EKF-Based IMU Orientation Estimation

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1 Introduction

We aim to estimate the 3-DoFs orientation of a rigid body using 3-axis gyroscope and 3-axis accelerometer measurement. The error of roll and pitch is bounded, but the error of yaw will increasing, as no other absolute measurement to render yaw observable.

2 Method

We use a EKF estimator. The state contains the orientation of the IMU in Global ENU frame: ${}^G_I\boldsymbol{R}$, and the 3-axis gyroscope bias $\boldsymbol{b}_g$.

2.1 Propagation

The IMU kinematics is

$$
\begin{aligned}
{}^G_I\dot{\boldsymbol{R}} &= {}^G_I\boldsymbol{R}\,[\boldsymbol{\omega}_m - \boldsymbol{b}_g - \boldsymbol{n}_w]\times && (1)\\
\dot{\boldsymbol{b}}_g &= \boldsymbol{0} + \boldsymbol{n}_g && (2)
\end{aligned}
$$

The orientation error is defined as:

$$
{}^G_I\boldsymbol{R} = {}^G_I\hat{\boldsymbol{R}}\mathrm{Exp}(\delta\boldsymbol{\theta}) \tag{3}
$$

So, the error-state kinematics is:

$$
\begin{aligned}
\dot{\delta\boldsymbol{\theta}} &= -[\boldsymbol{\omega}_m - \boldsymbol{b}_g]_\times\delta\boldsymbol{\theta} - \delta\boldsymbol{b}_g - \boldsymbol{n}_w && (4)\\
\delta\dot{\boldsymbol{b}}_g &= \boldsymbol{n}_g && (5)
\end{aligned}
$$

Use Euler-Integration, we get the discrete time kinematics:

$$
\begin{aligned}
\delta\boldsymbol{\theta} &= \mathrm{Exp}[(\boldsymbol{\omega}_m - \boldsymbol{b}_g)\Delta t]^T\delta\boldsymbol{\theta} - \delta\boldsymbol{b}_g\Delta t + \boldsymbol{\theta}_i && (6)\\
\delta\boldsymbol{b}_g &= \delta\boldsymbol{b}_g + \boldsymbol{\omega}_i && (7)
\end{aligned}
$$

Re-write as matrix form:

$$
\begin{bmatrix}\delta\boldsymbol{\theta}\\ \delta\boldsymbol{b}_g\end{bmatrix} = \underbrace{\begin{bmatrix}\mathrm{Exp}[(\boldsymbol{\omega}_m - \boldsymbol{b}_g)\Delta t]^T & -\boldsymbol{I}\Delta t\\ \boldsymbol{0} & \boldsymbol{I}\end{bmatrix}}_{\boldsymbol{F}_x}\begin{bmatrix}\delta\boldsymbol{\theta}\\ \delta\boldsymbol{b}_g\end{bmatrix} + \underbrace{\begin{bmatrix}\boldsymbol{I} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{I}\end{bmatrix}}_{\boldsymbol{F}_i}\begin{bmatrix}\boldsymbol{\theta}_i\\ \boldsymbol{\omega}_i\end{bmatrix} \tag{8}
$$

Covariance propagation:

$$
\boldsymbol{\Sigma}_{k+1} = \boldsymbol{F}_x\boldsymbol{\Sigma}_k\boldsymbol{F}_x^T + \boldsymbol{F}_i\boldsymbol{Q}\boldsymbol{F}_i^T \tag{9}
$$

2.2 Update

The gravity orientation in the ENU frame is $\boldsymbol{g} = [0\ 0\ 1]^T$. The normalized acceleration measurement is $\boldsymbol{a}$. Our measurement model is:

$$\boldsymbol{a} = {}^G_I\boldsymbol{R}^T\boldsymbol{g} \tag{10}$$

The Jacobian is:

$$\boldsymbol{H} = \begin{bmatrix} [{}^G_I\boldsymbol{R}^T\boldsymbol{g}]_\times & \mathbf{0} \end{bmatrix} \tag{11}$$

References

[1] Solà, Joan. Quaternion kinematics for the error-state Kalman filter[J]. 2017.