1. Data structure

```
class Graph
(1)
(2) {
(3)
         public:
(4)
             Graph();
(5)
             Graph(char, int);
(6)
             void E_construct(int, int, int);
(7)
             void test();
(8)
             void cyclebreak(fstream&);
(9)
             void sort_E();
(10)
            void MST_Kruskal();
(11)
            void dir_find_delete_edge(vector<Edge>&);
(12)
            void add_edge_back(vector<Edge>&);
(13)
            void output_u(fstream&);
(14)
            void output_d(fstream&);
(15)
        private:
(16)
            void Makeset(int);
(17)
            void Union(int, int);
            int Findset(int);
(18)
(19)
            void Link(int, int);
(20)
            void MergeSort(vector<Edge>&);
(21)
            void MergeSortSubVector(vector<Edge>&, int, int);
(22)
            void Merge(vector<Edge>&, int, int, int, int);
(23)
(24)
            bool DFS_cycle_detecting(Edge&);
            bool DFS_visit(int, int, vector<char>&);
(25)
(26)
(27)
            void DFS_cycle_detecting_check();
(28)
            void DFS_visit_check(int, vector<char>&);
(29)
(30)
            char type;
(31)
            int n;
(32)
            // weight[i][j] means the weight of edge(i,j)(-100~100)
(33)
            // if no edges between (i,j), weight[i][j] = 666
(34)
            vector<vector<int> > weight;
(35)
            vector<vector<int> > A;
(36)
            vector<Edge> E;
(37)
            vector<vector<int> > adj;
(38)
            vector<Vertex> V;
(39)};
```

使用一個叫做 Graph 的資料結構,裏頭包含 vertex, edge,type 是用來表達這個 graph 是 undirected 還是 directed 的,n 則是用來表達 vertice 數目(有時候直接用 n 取代 V.size 比較方便),weight 則是用來表達 edge weight(如 comment),在我們進行 cycle breaking 後,會將刪除某些 edge 的結果儲存在 A,最後 output 的時候將 A 和 weight 進行比對來輸出刪掉的邊,最後又多創了一個 adj 的二維陣列,以便跑 DFS。Graph 裡的一大堆函式會在下方的 Algorithm 中解釋。

2. Algorithm

在本次作業中,我將幾乎所有的 function 都定義在 Graph 裡面,一方面是我如果我要在別的地方使用這個演算法,可以直接呼叫 graph 的 member function,感覺比較直覺,一方面是這樣是直接改 graph 裡的各種屬性,不需

要用外部 function 寫 call by referenc。,根據輸入建立好 Graph 後,直接跑 G.cyclebreak(fout) 開始進行拆解 cycle。

```
1. void Graph::cyclebreak(fstream& fout){
2. if(type == 'u'){
3.
            // undirected:
4.
            // (1) find the maximum spanning tree
            // (2) compare the MST to origin edge and find the deleted edge
5.
6.
            MST_Kruskal();
7.
            output_u(fout);
8.
9.
        else if(type == 'd'){
10.
           // directed:
            // (1) treat all edges as undirected edges and find the minimum spanning tree
12.
           // (2) compare the MST to origin edge and find the deleted
13.
            // (3) trying add back edges with "positive weight" in decreasing order
14.
            // (4) run DFS to check whether the edge added will cause cycle in the Graph
15.
            // (5) if not, add it back
16.
            // (6) compare the final result to origin edge and find the deleted edge
17.
18.
            MST Kruskal();
             vector<Edge> d;
19.
            dir_find_delete_edge(d);
20.
21.
             add_edge_back(d);
22.
            output_d(fout);
23.
        else{
24.
25.
            cout << "type error" << endl;</pre>
26.
27. }
```

(1) Undirected graph

Undirected graph 的部分,要求刪掉的 edge weight 總和越小越好,並且所有頂點都要被連接,因此我將課本中的 MST-Kruskal 稍微修改,改成找 Maximum spanning tree,這樣代表要刪掉的的邊 weight 總和會是 min,改動的地方只有將 edge 根據 weight sort 時的順序改為由大到小(先嘗試把大的加進 MST 中以找到 Maximum spanning tree),將要刪掉的邊記在上述的 A 矩陣裡再和 weight 比對就可以找到哪些邊是要被刪掉的。

(2) Directed graph

一開始想要使用的方法是跑 DFS 去 detect cycle, 並刪掉這個 cycle 中最小weight 的 edge, 但是後來覺得這個方法似乎有點不切實際, 因為也許會找到很多 cycle, 每個 cycle 又要繞整圈去找,感覺比較麻煩,因此不採用這個方法。

後來選用如上方 code 所示的方法,先將所有的 directed edge 都當作是無向的去跑 MST (max),跑完後的結果,可以確保剩下來的 edge 絕對不會造成 cycle,接下來將這些被刪掉的 edge,由大排到小開始嘗試一一把 weight>0 的 edge 加進去,加進一條邊後,check 他加進去會不會造成新的 cycle (偵測辦法:若(u, v)加入後會造成 cycle,這個 cycle 一定會包含(u, v),因此從 v 開始跑 DFS,如果跑得到 u 的話就代表有 cycle 存在!),如果不會的話就代表我可

以安心的把這個 edge 加進去(並更新上述 A 矩陣以及 adj 矩陣),當把所有 positive edge 都加進去後,比對 A 矩陣和 weight 就可以寫 output 了。

3. Findings

這次作業比較困難的部分處理有向圖的部分,但我認爲只要想到一個合理的演算法就可以交了,因爲其實我覺得最後刪掉的總 weight,其實和刪除或者加入edge 的順序有關係,我嘗試過將上方紅字的這個排序改成quicksort(unstable),讓同樣 weight 的東西先後順序可能改變,導致 directed case 中有些會變好,有些結果則會變差,但很難去評斷這個順序到底怎麼排比較好,因此最後就隨便取一種 sort 方法來做,並祈禱測資對我有利。

我認爲另一個難題是這個 graph 到底要怎麼建立,課本中的演算法多半寫的很簡單,看看就過去了,但實際上要寫成 code 時卻會先卡一陣子這些資料結構到底要怎麼實現,我覺得我這次寫的 graph 的 coding 沒有很漂亮,因爲有些private member 其實是可以移除的,但是我後來寫法是需要什麼就把它加進去(比方說 DFS 需要 adj matrix,就加,MST 需要 vertex.rank, p 這些東西,就寫一個 class vertex,加入 graph 中),但感覺如果客家一點的話應該是有很多東西可以捨去的,總之最後有算出答案了就先維持這樣吧 XD,有空在來修的好看一點。

最後,因爲這次作業比較難用肉眼驗證正確性,因此我自己大爆寫了一個確定答案是否正確的 cpp 檔,讀入原本的 input file,以及算出來的 output file,可以確認是否有 cycle,是否所有 vertex 都走的到,以及是否誤刪根本沒有的邊,也有把這個 check 的檔案傳給幾個同學並請他們試用看看有沒有 bug,但目前還不知道這個 checker 到底會不會有問題 XD,只能說我跑的結果丟去 checker 是沒有問題的。爲了展現誠意,把 checker 丟在後面給看報告的人過目(其實可以看前面註解就好),我覺得我在做的事情很符合工程師該做的事情,設計了一個東西不知道其對或錯,因此再設計一個東西來驗證其正確性,感覺蠻酷的!

```
1. // this is the checker for PA3, NTUEE, algorithm, 2020 fall
2. // produced by b07901021 SHIH-HSUAN PAN in 2020/12/20
3. // compile: g++ correct.cpp -o correct
4. // run: ./correct <original input file> <output_file you_produced>
5. //
6. // (1) the program will first read the original input file and construct graph
7. // (only consider the edge without considering the w
8. // (2) then the output file you produced will be readed
           (only consider the edge without considering the weight)
9. // (3) according to your output file, some edges in the graph will be removed
10. //
           also, the program will check whether the edge you removed is in the original in
  put file
11. //
           however, the order of undirected edges doesnt follow the order in the original
    input file
12. // (4) then the program will run DFS-
   smiliary algorithm to check the existence of cycles
13. // (5) treating all directed edges as undirected, then start traverse all vertices to
    see
14. //
           whether all vertices are weakly connected
15. //
16. // summary:
17. // the program will check
18. // (1) whether you delete some edges not in the original input file
19. // (2) whether all vertices are weakly connected
20. // (3) whether there is any cycle in your graph aftering removing edges you specified
21. //
22. // if there is any problem about this checker, please send email to me to discuss abou
   t it
23. // email: b07901021@ntu.edu.tw
24.
25. #include <cstring>
26. #include <fstream>
27. #include <vector>
28. #include <iostream>
29. #include <algorithm>
30.
31. using namespace std;
32.
33. class Graph
34. {
35.
        public:
            Graph();
36.
37.
             Graph(char , int);
             void E_construct(int, int);
38.
39.
             void E_delete(int, int);
40.
            void DFS_cycle_detecting_check();
41.
             void DFS_visit_check(int, vector<char>&, vector<int>&);
42.
             void weakly_connected_check();
            void traverse(int, vector<char>&, vector<vector<int> >&);
43.
44.
        private:
45.
             char type;
46.
             int n;
47.
             vector<vector<int> > adj;
48. };
49.
50. Graph::Graph(char t, int num){
51.
        type = t:
52.
        n = num;
53.
        for(int i = 0; i < n; i++){</pre>
54.
         vector<int> blank;
55.
             adj.push_back(blank);
56.
57. }
58. void Graph::E_construct(int u, int v){
        adj[u].push_back(v);
60. if(type == 'u') adj[v].push_back(u);
```

```
61. }
62.
63. void Graph::E_delete(int u, int v){
        if(type == 'd'){
64.
             cout << "deleting edge(" << u << ", " << v << ")" << endl;</pre>
65.
66.
             vector<int>::iterator it = find (adj[u].begin(), adj[u].end(), v);
67.
             if (it != adj[u].end()) adj[u].erase(it);
68.
             else cout << "wrong!! your output file deletes edge not in original input file</pre>
     << endl;
         }
69.
70.
         else{
71.
             vector<int>::iterator it1 = find (adj[u].begin(), adj[u].end(), v);
72.
             vector<int>::iterator it2 = find (adj[v].begin(), adj[v].end(), u);
73.
             if (it1 != adj[u].end() && it2 != adj[v].end()) {
74.
                 adj[u].erase(it1);
75.
                 adj[v].erase(it2);
                 // cout << "deleting edge(" << u << ", " << v << ")" << endl;
76.
77.
78.
             else{
79.
                 cout << "wrong!! your output file deletes edge not in original input file"</pre>
      << endl;
80.
81.
82. }
83.
84. void Graph::DFS_cycle_detecting_check(){
         vector<char> color(n, 'w');
85.
86.
         vector<int> pi(n, -1);
         for(int i = 0; i < n; i++){</pre>
87.
88.
             if(color[i] == 'w')
89.
                 DFS_visit_check(i, color, pi);
90.
91. }
92. void Graph::DFS_visit_check(int u, vector<char>& color, vector<int>& pi){
93.
         color[u] = 'g';
94.
         for(int i = 0; i < adj[u].size(); i++){</pre>
95.
             int v = adj[u][i];
96.
             if(color[v] == 'w'){
97.
                 pi[v] = u;
98.
                 DFS_visit_check(v, color, pi);
99.
100.
             else if(color[v] == 'g' && type == 'd'){
101.
                 cout << "wrong!! there are cycles in your graph" << endl;</pre>
102.
103.
             else if(color[v] == 'g' && type == 'u'){
104.
                 if(v != pi[u])
105.
                      cout << "wrong!! there are cycles in your graph" << endl;</pre>
106.
107.
108.
         color[u] = 'b';
109.}
110.void Graph::weakly_connected_check(){
111.
         vector<vector<int> > adj_matrix;
112.
         vector<char> color(n, 'w');
113.
         for(int i = 0; i < n; i++){</pre>
114.
             vector<int> temp(n, 0);
115.
             adj_matrix.push_back(temp);
116.
117.
         for(int i = 0; i < n; i++){</pre>
118.
             for(int j = 0; j < adj[i].size(); j++){</pre>
119.
                 int u = i;
120.
                 int v = adj[i][j];
121.
                 adj_matrix[u][v] = 1;
122.
                 adj_matrix[v][u] = 1;
123.
             }
124.
```

```
125.
         traverse(0, color, adj_matrix);
126.
        for(int i = 0; i < n; i++){</pre>
127.
             if(color[i] == 'w'){
128.
                cout << "wrong!! all vertices are not weakly connected" << endl;</pre>
129.
                 break;
130.
131.
        }
132.
133.}
134.void Graph::traverse(int u, vector<char>& color, vector<vector<int> >& m){
        color[u] = 'g';
135.
136.
         for(int i = 0; i < m[u].size(); i++){</pre>
137.
             if(m[u][i] == 1){
138.
                int v = i;
139.
                 if(color[v] == 'w'){
140.
                     traverse(v, color, m);
141.
142.
143.
144.
        color[u] = 'b';
145.}
146.
147.int main(int argc, char* argv[])
148.{
149.
        if(argc != 3) {
150.
           cout << "wrong arguments" << endl;</pre>
151.
            return 0;
152.
153.
154.
        /////// read the input file /////////
155.
156.
        cout << "reading..." << endl;</pre>
157
        char buffer[200];
158.
        fstream fin(argv[1]);
159.
        fstream fin_d(argv[2]);
        char type; // 'u' means undirected edge
160.
161.
                    // 'd' means directed edge
162.
        fin >> type;
163.
        int m, n; // n: total number of vertices
164.
                  // m: total number of edges
165.
        fin >> n >> m;
166.
167.
        vector<int> data;
168.
        int num;
169.
        while (fin >> num)
170.
             data.push_back(num); // data[3i] will be the start point of an edge.
171.
                                   // data[3i+1] will be the end point of an edge.
172.
                                  // data[3i+2] will be the weight.
173.
174.
        cout << "processing original input file..." << endl;</pre>
175.
        Graph G(type, n);
176.
        for(int i = 0; i < m; i++){</pre>
177.
             int u = data[3*i];
178.
             int v = data[3*i+1];
179.
             int w = data[3*i+2];
            G.E_construct(u, v);
180.
181.
182.
183.
        cout << "processing output file..." << endl;</pre>
184.
        vector<int> data_d;
        int num_d, total;
185.
186.
        fin d >> total;
        cout << "sum of deleted edge weight = " << total << endl;</pre>
187.
188.
        while (fin_d >> num_d){
189.
            data_d.push_back(num_d);
190.
```

```
191.
        for(int i = 0; i < data_d.size(); i+=3){</pre>
193.
           int v = data_d[i+1];
194.
          int w = data_d[i+2];
195.
           G.E_delete(u, v);
196. }
197.
198.
       cout << "checking cycle existance" << endl;</pre>
199.
       G.DFS_cycle_detecting_check();
200.
201.
       cout << "checking connected" << endl;</pre>
202.
       G.weakly_connected_check();
203.
204.
       cout << "checking finished" << endl;</pre>
205.}
206.
```