Real Random Variables

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1 Real random variables

We consider a random experience of universe $\Omega = \{e_1; e_2; e_3; ...; e_r\}$ is finite and a law of probability p over Ω

Definition: Real random variable (discrete) An RRV X over Ω is a function that associates a real for each issue of Ω. We note $\{X = a\}$ the event X taking the value a and $p(X = a)$ its probability.

Definition: Let X a RRV over Ω with the values $x_1, x_2, ..., x_n$. When each value x_i, we associate the probability $p_i = p(X = x_i)$ we define the law of probability of X.

2 Expectation - Variance - Standard gap

2.1 Definitions

Definition: Expectation Expectation of X is the Real noted $E(X)$ defined by:

$$E(X) = p_1x_1 + p_2x_2 + ... + p_nx_n$$

Definition: Variance Variance of X noted $V(X)$ defined by:

$$V(X) = p_1(x_1 - E(X))^2 + p_2(x_2 - E(X))^2 + ... + p_n(x_n - E(X))^2$$

Definition: Standard gap The standard gap of X is the real noted $\sigma(X)$ defined by:

$$\sigma(X) = \sqrt{V(X)}$$

2.2 Properties of the indicators

Property: Formula of König-Huygens

$$V(X) = p_1(x_1)^2 + p_2(x_2)^2 + ... + p_n(x_n)^2 - (E(X))^2$$

Definition: Random variable aX+b For every real a and b, we can associate a new random variable by associating each issue giving the value x_i, the real $ax_i + b$. Named $aX + b$

Property: E(aX+b) and V(aX+b) Let a and b be two reals. We have:

- $E(aX + b) = aE(X) + b$
- $V(aX + b) = a^2V(X)$
- $\sigma(aX + b) = |a|\sigma(X)$

Property: Expectation and simulation With a sufficiently big sample of values taken by a random variable, the average of its values is close to the value of the expectation of this random variable.