54.

a) $\dfrac{91\pi}{4} \Rightarrow \dfrac{3\pi}{4}$

b) $\cos\left(\dfrac{91\pi}{4}\right) = \dfrac{-\sqrt{2}}{2}$, $\sin\left(\dfrac{91\pi}{4}\right) = \dfrac{\sqrt{2}}{2}$

2.

a) $\cos\left(-\dfrac{13\pi}{6}\right) = \cos\left(\dfrac{-\pi}{6}\right) = \dfrac{\sqrt{3}}{2}$

b) $\sin\left(\dfrac{81\pi}{2}\right) = \sin\left(\dfrac{\pi}{2}\right) = 1$

3.

a) $\cos\left(\dfrac{25\pi}{3}\right) = \cos\left(\dfrac{\pi}{3}\right) = \dfrac{1}{2}$

$\sin\left(\dfrac{25\pi}{3}\right) = \dfrac{\sqrt{3}}{2}$

b) $\sin\left(\dfrac{45\pi}{6}\right) = \sin\left(\dfrac{9\pi}{6}\right) = \sin\left(\dfrac{3\pi}{2}\right) = -1$

$\cos\left(\dfrac{3\pi}{2}\right) = 0$

57.

1 $-\pi < -\dfrac{55\pi}{4} + 2k\pi$

$-\pi < \dfrac{3\pi}{4} + 2k\pi$

$-1 < \dfrac{3}{4} + 2k$

46.

$\frac{2\pi}{3}$ tours per minute, sur 10 min $\frac{20\pi}{3}$

N $\quad -\frac{\pi}{3}$ $\qquad \frac{-\pi}{3} + \frac{20\pi}{3} = \frac{19\pi}{3} \Rightarrow \frac{\pi}{3}$

2. $\frac{4\pi}{3}$ per minute, choison = 1 min, début P

a) 3 min? 4π, $P = \frac{-\pi}{6}$ $\qquad \frac{-\pi}{6} + 4\pi = \frac{23\pi}{6} \Rightarrow \frac{11\pi}{6}$ N

b) 4 min? $\frac{16\pi}{3}$, $\frac{-\pi}{6} + \frac{32\pi}{6} = \frac{31\pi}{6} \Rightarrow \frac{7\pi}{6}$ G

c) 5 min $\frac{20\pi}{3}$, $\frac{-\pi}{6} + \frac{40\pi}{6} = \frac{39\pi}{6} \Rightarrow \pi$ F

d) 10 min $\frac{40\pi}{3}$, $\frac{-\pi}{6} + \frac{80}{6} = \frac{79\pi}{6} \Rightarrow \frac{7\pi}{6}$ G

51.

1. CRYPTER $\Rightarrow$ $\frac{2\pi}{13}$ $\frac{17\pi}{13}$ $\frac{24\pi}{13}$ $\frac{15\pi}{13}$ $\frac{19\pi}{13}$ $\frac{4\pi}{13}$ $\frac{12\pi}{13}$

$\frac{6\pi}{13}$ $\frac{5\pi}{13}$ $\frac{7\pi}{13}$ $\frac{45\pi}{13}$ $\frac{53\pi}{13}$ $\frac{12\pi}{13}$ $\frac{51\pi}{13}$

$\frac{11\pi}{13}$ $\frac{\pi}{49}$ $-\frac{\pi}{13}$ $\frac{\pi}{13}$ $\frac{\pi}{13}$ $-\frac{\pi}{13}$ $\frac{\pi}{13}$

L $\qquad$ M $\qquad$ I $\qquad$ G $\qquad$ S $\qquad$ X $\qquad$ M

2. ERKPI $\Rightarrow$ $\frac{4\pi}{13}$ $\frac{17\pi}{13}$ $\frac{10\pi}{13}$ $\frac{15\pi}{13}$ $\frac{8\pi}{13}$

$-\frac{\pi}{13}$ $\frac{12\pi}{13}$ $\frac{5\pi}{13}$ $\frac{10\pi}{13}$ $\frac{3\pi}{13}$

$-\frac{\pi}{39}$ $\frac{4\pi}{13}$ $\frac{5\pi}{39}$ $\frac{10\pi}{39}$ $\frac{\pi}{13}$

16. $g(x) = \cos(3x) + 1$

1. $g\left(\frac{2\pi}{3} + x\right) = \cos(2\pi + 3x) + 1 = \cos(3x) + 1 = g(x)$

2. $0 \leq g(x) \leq 2$

   $-1 + 1 \leq \cos(x) + 1 \leq 1 + 1$

Exos Application

25. $\Pi_D = \frac{-\pi}{4}$ $\Pi_E = \frac{\pi}{2}$ $\Pi_F = \pi$ $\Pi_6 = \frac{-\pi}{2}$ $\Pi_4 = 0$
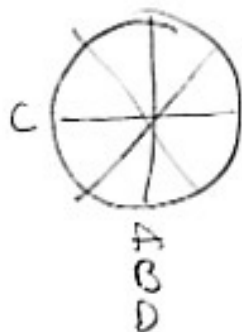
27.
1.

a) $\frac{\pi}{2} \Rightarrow A$ b) $\frac{5\pi}{2} \Rightarrow B$ c) $\frac{11\pi}{2} \Rightarrow C$

d) $-\frac{13\pi}{2} \Rightarrow D$

AB

C
D

2.

a) $\frac{67\pi}{2} \Rightarrow A$ b) $\frac{-37\pi}{2} \Rightarrow B$ c) $\frac{498\pi}{2} = 249\pi \Rightarrow C$

d) $\frac{-117\pi}{2} \Rightarrow D$

C

A
B
D

---

28. à partir de $\cos\left(\frac{\pi}{3}\right)$ det $\cos\left(\frac{-\pi}{3}\right)$ et $\cos\left(\frac{2\pi}{3}\right)$

$\cos\left(\frac{-\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right)$ $\cos\left(\frac{2\pi}{3}\right) = -\cos\left(\frac{\pi}{3}\right)$

2 $\sin\left(\frac{2\pi}{3}\right) = -\sin\left(\frac{-\pi}{3}\right)$ $\cos\left(\pi - \frac{\pi}{3}\right)$

33.

a) $\cos\left(\frac{10\pi}{3}\right) \Rightarrow$ négatif

c) $\cos\left(\frac{17\pi}{16}\right) \Rightarrow$ négatif

b) $\sin\left(\frac{\pi}{14}\right) \Rightarrow$ positif

d) $\sin\left(\frac{-7\pi}{8}\right) \Rightarrow$ négatif

34.

1. Résoudre sur $[0; 2\pi[$ $\cos(x) = \frac{\sqrt{3}}{2}$ $x = \frac{\pi}{6}$

2. $\sin(x) = \frac{\sqrt{2}}{2}$ $x = \frac{\pi}{4}$

38.

1. $A = \frac{-3\pi}{4}$ $B = -\frac{\pi}{4}$ 2. $]\pi; \pi]$ $\sin(x) = \frac{-\sqrt{2}}{2}$

3. $\sin(x) \geqslant \frac{-\sqrt{2}}{2}$ $< \frac{3\pi}{4}$ $x = \frac{-\pi}{4}; \frac{-3\pi}{4}$

$x \geqslant \frac{-\pi}{4}$

39.

1. $f(x) - f(-x)$ où f est paire

$= 0$

2. $f(x) = x^2 + \cos(x)$

x^2 est pair, $\cos(x)$ est pair

42. f, g, h $f(x) = \sin(2x)$ $g(x) = 2\sin(x)$ $h(x) = \sin^2(x)$

1. $f(-x) = -\sin(2x) = -f(x)$

$g(-x) = -\sin(x) = -g(x)$

2. $h(-x) = (-\sin(x))^2 = h(x)$

3. $\mathcal{C}_1 = h$ $\mathcal{C}_2 = g$ $\mathcal{C}_3 = f$

① Se repérer sur un cercle trigo

1) $x = \frac{\pi}{2}$

2. $\frac{\pi}{3} \to 60°$ $\frac{-\pi}{6} \to -30°$

1.

$\frac{\pi}{6} = 30°$ $\frac{\pi}{4} = 45°$ $\frac{-\pi}{3} = -60°$

$\frac{\pi}{2} = 90°$

2.

B_α $x = 60° \Rightarrow \frac{\pi}{3}$

C_α $x = 150° \Rightarrow \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

---

3. $\cos\left(\frac{\pi}{4}\right)$ det $\cos\left(\frac{3\pi}{4}\right)$ et

$\cos\left(\frac{-3\pi}{4}\right)$

$-\cos\left(\frac{\pi}{4}\right) = \cos\left(\frac{3\pi}{4}\right) = \cos\left(\frac{-3\pi}{4}\right)$

---

4. $\sin\left(\frac{\pi}{6}\right)$, det $\sin\left(\frac{-\pi}{6}\right)$ et $\sin\left(\frac{-5\pi}{6}\right)$

$\sin\left(\frac{\pi}{6}\right) = -\sin\left(\frac{-\pi}{6}\right)$

$= -\sin\left(\frac{-5\pi}{6}\right)$

---

②

1 det $\sin\left(\frac{-2\pi}{3}\right)$ à partir de $\sin\left(\frac{\pi}{3}\right)$

(comme opposés (symétrie)
par rapport à 0)

$\sin\left(\frac{-2\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right)$

2. det $\cos\left(\frac{3\pi}{4}\right)$ à partir de $\sin\left(\frac{3\pi}{4}\right)$

$\cos^2\left(\frac{3\pi}{4}\right) + \sin^2\left(\frac{3\pi}{4}\right) = 1$

$\cos^2\left(\frac{3\pi}{4}\right) = 1 - \sin^2\left(\frac{3\pi}{4}\right)$

$\cos\left(\frac{3\pi}{4}\right) = \sqrt{1 - \sin^2\left(\frac{3\pi}{4}\right)}$

$\cos\left(\frac{3\pi}{4}\right) = \sqrt{1 - \left(\frac{\sqrt{2}}{2}\right)^2} = \sqrt{\frac{1}{2}} = \frac{-\sqrt{2}}{2}$

---

③

1. sur $]-\pi; \pi]$, $\cos(\alpha) = \frac{-\sqrt{3}}{2}$ $\alpha \left\{\frac{-\pi}{6}; \frac{\pi}{6}\right\}$

7 sur $[0; 2\pi[$, $\sin(\alpha) \geqslant \frac{\sqrt{2}}{2}$ $S = \left[\frac{-\pi}{4}, \frac{3\pi}{4}\right)$

5. $\cos(\alpha) = \frac{1}{2}$ $\alpha = \frac{\pi}{3}; \frac{-\pi}{3}$

6 $\cos(\alpha) \leqslant \frac{\sqrt{2}}{2}$ $\alpha = \frac{\pi}{4}; \frac{-\pi}{4}$

7. $\sin(\alpha) = \frac{-\sqrt{3}}{2}$ $\alpha = \frac{-\pi}{3}; \frac{-2\pi}{3}$

8. $\sin(\alpha) < \frac{1}{2}$ $\alpha \frac{5\pi}{6}; \frac{\pi}{6}$

9. $\begin{cases} \cos(x) \leqslant \frac{\sqrt{2}}{2} \\ \sin(x) \leqslant \frac{1}{2} \end{cases}$ $x = \frac{\pi}{6}$

10. $\begin{cases} \cos(x) \leqslant \frac{-\sqrt{2}}{2} \\ \sin(x) \geqslant \frac{-\sqrt{3}}{2} \end{cases}$ $x = \frac{-3\pi}{4}$

11. $\begin{cases} \cos(x) \leqslant \frac{-\sqrt{2}}{2} \\ \sin(x) \geqslant 0 \end{cases}$ $x = \frac{5\pi}{6}$

12. $\begin{cases} \cos(x) \geqslant 0 \\ \sin(x) \geqslant \frac{-\sqrt{3}}{2} \end{cases}$ $x \frac{-\pi}{3}$

1 $g(-x) = \cos(-x) \vee \sin(-x) = \cos(x) \times -\sin(x) = -g(x)$

2.

$g(\pi+x) = \cos(\pi+x) \times \sin(\pi+x) = -\cos(x) \times -\sin(x)$
$= g(x)$

3. Vert pas périodique sur π

$g(\frac{\pi}{4}) = \cos(\frac{\pi}{4}) \times \sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{2}{4} = \frac{1}{2}$ donc bleue

---

13 $g(x) = \cos(4x)\sin^2(4x)$

1 Paire quand $g(x) = g(-x)$

$g(-x) = \cos(-4x)\sin^2(-4x) = \cos(4x)(-\sin(4x))^2$
$g(x) = \cos(4x)\sin^2(4x)$

2. $g(\frac{\pi}{2}+x) = \cos(\frac{4\pi}{2}+4x) \times \sin^2(\frac{4\pi}{2}+4x)$

$= \cos(2\pi+4x) \times \sin^2(2\pi+4x)$
$= \cos(4x) \times \sin^2(4x)$

14. $g(x) = \frac{1}{4}(3\sin(x) - \sin(3x))$

2. Impaire, 2π

3 $g(-x) = \frac{1}{4}(-3\sin(x) + \sin(3x))$

$g(-x) + g(x) = -\frac{3\sin(x)}{4} + \frac{\sin(3x)}{4} + \frac{3\sin(x)}{4} - \frac{\sin(3x)}{4}$

$= 0$

4. $g(2\pi+x) = \frac{1}{4}(-3\sin(6\pi+x) + \sin(6\pi+3x))$

$= \frac{1}{4}(-3\sin(x) + \sin(3x)) = g(x)$

15. $g(x) = \cos(x) + \sin(x)$

1. $g(-x) = \cos(x) - \sin(x) \neq g(x) \neq -g(x)$

2. $g(\pi+x) = \cos(x) + \sin(x) = g(x)$

3. $\cos -1 \leq \frac{\cos(x)}{\sin(x)} \leq 1$ donc $-2 \leq \cos(x)+\sin(x) \leq 2$
$g(x)$