CSCI3230 / ESTR3108 2023-24 First Term Assignment 1

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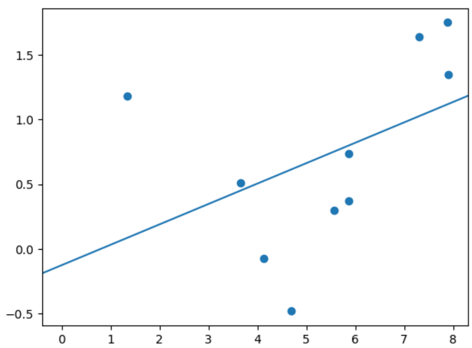
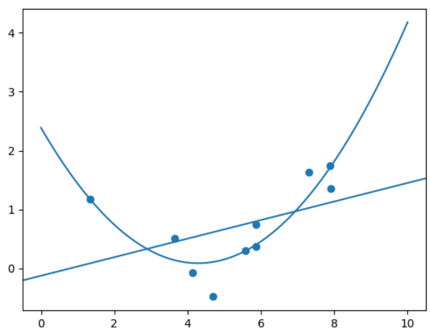
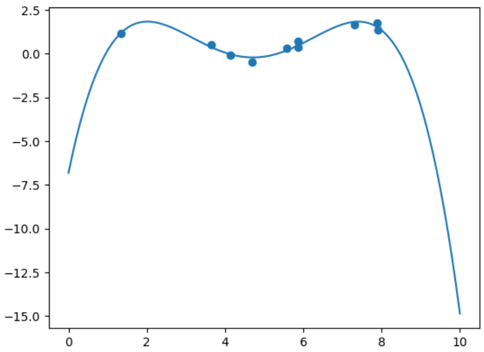


1a's figure.



1b's figure.



1c’s figure.

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1.a. $\hat{\theta} = (X^T X)^{-1} X^T Y$ since X is invertible

$$= \left[\begin{pmatrix} 5.86 & \cdots & 7.89 \end{pmatrix} \begin{pmatrix} 5.86 \\ \vdots \\ 7.89 \end{pmatrix} \right]^{-1} \begin{pmatrix} 5.86 & \cdots & 7.89 \end{pmatrix} \begin{pmatrix} 0.74 \\ \vdots \\ 1.75 \end{pmatrix}$$

$$= \left[\begin{pmatrix} 1 & \cdots & 1 \\ 5.86 & \cdots & 7.89 \end{pmatrix} \begin{pmatrix} 1 & 5.86 \\ \vdots & \vdots \\ 1 & 7.89 \end{pmatrix} \right]^{-1} \begin{pmatrix} 1 & \cdots & 1 \\ 5.86 & \cdots & 7.89 \end{pmatrix} \begin{pmatrix} 0.74 \\ \vdots \\ 1.75 \end{pmatrix}$$

$$\simeq \begin{pmatrix} -0.125 \\ 0.1576 \end{pmatrix}$$

b. Answer $\simeq \begin{pmatrix} 2.389 \\ -1.0704 \\ 0.125 \end{pmatrix}$

c. Answer $\simeq \begin{pmatrix} -6.808 \\ 10.917 \\ -4.605 \\ 0.732 \\ -0.039 \end{pmatrix}$

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1d. Q1a's may have underfitting
Q1c's may have overfitting
Q1b's may be a relatively good one

1a's RSS ~~prediction error~~ ≈ 9.4156
1b's RSS ≈ 5.0134
1c's RSS ≈ 19.5045
1a's expected prediction error ≈ 0.94156
1b's ≈ 0.50134
1c's ≈ 1.95045

2b. $\theta_0 = \bar{Y} - \bar{X}\theta$

$$J(\theta, \theta_0) = \sum_{i=1}^{m} \left(\quad \right)^2 + \lambda \|\theta\|_2^2$$

$$\left[(X^{(i)} - \bar{X})\theta + \theta_0 + \bar{X}\theta - Y^{(i)} \right]$$

$$= \sum_{i=1}^{m} \left(X_c^{(i)}\theta + \bar{Y} - Y^{(i)} \right)^2 + \lambda \|\theta\|_2^2$$

$$= \sum_{i=1}^{m} \left(X_c^{(i)}\theta - Y_c^{(i)} \right)^2 + \lambda \|\theta\|_2^2$$

$$= J_c(\theta)$$

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3 a. $P(\hat{y}=1 \mid x_1, x_2)$

$$= \frac{1}{1+e^{-(1.5x_1+0.5x_2-1)}}$$

$$= \frac{1}{1+e^{(1-1.5x_1-0.5x_2)}}$$

Cross-entropy error function:

$$= \sum_{i=1}^{m} -y^{(i)} \ln\left(\frac{1}{1+e^{(1-1.5x_1^{(i)}-0.5x_2^{(i)})}}\right) - (1-y^{(i)}) \ln\left(1-\frac{1}{1+e^{(1-1.5x_1^{(i)}-0.5x_2^{(i)})}}\right),$$

$y \in \{0,1\}$

b. $E(\theta)$ = cross-entropy error function. $\theta = \begin{pmatrix} -1 \\ 1.5 \\ 0.5 \end{pmatrix}$

$$\nabla E(\theta) = \sum_{i=1}^{m} \left(\frac{1}{1+e^{(1-1.5x_1^{(i)}-0.5x_2^{(i)})}} - y^{(i)}\right) \begin{pmatrix} 1 \\ x_1^{(i)} \\ x_2^{(i)} \end{pmatrix}$$

new $\theta = \theta - 0.1 \nabla E(\theta)$

$$\approx \theta - 0.1 \cancel{\begin{pmatrix} 1.386 \\ -1.09 \\ -0.757 \end{pmatrix}} \begin{pmatrix} 0.18998 \\ -0.32105 \\ -0.1161 \end{pmatrix}$$

$$\approx \cancel{\begin{pmatrix} -1.136 \\ 1.609 \\ 0.5757 \end{pmatrix}} \begin{pmatrix} -1.0019 \\ 1.5321 \\ 0.50118 \end{pmatrix}$$

c. True positive = 3   False Positive = 2   Accuracy = $\frac{2}{3}$ Recall = 1

True Negative = 1   False negative = 0   Precision = $\frac{3}{5}$

~~4a. for j=i,~~ $\cancel{\dfrac{\partial P(\hat{y}_i=1|X)}{\partial X^T\theta_j} = \dfrac{e^{X^T\theta_i}}{\sum_{k=1}^{K} e^{X^T\theta_k}} \times \left(1-\dfrac{e^{X^T\theta_i}}{\sum_{k=1}^{K} e^{X^T\theta_k}}\right)}$

$$\cancel{= \left(\frac{e^{X^T\theta_i}}{\sum_{k=1}^{K} e^{X^T\theta_k}}\right) - \left(\frac{e^{X^T\theta_i}}{\sum_{k=1}^{K} e^{X^T\theta_k}}\right)^2}$$

4a. for $j=i$,

$$\frac{\partial}{\partial X^T\theta_j}\left(\frac{e^{X^T\theta_i}}{\sum_{k=1}^{K} e^{X^T\theta_k}}\right) = \frac{\partial}{\partial X^T\theta_j}\left[\left(e^{X^T\theta_i}\right)\left(\sum_{k=1}^{K} e^{X^T\theta_k}\right)^{-1}\right]$$

$$= \frac{e^{X^T\theta_i}}{\sum_{k=1}^{K} e^{X^T\theta_k}} + e^{X^T\theta_i} \times \frac{1}{\sum_{k=1}^{K} e^{X^T\theta_k}} e^{X^T\theta_i} \times (-1)$$

$$= P(\hat{y}_i=1|X)(1-P(\hat{y}_j=1|X))$$

for $j \neq i$,

$$\frac{\partial}{\partial X^T\theta_j}\left(\frac{e^{X^T\theta_i}}{\sum_{k=1}^{K} e^{X^T\theta_k}}\right) = 0\left(\sum_{k=1}^{K} e^{X^T\theta_k}\right)^{-1} + \frac{e^{X^T\theta_i} \times e^{X^T\theta_j} \times (-1)}{\sum_{k=1}^{K} e^{X^T\theta_k}}$$

$$= -P(\hat{y}_i=1|X)P(\hat{y}_j=1|X)$$

4b $$\frac{\partial P(\hat{y}_i=1|X)}{\partial \theta_j} = \frac{\partial P(\hat{y}_i=1|X)}{\partial X^T\theta_j}\frac{\partial X^T\theta_j}{\partial \theta_j}$$

$$= -P(\hat{y}_i=1|X)P(\hat{y}_j=1|X)X \quad \text{if } i \neq j$$

$$= P(\hat{y}_i=1|X)(1-P(\hat{y}_j=1|X))X \quad \text{if } i=j$$

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4(. $\frac{\partial E(\theta)}{\partial \theta_j} = \frac{\partial E(\theta)}{\partial P(\hat{Y}_i=1|X)} \frac{\partial P(\hat{Y}_i=1|X)}{\partial X^T\theta_j} \frac{\partial X^T\theta_j}{\partial \theta_j}$

$\cancel{= -Y_j \frac{1}{P(\hat{Y}_i=1|X)} \frac{\partial P(\hat{Y}_i=1|X)}{\partial X^T\theta_j} \frac{\partial X^T}{\partial \theta}}$

$= \frac{\partial}{\partial \theta_j}\left(-\sum_{i=j} Y_i \ln P(\hat{Y}_i=1|X)\right)$

$+ \frac{\partial}{\partial \theta_j}\left(-\sum_{i\neq j} Y_i \ln P(\hat{Y}_i=1|X)\right)$

$= -\sum_{i=j} Y_i \frac{1}{P(\hat{Y}_i=1|X)} \times P(\hat{Y}_i=1|X)(1-P(\hat{Y}_j=1|X))X$

$+ \sum_{i\neq j} Y_i \frac{1}{P(\hat{Y}_i=1|X)} \times P(\hat{Y}_i=1|X)(P(\hat{Y}_j=1|X))X$

$= \sum_{i=j} (P(\hat{Y}_i=1|X)-1)Y_i X$

$+ \sum_{i\neq j} P(\hat{Y}_j=1|X)Y_i X$

$= P(\hat{Y}_j=1|X)Y_j X - Y_j X^T + \sum_{i\neq j} P(\hat{Y}_j=1|X)Y_i X$

$= -Y_j X + \sum_{k=1}^{K} P(\hat{Y}_j=1|X)Y_i X$

$= -Y_j X + 1\times P(\hat{Y}_j=1|X) X$

$= XP(\hat{Y}_j=1|X) - XY_j$

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2c. $\cancel{\frac{d J_c(\theta)}{d\theta} = \sum_{i=1}^{m} X_c^{(i)} + 2\lambda \|\theta\|_2}$

$$J_c(\theta) = \sum_{i=1}^{m} (X_c^{(i)} \theta - Y_c^{(i)})^2 + \lambda \|\theta\|_2^2$$

$$= \|x_c \theta - y_c\|^2 + \lambda \|\theta\|_2^2, \quad x_c = \begin{pmatrix} X_c^{(1)} \\ \vdots \\ X_c^{(m)} \end{pmatrix} \quad y_c = \begin{pmatrix} Y_c^{(1)} \\ \vdots \\ Y_c^{(m)} \end{pmatrix}$$

$$= (x_c \theta - y_c)^T (x_c \theta - y_c) + \lambda \theta^T \theta$$

$$= \theta^T x_c^T x_c \theta - y_c^T x_c \theta - \theta^T x_c^T y_c - y_c^T y + \lambda \theta^T \theta$$

$$\frac{\partial J_c(\theta)}{\partial \theta} = 2 x_c^T x_c \theta - x_c^T y_c - x_c^T y_c + 2 \lambda I \theta = 0$$

$$(x_c^T x_c + \lambda I)\theta - x_c^T y_c = 0$$

$$\theta = (x_c^T x_c + \lambda I)^{-1} (x_c^T y_c)$$

$$\frac{\partial^2 J_c(\theta)}{\partial \theta} = (x_c^T x_c + \lambda I) > 0 \text{ and } J_c(\theta) \text{ is convex}$$

$\therefore J_c(\theta)$ is in global min when $\frac{\partial J_c(\theta)}{\partial \theta} = 0$,

$$\min_{\theta}(J_c(\theta)) = (x_c^T x_c + \lambda I)^{-1} (x_c^T y_c)$$

2a. $\frac{\partial J(\theta, \theta_0)}{\partial \theta_0} = \sum_{i=1}^{m} 2\left[(X^{(i)} - \bar{x})\theta + \theta_0 + \bar{x}\theta - Y^{(i)}\right]$

$$= 2\sum_{i=1}^{m}\left[X^{(i)}\theta + \theta_0 - Y^{(i)}\right] = 0$$

$$m\theta_0 + \sum_{i=1}^{m} X^{(i)}\theta - \sum_{i=1}^{m} Y^{(i)} = 0$$

$$\theta_0 = \bar{Y} - \bar{x}\theta$$

Since $J(\theta, \theta_0)$ is convex, $\theta_0 = \bar{Y} - \bar{x}\theta$,

~~can make~~ $J(\theta, \theta_0)$ is in global minimum.

and $\frac{\partial^2 J(\theta, \theta_0)}{\partial \theta_0^2} = m \geq 0$.

So when

Q1

```
import numpy as np;
import matplotlib.pyplot as plt;

def OLS(XT,y):
    temp=np.matmul(XT,XT.T);
    temp=np.linalg.inv(temp);
    temp=np.matmul(temp,XT);
    temp=np.matmul(temp,y);
    print("coefficient is: ");
    print(temp);
    return temp;

def render(x1,y,Coefficient):
    plt.scatter(x1,y);
    plotx= np.linspace(0, 10, 100);
    ploty = np.polyval(Coefficient, plotx)
    plt.plot(plotx,ploty);
    plt.show();

def test(testx,testy,CoefficientInReverse):
    predictArray=np.zeros(sample_size,dtype=float);
    multiple=np.ones(sample_size,dtype=float);
    for coefficient in CoefficientInReverse:
        predictArray+=coefficient*multiple;
        multiple*=testx;
    print("predict is: ");
    print(predictArray)
    predictArray=predictArray-testy;
    predictArray=predictArray*predictArray;
    error=predictArray.sum();
    print("error is: ");
    print(error);

sample_size=10;
x1 = np.array([5.86,1.34,3.65,4.69,4.13,5.87,7.91,5.57,7.3,7.89]);
y = np.array([0.74,1.18,0.51,-0.48,-0.07,0.37,1.35,0.3,1.64,1.75]);
```

```
x2= x1*x1;
x3=x2*x1;
x4=x3*x1;
XT=np.vstack(([1,1,1,1,1,1,1,1,1,1],x1,x2,x3,x4));
print(XT);
render(x1,y,OLS(XT,y)[::-1])


testx=([5.8,0.57,4.3,6.55,0.82,3.72,5.8,3.26,6.75,4.77]);
testy=([0.93,1.87,-0.06,1.6,1.22,0.9,0.93,1.53,1.73,-0.51]);
test(testx,testy,OLS(XT,y));
```

Q3

```
import numpy as np
import math
import matplotlib.pyplot as plt;
def sig(X):
    return 1/ (1+  math.e**(-X));
def df(theta,trainX,trainY):
    total=np.zeros(dim);
    for X, Y in zip(trainX,trainY):
        temp=sig((np.dot(X,theta)))-Y;
        total+=temp*X;
    print("df: ");
    print(total);
    return total;
def test(theta,testX,testY):
    print("Beginning Testing: ")
    TP=0;TN=0;FP=0;FN=0;
    testResult=sig((np.dot(testX,theta)));
    testResult=np.where(testResult>=0.5,1,0);
    print(testResult);
    for X, Y in zip(testResult,testY):
        if X==1 and Y==1:
            TP=TP+1;
        if X==0 and Y==0:
            TN=TN+1;
        if X==1 and Y==0:
            FP=FP+1;
        if X==0 and Y==1:
            FN=FN+1;
    print("TP %d TN %d FP %d FN %d"%(TP,TN,FP,FN))
dim=3;
theta=np.array([-1,1.5,0.5]);
trainX=np.array([[1,0.346,0.78],[1,0.303,0.439],[1,0.358,0.729],[1,0.60
2,0.863],[1,0.790,0.753],[1,0.611,0.965]]);
trainY=np.array([0,0,0,1,1,1]);
theta=theta-0.1*df(theta,trainX,trainY);
```

```
testX=np.array([[1,0.959,0.382],[1,0.75,0.306],[1,0.395,0.76],[1,0.823,
0.764],[1,0.761,0.874],[1,0.844,0.435]])
testY=np.array([0,0,0,1,1,1]);
test(theta,testX,testY);
```