Exercices Spotted Geometry

①

1. $3x - 2y - 1 = 0$ $\vec{v}\begin{pmatrix}3\\-2\end{pmatrix}$

2. $-4x - y + 3 = 0$ $\vec{v}\begin{pmatrix}-1\\-4\end{pmatrix} = \begin{pmatrix}4\\1\end{pmatrix}$

3. $\vec{v}\begin{pmatrix}5\\2\end{pmatrix}$ $-b = 5$ $a = 2$ $\vec{v}\begin{pmatrix}2\\-5\end{pmatrix}$

1.

a) $-4x + 3y = 0$ $\vec{v}\begin{pmatrix}-4\\3\end{pmatrix}$

b) $y = 3x - 1$ $\vec{v}\begin{pmatrix}-3\\-1\end{pmatrix}$

c) $y = 3$ $\vec{v}\begin{pmatrix}0\\0\end{pmatrix}$

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2. $C(3;-2)$ $D(-1;-5)$ $\vec{v}\begin{pmatrix}-4\\-3\end{pmatrix}$ $-b = -4$ $a = -3$ $\vec{v}\begin{pmatrix}-3\\4\end{pmatrix}$

$C(-2;1)$ $D(-3;-5)$ $\vec{v}\begin{pmatrix}-1\\-6\end{pmatrix}$ $-b = -1$ $a = -6$ $\vec{v}\begin{pmatrix}-6\\1\end{pmatrix}$

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②

$A(2;3)$ $\vec{n}\begin{pmatrix}-2\\1\end{pmatrix}$ $-2x + y + 1 = 0$

$-2 \times 2 + 3 + c = 0$

$c = 1$

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3. $K(0;2)$ $\vec{n}\begin{pmatrix}1\\3\end{pmatrix}$

$x + 3y + c = 0$

$6 + c = 0$

$c = -6$

$x + 3y - 6 = 0$

4. $D(3;-2)$ $\vec{n}\begin{pmatrix}1\\-3\end{pmatrix}$

$x - 3y + c = 0$

$3 + 6 + c = 0$

$c = -9$

$x - 3y - 9 = 0$

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③ $d: 4x - 5y - 1 = 0$ $A(-1;2)$

$\vec{n}\begin{pmatrix}4\\-5\end{pmatrix}$ $d': 5x + 4y + 3 = 0$

pour A $5 - 8 + c = 0$

$c = 3$

$$\begin{cases} 4x - 5y - 1 = 0 \\ -5x - 4y + 3 = 0 \end{cases}$$

$$\begin{cases} (20x - 25y - 5) + (-20x - 16y + 12) = 0 \\ (16x - 20y - 4) - (-25x - 20y + 15) = 0 \end{cases}$$

$$\begin{cases} -41y + 7 = 0 \\ 41x - 19 = 0 \end{cases} \quad y = \frac{7}{41} \quad x = \frac{19}{41}$$

$H\left(\frac{19}{41}; \frac{7}{41}\right)$

5. $B(3;-4)$ $2x-y+4=0$ $\vec{n}\begin{pmatrix}2\\-1\end{pmatrix}$ d' $-x-2y-3=0$

$-3+12+c=0$

$\begin{cases}2x-y+4=0\\-x-2y-3=0\end{cases} \Rightarrow \begin{cases}-5y-32=0\\-5x-15=0\end{cases}$ $B'\left(-3;-\frac{32}{5}\right)$ $c=-3$

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6. $C(1;-2)$ $-3x-y+5=0$ $\vec{n}\begin{pmatrix}3\\-1\end{pmatrix}$ d' $-x+3y+c=0$

$-1-6+c=0$

$\begin{cases}-3x-y+5=0\\-x+3y+7=0\end{cases} \Rightarrow \begin{cases}8y+26=0\\-10x+22=0\end{cases}$ $C'\left(\frac{11}{5};-\frac{13}{4}\right)$ $c=7$

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7. $K(-2;-3)$ $4x+3y-2=0$ $\vec{n}\begin{pmatrix}4\\3\end{pmatrix}$ d' $3x-4y+c=0$

$\begin{cases}4x+3y-2=0\\3x-4y-6=0\end{cases} \Rightarrow \begin{cases}(12x+9y-6)-(12x-16y-24)=0\\(16x+12y-8)+(9x-12y-18)=0\end{cases}$

$\begin{cases}25y+18=0\\25x-26=0\end{cases}$ $K'\left(\frac{26}{25};\frac{25}{-18}\right)$

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8. $V(-1;0)$ $-x-5y+7=0$ $\vec{n}\begin{pmatrix}-1\\-5\end{pmatrix}$ d' $-5x+y+c=0$

$5+c=0$

$\begin{cases}-x-5y+7=0\\-5x+y-5=0\end{cases} \Rightarrow \begin{cases}-26x-13=0\\26y-40=0\end{cases}$ $V'\left(\frac{-9}{13};\frac{20}{13}\right)$ $c=-5$

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9. $G(2;-1)$ $x-3y+1=0$ $\vec{n}\begin{pmatrix}1\\-3\end{pmatrix}$ d' $-3x-y+c=0$

$-6+1+c=0$

$\begin{cases}x-3y+1=0\\-3x-y+5=0\end{cases} \Rightarrow \begin{cases}10x-14=0\\-10y+18=0\end{cases}$ $G'\left(\frac{7}{5};\frac{9}{5}\right)$ $c=5$

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10. $O(0;0)$ $x+y-4=0$ $\vec{n}\begin{pmatrix}1\\1\end{pmatrix}$ d' $x-y+c=0$

$\begin{cases}x+y-4=0\\x-y=0\end{cases} \Rightarrow \begin{cases}2x-4=0\\-2y+4=0\end{cases}$ $O(2;2)$

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11. $T(-2;3)$ $-\frac{2}{3}x+y-5$ $\vec{n}\begin{pmatrix}-\frac{2}{3}\\1\end{pmatrix}$ d' $x+\frac{2}{3}y+c=0$

$-2+2$

$c=0$

$\begin{cases}-\frac{2}{3}x+y-5\\x+\frac{2}{3}y=0\end{cases} \Rightarrow \begin{cases}\frac{13}{5}y-5=0\\\frac{13}{5}x+\frac{10}{3}=0\end{cases}$ $T'\left(\frac{30}{13};\frac{25}{13}\right)$

12. $D(6;1)$ $\frac{3}{4}x + y + \frac{1}{2} = 0$ $\vec{n}\begin{pmatrix}\frac{3}{4}\\1\end{pmatrix}$ d $x - \frac{3}{4}y + c = 0$

$\begin{cases} \frac{3}{4}x + y + \frac{1}{2} = 0 \\ x - \frac{3}{4}y - \frac{11}{4} = 0 \end{cases} \Rightarrow \begin{cases} \frac{25}{16}y + \frac{71}{16} = 0 \\ \frac{25}{16}x - \frac{39}{8} = 0 \end{cases}$ $D'\left(\frac{78}{25}; -\frac{71}{25}\right)$

$6 + \frac{3}{4} + c = 0$

$c = -\frac{41}{4}$

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13. $N(-5;-2)$ $\frac{x}{3} + y - \frac{2}{3} = 0$ $\vec{n}\begin{pmatrix}1\\3\end{pmatrix}$ d' $3x - y + c = 0$

$x + 3y - 2 = 0$

$-15 + 2 + c = 0$

$c = 13$

$\begin{cases} x + 3y - 2 = 0 \\ 3x - y + 13 = 0 \end{cases} \Rightarrow \begin{cases} 10x + 37 = 0 \\ -10y + 19 = 0 \end{cases}$ $N'\left(-\frac{37}{10}; \frac{19}{10}\right)$

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(4)

1. $x^2 - 4x = (x-2)^2 - 4$

$y^2 + 2y = (y+1)^2 - 1$

$(x-2)^2 - 4 + (y+1)^2 - 1 = 0$

$(x-2)^2 + (y+1)^2 - 5$

Cercle de rayon $\sqrt{5}$

2. Rayon $\sqrt{5}$, centre $C(2;-1)$

3. $A(0;-2)$, oui appartient au cercle

$4 + 1 = 5$

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14 $x^2 - 3x = \left(x - \frac{3}{2}\right)^2 - \frac{9}{4}$ $y^2 + 5y = \left(y + \frac{5}{2}\right)^2 - \frac{25}{4}$

$\left(x - \frac{3}{2}\right)^2 - \frac{9}{4} + \left(y + \frac{5}{2}\right)^2 - \frac{25}{4} - 1 = 0$

$\left(x - \frac{3}{2}\right)^2 + \left(y + \frac{5}{2}\right)^2 = \frac{19}{2}$ rayon $\sqrt{\frac{19}{2}}$

centre $\left(\frac{3}{2}; \frac{-5}{2}\right)$

15. $x^2 + 7x = (x + \frac{7}{2})^2 - \frac{49}{4}$ $\quad y^2 + 2 = (y + 0)^2 + 2$

$(x + \frac{7}{2})^2 - \frac{49}{4} + (y + 0)^2 + 2 = 0$

$(x + \frac{7}{2})^2 + (y + 0)^2 = \frac{41}{4}$ $\quad r = \sqrt{\frac{41}{4}}$

Centre $(-\frac{7}{2} ; 0)$

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16. $x^2 + x = (x + \frac{1}{2})^2 - \frac{1}{4}$ $\quad y^2 - y = (y - \frac{1}{2})^2 - \frac{1}{4}$

$(x + \frac{1}{2}) - \frac{1}{4} + (y - \frac{1}{2})^2 - \frac{1}{4} = 0$

$(x + \frac{1}{2}) + (y - \frac{1}{2})^2 = \frac{1}{2}$ $\quad r = \sqrt{\frac{1}{2}}$

Centre $(-\frac{1}{2} ; \frac{1}{2})$

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17. $x^2 + 4x = (x + 2)^2 - 4$ $\quad y^2 - 5y = (y - \frac{5}{2})^2 - \frac{25}{4}$

$(x + 2)^2 - 4 + (y - \frac{5}{2})^2 - \frac{25}{4} = 0$

$(x + 2)^2 + (y - \frac{5}{2})^2 = \frac{41}{4}$ $\quad r = \sqrt{\frac{41}{4}}$

Centre $(-2 ; \frac{5}{2})$

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18. $x^2 = (x + 0)^2$ $\quad y^2 - 6y = (y - 3)^2 - 9$

$(x + 0)^2 + (y - 3)^2 - 9 + 18 = 0$

$(x + 0)^2 + (y - 3)^2 = -3$

Pas un cercle

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19. $x^2 - 2x = (x - 1)^2 - 1$

$(x - 1)^2 - 1 + (y - 3)^2 - 9 + 10 = 0$

$(x - 1)^2 + (y - 3)^2 = 0$ $\quad r = 0$

Centre $(1, 3)$