Lab10-Turing Machine

CS214-Algorithm and Complexity, Xiaofeng Gao & Lei Wang, Spring 2021.

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1. Design a one-tape TM M that computes the function $f(x, y) = \lfloor x/y \rfloor$, where x and y are positive integers ($x > y$). The alphabet is $\{1, 0, \Box, \rhd, \lhd\}$, and the inputs are x "1"s, $\Box$ and y "1"s. Below is the initial configuration for input $x = 7$ and $y = 3$. The result $z = f(x, y)$ should also be represented in the form of z "1"s on the tape with pattern of $\rhd 111 \cdots 111 \lhd$, which is $\rhd 11 \lhd$ for the example.

Initial Configuration

	$\rhd$	1	1	1	1	1	1	1	$\Box$	1	1	1	$\lhd$
	$\uparrow$ q_S												

   (a) Please describe your design and then write the specifications of M in the form like $\langle q_S, \rhd \rangle \to \langle q_1, \rhd, R \rangle$. Explain the transition functions in detail.

   (b) Please draw the state transition diagram.

   (c) Show briefly and clearly the whole process from initial to final configurations for input $x = 7$ and $y = 3$. You may start like this:

   $(q_s, \underline{\rhd}1111111\Box111\lhd) \vdash (q_1, \rhd\underline{1}111111\Box111\lhd) \vdash^* (q_1, \rhd1111111\underline{\Box}111\lhd) \vdash (q_2, \rhd1111111\Box\underline{1}11\lhd)$

   (Note that for simplicity, we write $(q_1, \rhd\underline{1}111111\Box111\lhd) \vdash^* (q_1, \rhd1111111\underline{\Box}111\lhd)$ if the corresponding transaction repeats on multiple inputs with the same state.)

Solution. (a) Start state:

$$\langle q_S, \rhd \rangle \to \langle q_1, \rhd, R \rangle$$

Go to the first "1" of y:

$$\langle q_1, 1 \rangle \to \langle q_1, 1, R \rangle$$
$$\langle q_1, \Box \rangle \to \langle q_2, \Box, R \rangle$$

Change all "1" of y into 0, while delete the same number of "1" in x:

$$\langle q_2, 1 \rangle \to \langle q_3, 0, L \rangle$$
$$\langle q_3, 0 \rangle \to \langle q_3, 0, L \rangle$$
$$\langle q_3, \Box \rangle \to \langle q_3, \Box, L \rangle$$
$$\langle q_3, 1 \rangle \to \langle q_2, \Box, R \rangle$$
$$\langle q_2, \Box \rangle \to \langle q_2, \Box, R \rangle$$
$$\langle q_2, 0 \rangle \to \langle q_2, 0, R \rangle$$

Write a "1" in the right side of $\lhd$, and turn all "0" back to "1":

$$\langle q_2, \lhd \rangle \to \langle q_4, \lhd, R \rangle$$
$$\langle q_4, 1 \rangle \to \langle q_4, 1, R \rangle$$
$$\langle q_4, \Box \rangle \to \langle q_5, 1, L \rangle$$
$$\langle q_5, 1 \rangle \to \langle q_5, 1, L \rangle$$
$$\langle q_5, \lhd \rangle \to \langle q_5, \lhd, L \rangle$$
$$\langle q_5, 0 \rangle \to \langle q_5, 1, L \rangle$$
$$\langle q_5, \Box \rangle \to \langle q_2, \Box, R \rangle$$

If the remain "1" in x is less than the number of "1" in y, end the loop and get the answer:

$$\langle q_3, \triangleright\rangle \to \langle q_6, \square, R\rangle$$
$$\langle q_6, 1\rangle \to \langle q_6, \square, R\rangle$$
$$\langle q_6, \square\rangle \to \langle q_6, \square, R\rangle$$
$$\langle q_6, 0\rangle \to \langle q_6, \square, R\rangle$$
$$\langle q_6, \triangleleft\rangle \to \langle q_7, \triangleright, R\rangle$$
$$\langle q_7, 1\rangle \to \langle q_7, 1, R\rangle$$
$$\langle q_7, \square\rangle \to \langle q_H, \triangleleft, R\rangle$$

(b) Below is the state transition diagram.

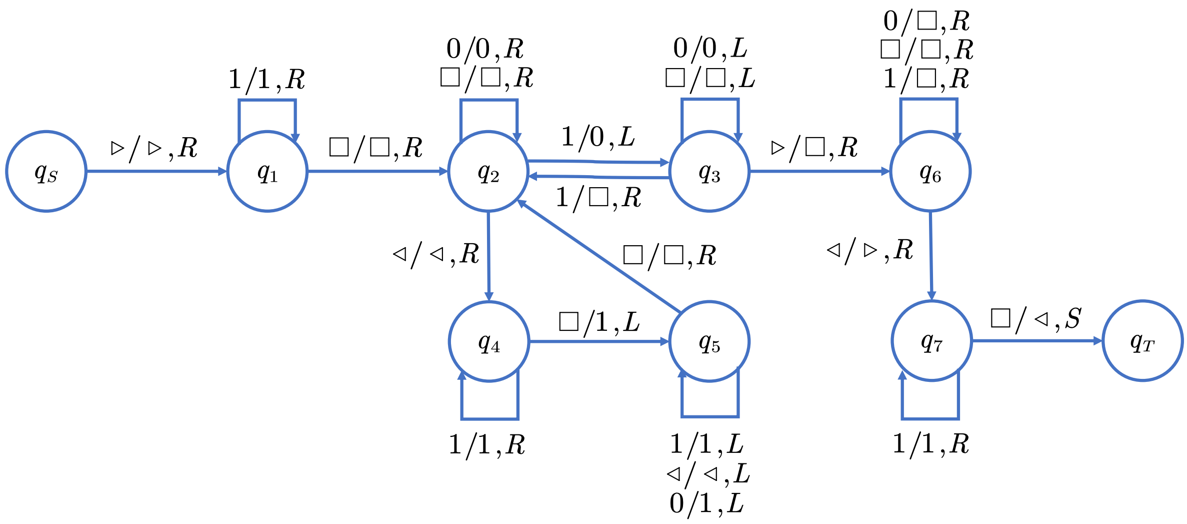

Figure 1: The state transition diagram

(c)

$$(q_s, \underline{\triangleright}1111111\square111\triangleleft)$$
$$\vdash(q_1, \triangleright\underline{1}111111\square111\triangleleft)$$
$$\vdash^*(q_1, \triangleright1111111\underline{\square}111\triangleleft)$$
$$\vdash(q_2, \triangleright1111111\square\underline{1}11\triangleleft)$$
$$\vdash(q_3, \triangleright1111111\underline{\square}011\triangleleft)$$
$$\vdash(q_3, \triangleright111111\underline{1}\square011\triangleleft)$$
$$\vdash(q_2, \triangleright111111\square\underline{\square}011\triangleleft)$$
$$\vdash(q_2, \triangleright111111\square\square\underline{0}11\triangleleft)$$
$$\vdash(q_2, \triangleright111111\square\square0\underline{1}1\triangleleft)$$
$$\vdash^*(q_2, \triangleright1111\square\square\square\square00\underline{0}\triangleleft)$$

$$
\begin{aligned}
&\vdash (q_2, \rhd 1111\square\square\square\square 000\underline{\lhd})\\
&\vdash (q_4, \rhd 1111\square\square\square\square 000 \lhd \underline{\square})\\
&\vdash (q_5, \rhd 1111\square\square\square\square 000\underline{\lhd}1)\\
&\vdash (q_5, \rhd 1111\square\square\square\square 00\underline{0} \lhd 1)\\
&\vdash (q_5, \rhd 1111\square\square\square\square 0\underline{0}1 \lhd 1)\\
&\vdash^* (q_5, \rhd 1111\square\square\square\underline{\square}111 \lhd 1)\\
&\vdash (q_2, \rhd 1111\square\square\square\square\underline{1}11 \lhd 1)\\
&\vdash^* (q_2, \rhd 1\square\square\square\square\square\square\square\underline{1}11 \lhd 11)\\
&\vdash^* (q_3, \underline{\rhd}\square\square\square\square\square\square\square\square 111 \lhd 11)\\
&\vdash (q_6, \square\underline{\square}\square\square\square\square\square\square\square 111 \lhd 11)\\
&\vdash^* (q_6, \underline{1}11 \lhd 11)\\
&\vdash (q_6, \square\underline{1}1 \lhd 11)\\
&\vdash^* (q_6, \underline{\lhd}11)\\
&\vdash (q_7, \rhd\underline{1}1)\\
&\vdash (q_7, \rhd 1\underline{1})\\
&\vdash (q_7, \rhd 11\underline{\square})\\
&\vdash (q_T, \rhd 11\underline{\lhd})
\end{aligned}
$$

$\square$

2. Given the alphabet $\{1, 0, \square, \rhd, \lhd\}$, design a time efficient 3-tape TM M to compute $f : \{0,1\}^* \to \{0,1\}$ which verifies whether the number of 0 and the number of 1 are the same in an input consisting of only 0's and 1's. M should output 1 if the numbers are the same, and 0 otherwise. For eample, for the input tape $\rhd 001101 \lhd$, M should output 1

   (a) Please describe your design and then write the specifications of M in the form like $\langle q_S, \rhd, \rhd, \rhd\rangle \to \langle q_1, \rhd, \rhd, R, R, S\rangle$. Explain the transition functions in detail.

   (b) Show the time complexity for one-tape TM M' to compute the same function f with n symbols in the input and give a brief description of such M' .

Solution. (a) Start state:

$$\langle q_S, \rhd, \rhd, \rhd\rangle \to \langle q_1, \rhd, \rhd, R, R, S\rangle$$

Go right, write a "1" in the second tape if it reads a "1" in the first tape, and do nothing if reads "0" in the first tape:

$$
\begin{aligned}
\langle q_1, 0, \square, \rhd\rangle &\to \langle q_1, \square, \rhd, R, S, S\rangle\\
\langle q_1, 1, \square, \rhd\rangle &\to \langle q_1, 1, \rhd, R, R, S\rangle\\
\langle q_1, \lhd, \square, \rhd\rangle &\to \langle q_2, \square, \rhd, L, L, S\rangle
\end{aligned}
$$

Go left, move to left in the second tape if it reads a "0" in the first tape, and do nothing if reads "1" in the first tape:

$$
\begin{aligned}
\langle q_2, 1, 1, \rhd\rangle &\to \langle q_2, 1, \rhd, L, S, S\rangle\\
\langle q_2, 0, 1, \rhd\rangle &\to \langle q_2, 1, \rhd, L, L, S\rangle
\end{aligned}
$$

If the first tape and the second tape meet $\triangleright$ together, then the number of 0 and the number of 1 are the same:

$$\langle q_2, \triangleright, \triangleright, \triangleright\rangle \to \langle q_4, \triangleright, \triangleright, S, S, R\rangle$$
$$\langle q_4, \triangleright, \triangleright, \square\rangle \to \langle q_5, \triangleright, 1, S, S, R\rangle$$
$$\langle q_5, \triangleright, \triangleright, \square\rangle \to \langle q_H, \triangleright, \triangleleft, S, S, S\rangle$$

Otherwise, then the number of 0 and the number of 1 are not the same:

$$\langle q_2, 0, \triangleright, \triangleright\rangle \to \langle q_6, \triangleright, \triangleright, S, S, R\rangle$$
$$\langle q_2, 1, \triangleright, \triangleright\rangle \to \langle q_6, \triangleright, \triangleright, S, S, R\rangle$$
$$\langle q_2, \triangleright, 1, \triangleright\rangle \to \langle q_6, 1, \triangleright, S, S, R\rangle$$
$$\langle q_6, 0, \triangleright, \square\rangle \to \langle q_7, \triangleright, 0, S, S, R\rangle$$
$$\langle q_6, 1, \triangleright, \square\rangle \to \langle q_7, \triangleright, 0, S, S, R\rangle$$
$$\langle q_6, \triangleright, 1, \square\rangle \to \langle q_7, 1, 0, S, S, R\rangle$$
$$\langle q_7, 0, \triangleright, \square\rangle \to \langle q_H, \triangleright, \triangleleft, S, S, S\rangle$$
$$\langle q_7, 1, \triangleright, \square\rangle \to \langle q_H, \triangleright, \triangleleft, S, S, S\rangle$$
$$\langle q_7, \triangleright, 1, \square\rangle \to \langle q_H, 1, \triangleleft, S, S, S\rangle$$

(b) The M' can use the similar algorithm with the M. Each time the reading head reads a "1", it moves to the right side and write a flag; after it goes to the rightest side, then turn left. In this turn, each time the reading head reads a "0", it moves to the right side and delete a flag. Then it can make a compare according to the number of flags.

Because the time complexity of moving right is $O(n)$ and there are n symbols in the input, the final time complexity is $O(n^2)$.

□

3. Define the corresponding decision or search problem of the following problems and give the "certificate" and "certifier" for each decision problem provided in the subquestions or defined by yourself.

(a) *3-Dimensional Matching.* Given disjoint sets X, Y, Z all with the size of n, and a set $M \subseteq X \times Y \times Z$. Is there a subset M' of M of size n where no two elements of M' agree in any coordinate?

(b) *Travelling Salesman Problem.* Given a list of cities and the distances between each pair of cities, find the shortest possible route that visits each city exactly once and returns to the origin city.

(c) *Job Sequencing.* Given a set of unit-time jobs, each of which has an integer deadline and a nonnegative penalty for missing the deadline. Does there exist a job sequence that has a total penalty $w \leqslant k$?

Solution. (a) This is a decision problem.

The search problem is: Find a subset M' of M of size n where no two elements of M' agree in any coordinate.

Certificate: A subset M' of M of size n.

Certifier: Check that in any coordinate there are no two elements of M' agree.

(b) This is a search problem.

The decision problem is: Is there a route that visits each city exactly once and returns to the origin city and its length $l \leqslant k$.

Certificate: A route r_0 that visits each city exactly once and returns to the origin city.

Certifier: Assume that the length of r_0 is l_0. Check that $l_0 \leqslant k$.

(c) This is a decision problem.

The search problem is: Find a job sequence with a smallest total penalty.

Certificate: A job sequence s.

Certifier: Check that the total penalty w of s is less or equal to k.

□

Remark: Please include your .pdf, .tex files for uploading with standard file names.