# Trigonometrical Function

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# 1 Tracking on the Trigonometrical circle

**Definition: Trigonometrical circle**   Trigonometrical circle, noted $C$ of center and origin $O$. With a radius $r = OI = 1$

**Property: Tracking**   We choose an orientation, circle trigo $C$ - direct orientation, reverse watch direction - indirect orientation, watch direction

**Property: Radian**   Point $M$ on the Trigonometrical circle, associated with a real $\mathbb{R}$. Where $x$ is the abscisse of a point on the axe which superposes $M$. This point $\rightarrow$ image point of $x$ on the Trigonometrical circle.

**Radian**   $C =$circle, $M =$point on $C$

Measure in radian of $\angle OIM$ and the length of the arc $IM$. Associated symbol $rad$ or $rd$.

# 2 Coordinates of a point in the Trigonometrical circle

## 2.1 Sinus and Consinus

**Definition: Sinus and cosinus**   For a real $x \in \mathbb{R}$ $\cos x$ and $\sin x$ are the coordinates of $M_x = (\cos x; \sin x)$.

**Properties: Sinus and Consinus**   For $x \in \mathbb{R}$:

$$(\cos x)^2 + (\sin x)^2 = 1$$

- $-1 \leq \cos x \leq 1$
- $-1 \leq \sin x \leq 1$

## 2.2 Remarkable values

**Property:** **Remarkable values**  $M_x$ point of $C$, image of a real $x$, So:

| $\angle OIM$ | $O^o$ | $30^o$ | $45^o$ | $60^o$ | $90^o$ |
|---|---|---|---|---|---|
| Real $x$ (rad) | $0$ | $\dfrac{\pi}{6}$ | $\dfrac{\pi}{4}$ | $\dfrac{\pi}{3}$ | $\dfrac{\pi}{2}$ |
| $\cos x$ | $0$ | $\dfrac{\pi}{6}$ | $\dfrac{\pi}{4}$ | $\dfrac{\pi}{3}$ | $\dfrac{\pi}{2}$ |
| $\sin x$ | $0$ | $\dfrac{1}{2}$ | $\dfrac{\sqrt{2}}{2}$ | $\dfrac{\sqrt{3}}{2}$ | $1$ |

## 2.3 Associated angles

**Property: Associated angles**

$$\cos -a = \cos a$$

$$\sin -a = -\sin a$$

$$-\cos a = \cos \pi - a$$

$$\sin a = \sin \pi - a$$

$$-\cos a = \cos a + \pi$$

$$-\sin a = \sin a + \pi$$
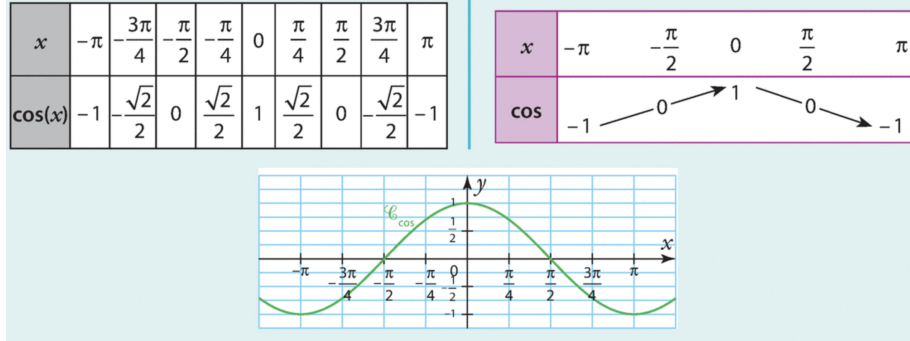
$$\cos \frac{\pi}{2} - a = \sin a$$

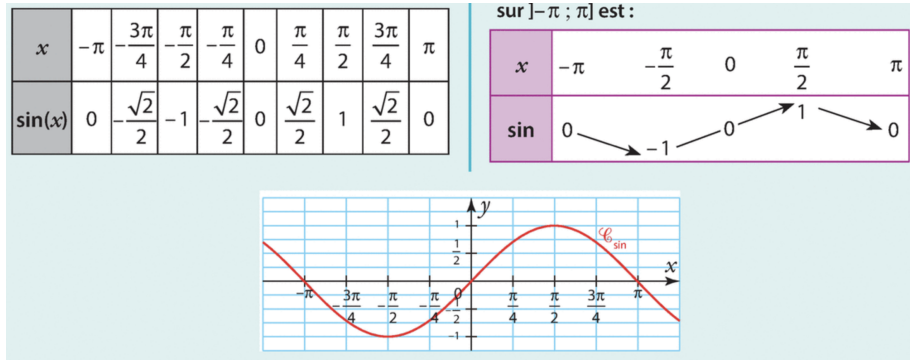$$\sin \frac{\pi}{2} - a = \cos a$$

$$cos\frac{\pi}{2} + a = -\sin a$$

$$\sin \frac{\pi}{2} + a = \cos a$$

# 3 Cosinus and Sinus functions

**<u>Definition</u>: Cosinus function**   Cosinus noted cos defined on $\mathbb{R}$

| $x$ | $-\pi$ | $-\dfrac{3\pi}{4}$ | $-\dfrac{\pi}{2}$ | $-\dfrac{\pi}{4}$ | $0$ | $\dfrac{\pi}{4}$ | $\dfrac{\pi}{2}$ | $\dfrac{3\pi}{4}$ | $\pi$ |
|---|---|---|---|---|---|---|---|---|---|
| $\cos(x)$ | $-1$ | $-\dfrac{\sqrt{2}}{2}$ | $0$ | $\dfrac{\sqrt{2}}{2}$ | $1$ | $\dfrac{\sqrt{2}}{2}$ | $0$ | $-\dfrac{\sqrt{2}}{2}$ | $-1$ |

| $x$ | $-\pi$ | $-\dfrac{\pi}{2}$ | $0$ | $\dfrac{\pi}{2}$ | $\pi$ |
|---|---|---|---|---|---|
| $\cos$ | $-1$ | $0$ | $1$ | $0$ | $-1$ |

**<u>Definition</u>: Sinus function**   Sinus noted sin defined on $\mathbb{R}$

| $x$ | $-\pi$ | $-\dfrac{3\pi}{4}$ | $-\dfrac{\pi}{2}$ | $-\dfrac{\pi}{4}$ | $0$ | $\dfrac{\pi}{4}$ | $\dfrac{\pi}{2}$ | $\dfrac{3\pi}{4}$ | $\pi$ |
|---|---|---|---|---|---|---|---|---|---|
| $\sin(x)$ | $0$ | $-\dfrac{\sqrt{2}}{2}$ | $-1$ | $-\dfrac{\sqrt{2}}{2}$ | $0$ | $\dfrac{\sqrt{2}}{2}$ | $1$ | $\dfrac{\sqrt{2}}{2}$ | $0$ |

sur $]-\pi\,;\,\pi]$ est :

| $x$ | $-\pi$ | $-\dfrac{\pi}{2}$ | $0$ | $\dfrac{\pi}{2}$ | $\pi$ |
|---|---|---|---|---|---|
| $\sin$ | $0$ | $-1$ | $0$ | $1$ | $0$ |

**<u>Property</u>: Superposition of images-points**   let $x$ a real and $M_x(\cos x; \sin x)$. $\overline{M_x}$ and $M_{x+2\pi}(\cos x + 2\pi; \sin x + 2\pi)$ are confused.

**<u>Property</u>: Periodicity**   sin and cos are periodic of period $2\pi$

$$\cos x = \cos 2\pi + x$$

$$\sin x = \sin 2\pi + x$$

**<u>Property</u>: Parity of sin and cos**   Let $x$ a Real:

- sinus is unpair because central symetric with origin
- cosinus is pair because axial symetric with ordinate axis