Set Theory Notes

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These notes are not necessarily correct, consistent, representative of the course as it stands today or, rigorous. Any result of the above is not the author's fault.

0 Notation

We commonly deal with the following concepts in Set Theory which I will abbreviate as follows for brevity:

Term	Notation

Contents

1 Fundamentals

1.1 The Set ω

We have the set of natural numbers, $\mathbb{N} = \{0, 1, 2, \ldots\}$, and from this, we define ω:

$$\omega = \{0, 1, 2, \ldots\},$$

where for some n in ω,

$$n = \{0, 1, 2, \ldots, n - 1\},$$

with 0_ω being the empty set. We can go beyond this definition, defining:

$$\begin{aligned}
\omega + 1 &= \{0, 1, 2, \ldots, \omega\}, \\
\omega + 2 &= \{0, 1, 2, \ldots, \omega, \omega + 1\}, \\
&\ldots \\
\omega + n &= \{0, 1, 2, \ldots, \omega, \omega + 1, \ldots \omega + n - 1\}.
\end{aligned}$$

1.2 Axiom of Extensionality

For two sets a and b, we have that $a = b$ if and only if for all x we have that:

$$x \in a \iff x \in b.$$

For two classes A and B, we have that $A = B$ if and only if for all x we have that:

$$x \in a \iff x \in b.$$

1.3 Axiom of Pair Sets

For any sets x and y, there is a set $z = \{x, y\}$. This is the (unordered) pair set of x and y.

1.4 Axiom of the Powerset

For each set x, there exists a set which is the collection of the subsets of x, the powerset $\mathcal{P}(x)$.

For some set x, we have the powerset defined as follows $\mathcal{P}(x) = \{z \text{ such that } z \subseteq x\}$.

1.5 Axiom of the Empty Set

There exists a set with no members, the empty set $\varnothing$.

We have the empty set defined as follows $\varnothing = \{x \text{ such that } x \neq x\}$.