Independent and Conditional probabilities

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1 Conditional Probability

We consider $p(A) \neq 0$

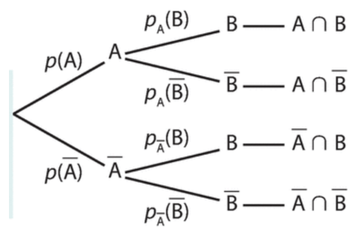
Definition: Conditional Probability $p_A(B)$ defines the prob of event B is realized knowing that A is realized.

Property: Conditional probability and intersection $p(A \cap B) = p(A) \times p_A(B)$ and $p_A(B) = \frac{p(A \cap B)}{p(A)}$

2 Weighted trees

2.1 Weighted trees and conditional probability

Property: Weighted tree and rule of the product Let A such as $p(A) \neq 0$ and $p(\overline{A}) \neq 0$



Property: Parition of universe Let n with $n \in \mathbb{N}$ and $n \geq 2$ events of prob $\neq 0$: $A_1, A_2, A_3, ..., A_n$ These events format a partition of universe Ω if:

- disjointed two by two $p(A_i \cap A_j) = \emptyset$ if $i \neq j$
- $A_1 \cup A_2 \cup ... \cup A_n = \Omega$

Property: Weighted tree and partition We can build trees with more than 2 branches from the same node as long as all the linked events form a partition of the universe.

2.2 Total probability formula

Property: Total probability formula (peculiar cases) Let A such as $p(A) \neq 0$ and $p(\overline{A}) \neq 0$. Probability of B is

$$p(B) = p(A \cap B) + p(\overline{A} \cap B) = p(A) \times p_A(B) + p(\overline{A}) \times p_{\overline{A}}(B)$$

Property: Total probability formula (general case) Let $A_1, A_2, ..., A_n$ and $B_1, B_2, ..., B_m$ two universe partitions. For $i \in [1; m]$ probability of B_i is:

$$p(B_i) = p(A_1 \cap B_i) + p(A_2 \cap B_i) + ... + p(A_n \cap B_i)$$

$$p(B_i) = p(A_1) \times p_{A_1}(B_i) + p(A_2) \times p_{A_2}(B_i) + ... + p(A_n) \times p_{A_n}(B_i)$$

3 Notion of independence

3.1 Independence of two events

A and B such as $p(A) \neq 0$ and $p(B) \neq 0$

Definition: Independence of two events A and B are independent if $p_A(B) = p(B)$

Property: Independence and intersection A and B are independent, if and only if, $p(A \cap B) = p(A) \times p(B)$

Property: Independence and contrary events If A and B are independent, so $\overline{A}$ and B also are, as well as $\overline{B}$ and A and $\overline{A}$ and $\overline{B}$

3.2 Succession of independent trials

N trials can be represented by a weighted tree and by an n entries array.