharge for Regime Theory. This distinction is more than semantically important, as mean annual discharge is a somewhat arbitrary flow that is distanced from the steady regime discharge in canals used in Regime Theory (Clifford, 2004). While HG would later be changed to include bankfull discharge as the dependent variable, its development from empirical observations of a convenient field parameter rather than a rationally conceived parameter was an important and noted deviation from the rational investigation of years prior.

This deviation made the geology community uneasy, an uneasiness best expressed by Mackin (1963) and illustrated by his concerns over the notion that mean velocity increases downstream (as with a positive exponent in DHG per equation 1). Visual observation of rivers and the known grain size/velocity relationship both indicate that velocity should in fact decrease downstream along a river (Mackin, 1963). As will be shown, DHG has been repeatedly verified since 1953, so Leopold and Maddock's finding remains secure: mean annual velocity does in fact tend to increase downstream. However, the larger point raised by Mackin is the utility and origin of mean annual discharge, a quantity with little geomorphic meaning that Leopold and Maddock calculated from discharge and area at-a-station. Mackin wondered that this variable should have ever appeared as a dependent variable and argued that "shotgun empiricism" grounded in engineering practice should not replace rational thought on landscape evolution then common in geology and geomorphology, yet he stopped short of an outright condemnation of Leopold and Maddock's ideas. Leopold responded to this criticism (Leopold and Langbein, 1963) by stating that his empirical and probabilistic approach may better represent

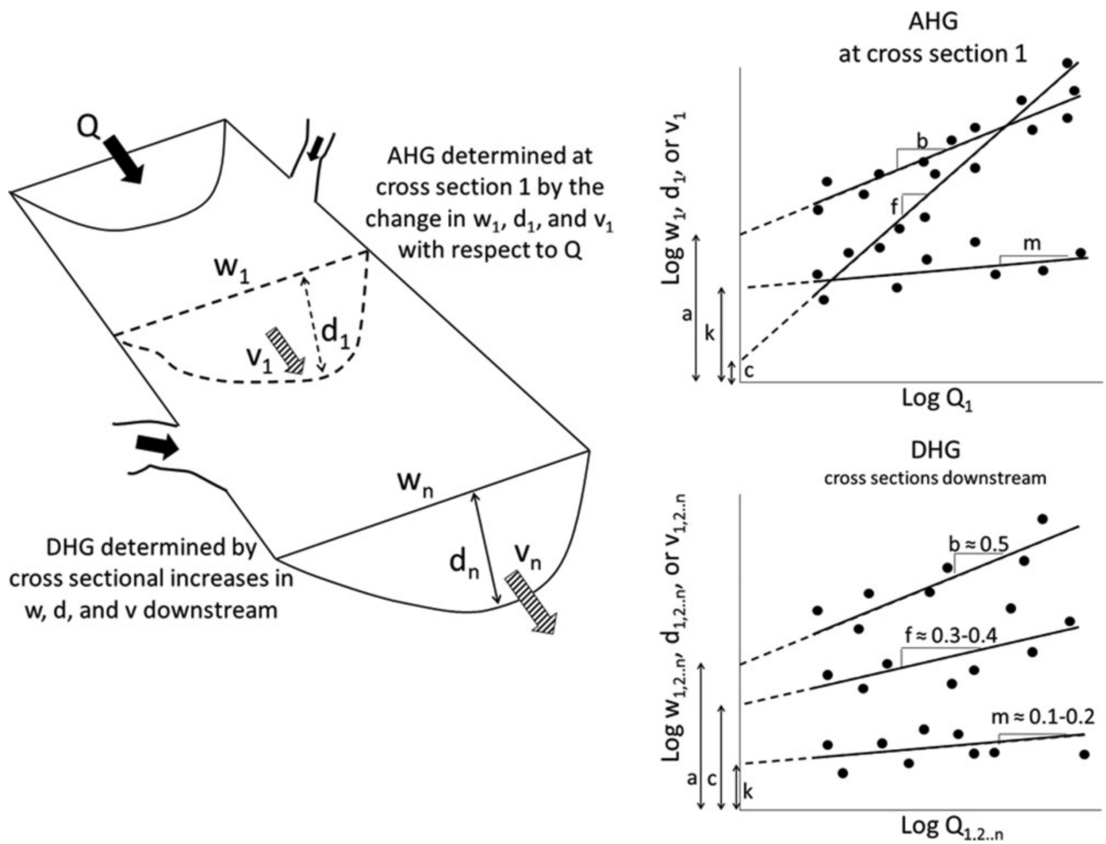

Figure 1. Hypothetical AHG and DHG relationships are given for this example channel with two tributaries. AHG is a function of cross sectional geometry (Ferguson, 1986), so the values for *b, f, and m* at cross section 1 are determined by the shape of the channel at that point. DHG is still not fully understood, but bed material, bank strength, and sediment transport (not pictured) are all considered important in determining DHG. The DHG relationships shown here give typical exponents that have been repeatedly observed over many empirical investigations. For both AHG and DHG, hypothetical datapoints representing *w, d, v, and Q* are given, and regressions of these data into power law HG are shown. The dotted lines indicate how the coefficients *a, c, and k* are calculated.

variation in natural systems, and this variation may become more important in landscape process investigation than exact physical laws. In addition, Leopold and Langein noted that empirical investigation of this sort can also generate hypothesis of landscape process never before possible from rational observation alone. However, this tension between rational and empirical paradigms for understanding DHG continues to the present day, as is noted in section "Towards further understanding of DHG" below.

HG remains largely unexplained. While Ferguson (1986) satisfactorily explained why AHG behavior is observed using rational deduction, he was unable to derive explicit formulations for the coefficients and exponents of the AHG formulae. Likewise, no study has yet proposed a universally accepted rational theory for why DHG occurs, nor defined explicit, universal formulations for its parameters. In the 60-plus years since Leopold and Maddock's original discovery of HG, scientists still do not agree on the fundamental principles that drive it. However, numerous researchers have tackled the problem from different vantage points, encapsulated in a series of reviews over the years (Clifford, 1996; Eaton, 2013; Ferguson, 1986; Huang and Nanson, 2000; Park, 1977; Rhodes, 1977; Thorne et al., 1998). This particular review spans publications from the past six decades, surveying empirical, rational, and extremal attempts to verify the existence of HG (i.e. to confirm that Leopold and Maddock's formulae reasonably match empirical data) and, later, to attempt to derive its underlying physical principles. In addition, several applications of HG are discussed, as are other forms of HG not proposed by Leopold and Maddock. Throughout this survey of HG, this paper highlights how HG has been expanded beyond its original construction and examines a range of research that invokes HG in novel ways.

II Towards further understanding of AHG

1 Early empirical investigation of AHG

Following Leopold and Maddock's seminal publication, most AHG research pursued empirical verification that these relationships indeed exhibited strong goodness of fit for different physiographic sites and settings (e.g. Ackers, 1964; Brush, 1961; Fahnestock, 1963; Leopold and Miller, 1956; Lewis, 1966; Miller, 1958; Stall and Fok, 1968; Wolman, 1955). The results of these investigations were compiled separately by both Park (1977) and Rhodes (1977, 1978, 1987), who independently introduced the ternary *b-f-m* diagram to show the scatter of AHG exponents derived from field data. By plotting *b*, *f*, and *m* exponents on a ternary diagram, cross sections may be placed in context with one another to understand how these exponents relate across different rivers in different settings (Figure 2). Both Park and Rhodes sought to find correlations between rivers based on their AHG exponents: Park segregated data based on natural breaks between the exponents themselves and Rhodes separated his diagram based on a river classification scheme. Despite these differences in grouping and classification, both authors concluded that AHG exponents show little similarity across rivers or physiographic settings, which, in the words of Park, "casts doubt on the use of mean values of samples of exponents to characterize the hydraulic geometry of particular areas." This was a major departure from AHG work at the time, as Leopold and Maddock (1953) had grouped their limited field data and found that streams of similar classifications bore similar AHG exponents, and had therefore logically concluded that a mean value for these exponents was suitable for describing a river

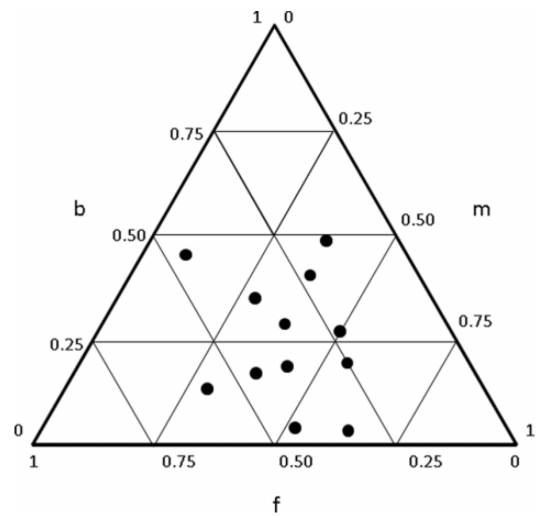

Figure 2. This figure shows a hypothetical ternary diagram as developed by both Park (1977) and Rhodes (1977). Each symbol within the diagram shows the *b, f, and m* values for a specific cross section for some hypothetical river. Both Park and Rhodes sought to compile previously collected AHG data within these ternary diagrams in a search for commonality across scales and physiographic settings, and both found none. Note that this diagram does not allow plotting of AHG coefficients—furthering the perception that only exponents are worthy of study.

system or region (see also Odemerho, 1984). The conclusions of Park and Rhodes were in keeping with a spirit of challenging Leopold and Maddock that began in the 1970s and differed from research of the previous decades: researchers began to look more critically at *why* AHG was occurring, rather than *if* it was occurring.

An early example of the critical nature of inquiry in the 1970s is exemplified by Richards' 1973 paper on HG and channel roughness. In discussing AHG and the empirical work of the decades prior, Richards notes the success of applying power laws as proposed by Leopold and Maddock, yet also notes that this practice lacked a solid theoretical justification beyond the repeated empirical finding that power laws effectively described discharge-geometry relationships for many rivers. His criticism is stated in his 1973 paper: "however, the point that appears to have been overlooked is that there is no a priori reason why power functions should necessarily represent the relationship between the dependent variables and the discharge." Richards goes on to show that polynomial functions describe HG better than linear functions (in log-log space), and notes especially the curvature of depth and velocity HG relationships for many rivers. This investigation was important in that it directly challenged one of the original tenets of AHG—i.e. that the power law is the optimal model for describing observed relationships between discharge and physical variables.

This line of inquiry was furthered by Knighton (1974, 1975), who noted that power law behavior is also observed between discharge and suspended sediment (Leopold and Maddock also made this connection), flow resistance, bed slope, Manning's n, and the Darcy-Weisbach f. In addition, Knighton (1975) found that AHG evolves over time by noting the change in slope of the AHG relationships within long-term datasets. While not a direct challenge to Leopold and Maddock, this work represents recognition that AHG as proposed by these original authors might not tell the entire story of channel geometry and flow hydraulics.

While Knighton challenged the idea that AHG was constant through time, Phillips and Harlin (1984) further lent credence to the conclusions of Park (1977) that AHG is also not constant through space. Phillips and Harlin studied a river that diverged into two stable channels before rejoining into a single channel within a small meadow. Despite identical geology and climatology between the two channels, they found wildly divergent AHG values, and concluded that AHG parameters neither tend towards stable parameter values nor are transferrable within the same physiography. These papers, along with similar investigations of DHG (covered in section "Towards further understanding of DHG"), showed that the scientific community, while generally accepting of Leopold and Maddock's ideas, were unsatisfied with the simple empirical formulation of AHG.

2 Hydraulics and geometry: AHG post Ferguson (1986)

This growing body of research that challenged the precepts of AHG, while at the same time accepting that it was indeed expressive of underlying physical principles, culminated in a key paper by Ferguson (1986). So authoritative were Ferguson's investigation and conclusions that, after the publication of this paper, research into the causes of AHG essentially ceased. Ferguson's great breakthrough was to reduce AHG to "hydraulics and geometry" through a rational approach; namely, the use of widely accepted flow resistance equations (i.e. Manning, Keulegan, and Darcy-Weisbach) to calculate width, depth, velocity, and discharge. By using these flow equations (which are themselves empirical), Ferguson was able to *derive* AHG for different channel geometries. Ferguson stated that if either Manning's n or the Darcy-Weisbach f (empirical constants describing river bed roughness) are constant or predictably variable at a cross section, then velocity may be written as a function of depth at that cross section. With this velocity-depth function in hand, Ferguson then sought to calculate velocity using fixed values for slope, bed grain size, and other parameters needed to employ a deep flow approximation of the Keulegan flow law. These velocity and velocity-depth functions were then imposed on different channel geometries, which yielded width as a function of depth. From these interrelationships, Ferguson was able to recreate the field data that would normally be collected to determine AHG: instantaneous width, depth, velocity, and discharge at-a-station (Figure 3). His rationally deduced conclusion following these data was that AHG exponents are a function of cross sectional channel shape. In addition, Ferguson concluded that the power law form of AHG is merely coincidental with the wealth of field data collected in the commonly found parabolic channel shape typical of non-meander bend river cross sections, and proved the misgivings of Richards (1973) that there is no reason why the variables in AHG should take the form of a power law. With this authoritative explanation of AHG, the volume of scientific research investigating the causes of AHG was significantly reduced thereafter.

Researchers nevertheless continued to broaden the knowledge base regarding the appearance of AHG in natural systems in the decades following Ferguson's influential paper. Bates (1990) asserted that a piecewise linear model (in log-log space) more accurately describes the AHG at cross sections that do not present power law behavior (i.e. when distinct sub-bankfull geometric sections are prevalent). Bates' finding was also anticipated by Richards (1973) in the latter author's assertion that natural systems often deviate from power law behavior while still exhibiting strong discharge—geometry relationships through time. Cao and Knight (1998) found that for canal design, particular geometries beget stable channels, agreeing with Ferguson's conclusions on the importance of cross sectional shape. Buhman et al. (2002) used a stochastic approach to study large spatial trends in AHG, while Turowski et al. (2008) concluded that AHG is particularly stable and well defined for bedrock channels. Despite Ferguson's reduction of AHG to hydraulics and geometry, researchers remain interested in verifying AHG's existence in new environments and its change over time (e.g. Baki et al., 2012; Ran et al., 2012).

Ferguson's conclusions were not universally accepted, however. Phillips (1990) challenged them on the basis that ignoring changes in morphology over time and feedbacks between AHG parameters is impractical (Ferguson acknowledged as much in his original paper but chose not to include feedbacks in his analysis). Drawing strong parallels to Hey (1978), Phillips used a four variable system within an eigenvector analysis of channel change to address AHG based on velocity, hydraulic radius, bed slope, and friction factor and concluded that "there is no single approach or hypothesis which is universally applicable for predicting [AHG]." Likewise, Miller (1991) reopened the extremal hypothesis of minimum variance as an explanation for AHG, which Ferguson rejected. Miller justified his reanalysis by stating that minimum variance is simply a particular expression of the principle of equable change, yet his ideas have not been widely adopted. Ridenour and Giardino (1991) challenged Ferguson (and indeed Leopold and Maddock) by noting that the unity of coefficients and exponents of AHG is not always observed in natural data due to measurement and fitting error. They sought to apply statistical techniques for compositional data on the wealth of published datasets in hopes of achieving universal understanding of AHG. Despite these challenges, Ferguson's formulation of AHG seems to have held, as by the mid-2000s studies shifted to resemble the more empirically based studies of the 1960s and before (e.g. Darby, 2005; Nanson et al., 2010; Pinter and

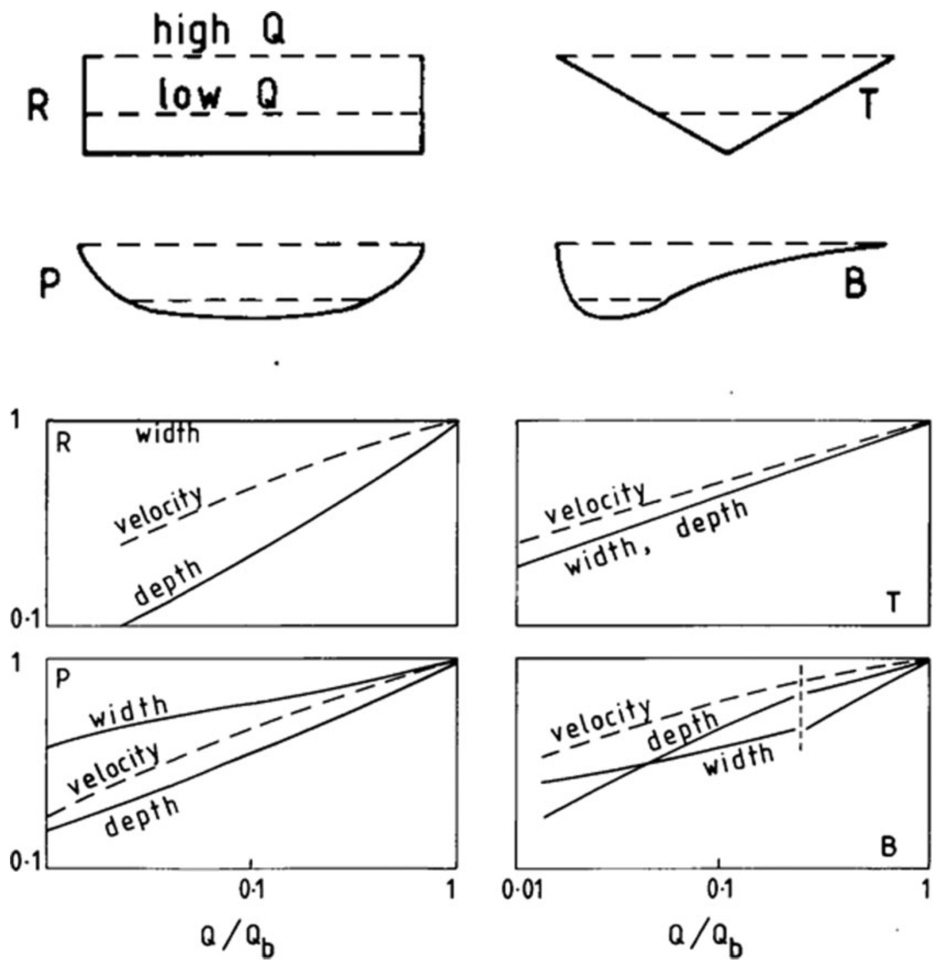

Figure 3. (Reprinted from Ferguson, 1986). Ferguson (1986) created two separate figures (his Figures 3 and 5) that have been combined here to illustrate his finding that AHG is a function of channel shape. Assuming constant bed friction (and therefore a velocity-depth function), Ferguson took fixed values for slope, bed grain size, and other parameters needed to employ a deep flow approximation of the Keulegan flow law, enabling calculation of velocity. By imposing these velocity and velocity-depth equations on the four different channel shapes seen in the top panel width, depth, velocity, and discharge, and therefore AHG, could be calculated. The bottom panel shows the corresponding AHG curves for each channel geometry, where the y-axis is a log of each hydraulic parameter as it increases to bankfull, and the x-axis is a log of discharge relative to bankfull discharge. It may be seen that AHG is only linear (and therefore a power law as per Leopold and Maddock) when channel geometry permits, and that breaks in slope are possible. Ferguson's breakthrough work showed that the power law is not a necessary form of HG and that AHG is a function of cross sectional shape, thus reducing AHG to "hydraulics and geometry."

Heine, 2005; Reid et al., 2010; Ridenour and Giardino, 1995).

Perhaps the most compelling evidence that Ferguson's conclusions have taken root came from Dingman (2007), who sought to "address some of the questions that Ferguson (1986) raised but did not fully answer." Dingman is here referring to the fact that Ferguson did not provide explicit general formulations explaining the interactions between AHG, channel geometry, and other factors. Dingman controlled for nearly all factors of channel geometry (slope, friction factor, geometry, sediment load) at-a-station, giving multiple channel scenarios for experimentation. For each of these scenarios, Dingman followed a similar procedure to Ferguson to calculate the AHG parameters (this study was the first to explicitly treat the coefficients of AHG as important variables for investigation, see section "Non-Leopoldian HG" below). He concluded that both coefficients and exponents depend in complex ways on other AHG parameters and also channel geometry, and could not find any explicit functional form for either coefficients or exponents.

As of today, Leopold and Maddock's AHG is still widely employed in its power law form, despite numerous verifications that there is no theoretical reason to do so. AHG remains a widely used tool for assessing discharge at-a-station, and understanding of these relationships has not significantly advanced beyond the explanation given by Ferguson in 1986. Further discussion of the utility of power law AHG and non-Leopoldian forms of AHG are given in sections "Applications of HG" and "Non-Leopoldian HG" below.

III Towards further understanding of DHG

1 Empirical investigations of DHG

Empirical investigation of DHG, like empirical investigation of AHG, began with studies intended to affirm that DHG existed in other field datasets besides those presented by Leopold and Maddock. However, unlike AHG, this vein of empirical research has continued, uninterrupted, to the present (e.g. Allen et al., 1994; De Rose et al., 2008; Doll et al., 2002; Kolberg and Howard, 1995; Rhoads, 1991; Thornes, 1970; Wohl and Wilcox, 2005) as no authoritative, universal, rational explanation for the existence of DHG has yet been found. Thus, empirical investigation remains a key component of DHG research.

Empirical data are still used to verify the existence of DHG in diverse environments and to propose constraints for its usage. Wohl (2004) proposed a stream power/grain size threshold below which DHG is not observed for steep mountain streams, while also finding (Wohl et al., 2004) that only the width-based DHG was statistically significant in their study. Ellis and Church (2005) used acoustic Doppler current profiling to demonstrate that DHG exists across secondary channels of the same river, while Haucke and Clancy (2011) investigated how gaps in the discharge record of gauging stations can influence the formulation of power law DHG. Xu (2004) found differences in the DHG of braided vs meandering streams, while Fola and Rennie (2010) found that DHG exists in streams with non-alluvial cohesive bed material. These examples of DHG research indicate that workers are still interested in the basic power law form of DHG and applications in novel environments.

As with AHG, researchers have challenged Leopold and Maddock's original conception of DHG using empirical data. One of the first such challenges arose from Leopold and Maddock's finding that velocity increased uniformly downstream. This debate was ongoing until the early 1970s, when study of DHG took a step forward with the use of computers to fit the DHG relationships that had previously been made "by eye" (Carlston, 1969). Carlston concluded that velocity sometimes does and sometimes does not increase downstream, and that neither previous hydraulic theory nor DHG was sufficient to explain velocity trends on a theoretical basis. In addition to this early challenge to DHG, by the 1980s the very definition of DHG was beginning to change. Researchers questioned why DHG should rely on mean annual flow as its independent variable, especially since most hydraulic properties of a river useful for DHG investigation change predictably as discharge increases until flow exits the channel (at bankfull). In addition, bankfull flows are recognized to have a more formative influence on channel morphology than mean annual flows and produce the highest average shear stress on channel banks (De Rose et al., 2008; Haucke and Clancy, 2011). As a result, by the early 1980s bankfull discharge had replaced mean annual discharge in hydrology textbooks (e.g. Richards, 1982), and by the mid-2000s this change in definition was accepted orthodoxy, although use of mean annual flow is still acceptable practice (e.g. Stewardson, 2005; Sweet and Geratz, 2003). The use of bankfull variables allows DHG to be investigated even when bankfull flow is not observed in situ (as bankfull variables can be calculated given channel geometry: mean annual flow is highly dependent on observational data), and allows DHG to inform AHG by providing bankfull hydraulic limits at a given cross section. As such, the definition of what "bankfull" conditions represent becomes important: Johnson and Heil (1996) describe the different ways that bankfull conditions are established as well as the implications of choosing bankfull stage for DHG and stream restoration design.

Empirical research has also been used to challenge and expand the very construct of DHG: researchers have incorporated other variables into the DHG relationships, especially variables characterizing bank strength and vegetation. Lee and Julien (2006) empirically tested a previously developed theoretical form of DHG (Julien and Wargadalam, 1995; see subsection "Theoretical explanations for DHG: extremal hypotheses" for further description) that lists discharge, mean sediment diameter, and river slope or Shields' number as independent variables. Bank vegetation and bank strength are other key additional DHG parameters and were a point of emphasis in a series of empirical papers by Huang and others in the late 1990s (Huang and Nanson, 1997, 1998; Huang and Warner, 1995). These papers empirically developed a form of DHG that included a bank strength coefficient, Manning's n, and river slope as independent variables, arguing that the addition of these variables more accurately describes river geometry than Leopold and Maddock's DHG. Huang and Warner's (1995) bank strength coefficient was found to vary with bank vegetation type and bank sediment, and confirmed earlier investigations that bank vegetation affects DHG coefficients but not DHG exponents (Hey and Thorne, 1986). In addition, bank vegetation was found to affect channel width more than depth or velocity (Huang and Nanson, 1997; Allmendinger et al., 2005). More recently, Pietsch and Nanson (2011) showed that in-channel vegetation can also affect bank strength, and Anderson et al. (2004) found that the influence of bank vegetation on DHG is scale dependent. These authors have all used empirical data to propose and verify different formulations for DHG and to gain further understanding of how external (non-hydraulic) factors affect DHG.

However, it is difficult to determine the impact of vegetation and other external factors on DHG from empirical observation alone. This is explicitly clear in Merritt and Wohl (2003), who sought to disentangle the effects of a flood that drastically changed channel geometry in an arid environment. They concluded that while the flood changed the DHG, the DHG of the existing channel also shaped how the floodwaters were routed through the system. Similarly, while Huang and Nanson (1997) found that bank strength exerts strong control on channel geometry, these authors later found that channel geometry also exerts strong control on bank strength through interactions with bank vegetation and rooting depth (Huang and Nanson, 1998). Likewise, Anderson et al. (2004) found that the influence of vegetation on DHG depends on the size of the system, yet Moody and Troutman (2002) determined that the variability of river width and depth is log-normal across many orders of magnitude—suggesting that vegetation effects should be similar regardless of scale. These recursive arguments highlight the difficulty of arriving at a more fundamental understanding of DHG through empirical observations alone.

Empirical investigations have confirmed that additional predictive power can be gained when including additional terms beyond width, depth, velocity, and discharge into DHG. In particular, the influence of bank vegetation and bank strength has been repeatedly investigated in these empirical studies, and this mirrors theoretical developments of DHG discussed in subsection "Theoretical explanations for DHG: extremal hypotheses". In addition, researchers continue to verify Leopold and Maddock's DHG in diverse field and laboratory environments. As new theoretical developments for DHG are introduced (see next subsection), workers will undoubtedly continue to head to the field to test these theories with empirical data.

2 Theoretical explanations for DHG: rational deduction

Researchers have made several advances towards a rational understanding of DHG, though none have been able to derive DHG on a level with Ferguson's 1986 explanation of AHG. Any rational deduction of DHG must attempt to solve for width, depth, and velocity for a given river reach with respect to discharge and other fluvial properties (as this is what DHG describes for stable channels). To make the problem mathematically determinate, researchers must utilize a system of equations that propose interrelations between these variables (ideally three equations for these three unknowns, taking discharge as known). The first and most obvious such equation is continuity, $Q=wdv$. Beyond this equation, researchers also usually employ a flow resistance law, of which there are several well accepted empirical flows laws that may be invoked (e.g. Manning's or Keulegan). Sediment transport equations or bank strength relations can provide a third equation, but the introduction of both flow resistance and sediment transport equations often introduce additional variables to the DHG system (e.g. river slope, bed load discharge, sediment characteristics, and other properties) and preclude a unique solution. To combat this problem, researchers have sought to either constrain or further relate the variables in the equations mentioned above, or alternatively have invoked extremal hypotheses in order to solve for a determinate DHG. This subsection reviews rational, constraint-based deduction attempts to explain DHG, and the following subsection reviews extremal hypotheses.

Authoritative theoretical explanations for DHG in two specific environments were given by Parker (1978a, 1978b). These two papers follow one another closely in approach, but are each applicable only to either mobile or gravel bed rivers that meet a host of other geometrical and fluvial parameter assumptions. In developing a rational solution for the DHG of mobile bed rivers, Parker assumes a wide, straight channel with a symmetrical parabolic cross section under uniform flow with a non-cohesive suspendable sand or silt bed under constant discharge. Given these assumptions, Parker sought to balance suspended sediment transport load and lateral bed load within the channel, relating these processes to flow via shear stress (i.e. Shields' number). He then imposed a suite of boundary conditions, and ended with a 4th order system of equations controlling erosion and sediment transport and six boundary conditions, which, among other conclusions, yielded a relationship between depth at channel center and Reynolds' number, slope, and particle size. Given this key (and novel) relationship, Parker used a flow resistance equation to specify discharge and sediment load according to proposed channel geometry and assumptions, ending with a set of three equations requiring velocity, bed material, and any two of depth, width, discharge, sediment load, or slope to be known to solve for the DHG of the channel. Parker derived a similar solution for gravel bed rivers; however, bed material cannot be suspended in these rivers, thus changing many of the mathematics and requiring assumptions of turbulence and lateral flows to arrive at a determinate DHG solution. These two papers gave promising, rationally deduced DHG whose exponents matched field data, but Parker was keen to note that his assumptions were very limiting and explicitly warned against taking these equations as applicable for all rivers.

Other researchers have also sought to make DHG determinate by other means than Parker's relationships between depth and bed material properties (valid only for his assumptions given in both 1978a and 1978b). Hey (1978) also produced a system of DHG equations that invoked other channel properties, but ended without a determinate solution. More recently, Julian and Wargadalam (1995) used Keulegan flow resistance, Shields' number, and secondary flow to form a system of DHG equations. This system resulted in highly complex DHG relationships that Julian and Wargadalam simplified by lumping five independent parameters into a constant, arguing that variation in these parameters was negligible compared to the variability of discharge, sediment properties, and Shields' number. These authors solved this system of equations via a 3D stability analysis of non-cohesive particles under 2D flows to give determinate HG solutions. However, they report that "most" HG parameters were calculated to within 50–200% of field values, and thus their theoretical DHG lacks the congruence with field data seen in Parker (1978a, 1978b) and other work.

Most recently, Parker et al. (2007) and Wilkerson and Parker (2011) have introduced the concept of a bankfull sediment "yield" to close the DHG system of equations. Parker et al. (2007) begin by empirically determining a dimensionless form of power law DHG as discovered by pooling relevant dimensionless variables across morphologically similar gravel bed rivers, which the authors assumed was representative of underlying physics. Taking this dimensionless DHG back into dimensioned space reveals complicated forms of DHG that rely on many more parameters than traditional DHG, agreeing with the empirical work discussed above. Parker et al. then used four different equations (friction, gravel transport, channel forming Shields' number, and a relation between Shields' number for the onset of motion and gravel yield) to end with a solvable system of equations (i.e. that predict DHG) that the authors take pains to note is a "broad brush." The novel gravel yield concept they propose is based on Parker's (1978) gravel bed work and gives the sediment yield at bankfull discharge. Similarly, Wilkerson and Parker (2011) found a dimensionless DHG across sand bed rivers and also found a bankfull discharge/sediment yield relation in these rivers, despite the marked differences in sediment transport characteristics of sand and gravel bed rivers. Following this sediment yield relationships, Wilkerson and Parker were able to derive DHG for these sand bed rivers. In both of these cases, a rationally deduced DHG was able to match empirical data, but these solutions are strictly constrained to the assumptions inherent in the derivations and are not universally applicable.

In all of the above examples, researchers have been able to create determinate DHG solutions by imposing particular channel geometries and fluvial constraints on a chosen system of equations. In most cases, this theoretically derived DHG matches empirical DHG as found from empirical data meeting the assumptions and constraints of the theoretical solution. However, none of these solutions are universal, that is, they cannot speak to DHG in rivers outside of their assumptions. Since DHG is empirically observed over many orders of magnitude and across nearly all river morphologies, further theoretical development is needed to explain the enduring legacy of DHG as proposed by Leopold and Maddock: its simple empirical form and relatively few physical parameters have yet to be described by a universal theory of DHG.

3 Theoretical explanations for DHG: extremal hypotheses

Extremal hypotheses form a different epistemological thrust of investigation for DHG, and offer alternative explanations for DHG behavior. An extremal hypothesis refers to any hypothesis requiring that the self-organization of rivers and the existence of DHG occur within some minimum, maximum, or other optimum state of a river system. Usually, researchers choose one aspect of the fluvial system to optimize (e.g. sediment transport or shear stress), yet this practice has been criticized for lacking a sound physical basis. Additional review of these approaches as they apply to rivers (and not just HG) is given in Paik and Kumar (2010).

A first paper that furthered the concept of imposing an extremal hypothesis on DHG came from Leopold himself (Langbein and Leopold, 1964). This work draws on the tenet of quasi-equilibrium as borrowed from entropy theory to state that river systems exhibit DHG because of the tendency of all natural systems towards minimum work and the equal distribution of work. DHG, they argued, was the result of opposing forces of sediment transport and landscape evolution, and that changes in the morphology of a channel would gradually result in a return to the original, stable DHG. This was debunked by both Slingerland (1981) and Phillips (1990), who each used systems analysis of change over time to show that river systems do not necessarily return to previous equilibrium states after perturbation.

Other researchers have proposed extremal hypotheses for DHG, and then formulated equations that match these hypotheses and used them to solve for DHG. Smith (1974) found that he could arrive at Leopold and Maddock's "overall" DHG by setting parameters in his system of equations to an arbitrary value for rivers operating at some extreme of natural behavior. Pickup (1976) proposed the idea of maximum sediment transport capacity as the driver of DHG, yet his results failed to yield satisfactory congruence with observed DHG. In seeking a pre-Ferguson explanation for AHG, Yang et al. (1981) proposed the concept of the minimum rate of energy dissipation, following Yang and Song (1979) and Song and Yang (1980). These authors state that a system is in equilibrium when its rate of energy dissipation is at a minimum, and that any system that tends towards equilibrium will tend towards this minimum energy condition. However, their published values for HG parameters are also not congruent with the wealth of observed HG data, and some of their assumptions have been debunked (Ferguson, 1986; Lecce, 1997).

Despite these challenges, the idea that DHG results from some extreme behavior of a natural system persists to the present day, yet in a different paradigm. Whereas Parker and others found relationships between depth and sediment properties or between bankfull discharge and sediment yield to rationally close an indeterminate DHG, these extremal works start from the same indeterminate DHG and apply an extremal hypothesis to close it, an assumption that is more universal but more controversial than the carefully constrained theoretical work discussed above. Such extremal research often includes a bank strength criterion (see also empirical investigations of bank strength and DHG above) as the key factor in both closing the equations and providing a physical basis for a particular extremal hypothesis. Huang and Nanson (2000) offer the extremal hypothesis of maximum flow efficiency to explain DHG as a companion to their empirical work (Huang and Nanson, 1997; Huang and Warner, 1995). They argue that rivers self-organize (e.g. exhibit DHG) as a result of both maximizing sediment transport and minimizing stream power, and Huang et al. (2002) further these arguments. If this extremal hypothesis is correct, then Huang and Nanson (2000) state that only continuity, flow resistance, and sediment transport equations are needed to solve for DHG. Doing so for a theoretical river with rectangular cross sections, straight channels, and steady and uniform flow yielded a DHG with b=f=0.44. This is close to, but not congruent with, the repeatedly observed empirical values of b =0.5 and f = 0.3–0.4, respectively. Huang and Nanson found they could achieve these expected DHG exponent values by maintaining a constant ratio between discharge and sediment load, which agrees with the theoretical work of Parker discussed above that sediment load and bankfull discharge are linked. Huang and Nanson take care to note that their extremal hypothesis has a physical basis, which Millar (2005) explains as the fact that excess stream power creates excess erosion and therefore channel instability, and this quantity must be minimized if a channel is observed to be stable over time.

Eaton et al. (2004) give a different extremal hypothesis to close a DHG system of equations: that the fluvial system, not the channel per se, maximizes its resistance to flow to achieve stability over time. Using a trapezoidal channel model developed by Millar and Quick (1993), Eaton et al. show that a system of equations for flow resistance, sediment transport, and, importantly, bank strength can be optimized with regard to flow resistance to choose from non-unique solutions of their DHG system of equations. They found that channels having similar bank strength are characterized by the same dimensionless function of shear stress and width/depth ratio that in turn specifies slope and discharge (and therefore DHG), and that this function matches well against flume data. Eaton and Millar (2004) furthered these conclusions, and argue that bank strength is essential to a physical basis for any extremal hypothesis. Finally, Eaton and Church (2007) tested the system-wide maximization of flow resistance given in Eaton et al. (2004) against empirical and modeled datasets, and found that their bank strength explicit DHG system is able to predict channel widths and depth as accurately as traditional DHG. In sum, these papers are a strong argument for bank strength as a key driver for DHG and for an extremal hypothesis to explain DHG.

Despite these successes, extremal hypotheses have been dogged by criticism since their adoption in fluvial systems. One such criticism was leveled by Griffiths (1984), who called extremal hypotheses for DHG "an illusion of progress," a sentiment echoed by Ferguson (1986). These and other authors note that while extremal hypotheses are sometimes able to recreate DHG, there is no physical basis to impose such extrema on DHG. This is especially true of early extremal work, where extremal hypotheses begat new equations that justified a DHG solution. Recent work uses extremal hypotheses to close indeterminate DHG systems of equations, and Huang, Eaton, Millar, and others have argued in favor of the physical bases for their particular extremal hypotheses and met with success in closing rational DHG systems. However, theoretical work by Parker et al. (2007) and Wilkerson and Parker (2011) is explicitly anti-extremal, and is dismissive of these hypotheses. Likewise, Dingman (2007) noted that explicit formulations for AHG are not yet developed, but "metaphysical explanations" (i.e. extremal hypotheses) for HG should not be adopted without first attempting deduction from first principles.

As extremal hypotheses continue to be invoked to choose a solution from an indeterminate rational system of DHG equations rather than introduce equations (as in early work), the rational deduction and extremal hypotheses literature should continue to converge. Furthermore, constraints related to threshold of sediment motion (i.e. the Shields' number) or bankfull relations between hydraulic parameters found in many rationally deduced DHG analyses (e.g. Diplas and Vigilar, 1992; Parker 1978a, 1978b; Wilkerson and Parker, 2011) are themselves extrema, as the onset of sediment motion and bankfull flow represent two critical values of fluvial operations (while not requiring an extremal hypothesis as to why they occur). It remains to be seen if any of the above approaches, alone or in combination with constraint-based deduction, will yield a universal theory of DHG.

DHG, like AHG, therefore remains an empirical relationship that is yet universally explained by rational means. Numerous researchers have given satisfying rational solutions for DHG, but these are only applicable given the myriad constraints and assumptions used to derive them, and other researchers have been able to replicate DHG by using extremal hypotheses to close systems of DHG equations that lack a proven physical basis. Future research on DHG will likely continue in empirical, rational, and extremal veins as workers move towards universal understanding of DHG.

IV Applications of HG

Researchers have employed HG in a variety of ways that are motivated by interests other than discovering the underlying physical principles of HG. This is very much in keeping with Leopold and Maddock's original reporting of their results, and sidesteps the body of work discussing the underlying physical principles of HG without ignoring their conclusions. Such research raises the interesting point that physical phenomena can be useful even if they are not fully understood: a concise summary of the general argument in favor of power law HG despite the many criticisms leveled against it. Perhaps the most famous application of HG is the derived USGS's (and other agency) use of AHG to publish daily discharge data for the streams and rivers. While agencies surely identified stage-discharge relationships before AHG was first published in 1953, Leopold and Maddock's seminal work allowed wider use of the rating curve and instilled more confidence in its usage. It is fair to say that without the AHG "rating curve," continuous time series of streamflow data would not be available to the public.

Commonly, researchers seek to characterize entire basins or regions by the different HG parameter values found therein (e.g. Arp et al., 2007; Johnson and Fecko, 2008). Even though researchers have shown (see previous sections) that AHG is unique to each cross section's channel geometry and bed properties, and DHG likely results from a complex set of site specific interactions along a specific river, there is no reason why these conditions should not be similar throughout a basin, thus making aggregate HG parameters useful for numerous applications. It is important to note that in such cases, authors must take care not to interpret commonality as indicative of the underlying principles of HG (as did Leopold and Maddock). This regionalization of HG, where representative HG parameters are given for large areas that include multiple rivers, is seen in Castro and Jackson (2001) who explicitly state that such regional HG equations only exist as the streams within each area are similar and are not expressions of a regional mean. Likewise, Pistocchi and Pennington (2006) sought amalgamated HG for large European rivers, while Mosley (1981) compiled data for 72 New Zealand rivers to arrive at lumped HG equations. Care must also be taken on where boundaries for regional HG studies are drawn, as Mulvihill and Baldigo (2012) found that drawing different physiographic boundaries affected their grouped HG curves.

Another common application of HG (often related to basin-wide generalizations, e.g. Singh and McConkey-Broeren (1989)) is its use in habitat assessment and in-stream use. Authors seeking to use HG in this capacity have linked it with various indices popular in fisheries management and other sciences that allow the easy-to-observe HG to stand in for these metrics. Examples of such research include Mosley

(1982), Lamouroux and Souchon (2002), and Rosenfeld et al. (2007).

Finally, researchers have found other, disparate ways to employ HG in various fields of study. Tinkler and Pengelly (1995) used measurements of paleo-floodplains to estimate the discharge from bursting ice lakes in the distant past via HG, while Nienow et al. (1996) discovered that HG is not universally applicable in subglacial closed channels due to hysteresis. HG is also used for hydrologic modeling, for example as the basis for a discharge wave propagation model in the Colorado River (Wiele and Smith, 1996), or to calculate runoff routing times following precipitation events (Paik and Kumar, 2004). There are also environment-specific applications, as Best et al. (2005) found that river ice affects HG in the arctic, while Comiti et al. (2007) and Ferguson (2007) investigated AHG behavior in high gradient mountain streams to understand further flow resistance in these streams. These examples highlight that HG is useful for monitoring and understanding river systems even if it is not fully understood.

V Non-Leopoldian HG

While most HG research has sought to understand or apply Leopold and Maddock's original formulations, researchers have found numerous other relationships that resemble HG in form: i.e. power laws between various hydraulic and geomorphic variables existing either at-a-station or downstream. These power laws may be an expression of landscape allometry as defined by Hood (2002), or they may underscore some yet to be understood physical principle of river behavior and of HG. Whatever the case, numerous hydrologic parameters distinct from width, depth, and velocity also display power law correlations with river discharge that may be useful for further understanding of HG.

For instance, researchers have discovered power law correlations between surface velocity and outlet width for sloughs entering a delta (Hood, 2002), between marsh area and channel depth, top width, and tidal prism (Williams et al., 2002), between suspended sediment concentration and discharge (Syvitski et al., 2000), and between grain resistance and other sediment properties in steep mountain streams (David et al., 2010; Wohl et al., 2004). Pizzuto (1992) found that basin magnitude and mean grain diameter, an important parameter for sediment transport models that are used to close DHG systems, also exist in a power law relationship. Finally, Magnusson et al. (2012) found that in proglacial streams HG can be written as a function of temperature—water gains heat as it moves away from its glacial source in a power law fashion. All of these relationships could prove useful in providing additional closure to DHG systems of equations by giving additional interrelationships between DHG relevant parameters (see the discussion in section "Towards further understanding of DHG" above).

HG behavior has also been observed in braided rivers, yet with a slight change in variable definition. Smith et al. (1996) first reported that effective width (inundation area divided by reach length) exhibited HG behavior with discharge for braided rivers in Alaska. This has been confirmed in numerous papers, including Ashmore and Sauks (2006) and Smith and Pavelsky (2008).

Another form of HG has been discovered and termed reach averaged hydraulic geometry (RHG). First introduced by Jowett (1998), this concept applies HG in an identical manner as AHG or DHG, but uses reach averaged variables instead of single cross sections. Jowett included this reach averaging step for practical reasons—both for ease of measurement and for his application of habitat assessment, but in doing so he invented another form of HG. Wohl and Merritt (2008) also found RHG to exist within mountain streams. RHG has seen few attempts to understand its underlying physical principles, but Harman et al. (2008) quantified the measurement and model error of using RHG for Australian rivers, and Navratil and Albert (2010) found that RHG existed for rivers in their study but exhibited a break in slope at certain discharges. It remains to be seen if RHG is simply an approximation for AHG where reaches are small compared to total river scale or whether RHG is a separate entity. In either case, RHG remains closely associated with habitat assessment applications that seek to predict the suitability of particular river reaches for particular species of interest given RHG predicted hydraulics (e.g. Jowett, 1998; Lamouroux and Souchon, 2002; Stewardson, 2005).

In all of the above examples, researchers have found power laws that link various hydraulic parameters within alluvial systems. The preponderance of the power law is quite interesting, and an unanswered question is whether or not AHG and DHG are simply particular expressions of an underlying phenomenon that exists for all aspects of a fluvial system (see Stumpf and Porter (2012) for a discussion of power laws in scientific data). The existence of RHG and of HG from braided rivers using effective width is particularly supportive of such a hypothesis. This interpretation would also seem to lend support to extremal hypotheses for explaining HG behavior, yet the many criticisms of these hypotheses (e.g. Griffiths, 1984) should be taken into consideration when attempting to uncover such a phenomenon.

1 At-many-stations HG

Several researchers have investigated AHG as changes downstream. As stated by Rhodes (1977), ''all cross sections of a given stream system are interrelated. Thus, it seems reasonable to expect some relationship to exist between the [AHG] of channels in a downstream direction. This idea differs from [DHG].'' Similarly, Orlandini and Rosso (1998) explicitly linked AHG and DHG under the idea that the same runoff events influence both at-a-station and downstream flows by linking stations to the outlet of the entire network. In doing so, they proposed an equation where two constants (S and A/A_{outlet}) link variable parameters that change with location along the river and depend on AHG b exponents and outlet discharge as follows

$$\log(S) = \log(W_{outlet} Q_{outlet}^{-b_{DHG}}) + (b_{DHG} - b_{AHG}) \log(A/A_{outlet}) \tag{4}$$

(Modified from Orlandini and Rosso's equation 35)

Dodov and Foufoula-Geogiou (2004a, 2004b) also link drainage area to AHG parameters, but with unconvincing goodness of fit, while Tabata and Hickin (2003) found that plotting all stations among anabranches of the Columbia river yielded an ''Interchannel'' HG. This analysis culls all stations together to create an overall relationship, similar to how Parker et al. (2007) found a dimensionless DHG across morphologically similar sand bed rivers. Interchannel HG is quite interesting in that spatially distributed data form a sort of DHG, but these data come from different channels and therefore represent a very different phenomenon than DHG.

Recently, Gleason and Smith (2014) found that the paired coefficients and the exponents of AHG relationships along a river exhibit very strong semi-log relationships (a straight line when one variable is logged and the other variable is not) over surprisingly long river reaches (up to ~3000 km). They termed this finding at-many-stations hydraulic geometry (AMHG), and demonstrated its existence from USGS and other agency field measurements and gauge data (Figure 4). Such a finding indicates that AHG coefficients and exponents (e.g. a and b) are functionally related and dependably predictable from one station to the next, and that one AHG parameter may be calculated from the other given the AMHG equation.

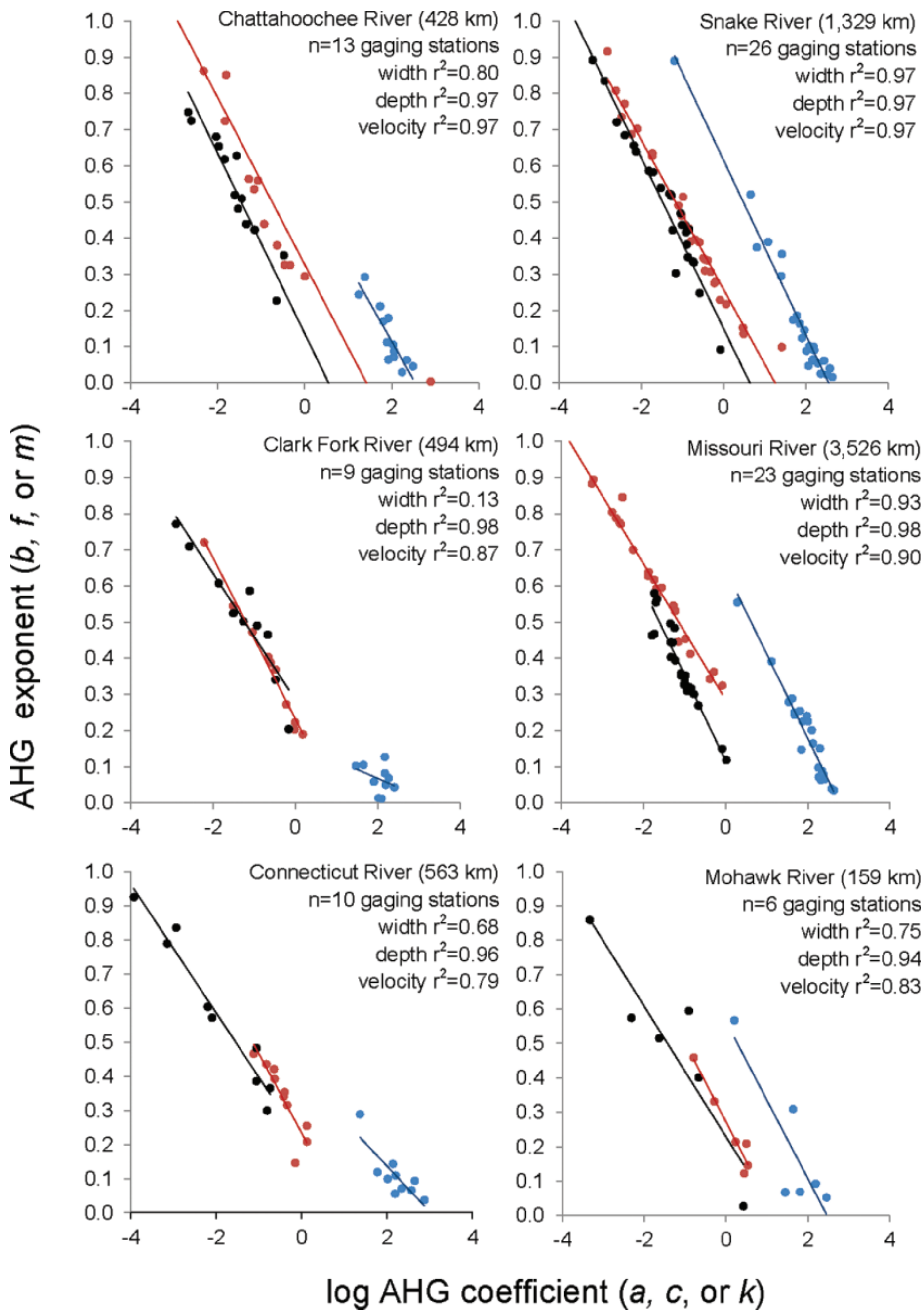

Figure 4. (Reprinted from Gleason and Smith, 2014). AHG a, c, and k coefficients interact predictably with their corresponding b, f, and m exponents as revealed here from thousands of in situ measurements of flow width, depth, and velocity collected at 88 USGS gauging stations along six US rivers over the period 2004–2013 The discovery of these strong semi-log trends (blue, red, and black lines), are called at-many-stations hydraulic geometry or AMHG. Blue=width AMHG (a vs b), Red=depth AMHG (c vs f), Black=velocity AMHG (k vs m). These relationships indicate predictable linkages between AHG parameters through space.

Geomorphically, AMHG suggests that there are a set of width-discharge, velocity-discharge, and depth-discharge values that are shared by all cross sections within a river. This may be seen in the following mathematical analysis.

At any station, the width-based AHG can be written as

$$log(w) = b\ log(Q) + log(a) \tag{5}$$

If there are values of w and Q that are common to all stations, then

$$b_1\log(Q) + \log(a_1) = b_2\log(Q) + \log(a_2) \tag{6}$$

where the subscripts 1 and 2 refer to any two cross sections of a river. Solving Equation 6 for Q yields

$$\log(Q) = \frac{\log(a_2) - \log(a_1)}{b_1 - b_2} \tag{7}$$

suggesting that the ratio of the difference in the log of the AHG coefficients to the difference in AHG exponents must be constant, which is precisely what Gleason and Smith observed in AMHG: in defining AMHG, Gleason and Smith observed that the relationship between $\log(a)$ and b is linear with a slope equivalent to

$$slope = \frac{\Delta b}{\Delta\log(a)}. \tag{8}$$

Equations 7 and 8 are equivalent (noting that the AMHG slope has always been observed to be negative; Gleason and Smith, 2014), affirming that there is a theoretical w-Q pair (and also d-Q and v-Q pairs) that is shared by all cross sections in a river. This is geomorphically puzzling, especially since the values of w, d, v, and Q that are shared by all cross sections are sometimes far outside the range of observed values of these quantities in the data reported by Gleason and Smith (2014). In addition, the strong goodness of fit reported by Gleason and Smith suggests mathematical artifice in AMHG. However, AMHG cannot be purely a mathematical artifact of using a power law for AHG, as the AMHG built from completely independent datasets of the same river reach covering much different ranges of data agree with one another (Gleason and Smith, 2014). It is important to note also that the constant slope values reported by Gleason and Smith cannot correspond to either mean annual or bankfull discharge, as these quantities change downstream (as observed in DHG).

AMHG seems at odds with Phillips' (1990) assertion that "there is no single approach or hypothesis which is universally applicable for predicting at a station HG." However, these findings, as well as the work of Rhodes (1977) and Orlandini and Rosso (1998), indicate that there are facets of HG that have yet to be uncovered and that DHG and AHG may be more closely linked than previously thought. In addition, AMHG in particular proves the premonitions of Dingman (2007) that much more attention should be paid to the coefficients of HG.

VI Conclusions

After 60 years of research, Leopold and Maddock's HG, a simple formulation of power law functions between channel geometric parameters and discharge, remains an important idea for monitoring river systems worldwide. The utility of HG has only increased since their original publication: USGS (and other agency) rating curves based on HG provide critical discharge monitoring capacity for the world's rivers, and HG has been successfully applied to calculate flood risk, investigate flow conditions in the distant past, and understand habitat conditions worldwide. HG is observed in myriad environments and DHG exponents tend towards similar values across these environments, suggesting a measure of universality. However, the underlying physical principles that produce HG behavior have yet to be satisfactorily uncovered, despite numerous attempts by prominent researchers; and no explicit, rational formulations for the coefficients and exponents of DHG relationships have been proposed. There has been, however, considerable progress towards an understanding of HG. Ferguson (1986) successfully reduced AHG to "hydraulics and geometry" despite being unable to develop explicit formulations for AHG coefficients and exponents. Numerous researchers have proposed rational solutions for DHG parameters, but these explanations are valid only for those rivers which meet the assumptions under which the solutions were developed. In addition, recent work has shown that new facets of HG may be uncovered by examining both AHG and DHG in novel ways beyond their original construction (e.g. Gleason and Smith, 2014; Tabata and Hickin, 2003). The utility of HG is unquestioned, but its elevation from empirically observed relationship to physical principle is not complete, despite universal acceptance of its existence. It may be that such a transition is never made, in which case future research of HG will focus almost exclusively on its applications.

This begs the question: where will the next advances in understanding of HG come from? Extremal hypotheses and rational deduction seem to be converging towards theories of DHG that are more and more universal, but are still plagued by restrictive assumptions or controversial physical bases for particular extremal hypotheses. Another avenue that may yield further progress towards a theory of HG could come from ideas that break from Leopold and Maddock's original formulations of HG. AMHG, Interchannel HG, RHG, and HG from effective width in braided rivers all suggest that there is more to discover about HG, and that DHG and AHG may be more closely linked than previously thought. Whether or not this speculation will prove correct will occur in future work, as all of these novel HG variants have been proposed via empirical observation without rational explanation. No matter what form future investigation takes, an emphasis on the power law form of HG could obfuscate understanding of HG—researchers as early as Richards (1973) have shown that the power law, replete with its two parameters, is not the best model to describe HG. If future researchers insist on using the Leopold and Maddock construction of HG, then the coefficients of both AHG and DHG *must* be given more thorough treatment and not simply viewed as statistical artifacts of the fitting process. These arguments suggest that a paradigm shift in thinking about HG will be beneficial to further understanding.

A final consideration is the intriguing possibility of a unifying HG theory, if one should be found to exist. Such a theory would need to satisfactorily explain both AHG and DHG. Since DHG relies on bankfull variables at a cross section and also relies on cross sectional shape (which determines AHG) to define its bank strength and sediment transport properties, DHG and AHG are implicitly linked. Furthermore, non-Leopoldian linking HG variants, like AMHG and Interchannel HG discussed herein, suggest further, explicit linkages and dependencies between the two. By broadening perspective on HG research to include such linkages, further insights about HG could be gained. Whether or not the elusive universal rationalization of HG is uncovered, HG, in all its forms, remains a viable field for future research.

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