

In this project, a simulation of a multi-server system was implemented in Python. The code was generate under the Python 3.5 with pylab installed.

## 1. Introduction of Design

According to the instruction of the multi-server system, preprocessor and subserves obey the first-come-first-serve queueing discipline and all of them has infinite buffer, which means during the simulation period, no task would be decline. So a simple way to implement the simulation would be just handle each arrival request and calculate the finish time. If any request go over the hole simulation time, the request will not take into consideration as it is not finished. In other word, if we handle the previous finish time of preprocessor and subserves, each request can be calculate independently.

### Formula Design

Internal arrive time is defined as the sum of two part. The sequence  $a1_k$  is exponentially distributed with a mean arrival rate 0.85 requests per time unit. The sequence  $a2_k$  is uniformly distributed in the interval  $[0.05, 0.25]$ . I use the formula in `sim_mm1` :

**`-math.log(1-random.random())/rate`**

to generate exponentially distributed sequence where `random.random()` cloud generate random number in range $[0,1)$ :

**`random.uniform(0.05,0.25)`**

Previous function could generate uniformly distributed sequence, where uniform could generate uniform distribute in specific range. Inter\_arrival time is sum of those two random number

Subserves process time is the inverse function of the The probability density function  $f(ts)$ . First the integral of the function should be calculated and then we could write the formula as following:

**`(10.3846/(n**1.65))/((1-random.uniform(0,1))**(1/2.08))`**

Here `random.uniform(0,1)` is used to simulate Pareto distribution in python, as there is no build function to generate it. But it could be implement by uniform distribution between 0 and 1. Thus, I use `random.uniform(0,1)`.

## Work flow Design

When a request come into the system, a parameter is used to record the arrival time. While the arrival time is smaller than whole simulation period, we keep handle the request. Then we made a judgement whether the preprocessor is busy. Preprocessor was designed as an integer to record previous task's finish time. If the preprocessor is greater than current request's arrival time, preprocessor is busy. So current task will finish its preprocess in

$$\text{preprocessor} + \text{preprocess\_time}$$

Otherwise current task will finish in

$$\text{Arrival\_time} + \text{preprocess time}$$

After preprocess, the whole task is distribute into n sub\_tasks and send to each subserves. Here a python build in function **random.sample((0,10),n)** is used to generate a randomly list of subserves distribution without repeat(we assume preprocessor do not repeatedly forward to subserves for one request). Subserves are designed as Array in order to record their previous task's finish time:

$$\text{sub\_servers} = [0]*10$$

In each subserves we did same thing as what we had done for preprocessor. Comparing the value of subserves and renew the subserves. The finish time of a request is the longest finish time in each subserves. In this way, each request could be calculate independently.

The result of the simulation is stored in a list. The graph of the simulation or mean response time could be generate from the list.

In order to make the simulation reproducible, I use fake random which could generate same random number in different test.

## 2. Output Analysis

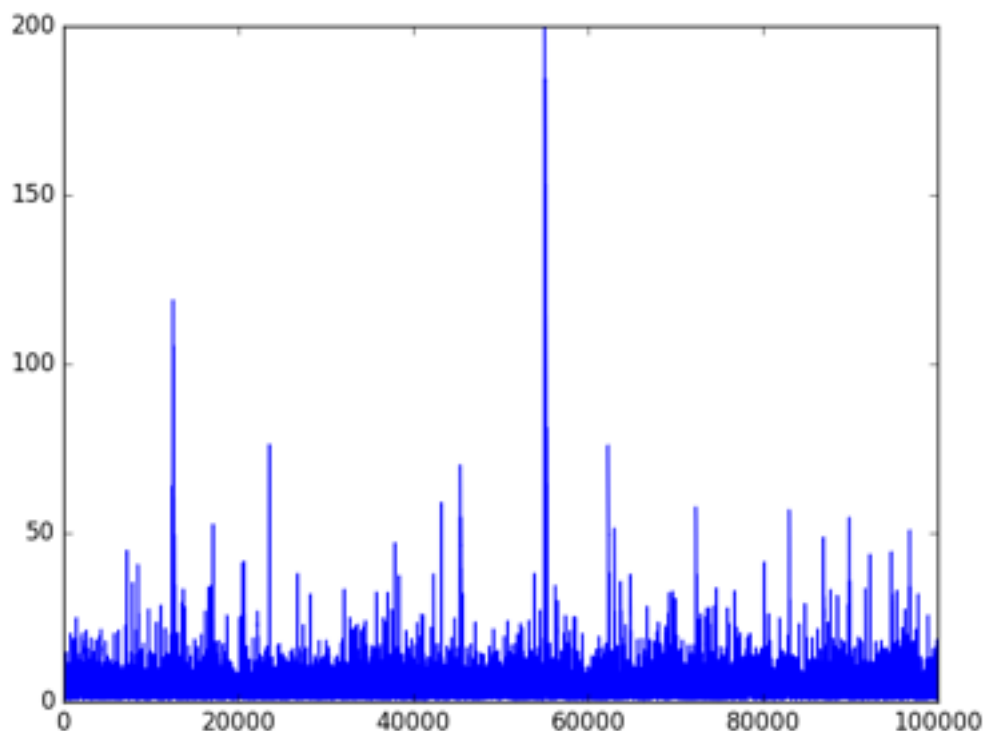
When set the whole simulation time as 10000, 80000, 90000, 100000 and 200000 time unit, we could get following data set:

n	10000	80000	90000	100000	200000	300000
1	1584.1545225 50526	13646.969993 770357	15397.080720 752654	17192.30246 064073	34323.36268 422478	51349.6708340

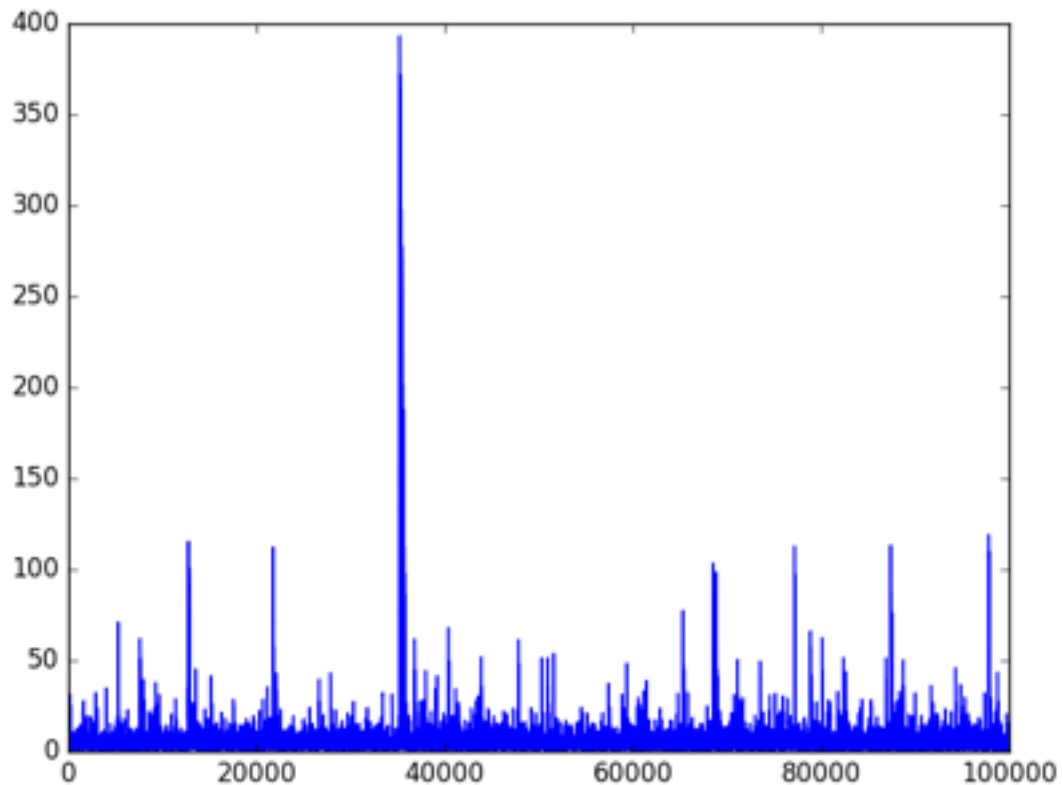
2	142.91670579 648658	300.32539670 816595	273.62034990 62273	258.0213212 923813	280.3624003 812613	328.2582478 097392
3	28.987195470 80968	31.587639270 24156	31.014511132 401942	31.65077172 174367	1.34015238353	33.31181835 073347
4	14.369633845 52062	15.931379480 487541	16.040043226 215605	17.07701258 357501	16.18986494 608148	17.19878589 5210353
5	10.571561664 603573	18.517094002 257593	9.9846227384 16883	9.743526070 379309	12.70911726 5624542	28.49081981 3913117
6	7.6021382108 54852	8.1375035206 67755	5.591854178997	8.358498062 145816	10.84551657 9330193	8.772087243 548873
7	7.1779874906 45033	8.8735829476 36786	7.1936460243 97184	7.445295893 778244	8.975120302 817741	8.981218331 644696
8	6.0424016658 03292	7.0025784368 98714	6.9499175056 44183	7.252334583 016475	3.668591299962	6.879488979 911506
9	6.933075440542	6.7698513984 25083	6.1956693247 03803	7.042908680 561254	8.000783066 363867	6.860541980 789675
10	6.3242722927 59348	7.3289622749 90749	8.7063468825 06097	8.512071879 948845	7.594867188 841812	7.348871239 605482

It is clear that although there are some fluctuations, when test time is long enough, the simulation will generate the solution that when broken into 8 or 9 sub-tasks, the mean response time would be least. Generally, when distributed tasks into too many subtasks, there is a bottleneck on the preprocessor as it will spend more time when breaking tasks. So there is a balance point for breaking tasks, too many subtasks or too small separation will lead to longer response time.

In order to make a solution of which distribution is better, we should do further test.



Take  $n=9$  and simulation time = 100000 time unit



Take  $n=8$  and simulation time = 100000 time unit

We could find there are very few results which are extremely huge. According to test result diagram we can set the confident interval as results which is less than 50 as most of the data distribute in this area. So we could generate new data set for  $n=8$  and  $n=9$  to analysis which choice is better:

n	10000	80000	900000	100000	200000	300000
8	5.824452698 099063	5.742732056 222329	5.737556093 064794	5.791103215 949062	5.748870647 473025	5.867649837 149568
9	5.838954234 828516	6.079313128 693893	5.946730086 436914	6.039459659 539555	5.908259323 114307	6.013586517 2902525

New test result shows that when we ignore result greater than 50, each test case shows that the  $n=8$  is a better choice. In this case we could say that distribute request into 8 sub-tasks will be a better choice