

Correction of flux measurements for density effects due to heat and water vapour transfer

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SUMMARY

When the atmospheric turbulent flux of a minor constituent such as CO₂ (or of water vapour as a special case) is measured by either the eddy covariance or the mean gradient technique, account may need to be taken of variations of the constituent's density due to the presence of a flux of heat and/or water vapour. In this paper the basic relationships are discussed in the context of vertical transfer in the lower atmosphere, and the required corrections to the measured flux are derived.

If the measurement involves sensing of the fluctuations or mean gradient of the constituent's *mixing ratio* relative to the dry air component, then no correction is required; while with sensing of the constituent's *specific mass content* relative to the total moist air, a correction arising from the water vapour flux only is required. Correspondingly, if in mean gradient measurements the constituent's *density* is measured in air from different heights which has been pre-dried and brought to a common temperature, then again no correction is required; while if the original (moist) air itself is brought to a common temperature, then only a correction arising from the water vapour flux is required.

If the constituent's *density* fluctuations or mean gradients are measured directly in the air *in situ*, then corrections arising from both heat and water vapour fluxes are required.

These corrections will often be very important. That due to the heat flux is about five times as great as that due to an equal latent heat (water vapour) flux. In CO₂ flux measurements the magnitude of the correction will commonly exceed that of the flux itself. The correction to measurements of water vapour flux will often be only a few per cent but will sometimes exceed 10 per cent.

1. INTRODUCTION

For some years there has been increasing interest in measuring the transfer of CO₂ and other minor gaseous constituents in the atmosphere. This paper discusses an important aspect which does not appear to have been generally recognized: when the turbulent flux of any constituent is measured by either the eddy covariance or the mean gradient technique, account may need to be taken of the simultaneous flux of any entity – in particular, heat or water vapour – which causes expansion of the air and thus affects the constituent's density. We are concerned here solely with the constituent density variations as such, and not with any buoyancy effects which may arise.

We consider the particular case of vertical fluxes through the atmospheric surface layer (lowest few metres) over an extensive uniform surface. Other contexts require separate consideration in some respects, but the underlying principles can suitably be described in the surface layer framework.

The analysis to be described shows that if the measurement involves sensing of the fluctuations or mean gradient of the constituent's *mixing ratio* relative to the dry air component, then no correction to the measured flux is required. But in the case of sensing of the constituent's *density* fluctuations or mean gradient in the *in situ* air, corrections arising from both heat and water vapour fluxes are required.

A preliminary partial discussion, limited to the effect of water vapour flux, has been presented elsewhere (Webb and Pearman 1977). Two short discussions including the effect of

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heat flux, by Bakan (1978)* and by Jones and Smith (1978), came to our notice respectively shortly before and shortly after submission of the present paper. We are not aware of any other previous treatments in either meteorology or physical chemistry (cf. Bird *et al.* 1960; Hirschfelder *et al.* 1964) which deal with the density effects in turbulent transfer in the way required here. We note that the molecular 'thermal diffusion' process in gases, with which corresponds the 'Soret effect' in liquids, and which is discussed by Grew and Ibbs (1952), Monchick and Mason (1967), and Grew (1969), is quite different in nature from (and more complicated than) the effect of a mean temperature gradient on the transfer relationship for a constituent as discussed in the present paper.

Our initial motivation in this topic arose in the evaluation of CO_2 flux from the flux-gradient relationship, with the mean CO_2 concentration at different heights measured by use of an infrared gas analyser (IRGA). It is common practice (Monteith and Szeicz 1960; Denmead 1969; Uchijima 1970; Pearman and Garratt 1973) to pre-dry the air, in order to avoid the difficulty of infrared absorption by the water vapour bands. Since the volume of any sample of air is reduced by drying, the density of the CO_2 component is increased, and there have been suggestions that a correction should be applied to recover the original CO_2 density of the moist air (Parkinson 1971; Ripley and Saugier 1972; Pearman 1975; Spittlehouse and Ripley 1977; Saugier and Ripley 1978). But the conclusion from the present paper is that no such correction should be applied – with the dried air from the two heights brought to a common pressure and temperature, the measured CO_2 density difference is equivalent to the difference of CO_2 mixing ratio relative to the dry air component, and leads to the true CO_2 flux.

An alternative practice sometimes followed (Janac *et al.* 1971; Verma and Rosenberg 1976) is to insert an optical filter in the infrared system to eliminate the water vapour bands, and the air can then be sampled directly by the narrow band IRGA without pre-drying. With this arrangement, it turns out that a correction depending on the moisture flux is required, while if the air is sampled at its *in situ* temperature at each height, a correction depending on the heat flux is needed as well. These corrections are found to be very important, especially that from the heat flux, the effect of which not uncommonly can exceed the CO_2 flux itself. Thus, if the heat and moisture fluxes are not known accurately, the 'corrected' CO_2 flux can still be appreciably in error, so it seems preferable to avoid the density effect from the outset by pre-drying the air and bringing it to a common temperature.

CO_2 flux measurements by the eddy covariance technique have been made by Desjardins and Lemon (1974) using an IRGA modified to improve the response time, and by Ohlakti and Seo (1976) and Jones and Smith (1977) using an infrared sensing path open to the wind; Hyson and Hicks (1975) have used a similar open system for water vapour flux measurements. Analysis shows that the correction required for eddy covariance measurements is similar to that mentioned above for mean gradient measurements. No correction would be required if the air could be pre-dried and brought to a constant temperature, but of course this is not likely to be feasible in fast response eddy covariance measurements.

While the emphasis above is on CO_2 , the considerations of this paper are equally valid for any minor constituent, and for water vapour itself. Examples of other minor constituents for which fluxes evaluated from mean gradient measurements have been reported are ozone (Galbally 1971) and ammonia (Denmead *et al.* 1974).

Section 2 outlines some basic considerations, and section 3 gives the form of the mean vertical velocity, which plays an essential part. Sections 4 and 5 give the analysis for eddy covariance and mean gradient measurements, respectively, while the extension to relation-

* Bakan's argument (leading to his Eq. (6)) that the turbulent mass flux of air is zero, based on $w = 0$ at the surface, is incorrect – in fact it would equally imply that the turbulent flux of any quantity is zero. However, this is of no real consequence since the zero mass flux of air can itself be taken as a reasonable assumption.

ships explicitly in terms of heat and water vapour fluxes is given in section 6, and the practical quantitative effects are discussed in section 7. Section 8 gives an overview of the conclusions.

2. BASIC CONSIDERATIONS

We consider the vertical flux of a minor constituent C , and take the relative concentration of C to be too small to affect the total air density appreciably.

The effect of heat flux on eddy covariance measurements of the flux of C can be seen as follows. If the heat flux is upwards (positive) then rising air parcels are on average warmer and less dense than descending parcels, so that on the assumption of zero mean vertical mass flow of air there must exist a small *mean* upward velocity component. Thus, in measurements which include the fluctuations of C density and of vertical airspeed, w , about its mean \bar{w} , the contribution to the flux of C associated with \bar{w} is missed, and an appropriate correction having the same sign as the heat flux must be added. Similarly, a correction having the same sign as the water vapour flux must also be added to the measured flux of C . If the total vertical airspeed w , including \bar{w} , could be measured accurately, then the flux of C would not require correction, but in practice \bar{w} is far too small to be measured with sufficient accuracy.

In the flux-gradient method, the raw measured flux of C is akin to that from eddy covariance measurements – the fluctuations of C density and of w are directly related to the mean gradient of C density and to the turbulent transfer coefficient, respectively. Accordingly, similar corrections must be added in this method also.

Analysis of the problem hinges on two key principles. One is that the mean vertical airspeed plays an essential role. The other is that with the mean vertical airspeed initially unknown, it is necessary to specify some kind of governing constraint, the exact nature of which depends on the particular problem in hand; unless this is done, the problem is indeterminate. Here, the appropriate constraint adopted is that the vertical flux of dry air constituent should be zero, on the assumption that there is no source or sink of air at the ground.

Let E be the vertical flux of water vapour; F , the vertical flux of C ; ρ , the total density of air; ρ_a , the density of dry air constituent; ρ_w , the density of water vapour constituent; ρ_c , the density of C constituent; $S = \rho_c/\rho$, the specific content of C (mass per mass of air); and $s = \rho_w/\rho_a$, the mixing ratio of C (mass per mass of dry air constituent). Overbar and prime will denote mean and fluctuation therefrom, respectively. The total density of the air is $\rho = \rho_a + \rho_w + \rho_c$, and we assume that the relative density of C is very small and take with sufficient accuracy

$$\rho = \rho_a + \rho_w. \quad (1)$$

We take the ambient atmospheric pressure, p , to be constant throughout. By this we are for simplicity ignoring the change of mean pressure with height (though this can be taken into account by regarding all temperatures and densities as potential quantities), and we are also taking the turbulent fluctuations of pressure to be negligibly small (see Calder 1949; Kimball and Lemon 1970).

The individual gas laws for dry air (suffix 'a'), for water vapour (suffix 'w'), and for entity C (suffix 'c') are

$$p_a = (R/m_a)\rho_a T, \quad (2a)$$

$$p_w = (R/m_w)\rho_w T, \quad (2b)$$

$$p_c = (R/m_c)\rho_c T, \quad (2c)$$

where p_a , p_w , p_c are the respective partial pressures; m_a , m_w , m_c are the molecular masses ('weights'); R is the universal gas constant; and T is the absolute temperature.

Dalton's law of partial pressures is

$$p_a + p_v + p_e = p, \quad (3)$$

which, from Eqs. (2a, b) with the assumption $\rho_e \ll \rho_a + \rho_v$, may be written approximately

$$\rho_a/m_a + \rho_v/m_v = p/RT. \quad (4)$$

If the gas law for moist air is written

$$p = (R/m)\rho T, \quad (5)$$

then we find from Eqs. (4) and (5) (again neglecting the effect of C on the total density) that m is given by

$$m = \{(1-q)/m_a + q/m_v\}^{-1}, \quad (6)$$

where $q = \rho_v/\rho$ is the specific humidity. We note that this expression for m is in fact the same as one obtains by taking the weighted mean of m_a and m_v , with weighting according to the respective numbers of molecules per unit volume.

The concentration of a constituent has often been expressed in terms of relative volume assuming the constituents to be extracted and reduced to a standard temperature and pressure (STP). The conversion between this and the actual relative density (or mass) in a sample of air, as used in this paper, is seen from Eqs. (2a), (2c), and (5) to be

$$s = \rho_e/\rho_a = (m_e/m_a) \frac{\text{volume of C at STP}}{\text{volume of dry air constituent at STP}} \quad (7a)$$

$$S = \rho_e/\rho = (m_e/m) \frac{\text{volume of C at STP}}{\text{volume of moist air at STP}}. \quad (7b)$$

Eq. (4) gives the basic relationship connecting ρ_a , ρ_v , and T . If we expand $T^{-1} = (\bar{T} + T')^{-1}$ in powers of (T'/\bar{T}) , then Eq. (4), its mean, and its departure from the mean, are expressed by

$$\rho_a/m_a + \rho_v/m_v = \{p/R\bar{T}\} \{1 - T'/\bar{T} + (T'/\bar{T})^2 - (T'/\bar{T})^3 + \dots\} \quad (8a)$$

$$\bar{\rho}_a/m_a + \bar{\rho}_v/m_v = \{p/R\bar{T}\} \{1 + \bar{T}'^2/\bar{T}^2 - \bar{T}'^3/\bar{T}^3 + \dots\} \quad (8b)$$

$$\rho'_a/m_a + \rho'_v/m_v = \{p/R\bar{T}\} \{-(T'/\bar{T}) + (T'^2 - \bar{T}'^2)/\bar{T}^2 - (T'^3 - \bar{T}'^3)/\bar{T}^3 + \dots\}. \quad (8c)$$

An expression for ρ'_a in terms of ρ'_v and T' is obtained from Eq. (8c) with $p/R\bar{T}$ eliminated by use of Eq. (8b), which gives

$$\rho'_a = -\mu\rho'_v - \bar{\rho}_a(1 + \mu\sigma)(T'/\bar{T})^2 + \dots \{1 + \bar{T}'^2/\bar{T}^2 - \dots\}^{-1} \quad (9a)$$

where $\mu = m_v/m_a$ and $\sigma = \bar{\rho}_v/\bar{\rho}_a$. If we ignore terms of order $(T'/\bar{T})^2$ and smaller, Eq. (9a) becomes to a close approximation

$$\rho'_a = -\mu\rho'_v - \bar{\rho}_a(1 + \mu\sigma)T'/\bar{T}. \quad (9b)$$

In practice the scale magnitude of T'/\bar{T} is virtually always less than 10^{-2} , and we shall generally ignore terms of order $(T'/\bar{T})^2$ relative to unity.

We note also that the magnitude of $\rho'_v/\bar{\rho}_a$ or $\rho'_v/\bar{\rho}$ is generally comparable with or smaller than that of T'/\bar{T} . This is apparent since the ratio of the former to the latter is like $\bar{T} \bar{\rho}'_v/\bar{\rho}_v \bar{w} \bar{T}' = \bar{T} c_p/\lambda\beta \approx 0.12\beta^{-1}$, where c_p is the specific heat of air at constant pressure, λ is the latent heat of evaporation, and β is the Bowen ratio (cf. section 6) of which the magnitude is generally greater than 0.1. Further, it then follows from Eq. (9b) that the magnitude of $\rho'_a/\bar{\rho}_a$ is also generally comparable with that of T'/\bar{T} .

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Again, expressing $\partial\bar{\rho}_a/\partial z$ from Eq. (8b) and then eliminating $p/R\bar{T}$ by use of Eq. (8) and neglecting small terms of order $(T'/\bar{T})^2$ relative to unity, we have to a close approximation

$$\partial\bar{\rho}_a/\partial z = -\mu\partial\bar{\rho}_v/\partial z - (1 + \mu\sigma)(\bar{\rho}_a/\bar{T}) \partial\bar{T}/\partial z. \quad (10)$$

3. GOVERNING CONSTRAINT AND MEAN VERTICAL VELOCITY

The governing constraint that the mean vertical flux of dry air constituent should be zero is expressed by

$$\bar{w}\bar{\rho}_a = 0, \quad (11)$$

which, with the l.h.s. written as $\bar{w}\bar{\rho}_a + \bar{w}'\rho'_a$, gives the mean vertical velocity component

$$\bar{w} = -\bar{w}'\rho'_a/\bar{\rho}_a. \quad (12)$$

On replacement of $\bar{w}'\rho'_a$ using Eq. (9a) multiplied by \bar{w}' and averaged, Eq. (12) becomes

$$\bar{w} = \mu\bar{w}'\rho'_v/\bar{\rho}_a + (1 + \mu\sigma)\{\bar{w}'\bar{T}'/\bar{T} - \bar{w}'\bar{T}'^2/\bar{T}^2 + \bar{w}'\bar{T}'^3/\bar{T}^3 - \dots\} \times \{1 + \bar{T}'^2/\bar{T}^2 - \bar{T}'^3/\bar{T}^3 + \dots\}^{-1} \quad (13)$$

We ignore terms of order \bar{T}'^2/\bar{T}^2 relative to unity. We also ignore $\bar{w}'\bar{T}'^2/\bar{T}^2$ relative to \bar{w}'/\bar{T} - the ratio of these two is expected to be smaller in magnitude than T'/\bar{T} , since T'^2 is well correlated with w' than is T' . This becomes less true for the skewed T' distribution which occurs in convective conditions and which is illustrated in traces reproduced by Pries (1959, Fig. 19), Businger and Miyake (1968), and Webb (1977, Fig. 3); but the ratio remains no greater than a maximum of about the scale magnitude of T'/\bar{T} (Wyngaard Coté 1971, Fig. 11), and has been measured as only about 1.2×10^{-3} by Businger and Miyake. Thus, to a close approximation Eq. (13) becomes

$$\bar{w} = \mu\bar{w}'\rho'_v/\bar{\rho}_a + (1 + \mu\sigma)\bar{w}'\bar{T}'/\bar{T}.$$

4. EDDY COVARIANCE MEASUREMENTS

The flux of C is $F = \bar{w}\rho'_c$. With this written as

$$F = \bar{w}\bar{\rho}_c\bar{S} + (\bar{w}\rho'_c)S', \quad (14)$$

where $S = \rho'_c/\rho_c$, the assumption is frequently made that $\bar{w}\bar{\rho}_c = 0$. But this assumption is seen to be not quite correct in general, since Eq. (11) with Eq. (1) gives $\bar{w}\bar{\rho}_v = \bar{w}\bar{\rho}_a = I$

However, the flux F can be expressed in pure eddy covariance form (no mean flow term) by introduction of $s = \rho'_c/\rho_a$ and of the constraint Eq. (11). Thus we have $F = \bar{w}\bar{\rho}_a \bar{w}'\bar{\rho}_a' \bar{S} + (\bar{w}\rho'_a)S'$ i.e.

$$F = (\bar{w}\rho'_a)S'.$$

This expression is not likely to be useful in practice, as $(\bar{w}\rho'_a)S'$ is not a readily measurable quantity.

Usually the vertical component of flow is measured by means of a propeller-vane sonic anemometer, both of which respond to w itself. Other possibilities are measurement of ρw approximately by a hot-wire sensor and of ρw^2 by a pressure sensor. Here we assume throughout that it is w which is measured.

The pure covariance of w' with s' gives the flux F to a close approximation. This is shown either from the definition of s or alternatively via Eq. (16). First, for small fluctuations δ we have $\delta s = (\partial s/\partial \rho_c)\delta\rho_c + (\partial s/\partial \rho_a)\delta\rho_a$, i.e. to a close approximation

$$s' = (1/\bar{\rho}_a)\rho'_e - (\bar{\rho}_e/\bar{\rho}_a^2)\rho'_a, \quad (17)$$

$$\bar{w}s' = (1/\bar{\rho}_a)\bar{w}\rho'_e - (\bar{\rho}_e/\bar{\rho}_a^2)\bar{w}\rho'_a, \quad (18)$$

$$\text{We can write } F = \bar{w}'\rho'_e + \bar{w}\bar{\rho}_e, \quad (19)$$

from which, with Eqs. (12) and (18), we have to a close approximation

$$F = \bar{\rho}_a \bar{w}'s'. \quad (20)$$

(Neglected terms of higher order in the density fluctuations are small, as in the discussion preceding Eq. (14).)

Alternatively, Eq. (16) on replacement of $(w\rho_a)'$ by $w\rho_a = (\bar{w} + w')(\bar{\rho}_a + \rho'_a)$ becomes

$$F = \bar{w}\bar{\rho}_a s' + \bar{\rho}_a \bar{w}'s' + \bar{w}\rho'_a s'.$$

On the r.h.s. of this equation the middle term is dominant – the first term, on substitution for \bar{w} from Eq. (12), is seen to be much smaller, to the order of $(\rho'_a/\bar{\rho}_a)^2$, while the third term is also much smaller than the middle term as in the discussion preceding Eq. (14). Thus, we reproduce Eq. (20) to a close approximation.

For the special case of water vapour transfer, the foregoing manipulations are still valid with s replaced by the humidity mixing ratio $r = \rho_v/\rho_a$, leading to

$$E = \bar{\rho}_a \bar{w}'r'. \quad (21)$$

The important conclusion indicated by Eqs. (20) and (21) is that the covariance of w' and the mixing ratio relative to dry air constituent, s (or r as the case may be), gives the flux of C (or respectively of water vapour) to a close approximation, without need of correction for density variation effects.

Alternative expressions in terms of specific mass content S and specific humidity $q = \rho_v/\rho$, rather than the mixing ratios, are readily derived as follows. From $s = S/(1-q)$, noting that for small fluctuations $\delta s = (\partial s/\partial S)\delta S + (\partial s/\partial q)\delta q$, we have

$$s' = \{1/(1-q)\}\{S' + q'S/(1-q)\}$$

Replacement of the exterior $(1-q)$ approximately by $\bar{\rho}_a/\bar{\rho}$ (the validity of which becomes apparent on expansion of $q = \rho_v/\bar{\rho}(1+q'/\bar{q})$ as a power series in q'/\bar{q}), and insertion into Eq. (20), gives to a close approximation

$$F = \bar{\rho}[\bar{w}'S' + \{S/(1-q)\}\bar{w}'q']. \quad (22)$$

For the case of water vapour transfer, we have $r = q/(1-q)$ and thus approximately $r' = q'/(1-q)^2$, insertion of which into Eq. (21) gives to a close approximation

$$E = \bar{\rho}\bar{w}'q'/(1-q). \quad (23)$$

At present it is not likely that the fluxes of minor constituents could be determined by use of Eq. (20) or (22), in the absence of practical techniques for measuring the relative content s or S with sufficiently fast response. But in the case of water vapour flux, fast response psychrometry would probably provide suitable measurements of r or q for application of Eq. (21) or (23).

We now turn to the case where the fluctuations of actual density ρ_e of constituent C , or ρ_v of water vapour, are measured *in situ*. In Eq. (19), we substitute for \bar{w} from Eq. (14) and obtain immediately

$$F = \bar{w}'\rho'_e + \mu(\bar{\rho}_e/\bar{\rho}_a)\bar{\rho}_a \bar{w}'\rho'_e + (1+\mu\bar{\rho}_e/\bar{\rho}_a)\bar{\rho}_a \bar{w}'T'. \quad (24)$$

Eq 24 is the same as
Eq 42a and 44.

$$LE = LE_{raw} + \mu s K E_{raw} + (1+\mu s) \frac{\rho_v L_v w' T'}{T}$$

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Similarly, for the flux of water vapour, we have

$$E = (1+\mu\bar{\rho}_e/\bar{\rho}_a)\{w'\rho'_e + (\bar{\rho}_e/\bar{T})w'T'\}. \quad (25)$$

These give the fluxes in terms of measured covariances, incorporating the corrections which arise from the fluxes of both water vapour and heat.

5. MEAN GRADIENT MEASUREMENTS

For mean gradient (or difference) measurements there is a counterpart of each situation considered for covariance measurements. The connections between the two can be made via the formal mixing-length relationships.

Eq. (19) with substitution for \bar{w} from Eq. (12) gives

$$F = \bar{w}'\rho'_e - (\bar{\rho}_e/\bar{\rho}_a)\bar{w}'\rho'_a.$$

By introduction of the mixing length relationships $\rho'_e = -l\partial\bar{\rho}_e/\partial z$ and $\rho'_a = -l\partial\bar{\rho}_a/\partial z$ where l is the mixing length and z is height, this equation becomes

$$F = -\bar{w}'l\{\partial\bar{\rho}_e/\partial z - (\bar{\rho}_e/\bar{\rho}_a)\partial\bar{\rho}_a/\partial z\}$$

$$F = -\bar{\rho}_a K \frac{\partial}{\partial z} (\bar{\rho}_e/\bar{\rho}_a), \quad (26)$$

where $K = \bar{w}'l$ is the conventional turbulence transfer coefficient. (Here, and in the following, the gradient $\partial/\partial z$ can of course be replaced by the corresponding difference over a finite height range, with K appropriately redefined.) For the flux of water vapour, the same argument gives

$$E = -\bar{\rho}_a K \frac{\partial}{\partial z} (\bar{\rho}_v/\bar{\rho}_a). \quad (27)$$

An alternative approach starting from Eq. (20) gives the result in a very slightly different form. Since s is a ratio, the mixing-length relationship for s is in principle not exactly the same as that for an individual entity ρ_e , ρ_a , etc., but in practice the difference is likely to be quite negligible. Introducing $s' = -l\partial\bar{s}/\partial z$ in Eq. (20), we have

$$F = -\bar{\rho}_a K \partial\bar{s}/\partial z, \quad (28)$$

and similarly from Eq. (21)

$$E = -\bar{\rho}_a K \partial\bar{r}/\partial z. \quad (29)$$

To a very close approximation Eqs. (28) and (29) are the same as Eqs. (26) and (27). This can be seen since

$$\bar{s} = (\bar{\rho}_e/\bar{\rho}_a) = (\bar{\rho}_e/\bar{\rho}_a)(1 - \bar{\rho}_e/\bar{\rho}_a + \dots),$$

in which the second term in the right-hand brackets is very small compared with unity and also is not likely to vary strongly with height. Thus the gradient of either $\bar{\rho}_e/\bar{\rho}_a$ or \bar{s} can loosely be referred to as the gradient of mean mixing ratio. For most constituents the quantity measured in practice will be $\bar{\rho}_e/\bar{\rho}_a$ rather than \bar{s} .

Eqs. (26) and (27), or (28) and (29), are the counterparts of Eqs. (20) and (21). They indicate the conclusion that the flux of C (or of water vapour) can to a close approximation be evaluated directly from the vertical gradient or difference of mean mixing ratio of (or of water vapour) relative to dry air content, without need of correction for density variation effects. In the case of C , this ratio is effectively what is measured when the air is pre-dried before measurement of the content of C at a common temperature and pressure.

In terms of relative *volume* content, Eq. (26) converted by the application of Eq. (7a) is

$$F = -\bar{p}_N(m_e/m_a)K \frac{\partial}{\partial z} \left(\frac{\text{mean volume of C at STP}}{\text{mean volume of dry air at STP}} \right) \quad (30a)$$

or, from Eqs. (2a, c),

$$F = -\bar{p}_N K \frac{\partial}{\partial z} \left(\frac{\text{mean volume of C at STP}}{\text{mean volume of dry air at STP}} \right), \quad (30b)$$

where \bar{p}_N is the density of pure entity C at ambient temperature and pressure \bar{p}_a (i.e. pressure $p - \bar{p}_v$, which is *almost* the ambient total pressure p).

Conversion of Eqs. (26) and (27) to specific concentrations instead of mixing ratios is made analogously as in reaching Eqs. (22) and (23). Letting $r_m = \bar{p}_v/\bar{p}_a$, $q_m = \bar{p}_v/\bar{p}_a$, $s_m = \bar{p}_a/\bar{p}_a$ and $S_m = \bar{p}_v/\bar{p}_a$, we have $s_m = S_m/(1 - q_m)$ and hence we derive $\partial s_m/\partial z$, from which Eq. (26) becomes

$$F = -\bar{p} K \left\{ \frac{\partial}{\partial z} (\bar{p}_e/\bar{p}) + (\bar{p}_e/\bar{p}_a) \frac{\partial}{\partial z} (\bar{p}_v/\bar{p}) \right\}. \quad (31)$$

For the flux of water vapour we have $r_m = q_m/(1 - q_m)$ and hence $\partial r_m/\partial z$, from which Eq. (27) becomes

$$E = -\bar{p}(1 - \bar{p}_v/\bar{p})^{-1} K \frac{\partial}{\partial z} (\bar{p}_v/\bar{p}). \quad (32)$$

Turning now to the case where the vertical gradient (or difference) of the mean C density, \bar{p}_e , itself is measured *in situ*, we immediately derive the required correction by introducing into Eq. (24) the mixing-length expressions $\rho'_e = -l \partial \bar{p}_e/\partial z$, $\rho'_v = -l \partial \bar{p}_v/\partial z$ and $T' = -l \partial \bar{T}/\partial z$, which gives

$$F = -K \{ \partial \bar{p}_e/\partial z + \mu (\bar{p}_e/\bar{p}_a) \partial \bar{p}_v/\partial z + (1 + \mu \sigma) (\bar{p}_e/\bar{T}) \partial \bar{T}/\partial z \}. \quad (33)$$

The first term on the r.h.s. represents the raw (uncorrected) flux of C, while the other two terms represent the corrections for density variations associated with the mean humidity gradient and the mean temperature gradient. Again, the same argument for the particular case of water vapour transfer leads from Eq. (25) to the counterpart of Eq. (33) with \bar{p}_e replaced by \bar{p}_v , which is

$$E = -K(1 + \mu \sigma) \{ \partial \bar{p}_v/\partial z + (\bar{p}_v/\bar{T}) \partial \bar{T}/\partial z \}. \quad (34)$$

Suppose now that samples of the original moist air are brought to a common temperature T_1 and pressure p_1 in the chamber of an instrument (indicated by suffix 1) where the C density \bar{p}_{e1} is measured. From Eq. (5) and the invariance of m and of \bar{p}_e/\bar{p} for each sample, we have $\bar{p}_e = (p/p_1) (T_1/\bar{T}) \bar{p}_{e1}$, from which the substitution of $\partial \bar{p}_e/\partial z$ into Eq. (33) gives

$$F = -K(p/p_1)(T_1/\bar{T}) [\partial \bar{p}_{e1}/\partial z + \mu (\bar{p}_{e1}/\bar{p}_a) (\partial \bar{p}_v/\partial z + (\bar{p}_v/\bar{T}) \partial \bar{T}/\partial z)]. \quad (35a)$$

Here we see that the major part of the correction term in $\partial \bar{T}/\partial z$ has disappeared. Further, on substitution from Eq. (34) into Eq. (35a), we have

$$F = (p/p_1)(T_1/\bar{T}) \{ -K \partial \bar{p}_{e1}/\partial z + (\mu \bar{p}_{e1}/\bar{p}_a) (1 + \mu \sigma)^{-1} E \}, \quad (35b)$$

which gives the correction purely in terms of the water vapour flux, E .

For the case where both \bar{p}_{e1} and \bar{p}_{v1} are measured in the instrument at common temperature T_1 and pressure p_1 , we have also $\bar{p}_v = (p/p_1) (T_1/\bar{T}) \bar{p}_{v1}$, whence Eq. (35a) becomes

$$F = -K(p/p_1)(T_1/\bar{T}) \{ \partial \bar{p}_{e1}/\partial z + \mu (\bar{p}_{e1}/\bar{p}_a) \partial \bar{p}_{v1}/\partial z \}. \quad (36)$$

CORRECTION OF FLUX MEASUREMENTS

For the particular case of water vapour flux, with \bar{p}_{v1} measured in the instrument at common T_1 and p_1 , the counterpart of Eq. (36) is

$$E = -K(p/p_1)(T_1/\bar{T}) (1 + \mu \bar{p}_{v1}/\bar{p}_a) \partial \bar{p}_{v1}/\partial z. \quad (37)$$

We note in Eqs. (36) and (37) that the reduction of air samples to a common temperature has completely eliminated the correction in $\partial \bar{T}/\partial z$, leaving only the correction in terms of water vapour density gradient as measured. If the pressure and temperature in the instrument are not greatly different from those in the atmosphere, then Eqs. (36) and (37) reduce respectively to the forms of Eqs. (33) and (34) with their $\partial \bar{T}/\partial z$ terms removed. While Eqs. (36) and (37) are in a sense counterparts of Eqs. (31) and (32), the correspondence is not exact because \bar{p} still varies with z after reduction to a common instrument temperature and pressure.

6. RELATIONSHIPS IN TERMS OF FLUXES

It will often be useful to have the corrections in terms of the heat and water vapour fluxes (e.g. Eq. (35b)). First we must consider the heat flux H expressed in terms of eddy covariance quantities.

Recognizing the two parts of H carried by the dry air and water vapour constituent (specific heats at constant pressure denoted by c_{pa} and c_{pv} , respectively) we write

$$H = c_{pa} \overline{w \rho_a (T - T_b)} + c_{pv} \overline{w \rho_v (T - T_b)}. \quad (38)$$

Here T_b , taken as constant at any given height, represents roughly an assumed initial 'base' temperature from which each element of air is warmed (or cooled) during the vertical transfer of heat supplied (or removed) at the underlying surface. Even though T_b is not amenable to precise specification, it is included because the heat imparted to and carried by each parcel of air is represented by the temperature change, $T - T_b$, not the temperature itself. (T_b is different in concept and in value from the 'zero enthalpy temperature' defined by Montgomery (1948) to incorporate latent heat effects.)

In the dry air term of Eq. (38) we replace $T - T_b$ by T' (which is permissible since, in the term only, \bar{T} and T_b may be discarded in view of Eq. (11)), take $\rho_a = \bar{p}_a/\bar{p}_v$ with ρ'_a from Eq. (9a), and ignore terms of order \bar{T}^{-2}/\bar{T}^2 relative to unity. This leads to

$$H = (c_{pa} \bar{p}_a + c_{pv} \bar{p}_v) \overline{w' T'} + c_{pv} \overline{w' \bar{p}_v} (\bar{T} - T_b) - c_{pa} (\bar{p}_v/\bar{T}) (1 + \mu \sigma) \overline{w' T'^2} + (c_{pv} - \mu c_{pa}) \overline{w' T'}. \quad (39)$$

The first term on the r.h.s. of Eq. (39) is dominant, while in comparison the other three terms are small and can be neglected.* Introducing in the first term c_p , the specific heat of the (moist) air given by $c_p \bar{p} = c_{pa} \bar{p}_a + c_{pv} \bar{p}_v$, we have to a close approximation

$$H = c_p \overline{w' T'}. \quad (40)$$

i.e. the heat flux is closely represented by the uncorrected covariance alone.

It is worth noting that the absence of correction terms in Eq. (40) can be understood from the viewpoint of Eq. (20) - on considering the quantity of heat per unit mass of dry air constituent, one finds that the quantity $c_p \bar{p} T'$ in Eq. (40) is to a close approximation the counterpart of $\bar{p}_a s'$ in Eq. (20).

* On the r.h.s. of Eq. (39) the ratio of the second term to H is $c_{pv}/(c_p \bar{p}) \approx 0.76 \times 10^{-3} (T - T_b)/\beta$, where $\beta = H/(E F)$ is the Bowen ratio; by taking $T - T_b$ to be a little greater than T' , and thus having extreme values not greater than say 5°C for the largest heat fluxes with $\beta > 1$, say (or proportionately smaller $T - T_b$ for smaller β), we see that the second term will always be below 1% of H . The third term is small compared with H as discussed in connection with Eq. (33). The fourth term is generally smaller still, its ratio to the third term turning out to be roughly $0.031 \beta^{-1}$ when we take $(w p_1 T_1/\bar{p}_a) \bar{p}_a$ to be at most comparable with $(c_p \bar{p}) \bar{T}$.

From Eq. (40) the corresponding mean gradient form, which follows by introduction of the mixing-length relationship $T' = -l\partial T/\partial z$, is

$$H = -c_p \bar{\rho} K \partial \bar{T} / \partial z. \quad (41)$$

This likewise is valid to a close approximation without any correction term.

We can now express the corrections in terms of the fluxes H and E . Using Eq. (40) in Eq. (25) for eddy covariance measurements, or Eq. (41) in Eq. (34) for mean gradient measurements, we have in either case

$$E = (1 + \mu\sigma)\{E_{\text{raw}} + (\bar{\rho}_v/\bar{\rho})(H/c_p \bar{T})\}. \quad (42a)$$

This gives the corrected water vapour flux E from the raw uncorrected flux which is either $E_{\text{raw}} = \bar{w}'\bar{\rho}_v$ or $E_{\text{raw}} = -K\partial\bar{\rho}_v/\partial z$, as the case may be. An alternative form of Eq. (42a), with the correction appearing purely as a factor, is

$$E = (1 + \mu\sigma)\{1 + (\lambda/c_p)(\bar{\rho}_v/\bar{\rho})\bar{T}\}E_{\text{raw}}, \quad (42b)$$

where λ is the latent heat of evaporation and $\beta_{\text{raw}} = H/\lambda E_{\text{raw}}$ is the uncorrected Bowen ratio.

An expression for \bar{w} in terms of fluxes follows from Eq. (14) on substitution for $\bar{w}'\bar{\rho}_v$ and $\bar{w}'\bar{T}$ from Eqs. (42a) and (40):

$$\bar{w} = \{\mu/(1 + \mu\sigma)\}E/\bar{\rho}_a + H/c_p \bar{\rho} \bar{T}. \quad (43)$$

The expression for F then follows immediately from Eq. (19) on substitution for \bar{w} from Eq. (43), which gives

$$F = F_{\text{raw}} + (\bar{\rho}_v/\bar{\rho}_a)\{\mu/(1 + \mu\sigma)\}E + (\bar{\rho}_v/\bar{\rho})H/c_p \bar{T}. \quad (44)$$

Here the raw uncorrected flux of entity C is given by $F_{\text{raw}} = \bar{w}'\bar{\rho}_v$ or $F_{\text{raw}} = -K\partial\bar{\rho}_v/\partial z$ as the case may be, these two forms of F_{raw} being equivalent through the mixing-length relationship $\rho'_v = -l\partial\bar{\rho}_v/\partial z$.

7. QUANTITATIVE DISCUSSION

The correction to the flux of a minor constituent is given in terms of the fluxes of heat and water vapour by Eq. (44). When we take $m_v/m_a = 0.622$ with representative values $\bar{T} = 293$ K, $c_p = 1010$ J kg⁻¹ K⁻¹, and $\bar{\rho}_v/\bar{\rho} = 10$ mb/1000 mb, i.e. $\bar{\rho}_v/\bar{\rho}_a \approx 6.28 \times 10^{-3}$, then Eq. (44) becomes

$$F = F_{\text{raw}} + (\bar{\rho}_v/\bar{\rho}_a)(1.592E + 3.358 \times 10^{-6}H) \quad (45a)$$

or, with $\lambda = 2.453 \times 10^6$ J kg⁻¹,

$$F = F_{\text{raw}} + (\bar{\rho}_v/\bar{\rho}_a)(0.649 \times 10^{-6}E + 3.358 \times 10^{-6}H); \quad (45b)$$

here H and λE are in W m⁻² with F in kg m⁻² s⁻¹, while in Eq. (45a) E is in the same units as F .

For the particular case of CO₂ flux, F_{CO_2} , with the mean CO₂ content taken as 330 ppm by volume relative to dry air, which with $m_c/m_a = 44.01/28.96$ corresponds to $\bar{\rho}_c/\bar{\rho}_a = 0.5015 \times 10^{-3}$, Eqs. (45a) and (45b) become

$$F_{\text{CO}_2} = F_{\text{CO}_2\text{raw}} + (0.798 \times 10^{-3}E + 1.684 \times 10^{-9}H) \quad (46a)$$

$$F_{\text{CO}_2} = F_{\text{CO}_2\text{raw}} + (0.325 \times 10^{-9}E + 1.684 \times 10^{-9}H), \quad (46b)$$

again with H and λE in W m⁻² for F in kg m⁻² s⁻¹.

CORRECTION OF FLUX MEASUREMENTS

For the special case of water vapour flux, Eq. (42b) with the above values of physical quantities inserted gives

$$E = 1.010(1 + 0.051\beta_{\text{raw}})E_{\text{raw}}. \quad (47)$$

Note that for different values of the mean water vapour content, the decimal parts 0.01 and 0.051 representing the corrections in Eq. (47) must be adjusted proportionately, in accordance with Eq. (42b); on the other hand, the effect of altering the mean water vapour content is comparatively very minor in Eqs. (45a, b) and (46a, b), and in Eqs. (48a, b) below.

The mean vertical velocity component from Eq. (43), with the above values of physical quantities together with $\bar{\rho}_a = 1.2$ kg m⁻³, is given by

$$\bar{w} = 1326E + 2.798 \times 10^{-3}H \quad (48a)$$

$$\bar{w} = 0.541 \times 10^{-3}\lambda E + 2.798 \times 10^{-3}H. \quad (48b)$$

or

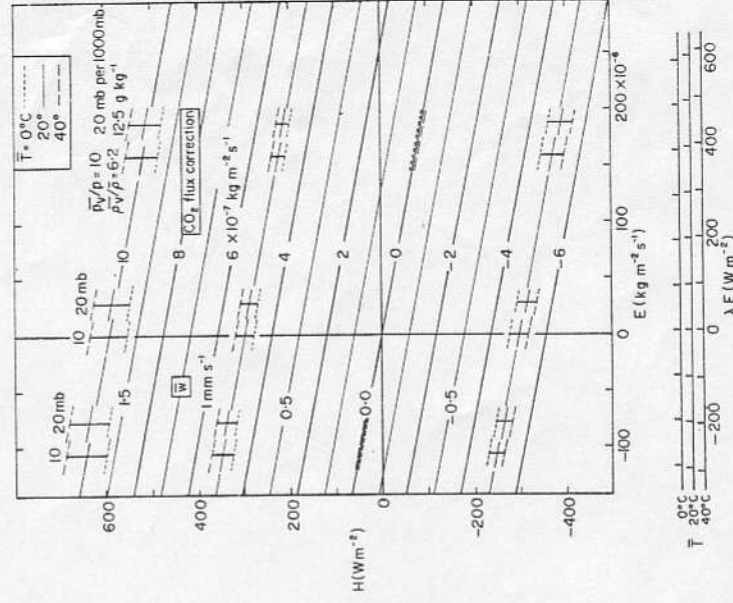


Figure 1. Isopleths showing the additive correction to CO₂ flux (labelled on r.h.s.), and also the mean vertical airspeed, \bar{w} (labelled on l.h.s.), for given fluxes of heat, H (ordinate) and water vapour, E , or latent heat, λE (abscissa). Main plotting is for mean temperature $\bar{T} = 20^\circ\text{C}$ and mean water vapour partial pressure $\bar{p}_v/\bar{p}_a = 10$ mb per 1000 mb total pressure; pairs of small bars show other combinations (some of which would imply supersaturation but are included to facilitate interpolation).

CO₂ flux correction is plotted from Eqs. (44) and (46a, b), and is applicable when flux is from measurements or mean gradient of CO₂ density *in situ*. Intercepts on abscissa axis also give corrections regardless of heat flux, for mean gradient measurements with air from different levels brought to a common temperature and pressure comparable with those *in situ* - Eq. (35b). Based on mean CO₂ mixing ratio 330 ppmv, i.e. $\bar{\rho}_c/\bar{\rho}_a = 0.5015 \times 10^{-3}$, applicable for other constituent mixing ratios by conversion corrections in proportion to $\bar{\rho}_c/\bar{\rho}_a$.

\bar{w} is plotted from Eqs. (43) and (48a, b) for dry air mean partial density $\bar{\rho}_a = 1.2$ kg m⁻³; for other densities take \bar{w} proportional to $\bar{\rho}_a^{-1}$.

Here \bar{w} is in mm s^{-1} , E in $\text{kg m}^{-2} \text{s}^{-1}$, and H and λE in W m^{-2} . Note that for different mean air densities, the coefficients in Eqs. (48a) and (48b) are proportional to \bar{p}_a^{-1} .

The correction required for CO_2 flux, given by the quantity added to F_{raw} in Eq. (44), is plotted in Fig. 1 for a variety of conditions of mean temperature and humidity. Here the mean CO_2 content is taken to be 330 ppm by volume relative to dry air, i.e. $\bar{p}_c/\bar{p}_a = 0.5015 \times 10^{-3}$. Fig. 1 may also be applied for minor constituents at other values of mean mixing ratio, by modifying the corrections in proportion to \bar{p}_c/\bar{p}_a .

The intercepts on the horizontal axis $H = 0$ in Fig. 1 also represent the case of mean gradient measurements with any value of heat flux, when air from different heights is brought to a common pressure and temperature not greatly different from the *in situ* values. This follows from Eq. (35b).

In Fig. 1 the isopleths also serve to indicate the mean vertical velocity component, \bar{w} , from Eq. (43). Values of \bar{w} are shown in Fig. 1 for $\bar{p}_a = 1.2 \text{ kg m}^{-3}$ (and of course have no connection with \bar{p}_c/\bar{p}_a); for other values of air density, modify \bar{w} in proportion to \bar{p}_a^{-1} .

The water vapour flux correction factor as given by Eq. (42b) is plotted in Fig. 2. The small intercepts on the ordinate axis reproduce Eq. (37), representing the case of mean gradient measurements with any value of heat flux, when air from different heights is brought to a common pressure and temperature not greatly different from the *in situ* values.

In the corrections and similarly in \bar{w} , it is apparent that the effect of heat flux is greater than that of a comparable latent heat flux by a factor of about five. From Eq. (44), the ratio of the two effects in the correction for F is $\mu^{-1}(1 + \mu\sigma)(\lambda/c_p T)(\bar{p}_a/\bar{p})\beta$ (where β is the Bowen ratio, $H/\lambda E$), and the same expression applies also in \bar{w} from Eq. (43). From Eq. (42b), the ratio in the case of E is $\mu^{-1}(\lambda/c_p T)(\bar{p}_a/\bar{p})\beta_{\text{raw}}$. Over a fairly wide range of conditions, these two expressions remain close to 5.2β and $5.1\beta_{\text{raw}}$, respectively.

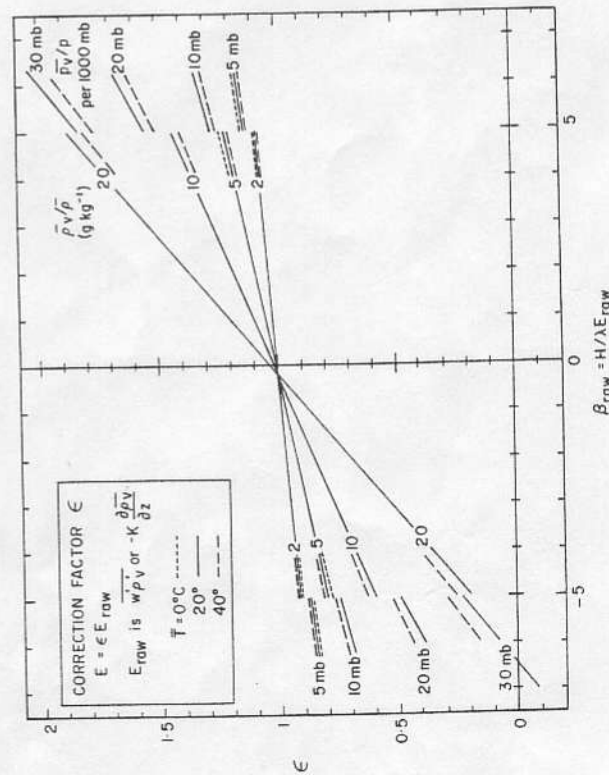


Figure 2. Correction factor for water vapour flux from measured fluctuations or mean gradient of water vapour density, plotted against measured Bowen ratio, β_{raw} , from Eqs. (42b) and (47). Intercepts on ordinate axis also represent correction factors, regardless of heat flux, for mean gradient measurements with air from different levels brought to a common temperature and pressure comparable with those *in situ* - Eq. (37).

CORRECTION OF FLUX MEASUREMENTS

The required additive correction to F is proportional to the mean CO_2 density \bar{p}_c , with the raw flux, F_{raw} , is itself proportional to the variations of ρ_c , i.e. to the change of \bar{p}_c with height or to the fluctuations, ρ'_c . Thus, the smaller the variations relative to the mean density the larger will be the correction relative to the raw flux of the entity concerned. For CO_2 the ratio of density variations to the mean is typically about one tenth of that for water vapour, and we can therefore expect that the relative corrections to CO_2 flux will commonly be an order of magnitude larger than those of water vapour flux.

The mean vertical speed, \bar{w} , associated with the flux corrections is very small indeed, commonly less than 1 mm s^{-1} (Fig. 1). In eddy covariance measurements, it is highly unlikely

TABLE 1. EXAMPLES OF CO_2 FLUXES AND CORRECTIONS
 CO_2 fluxes evaluated from mean gradient measurements

Reference	Date	Heat flux	Latent heat flux	Corresponding	CO_2 flux
Nature of surface	Time	H	λE	from H	including corrections
CO_2 measurement		(W m^{-2})	(W m^{-2})	$(\text{kg m}^{-2} \text{s}^{-1})$	$(\text{kg m}^{-2} \text{s}^{-1})$
Whether correction required					
Brown and Rosenberg (1971)	4 Sep. 66	110	210	$10^{-7} \times 1.9$	$10^{-7} \times 0.7$
Fig. 2 (c), approximately	08	130	330	2.2	1.1
from plotted points:	09	180	500	3.0	1.6
hourly values.	11	140	550	2.4	1.8
Irrigated sugar beet.	12	130	590	2.2	1.9
Air dried and (presumably)	13	50	650	0.8	2.1
brought to common	14	0	550	0.0	1.8
temperature.	15	-70	540	-1.2	1.7
CO_2 flux does not require	16	-90	320	-1.5	1.0
correction.	17	-40	90	-0.7	0.3
Paltridge <i>et al.</i> (1972),	30 Oct. 71	250	290	$10^{-7} \times 4.2$	$10^{-7} \times 0.9$
fluxes (not published) -	10	330	330	5.6	1.1
personal communication:	11	380	320	6.4	1.0
hourly values.	12	340	320	5.7	1.0
Wheat field.	13	260	260	4.4	0.8
Air dried and brought to	14				
common temperature,					
CO_2 flux does not require					
correction.					
Ripley and Saugier (1975)	11 May 71	85	42	$10^{-7} \times 1.43$	$10^{-7} \times 0.14$
Fig. 2, approximately from	21 May 71	42	60	0.71	0.20
plotted points: 10-day	31 May 71	49	78	0.83	0.25
means over daytime values.	10 June 71	30	86	0.51	0.28
Native grass land.	20 June 71	29	100	0.49	0.32
Air dried and brought to	30 June 71	33	98	0.56	0.32
common temperature, but	10 July 71	21	100	0.35	0.32
CO_2 content recalculated	20 July 71	20	125	0.34	0.34
as for original moist air,	30 July 71	53	94	0.89	0.31
CO_2 flux requires	9 Aug. 71	48	60	0.81	0.20
correction for λE	19 Aug. 71	49	48	0.83	0.16
	29 Aug. 71	57	37	0.96	0.12
	8 Sep. 71	46	32	0.77	0.10
	18 Sep. 71	26	30	0.44	0.10
	28 Sep. 71	39	19	0.66	0.06

* Corrections to CO_2 flux, from Eq. (46b) or Fig. 1, which would be required if mean or fluctuating CO_2 density were measured in unmodified air. In the case of mean gradient measurements, if the air is also pre-dried, then no correction for λE is required - Eqs. (35b) and (36); if the air is also pre-dried, then no correction for λE is required - Eq. (26).

that such small \bar{w} could be measured reliably, and it follows that there is virtually no prospect of dispensing with the need for a correction by measuring $\bar{w}\bar{p}_e$ instead of $\bar{w}\bar{p}_e$.

Table 1 lists some results from three published investigations in which daytime fluxes of CO_2 , heat and latent heat were all measured, and shows the corrections which would be required if the CO_2 density variations had been measured in the unmodified air. The daytime fluxes of CO_2 over the different surfaces range from zero to about $-10 \times 10^{-7} \text{ kg m}^{-2} \text{ s}^{-1}$. While the correction amounts typically to around one half of the CO_2 flux itself in the first set of data, it is comparable with or larger than the flux in the second set, where the heat flux is fairly large, and in the third set where the CO_2 flux is comparatively small. It is evident that the CO_2 flux correction is extremely important.

The data in Table 1 also exemplify the generally smaller relative corrections to the water vapour flux. The Bowen ratios range from less than 0.2 to just over 2, and with a moderate value of mean humidity assumed, Eq. (47) gives relative corrections to E ranging from under 2% up to 12% in these data. It is evident that the correction to E , even if negligible in some cases, will sometimes be significant.

8. CONCLUSIONS

The main conclusions concerning corrections for the density variation effects on the measured flux of a minor constituent (or of water vapour as a special case) are as follows. They are based on the governing constraint of zero mean vertical mass flux of dry air constituent, and are relevant for both eddy covariance and mean gradient types of measurements.

If the flux is evaluated from measured fluctuations or mean gradient of the *mixing ratio* of the constituent relative to the dry air component, then no correction is needed - Eqs. (20), (21), (26)-(29). This corresponds to the practice (common in CO_2 mean gradient measurements) of pre-drying air from different heights and bringing each to a common temperature before measuring the constituent's density. If the constituent's *specific mass content* relative to the total moist air is measured, then a correction arising only from the water vapour flux is required - Eqs. (22), (23), (31) and (32); this is also the requirement when the mean gradient of the constituent's density is measured in air brought to a common temperature without pre-drying - Eqs. (35a, b), (36) and (37).

It is when the measurement of the constituent's *density* fluctuations or mean gradients is made in unmodified *in situ* air that corrections arising from the fluxes of both heat and water vapour are required. These are given for eddy covariance measurements in Eqs. (24) and (25), for mean gradient measurements in Eqs. (33) and (34), and in terms of the *fluxes* of heat and water vapour in Eqs. (42a, b), (44), (45a, b), (46a, b) and (47), and in Figs. 1 and 2.

Associated with the density effects is a very small mean vertical airspeed, given by Eqs. (14), (43) and (48a, b), and in Fig. 1.

The effect due to heat flux is about five times as great as that due to an equal latent heat flux. The greater the mean density of a constituent compared with its transport-related fluctuations or mean gradient of density, the greater will be its required relative flux correction. In the case of CO_2 transfer, the magnitude of the correction will commonly exceed that of the CO_2 flux itself. The correction to the flux of water vapour will often amount to only a few per cent, but can well exceed 10% when the Bowen ratio is around 2 or greater.

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CORRECTION OF FLUX MEASUREMENTS

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REFERENCES

- Bakan, S. 1978 Note on the eddy correlation method for CO_2 flux measurements, *Boundary-Layer Met.*, **14**, 597-600.
- Bird, R. B., Stewart, W. E. and Lightfoot, E. N. 1960 *Transport phenomena*, Wiley, New York and London.
- Brown, K. W. and Rosenberg, N. J. 1971 Energy and CO_2 balance of an irrigated sugar beet (*vulgaris*) field in the Great Plains, *Agron. J.*, **63**, 201-207.
- Businger, J. A. and Miyake, M. 1968 Justification for neglecting the third moment in the state expression for the turbulent heat-flux near ground, *Quart. J. R. Met. Soc.*, **94**, 206-207.
- Calder, K. L. 1949 The criterion of turbulence in a fluid of variable density with particular reference to conditions in the sphere, *ibid.*, **75**, 71-88.
- Dennead, O. T. 1969 Comparative micrometeorology of a wheat field and a field of *Pinus radiata*, *Agric. Met.*, **6**, 357-371.
- Dennead, O. T., Simpson, J. R. and Freney, J. R. 1974 Ammonia flux into the atmosphere from a grazed pasture, *Science*, **185**, 609-610.
- Desjardins, R. L. and Lemon, E. R. 1974 Limitations of an eddy-correlation technique for determination of the carbon dioxide and sensible fluxes, *Boundary-Layer Met.*, **5**, 475-488.
- Galbally, I. E. 1971 Ozone profiles and ozone fluxes in the atmospheric boundary layer, *Quart. J. R. Met. Soc.*, **97**, 18-29.
- Grew, K. E. 1969 Thermal diffusion. In *Transport phenomena in fluid media*, H. J. M. Hanley, Marcel Dekker, New York and London, 333-376.
- Grew, K. E. and Ibbes, T. L. 1952 *Thermal diffusion in gases*, Cambridge University Press.
- Hirschfelder, J. O., Curtiss, C. F. and Bird, R. B. 1964 *Molecular theory of gases and liquids*, Wiley, New York and Sydney.
- Hyson, P. and Hicks, B. B. 1975 A single-beam infrared hygrometer for evaporation measurement, *J. Appl. Met.*, **14**, 301-307.
- Janae, J., Catsky, J., Jarvis, P. G. and others 1971 Infra-red gas analysers and other physical analyses for plant photosynthetic production, *manual of methods* (ed. Z. Sesak, J. Catsky and P. G. Jarvis), Dr W. N.V., The Hague, 111-197.
- Jones, E. P. and Smith, S. D. 1977 A first measurement of sea-air CO_2 flux by eddy-correlation, *J. Geophys. Res.*, **82**, 5990-5992.
- Kimball, B. A. and Lemon, E. R. 1978 The air density correction to eddy flux measurements, *Boundary-Layer Met.*, **15**, 357-360.
- Monchick, L. and Mason, E. A. 1970 Spectra of air pressure fluctuations at the soil surface, *Geophys. Res.*, **75**, 6771-6777.
- Monteith, J. L. and Szeicz, G. 1967 Free-flight theory of gas mixtures, *Phys. Fluids*, **10**, 1390.
- Montgomery, R. B. 1960 The carbon-dioxide flux over a field of sugar beet, *J. R. Met. Soc.*, **86**, 205-214.
- Ohtaki, E. and Seo, T. 1948 Vertical eddy flux of heat in the atmosphere, *J. Meteorol.*, **26**, 265-274.
- Paltridge, G. W., Dilley, A. C., Garratt, J. R., Pearman, G. I., Shepherd, W. and Connor, D. J. 1972 The Rutherglen experiment on Sherpa wheat: environmental and biological data, *CSIRO Aust. Div. Atm. Phys. Tech. Pap.* No. 22.
- Parkinson, K. J. 1971 Carbon dioxide infra-red gas analysis, *J. Exp. Bot.*, **169**-176.
- Pearman, G. I. 1975 A correction for the effect of drying of air samples on significance to the interpretation of atmospheric measurements, *Tellus*, **27**, 311-317.
- Pearman, G. I. and Garratt, J. R. 1973 Carbon dioxide measurements above a wheat crop: Observations of vertical gradients and concentrations, *Agric. Met.*, **12**, 13-25.

- Priestley, C. H. B. 1959 *Turbulent transfer in the lower atmosphere*, Univ. Chicago Press.
- Ripley, E. A. and Saugier, B. 1972 Micrometeorology: I. Description of sensors and measurement programme, Matador Project, *Technical Report No. 4*, Univ. of Saskatchewan, Saskatoon.
- 1975 Energy and mass exchange of a native grassland in Saskatchewan. In *Heat and mass transfer in the biosphere* (ed. D. A. de Vries and N. H. Afgan), Wiley, New York, London, Sydney, and Toronto, 311–325.
- Saugier, B. and Ripley, E. A. 1978 Evaluation of the aerodynamic method of determining fluxes over natural grassland, *Quart. J. R. Met. Soc.*, **104**, 257–270.
- Spittlehouse, D. L. and Ripley, E. A. 1977 Carbon dioxide concentrations over a native grassland in Saskatchewan, *Tellus*, **29**, 54–65.
- Uchijima, Z. 1970 Carbon dioxide environment and flux within a corn crop canopy. In *Prediction and measurement of photosynthetic productivity* (ed. I. Setlik), Centre for Agricultural Publishing and Documentation, Wageningen, 179–196.
- Verma, S. B. and Rosenberg, N. J. 1976 Carbon dioxide concentration and flux in a large agricultural region of the Great Plains of North America, *J. Geophys. Res.*, **81**, 399–405.
- Webb, E. K. 1977 Convection mechanisms of atmospheric heat transfer from surface to global scales. In *Second Australasian Conference on Heat and Mass Transfer*, University of Sydney, 523–539.
- Webb, E. K. and Pearman, G. I. 1977 Correction of CO₂ transfer measurements for the effect of water vapour transfer. In *Second Australasian Conference on Heat and Mass Transfer*, University of Sydney, 469–476.
- Wyngaard, J. C. and Coté, O. R. 1971 The budgets of turbulent kinetic energy and temperature variance in the atmospheric surface layer, *J. Atmos. Sci.*, **28**, 190–201.

An experimental study of the profiles of wind speed, shearing stress and turbulence at the crest of a large hill

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SUMMARY

Measurements have been made, under neutral conditions, of the turbulent wind structure on a 100-m tower at the crest of a hill of height $h = 170$ m. A local velocity maximum, or 'jet', was observed at about $h/5$, below which the ratio of increase in wind speed to the upwind value at the same height was 1.07, and almost independent of height near the surface.

The r.m.s. turbulence components σ_u , σ_v and σ_w were also approximately doubled near the surface, whereas σ_u and σ_v both decreased with height, the surface increase in σ_w extended over the entire height measurement. Reynolds stress was almost uniform above the 'jet', but increased sharply towards the surface to 3–5 times the upwind value. Departures from neutrality affected the profile shape and the magnitude of turbulence fluctuations very markedly.

Although the size of hill lay outside the range of validity of an analytical theory of Jackson and Hunt, their expressions for velocity and horizontal pressure gradient were in good agreement with the observations. Numerical theories by Frost, Harper and Fichtl and by Taylor, for hills of different shape, but similar slope to the experimental hill, also produced values of surface stress and velocity increase of the magnitude observed.

1. INTRODUCTION

During the past two decades steady progress has been made in efforts to formulate the behaviour and characteristics of the atmospheric surface layer under steady-state equilibrium conditions, in which, over a limited depth of the atmosphere, horizontal shear stress and the vertical fluxes of heat, moisture and momentum are assumed to be constant. Vertical gradients dominate over horizontal gradients; and pressure gradients are insignificant. Results from a number of field experiments, interpreted against the principles of Monin–Obukhov similarity, have established dimensionless forms for the flux–profile relationships and for the turbulence structure, as functions of thermal stability. Panofsky (1974) provided a comprehensive summary of the important aspects; Dyer (1974) and Yaglom (1977) commented on disagreement between various sets of experimental data.

These formulations provide a working basis for further studies of, for example, agricultural and forest meteorology, pollution dispersal, or building aerodynamics, provide that the basic requirement of an extensive, flat, horizontally homogeneous surface can be assumed. In many examples of practical importance, however, the crop, forest, building complex, or pollutant source is situated on a sloping site or amongst hilly topography, where one-dimensional assumptions are unjustified, and predictive models of atmospheric transport processes must take account of the influence of terrain on airflow.

Acceleration of the wind up a hill is a matter of common experience. Associated with the development of horizontal and vertical wind gradients, however, changes are expected in the fields of turbulence, shearing stress and surface drag, and in the creation of pressure gradients. Such modifications to the wind structure must clearly influence the mechanism of turbulent diffusion, heat and mass transfer, soil erosion and wind loads on structures. Building codes (Wyatt 1971) recognize the effect of acceleration up exposed slopes by the application of an arbitrary loading factor, but take no account of the generation of dynamic forces by increased turbulence. The US Federal Energy Administration Task Force Report