Vectors

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Contents

1 Scalar product, colinearity and orthogonality of vectors

Definition: Angle of 2 vectors let $\overrightarrow{u}$ and $\overrightarrow{v}$ two vectors, we note $\angle(\overrightarrow{u}, \overrightarrow{v})$ the geometrical angle $\angle ABC$ where $\overrightarrow{u} = \overrightarrow{AB}$ and $\overrightarrow{v} = \overrightarrow{AC}$

Definition: With the cosinus Scalar product is the $\mathbb{R}$ defined by:

$$\overrightarrow{u} \cdot \overrightarrow{v} = ||\overrightarrow{u}|| \times ||\overrightarrow{v}|| \times \cos(\angle \overrightarrow{u}, \overrightarrow{v})$$

If $\overrightarrow{v} = \overrightarrow{0}$ or $\overrightarrow{u} = \overrightarrow{0}$ so $\overrightarrow{u} \cdot \overrightarrow{v} = 0$

Property: Symmetry $\overrightarrow{u} \cdot \overrightarrow{v} = \overrightarrow{v} \cdot \overrightarrow{u}$

Property: Case of colinearity

- Same sense $||\overrightarrow{u}|| \times ||\overrightarrow{v}||$
- Opposite sense $-||\overrightarrow{u}|| \times ||\overrightarrow{v}||$

Property: With orthogonal projection let A, B, and C be three points and H orthogonal projection of C on (AB).

- Same sense $\overrightarrow{AB} \cdot \overrightarrow{AC} = AB \times AH$
- Opposite sense$\overrightarrow{AB} \cdot \overrightarrow{AC} = -AB \times AH$

Definition: * Definition: Orthogonality $\overrightarrow{u}$ and $\overrightarrow{v}$ are orthogonal if:

$$\overrightarrow{u} \cdot \overrightarrow{v} = 0$$

Property: Perpendicular lines Perpendicular only if underlying vectors are orthogonal.

2 Definitions and properties

Property: Scalar product with norms

$$\overrightarrow{u} \cdot \overrightarrow{v} = \frac{1}{2}(||\overrightarrow{u}||^2 + ||\overrightarrow{v}||^2 - ||\overrightarrow{u} - \overrightarrow{v}||^2)$$

Property: Scalar product with coordinates $\overrightarrow{u} \begin{pmatrix} x \\ \\ y \end{pmatrix}$ and $\overrightarrow{v} \begin{pmatrix} x' \\ \\ y' \end{pmatrix}$ so $\overrightarrow{u} \cdot \overrightarrow{v} = xx' + yy'$

Property: Orthogonality conditions $\overrightarrow{u} \begin{pmatrix} x \\ \\ y \end{pmatrix}$ and $\overrightarrow{v} \begin{pmatrix} x' \\ \\ y' \end{pmatrix}$ only if $\overrightarrow{u} \cdot \overrightarrow{v} = 0$

Property: Bilinearity, scalar square, sum's norm Let $\overrightarrow{u}$, $\overrightarrow{v}$ and $\overrightarrow{w}$ be 3 vectors and k a real.

1. $\overrightarrow{u} \cdot (\overrightarrow{v} + \overrightarrow{w}) = \overrightarrow{u} \cdot \overrightarrow{v} + \overrightarrow{u} \cdot \overrightarrow{w}$
2. $\overrightarrow{u} \cdot (k\overrightarrow{v}) = k(\overrightarrow{u} \cdot \overrightarrow{v})$
3. $\overrightarrow{u} \cdot \overrightarrow{u} = (\overrightarrow{u})^2 = ||\overrightarrow{u}||^2$
4. $||\overrightarrow{u} + \overrightarrow{v}||^2 = ||\overrightarrow{u}||^2 + ||\overrightarrow{v}||^2 + 2\overrightarrow{u} \cdot \overrightarrow{v}$

Property: Al-Kashi's formula In a triangle ABC:

$$BC^2 = AB^2 - 2 \times AB \times AC \times \cos(\angle BAC)$$

3 Study of a set of points

Property: Transformation of an expression Two points A, B and their middle I, we have: $\overrightarrow{MA} \cdot \overrightarrow{MB} = MI^2 - \frac{1}{4}AB^2$

Property: Circle Two points A and B, the set of M points where $\overrightarrow{MA} \cdot \overrightarrow{MB} = 0$ is the circle of diameter [AB]

Property: Circle and rectangle triangle Triangle ABC rectangle in C only if C belongs to the [AB] diameter circle. (different from A et B)