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Mobile robots path planning: Electrostatic potential field approach

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ABSTRACT

This paper deals with the mobile robots path planning problem in the presence of scattered obstacles in a visually known environment. Utilizing the theory of charged particles' potential fields and inspired by a key idea of the authors' recent work, an optimization based approach is proposed to obtain an optimal and robust path planning solution. By assigning a potential function for each individual obstacle, the interaction of all scattered obstacles are integrated in a scalar potential surface (SPS) which strongly depends on the physical features of the mobile robot and obstacles. The optimum path is then obtained from a cost function optimization by attaining a trade-off between traversing the shortest path and avoiding collisions, simultaneously. Hence, irrespective of any physical constraints on the obstacles/mobile-robot and the adjacency of the target to the obstacles, the achieved results demonstrate a feasible, fast, oscillation-free and collision-free path planning of the proposed method. Utilizing a scalar decision variable makes it extremely simple in terms of mathematical computations and thus practically feasible that can be applied to both static and dynamic environments. Finally, simulation results verified the performance and fulfillment of the mentioned objectives of the approach.

1. Introduction

Extensive developments of mobile robot (MR) applications in scientific and industrial fields require more accurate, optimum and safe path planning algorithms. Furthermore, in MR navigation, we are often provided with some kind of representation of the environment such as global visual images and/or local infrared/sonar information. Then, the goal is to plan a path from some initial robot location to a desired location. The MR autonomous path planning and accomplishing its mission without a human supervisor in real world applications is a substantial problem. This problem has drawn vast attractions from scientists and many successful algorithms have been represented such as roadmap (Raja & Pugazhenthi, 2012; Saha, 2006), cell decomposition (Lingelbach, 2004), A*, D* (Hart, Nilsson, & Raphael, 1968; Koenig & Likhachev, 2002a; 2002b) and many other approaches.

The problem of autonomous path planning is not confined to the field of (mobile) robotics and many interesting ideas and applications can be found in various scientific fields. According to this fact and inspired by a key idea of the authors' recent work in the field of non-convex data clustering (see Bayat, Mosabbeb, Jalali, & Bayat, 2010), in this paper, a new approach has been developed which finds an optimal robust traversable path in congruous with the physical constraints of the MR without any use of complicated mathematics. The optimality of the proposed method arises from the fact that the solution of the path planning problem is obtained via an optimization problem whose cost function contains the design objectives. Also, the robustness of the proposed method rooted in the fact that the design strategy follows the physical principles governing electric potential fields where the robot attempts to move in a direction that represents the resultant repulse of all particles (obstacles) to avoid collisions. Another feature of the proposed approach is that the size, color, orientation and number of the obstacles, and adjacency of the target to the obstacles do not affect the path planning procedure since the effect of all mentioned items are integrated in a scalar potential surface (SPS). As a consequence, the MR can navigate without being trapped in local minimums as demonstrated via broad challenging simulations.

The remainder of this paper is organized as follows. In Section 2, a comprehensive review of the related works is stated. Section 3 provides preliminary definitions used throughout this work, formulates the problem and introduces the theoretical framework of the proposed method. Subsequently, Section 4 evaluates the performance of the proposed approach. Finally, the conclusions and future works are drawn in Section 5.

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2. Related works

A significant number of research-works in the mobile robot path planning field has been done during the past decades. Regarding the ability of accessing the environmental information of the robot, the algorithms are classified into two categories: to be specific online (global) and off-line (local) (Mac, Copot, Tran, & De Keyser, 2016; Tavares, Martins, & Tsuzuki, 2011). Concerning off-line path planning, the prior information about the robot's environment, including static and dynamic obstacles locations, is available (Raja & Pugazhenthi, 2012). Therefore, the navigation path for the robot from the initial point to the target is decided at the start point (Leena & Saju, 2014). In the contrast, online path planning deals with completely unknown environments and the robot acquires information through sensors by navigating between obstacles. Nevertheless, on-line method starts the path planning in off-line mode and after detecting the new changes in environment switches to online mode (Raja & Pugazhenthi, 2012). The first formulated attitude toward the path planning problem is the Configuration space (C-space) presented by Lozano-Pérez and Wesley (1979). Yet, the idea of the approach was inspired from Udupa's doctoral thesis. According to this method, the robot is considered as a point and to compensate this consideration, obstacles are assumed to be bigger than their real size corresponding to the robot's dimensions. So the path planning problem becomes a 2D issue (Raja & Pugazhenthi, 2012). The C-space method plays a pivotal role in many classical path planning approaches such as road-map, cell decomposition, potential field and mathematical programming (Masehian & Sedighizadeh, 2007; Raja & Pugazhenthi, 2012). Taking the roadmap approach into account, the method deals with networks of collision free paths from the initial point to the target (Raja & Pugazhenthi, 2012). This attitude is a graph-searching problem and is also known as Retraction, Skeleton, or Highway approach. The most famous roadmap methods are visibility graph, Voronoi diagram, Silhouette, and the Subgoal Network (Masehian & Sedighizadeh, 2007). Considering visibility graph approach (Lozano-Pérez & Wesley, 1979), the graph of roads are built by connecting the vertices of polygonal obstacles to each other located between the start point and the target. The shortest road is considered the best path to reach to target. This method is more suitable for scattered environments and as it gets crowded with different obstacles, the number of roads increases (Raja & Pugazhenthi, 2012). The concept of Voronoi diagram (Ó'Dúnlaing & Yap, 1985; Wei, Mao, Guan, & Li, 2017), is to find the points that have the same distance from two or more obstacles and connect these points to reach the goal. The chosen path for the robot is the safest path and not necessarily the shortest one (Garrido, Moreno, Blanco, & Jurewicz, 2011; Masehian & Amin-Naseri, 2007; Raja & Pugazhenthi, 2012). As for Silhouette approach, it maps the obstacles from higher dimensional space to a lower and pursues the outline of the mapped ones which are called Silhouette. The Subgoal Network method constructs a list of reachable configurations from the initial point to the target and their attainability is determined by local operator algorithm (Mac et al., 2016; Masehian & Sedighizadeh, 2007). Concerning the cell decomposition method, the free C-space of the robot is decomposed into cells and a path to the target point is resulted by connecting a sequence of these cells together. The famous cell decomposition method is a griding approach in which the map of the environment is divided into grids. Application of multi-objective variable neighborhood search method is proposed in Hidalgo-Paniagua, Vega-Rodríguez, and Ferruz (2016) that optimizes the path length, the safeness and smoothness, simultaneously. A review of the optimization based and Neuro-Fuzzy approaches are provided in Parhi and Jha (2012). The concept of artificial potential field (APF) method was first developed in Khatib (1986) and then extended in Borenstein and Koren (1989). Since then, this idea found extensive applications in the literatures (Kim, 2009; Kuo, Li, Chen, Ho, & Lin, 2017; Mac et al., 2016; Montiel, Orozco-Rosas, & Sepulveda, 2015a; Montiel, Sepulveda, & Orozco-Rosas, 2015b; Valavanis, Hebert, Kolluru, & Tsourveloudis, 2000). According to this approach, artificial forces are generated in order to repulse the robot from obstacles and meanwhile attract it to the target (Kuo et al., 2017; Mac et al., 2016). This method was proved to be efficient in the presence of multiple robots (Pradhan, Parhi, Panda, & Behera, 2006) and dynamic obstacles (Hwang & Ahuja, 1992; Montiel et al., 2015a). However, authors in Borenstein and Koren (1991); Koren and Borenstein (1991) revealed some intrinsic demerits of the APF algorithm and then presented a new method named vector field histogram (VHF) to cope with limitations of the APF. During the past years, various modification of the APF method has been proposed to improve the implementation of this approach, e.g. escape-force algorithm (Kim, 2009), multi-resolution APF (Kim & Lee, 2005), trap recovery model (Lv & Feng, 2006), gravity chain approach (Tang et al., 2010), adaptive virtual target algorithm (Luh & Liu, 2008), finite-element based approach (Pimenta et al., 2006) and iterative APF method (Adeli, Tabrizi, Mazloomian, Hajipour, & Jahed, 2011) can be mentioned. Also, new algorithms are proposed in Antich and Ortiz (2005); Kim (2009) to escape from local minima, which is an important drawback of the APF algorithm. Furthermore, an approach is developed in Mac et al. (2016) in order to diminish oscillation.

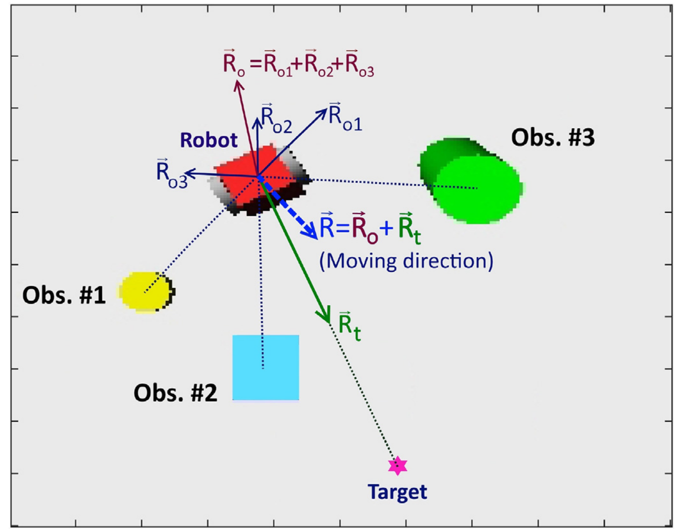


Fig. 1. Illustration of the APF approach.

3. Path planning algorithm

As already mentioned, the proposed algorithm in this study is basically inspired by the key idea of our previous research work in the field of non-convex data clustering (Bayat et al., 2010). It is emphasized that, the proposed method is fundamentally different from the APF approach. As shown in Fig. 1, the APF algorithm is constructed based on the concept of configuration space where the robot's configuration at the current time is assumed as $\vec{q}=(x,y)$. The target at $\vec{q}_t=(x_t,y_t)$ exerts an attractive force as $\vec{R}_t=(x-x_t,y-y_t)$, while the i-th obstacle at $\vec{q}_{oi}=(x_{oi},y_{oi})$ applies repulsive force as $\vec{R}_{oi}=(x-x_{oi},y-y_{oi})$ to the robot. Then, the moving direction of the robot is given by the resultant force vector $\vec{R}=\vec{R}_t+\sum_{i=1}^{N}\vec{R}_{oi}$, comprising the sum of the target attractive force $\vec{R}_t$ and the repulsive forces of all obstacles $\vec{R}_{oi}$, for a given robot position $\vec{R}$. However, in this paper, instead of considering a set of vector fields (forces) attracting or repelling the robot, which is computationally expensive, the influence of a scalar potential surface (SPS) caused by all obstacles on the robot's environment is employed. This modification reduces the computational complexity of the approach since in the real time operation a scalar function is evaluated rather a set of vector fields. In other words, a potential surface is constructed representing the resultant potential fields of all obstacles and the objective is that the robot avoids the high potential areas and finds an optimal and collision-free path to the target through the potential surface.

Definition 1. Space (D, $\|.\|$) in linear vectors set $D \subseteq \Re^n$ and real functions $\|.\| : D \to \Re_+$ are called a norm space if all following conditions fulfill:

1. $\|Z\| \geq 0, \forall Z \in D$
2. $\|Z\| = 0 \Leftrightarrow Z = \mathbf{0}, \; and \, Z \in D$
3. $\|\alpha Z\| = |\alpha|.\|Z\|, \; \forall Z \in D, \; \alpha \in \Re$
4. $\|Z_1 + Z_2\| \leq \|Z_1\| + \|Z_2\|, \; \forall Z_1, Z_2 \in D$

Definition 2. Assume $D = \{Z_i \in \Re^n | Z_i = (z_{i1}, \ldots, z_{in}), \; z_{ij} \in \Re\}$, where $i = 1, \ldots, N_o$, and $D \in \Re^{N_o \times n}$. Then the norm space (D, $\|.\|_2$) is called a limited norm space relative to $\|.\|_2$ if and only if there exists a scalar $0 < h < \infty$ such that $\|Z_i\|_2 \leq h$ for all $\mathbf{Z}_i \in D$ where $\|Z_i\|_2^2 = \sum_{j=1}^{n} z_{ij}^2$.

Definition 3. A scalar function $V(Z): \Re^n \to \Re$ is called a potential function if:

1. $V(Z)$ is a continuous smooth function in the given limited norm space (later we will find out that this space is in fact the robot's environmental space).
2. $V(Z)$ is isotopic, i.e. it has symmetric behavior and characteristics in all dimensions.
3. If $V_i(Z)$ is the potential function for component Z_i, increasing $\|Z - Z_i\|_2$ should cause $V_i(Z)$ to decrease and $V_i(Z) \to 0$ for each $\|Z - Z_i\|_2 \to \infty$.

Assumption 1. In what follows, it is assumed that the regarding space in the path planning problem is a limited norm space as such the potential functions and the associated scalar field could be extracted.

Associated with the i-th obstacle (see Fig. 2(a)) which is centered at $Z_i = (x_i, y_i)$, a potential function is defined as:

$$\begin{aligned} V_i(Z) &= e^{-\beta_i \|Z - Z_i\|_2^2} = e^{-\beta_i \left((x - x_i)^2 + (y - y_i)^2\right)} \\ Z &= [x, \; y]^T \in R^2, \quad Z_i = [x_i, y_i]^T \in R^2 \end{aligned} \tag{1}$$

where β_i is a positive scalar, and $V_i(Z)$ can be interpreted as the scalar potential function of an electric particle located in Z_i. Clearly, this candidate function satisfies all above mentioned properties. The impact of β_i on the potential functions are illustrated in Figs.2(c,e,g) for 5-th obstacle. Generally, as a consequence of being the largest obstacle among the others (by means of its area), the 5-th obstacle has a broad hillside and similarly, the 1-st obstacle, which is the smallest one, has a slender hillside. However, it is important to note that, setting up each β_i for path planning purpose requires not only to consider the i-th obstacle but also the effects of other obstacles' and the robot's dimensions should be taken into account simultaneously. In order to obtain an applied formula, it is necessary to pay attention to a few principles. First, the potential function $V_i(Z)$ in (1) is a decreasing function of β_i. So, the larger value of the β_i, the faster vanishing and tighter $V_i(Z)$, and vice versa. Second, when assigning a potential function to each obstacle, it is natural that, the potential function of an obstacle with larger size (area) compared to the average obstacles' area, must be wider, and vice versa. This equivalently means that $\beta_i \propto \frac{1}{A_i}$ and $\beta_i \propto \frac{1}{N_o} \sum_{j=1}^{N_o} A_j$ denoting the effects of i-th obstacle's area and the average area, respectively. In other words, when an obstacle has small area (A_i) compared to other ones, then its corresponding β_i should be large, and vice versa. Third, since the robot will traverse through the resultant potential surface, it is important to take the size of the robot (A_R) into account when adjusting the parameters β_i. In fact, in order to avoid the obstacle collision when the robot moves, as much as the size of the robot becomes larger, the potential functions have to be wider, and vice versa. This implies that $\beta_i \propto \frac{1}{A_R}$. In sum, an applied formula for adjusting β_i is given as:

$$\beta_i = \frac{K}{A_i A_R N_o} \sum_{j=1}^{N_o} A_j \tag{2}$$

where N_o is the number of obstacles, A_i and A_R denote the areas of the i-th obstacle and the robot, respectively. Also, K is an empirical positive tuning constant that is used to uniformly scale all potential functions. Note that, increasing K will decrease the robustness of the path planning in terms of the collision, because this increase will make all potential functions tighter which consequently allows the robot to get closer to the obstacles. On the other hand, this will increase the possibility of faster approaching to the target. The effect of variations in the parameter K is illustrated in Fig. 2(d,f,h). Using the information of all obstacles and the robot, the resultant scalar potential surface can be obtained. To this end, the average potential function of the environment is indicated by:

$$V_a(x, y) = \frac{1}{N_o} \sum_{i=1}^{N_o} V_i(x, y) \tag{3}$$

As shown in Fig. 2(d), where $V_a(x, y)$ has several local maximum, in a way that, each maximum represents the center of an obstacle.

Path planning implementation requires information about obstacles' position and their areas. To acquire them, a picture of the robot's environment is captured by a camera installed right above the field. By means of the image processing techniques, the MR and obstacles can be differentiated from each other (see Fig. 3), and subsequently, the obstacles' area and centroid can be obtained. Furthermore, a set of conceivable orientations for MR's next destination must be taken into account. The eight directions associated with eight cells are depicted in Fig. 4 and formulated as:

$$Z_w^* = Z + \begin{bmatrix} dx_w \\ dy_w \end{bmatrix} \tag{4}$$

where Z, Z_w^* are the robot's current and next desired locations, and dx_w, dy_w denote the optimal directions along x and y axes, respectively. Note that, $dx_w \in \{-dx, 0, +dx\}$ and $dy_w \in \{-dy, 0, +dy\}$ where dx and dy are the given resolutions along x and y axes, respectively.

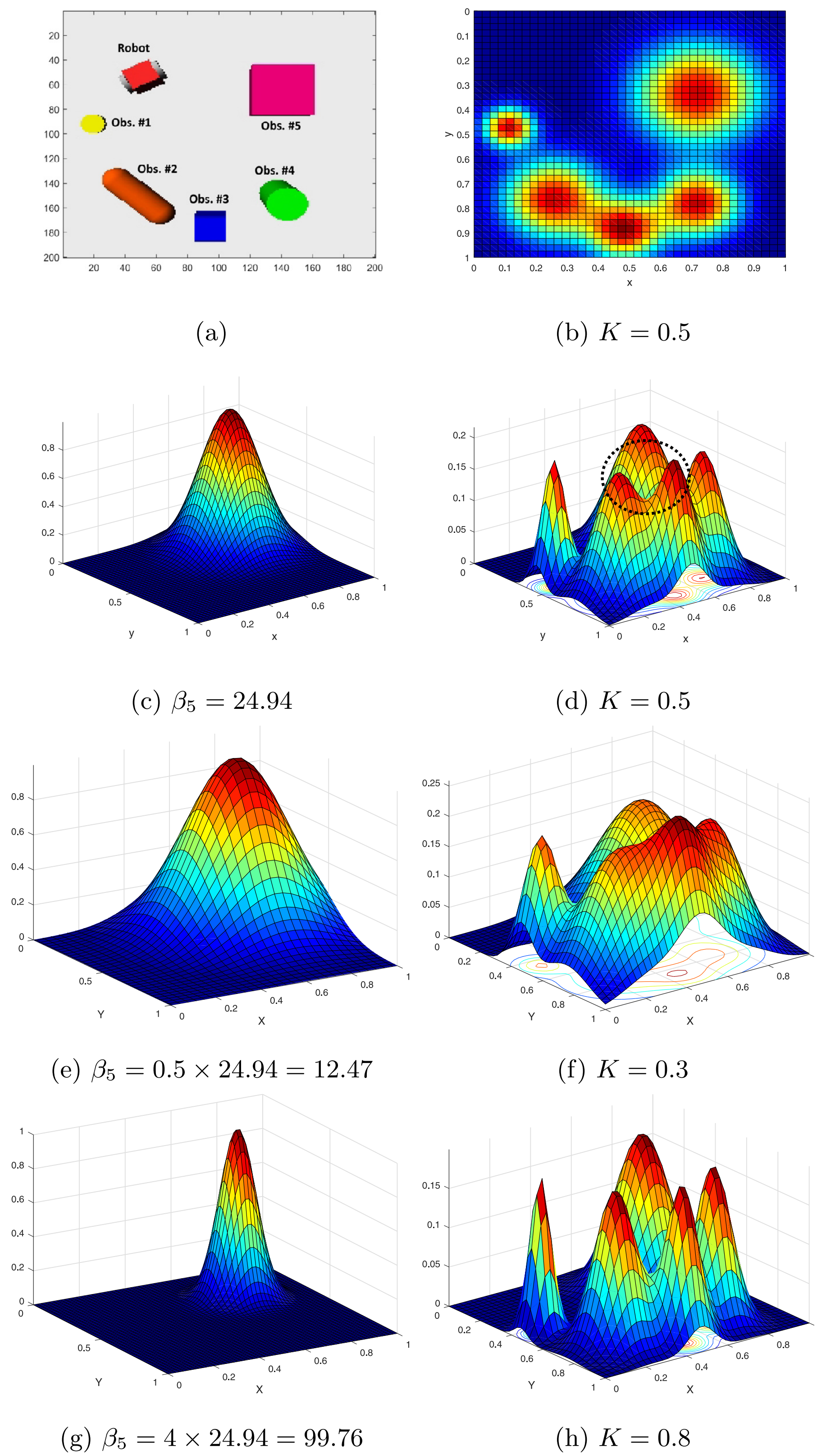
It is emphasized that, the only use of scalar potential function of the obstacles in the robot's environment can not lead to the optimal result. In the other words, the optimal path results from a trade-off between the obstacles collision avoidance goal (by avoiding the high potential districts) and choosing the shortest path, simultaneously. Hence, in the cost function it is required to obtain which cell has the lowest cost among the others that is determined as the MR's next destination. To elaborate the path planning problem, a cost function J_w is defined for each direction as:

$$J_w(Z_w, Z_T) = \alpha D(Z_w, Z_T) + (1 - \alpha) V_a(Z_w) \qquad \forall w = 1, \ldots, 8 \tag{5}$$

Where $0 \leq \alpha \leq 1$ and $Z_T = [x_T, y_T]^T$ denotes the target point and $D(Z_w, Z_T)$ is a distance functions defined as

$$D(Z_w, Z_T) = \|Z_w - Z_T\|_2 = \sqrt{(x_w - x_T)^2 - (y_w - y_T)^2} \quad \forall w = 1, \ldots, 8 \tag{6}$$

Then, the path planning problem is solved using the following optimization problem:



(a) (b) $K = 0.5$

(c) $\beta_5 = 24.94$ (d) $K = 0.5$

(e) $\beta_5 = 0.5 \times 24.94 = 12.47$ (f) $K = 0.3$

(g) $\beta_5 = 4 \times 24.94 = 99.76$ (h) $K = 0.8$

Fig. 2. Illustration of (a) robot's environment, (b) projection of the obstacles' potential surface, (c,e,g) potential function of obstacle 5 for different values of β, (d,f,h) average potential surface of all obstacles for different values of K.

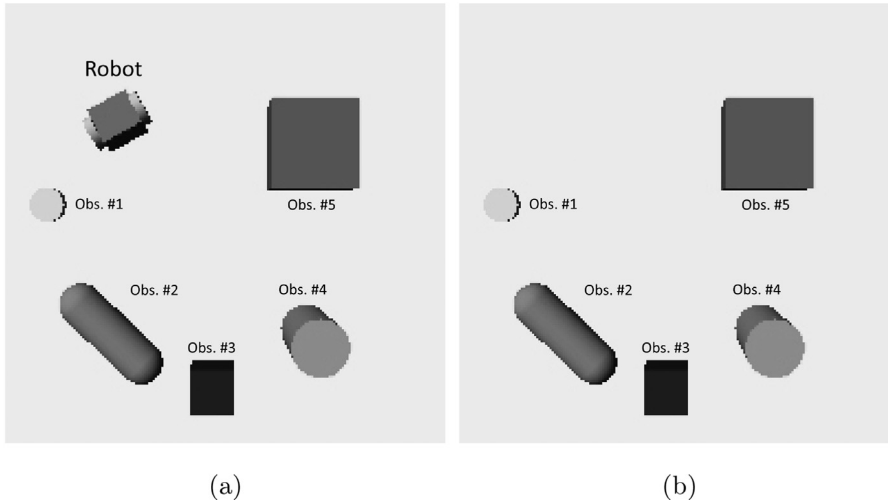


Fig. 3. The original image of the MR's environment and the image of obstacles after omitting the MR.

Table 1
Optimal directions of eight cells around the robot.

	$w^*=1$	$w^*=2$	$w^*=3$	$w^*=4$	$w^*=5$	$w^*=6$	$w^*=7$	$w^*=8$
dx_w	$-dx$	0	$+dx$	$+dx$	$+dx$	0	$-dx$	$-dx$
dy_w	$+dy$	$+dy$	$+dy$	0	$-dy$	$-dy$	$-dy$	0

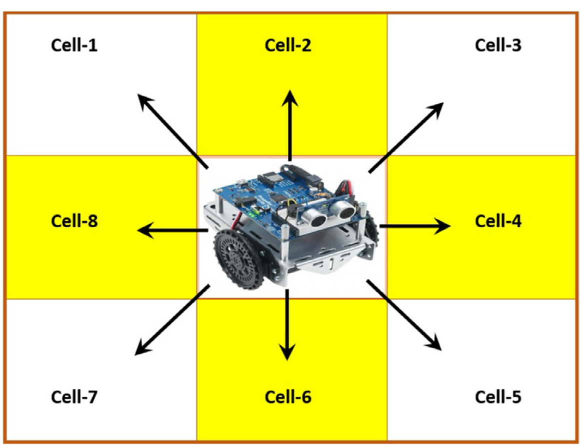


Fig. 4. Possible directions for the MR to navigate.

$$w^* = \arg \min_{w\in\{1,\ldots,8\}} \{J_w = \alpha D(Z_w, Z_T) + (1-\alpha)V_a(Z_w)\} \tag{7}$$

As far as the solution of (7) is obtained, the optimal direction can be calculated using Table 1. The implementation steps of the proposed method are presented in Algorithm 1.

Remark 1. The parameter $\alpha \in [0\ 1]$ is a trade-off knob that swaps the importance of two planning objectives, i.e. robustness and fast approaching, in the cost function. When α takes larger values it permits the robot to get closer to the obstacles and thus getting shorter path. However, it is important to note that if α is set to a small value then the problem can be infeasible when the target point is close to an obstacle. The idea is to adjust its value dynamically during the path planning to prevent collisions and handle those targets adjacent to the obstacles. In other words, at the beginning when the MR is far from the target point, the collision should be prevented by setting up the value of α to a small value (e.g. 0.5), and when the MR is about to reach to the target, the value of α should be increased to a bigger value (e.g. 0.7) to handle the adjacent non-reachable target.

Algorithm 1 Path Planning Procedure

INPUT: Reaching error bound ε, robot's physical dimensions and initial position Z_{R0}, axes resolutions dx, dy and target position Z_T.
OUTPUT: Optimal path to reach the target point.
Initialization: Set $\alpha = \alpha_0$ and $Z_R = Z_{R0}$.

(a) Capture the environmental image from camera.
(b) Extract the robot's and obstacles' necessary information, i.e. N_o, A_i, $(x_i, y_i), \forall i = 1, \ldots, N_o$, using standard or off-the-shelf image processing tools.
    FOR $i = 1$ **TO** N_o **DO**
    Calculate β_i and $V_i(x, y)$ using (2) and (1), respectively.
    END FOR
(c) Calculate $V_a(x, y)$ using (3).
(d) Evaluate the cost function $J_w(Z_w, Z_T)$ given in (5) and (6) for the current position of the robot.
(e) Solve the optimization problem (7) to obtain the optimal desired position.
(f) Update the robot position using (4), (7) and Table 1.
    IF $\|Z - Z_T\| \le \varepsilon$ **THEN**
    Target is reached. Go to step (g).
    ELSE IF the environment is dynamic **THEN**
    Go to step (a).
    ELSE
    Go to step (d).
    END IF
(g) Stop.

Remark 2. It is worthy to note that, setting α to a very small value (e.g. 0) is equivalent to remove the target approaching goal that may cause the target loss situation (for $\alpha = 0$) or very long route to the target (for small α). On the other hand, setting α to a very large value (e.g. 1) is equivalent to remove the obstacle avoidance goal and tempting to reach the target with shortest route. Conse-

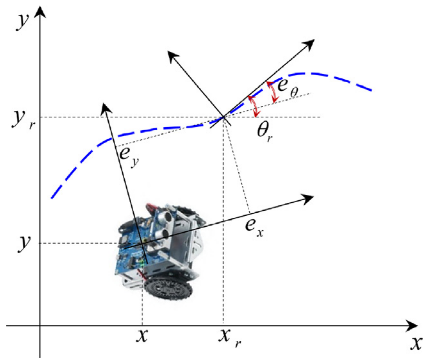


Fig. 5. MR's coordinate representation with respect to the reference path.

quently, this may cause the robot being trapped in a local minima (for $\alpha = 1$) since the robot just try to approach the target through the shortest rout, or possibility of the obstacles collision problem (for large α).

Remark 3. As discussed in Mac et al. (2016), one major disadvantage of the existing artificial vector field approaches, based on the attracting/repelling force vectors calculation, is that they are structurally less adaptive compared to other existing heuristic path planning approaches. However, the proposed approach in this paper which is based on a scalar potential surface can be simply adapted by adjusting the tuning parameters (α and β) to cope with the environmental variations.

4. Algorithm evaluation

In order to evaluate the performance of the proposed algorithm in a near realistic environment, first the MR's kinematic and dynamic equations are derived and a global path tracking controller is designed. Then, the MR's environment is designed in the widely used Webots software (see Michel, 2004) and by integrating the proposed path-planning algorithm and the path-tracking controller, the simulation results are provided to investigate the performance of the proposed approach.

4.1. Mobile robot modelling and controller design

Here, we assume a two degrees of freedom, wheeled and differentially driven mobile robot as shown in Fig. 5. The MR's dynamics are given as:

$$\dot{q}(t) = \begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} \cos\theta(t) & 0 \\ \sin\theta(t) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \nu(t) \\ \omega(t) \end{bmatrix} \tag{8}$$

where (x, y) denote the Cartesian coordinates of the MR's center of mass, and θ is the angle between the heading direction and the x axis (see Fig. 5). $q(t) = [x(t)\ y(t)\ \theta(t)]^T$ is the generalized coordinate of the MR, $q_r(t) = [x_r(t)\ y_r(t)\ \theta_r(t)]^T$ is the generalized coordinate of the reference robot. ν and ω are the control inputs denoting the linear velocity and angular velocity of the MR, respectively. Defining $e_q = [e_x\ e_y\ e_\theta]^T$, one gets:

$$e_q(t) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} (q_r(t) - q(t)) \tag{9}$$

Then, the error dynamics are obtained as

$$\dot{e}_q(t) = \begin{bmatrix} \dot{e}_x \\ \dot{e}_y \\ \dot{e}_\theta \end{bmatrix} = \begin{bmatrix} \cos e_\theta & 0 \\ \sin e_\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \nu_r \\ \omega_r \end{bmatrix} + \begin{bmatrix} -1 & e_y \\ 0 & -e_x \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \nu \\ \omega \end{bmatrix} \tag{10}$$

Table 2
Features of all Obstacles.

Obstacle	$A_i(m^2)$	$Z_i = (x_i, y_i)$	β_i
#1 (Yellow)	0.0087	(0.097, 0.458)	173.91
#2 (Orange)	0.0350	(0.241, 0.751)	43.23
#3 (Blue)	0.0208	(0.473, 0.875)	72.74
#4 (Green)	0.0261	(0.706, 0.770)	57.97
#5 (Pink)	0.0607	(0.703, 0.320)	24.93

where $\nu_r(t) = \sqrt{\dot{x}_r^2(t) + \dot{y}_r^2(t)}$, $\omega_r(t) = \frac{\dot{x}_r(t)\ddot{y}_r(t) - \dot{y}_r(t)\ddot{x}_r(t)}{\dot{x}_r^2(t) + \dot{y}_r^2(t)}$. The goal is to design $u = [\nu\ \omega]^T$ such that the mobile robot coordinate $q(t)$ perfectly follows the reference robot coordinate $q_r(t)$. To this aim, we will follow the approach presented in JIANGdagger and Nijmeijer (1997). According to the results in this paper, the following control laws guarantee the global asymptotic tracking of the closed loop system.

$$\begin{aligned} \nu(t) &= \bar{\nu}(t) - e_y(t)\omega(t) + \delta_1(e_x(t) - \delta_2\omega(t)e_y(t)) \\ \omega(t) &= \omega_r(t) + \delta_3 e_y(t)\nu_r(t) \int_0^1 \cos(e_\theta \tau)d\tau + \delta_4 e_\theta(t) \end{aligned} \tag{11}$$

where $\bar{\nu}(t) = e_y(t)\omega(t) + \nu_r(t)\cos(e_\theta) - \delta_2\dot{\omega}(t)e_y(t) + \delta_2\omega(t)[\omega(t)e_x(t) - \nu_r(t)\sin(e_\theta)]$, $\delta_i > 0, \forall i = 1, 2, 3, 4$. In order to verify the proper functionality of the presented control laws in (11), a complex eight-shaped reference path is considered in Fig. 6, where $x_r(t) = 0.5 + 0.3sin(\pi t/15)$ and $y_r(t) = 0.5 + 0.3sin(2\pi t/15)$. The closed-loop results associated with the MR's actual traversed path are illustrated in Fig. 6(a). The obtained result demonstrates that the asymptotic tracking goal is achieved even if the MR's initial position is out of the reference path. Meanwhile, it is important to note that, since in our case the path planning is started precisely from the MR's current position, then the MR is initially on the reference path and the perfect tracking is achieved as shown in Fig. 6(b). In the following by integrating the presented control law and the path planning algorithm, we will evaluate the performance of the proposed approach in the face of some challenging environments.

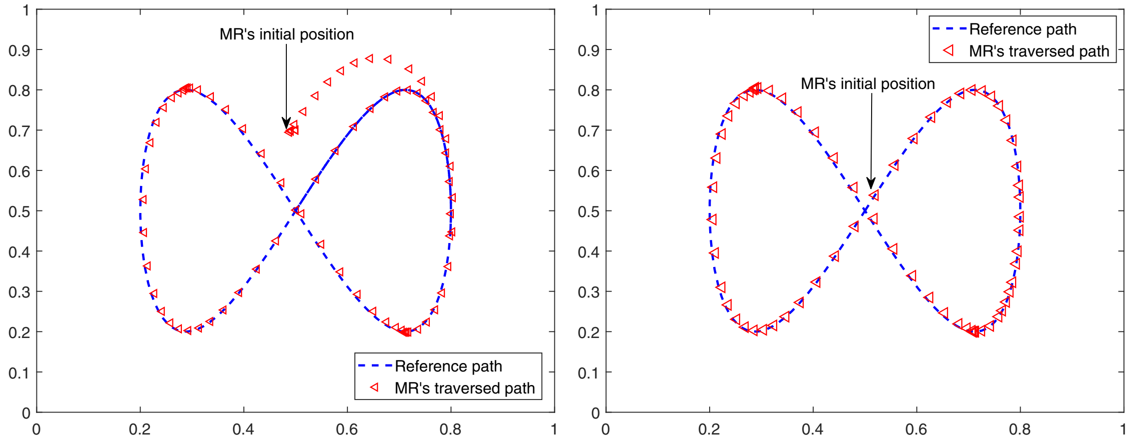
4.2. Performance evaluation in different environments

In the first test-case, as shown in Fig. 1(a), it is assumed that the obstacles are randomly situated and their locations and areas are not known a priori; yet, these information can be obtained through image processing procedure. In the following, the acquired data associated with the obstacles in Fig. 1(a) is presented. Concerning the red object in this picture, it is the MR located in its initial position and during the path planning procedure it will be float. Moreover, the mobile robot's area is 0.01 m^2. The extracted data associated with the colored obstacles are presented in Table 2. For example, for the first yellow obstacle (Obs.#1), the corresponding potential function with $K = 0.5$ is obtained as:

$$V_1(x, y) = e^{-173.85\left((x-0.097)^2 + (y-0.458)^2\right)} \tag{12}$$

The proposed path planning algorithm has been tested and verified on an experimental navigation system that is presented in Algorithm 1. It is implemented in a designed Matlab based graphical user interface (GUI) and the results are shown in Fig. 7.

Remark 4. As mentioned before and also presented in the Algorithm 1, the proposed method requires an image processing procedure carried out for extracting the environmental information. Although, one may argue that the image processing is usually a computationally demanding task, but it is important to note that, (1) nowadays, due to the rapid technological advancements, very powerful and at the same time inexpensive tools are available (e.g. GPU, FPGA, DSP, etc.) that allow us to perform image processing computations very fast and efficient, and (2) in our particular case,



(a) (b)

Fig. 6. The closed-loop performance (a) asymptotic tracking, (b) perfect tracking.

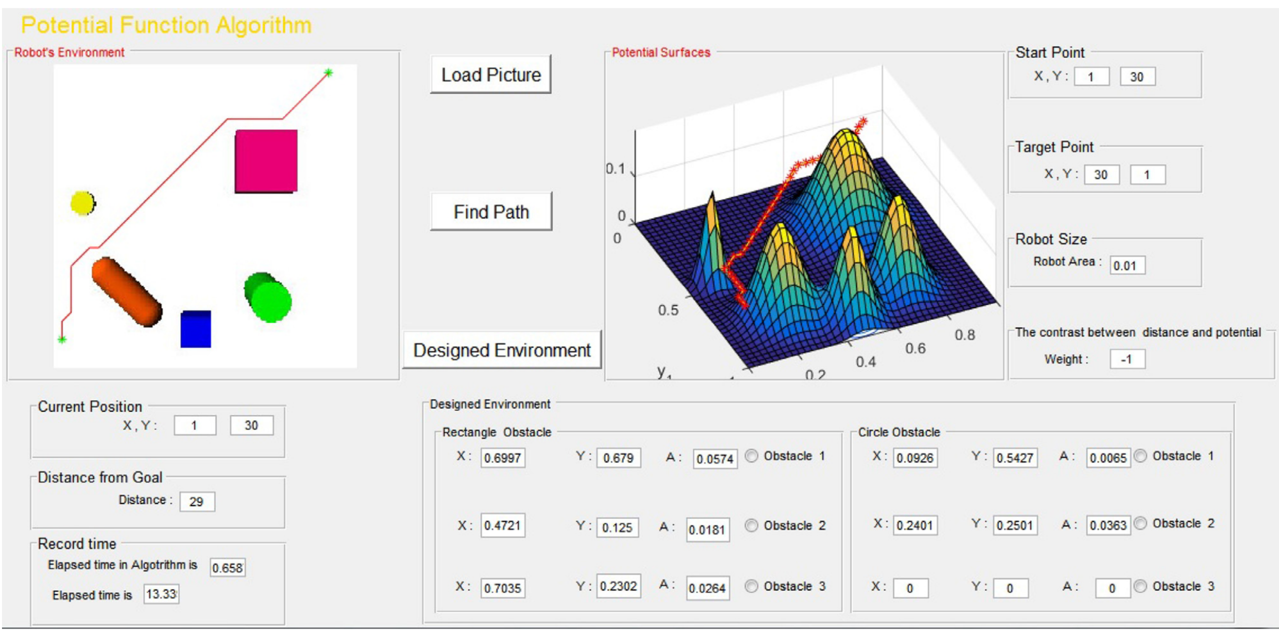


Fig. 7. Screen shot of the designed GUI.

i.e. mobile robot path planning problem, since in the most of the related applications the environment is either static or at least slowly varying (e.g. exploratory robots in unknown and high-risk environments for humans, rescue robots in the war, fire and earthquake incidents, etc.), then the image processing is only required to be carried out once at the beginning of the mission or at most from time to time based upon the rate of the environmental variation. In this case, the image processing is only used to obtain the environmental information (obstacles position, size, etc.) while the robot's real-time position is obtained using onboard mounted sensors.

4.3. Local minimum problem

Local minimum trap is one of the main quandaries in the path planning problem and failure to obviate it, implies the incompetence of an algorithm. Here we provide four different test-cases for evaluating the proposed approach in the face of the local minima problem. The first two cases shown in Fig. 8(a) and Fig. 8(b), illustrate the typical and widely referenced local minima traps in the APF based approaches (Kim, 2009; Montiel et al., 2015a). In these cases a combination of obstacles make an L-shape wall. The robot's traversed optimal path in each case is illustrated in the associated figure. It is noted that the resultant SPS has high potential values near the wall and this prevents the robot from approaching the wall and thus hinders it from being trapped in the local minima. Two extra complicated environments with possible local minima traps are shown in Fig. 8(c,d). Furthermore, the MR cannot path through some obstacles due to its physical constraints. In other words, the MR cannot choose the shortest path. The potential surface of Fig. 8(d) and its 2D projection are presented in Fig. 9(a,b). Accordingly, the overlapped surfaces imply that there are adjacent obstacles and inform the MR is possibly nearby of a trap. As it is indicated in Fig. 8(c), at the start point, the MR avoids the pink obstacle (Obs.#5) and leads toward the goal point but owing to the high potential value of a local minimum district. In other words, the MR changes its path (by changing the value of α in (5)) and finds another path to converge to the target. Similarly, in Fig. 8(d), the close obstacles at the start point cause a high potential value. Hence, the algorithm successfully steers the MR down

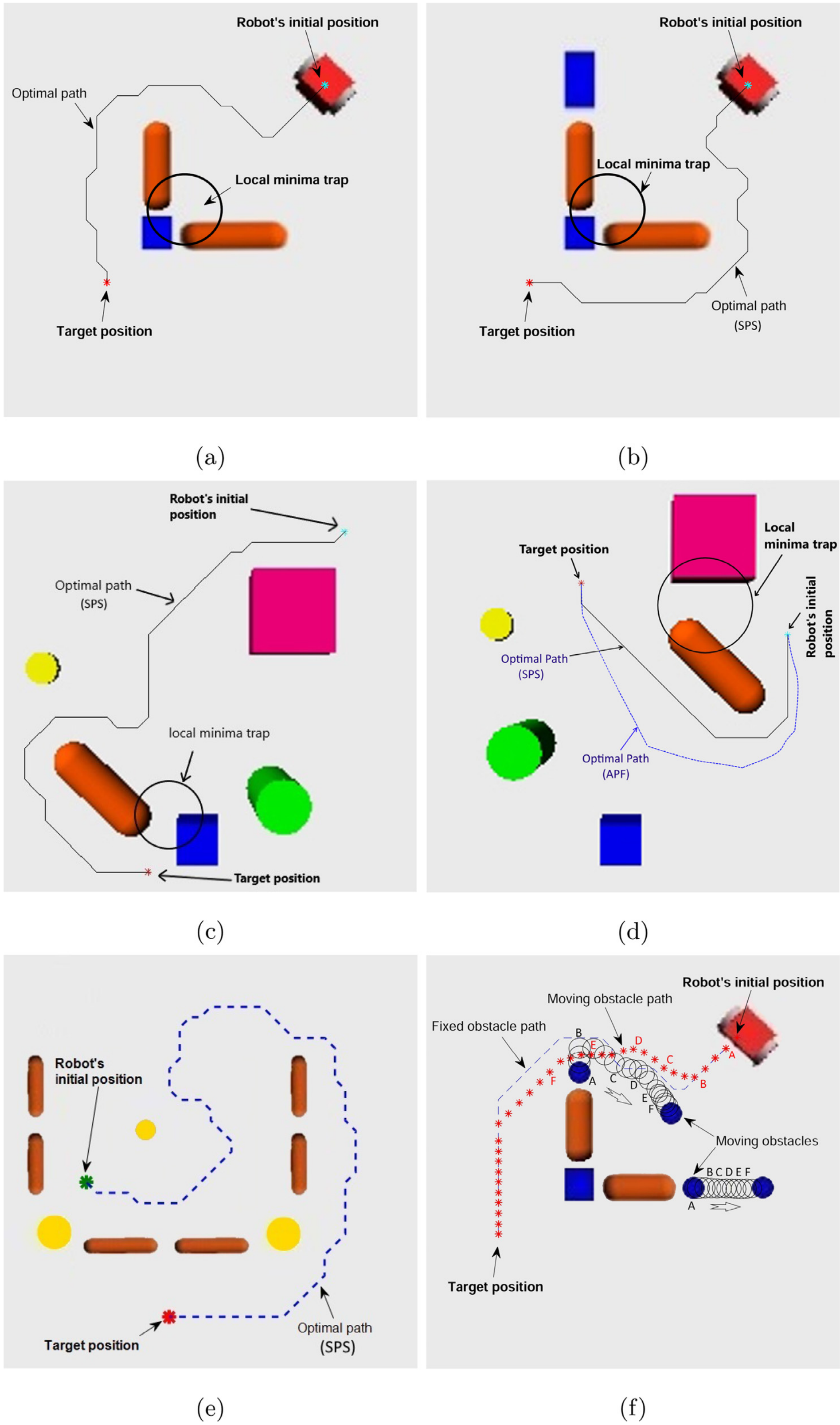


Fig. 8. Investigation of the local minima trap and dynamic environment in the proposed method.

to a lower potential region by selecting the amount of α in (5) approximately zero (i.e. ignoring the target point) until it reaches to a safe place. Then, the algorithm tries to find an optimum path to the target by compromising between both goals. Another challenging environment to evaluate the performance of the proposed approach is shown in Fig. 8(e) representing a tricky set of U-shaped obstacles. It is demonstrated that the proposed algorithm can efficiently steer the robot to the target point. Furthermore, in some practical robotic applications there might be some moving objects in the environment and it is important to evaluate the path planing

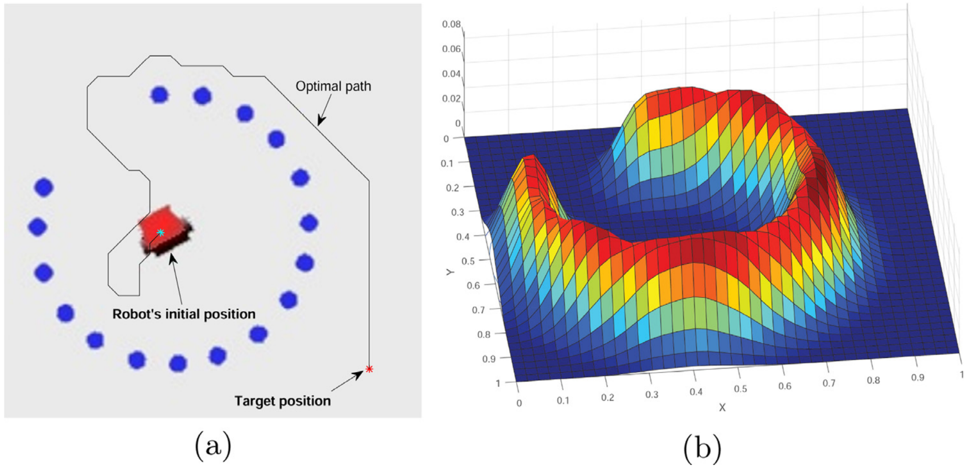


Fig. 9. Illustration of (a) a semi-closed environment, (b) the corresponding SPS.

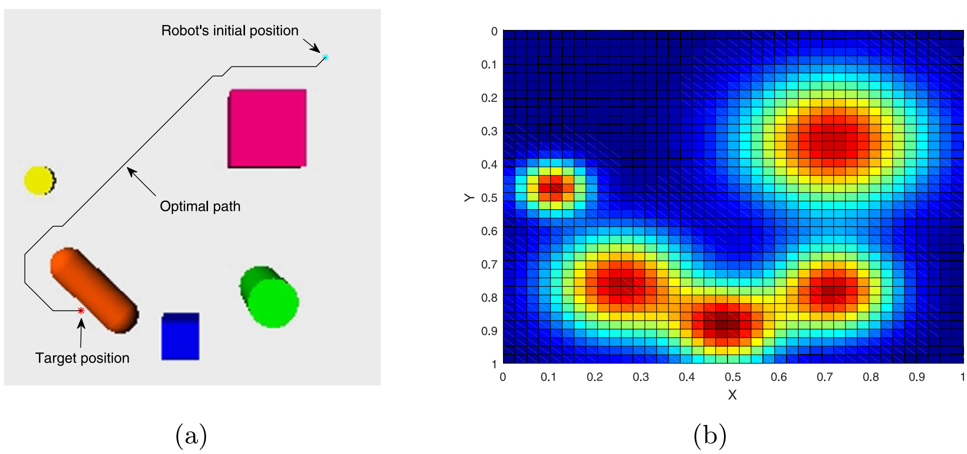


Fig. 10. Non-reachable target, (a) environment, (b) projection of the SPS.

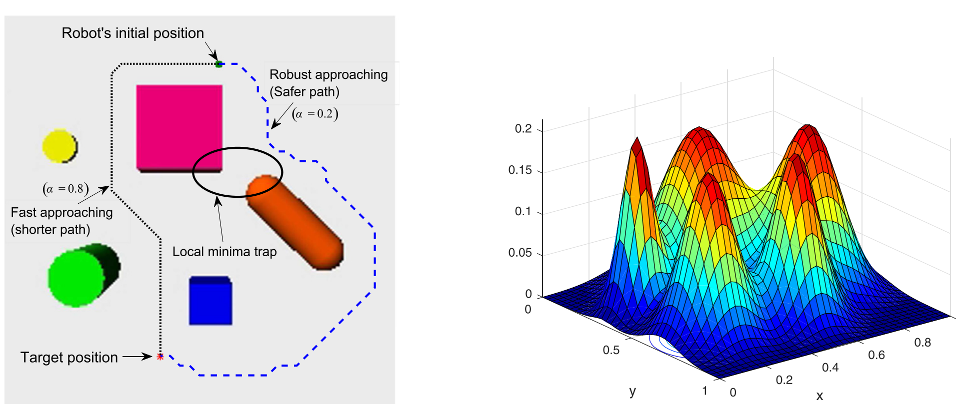


Fig. 11. Efficacy of α, (a) comparing the fast approaching (black solid line) and robust approaching (blue dashed line) objectives, (b) the average SPS.

algorithm in such dynamic environments. To this aim, as shown in Fig. 8(f), an environment with L-shaped obstacles are considered where two of the obstacles are moving. The path planing results are shown in Fig. 8(f) for two cases. The blue dashed line shows the path planning when all obstacles are fixed (static environment) and the red stared-line illustrates the path planning result when two blue circle obstacles are moving in the direction shown in this figure (dynamic environment). It can be seen that the algorithm corrects the path to avoid moving obstacle collision. Furthermore, for the sake of comparison, the test-cases shown in Fig. 8 are also investigated using the APF approach. The APF algorithm could only reach the target in the test-case shown in Fig. 8(d), while it was trapped in the local minima of the other test-cases.

In the next test-case, we plan to evaluate the performance of the algorithm in a semi-closed environment as shown in Fig. 9 which is a quite challenging problem. In this case the robot is located in a semi-closed circle shaped obstacles with only one exit way. The corresponding SPS is shown in Fig. 9(b) and the robot's traversed path is shown in Fig. 9(a). The obtained results verify the effectiveness of the proposed approach. In the case of having several alongside obstacles, the potential value of the target point and its adjoining districts are very high. As a result, the MR may not be able to find a way to the target (Fig. 10). In the proposed method, solving this problem does not need any further calculations and the solution is to change the value of α appropriately. To this aim, when the MR is about to reach to the target, the value of α in the cost function should be increased (i.e. $\alpha \to 1$) to intensify the importance of getting closer to the target. In this case, only the nearest cells to the goal are chosen and potential value would be less considered.

4.4. Effect of parameter α

Considering the central role of the parameter α in the objective function (5), one easily concludes that this parameter makes a linear compromise between the fastest ($\alpha \to 1$) and the safest ($\alpha \to 0$) path indicated by $D(Z_w, Z_T)$ and $V_a(Z_w)$, respectively. At the boundary points, only one of these goals are incorporated for path planning task. In other words, $\alpha = 1$ leads to the fastest path planning goal while $\alpha = 0$ leads to the safest path planning goal. Furthermore, $0 < \alpha < 1$ compromises between two goals and leads to an optimal path which is neither the fastest path nor the safest path. For example, as shown in Fig. 11, two different paths are found by choosing two different values $\alpha = 0.2$ and $\alpha = 0.8$, respectively. Also in this figure, there is a local minimum trap which is successfully avoided in both paths.

5. Conclusions and future work

In this paper, a novel approach toward the mobile robot path planning in the scattered environments has been presented which is quite feasible in real world applications. The concept of the proposed method is mainly based on the theory of electric field potential and the influence of the field on an electric charge. Regarding this and inspired by the authors' previous work on the non-convex data clustering, an optimization based path planning algorithm is constructed which makes an optimal trade-off between two design objectives, i.e. fast approaching to the target and safe/collision-free goals. To this aim, instead of studding the individual effect of all obstacles and their interplay with the moving robot, their entire effects are integrated in a scalar potential surface (SPS) and the effect of such scalar field is then considered in the path planning procedure. Using a scalar decision variable makes the proposed approach very simple in terms of mathematical computations and thereby makes it feasible for practical applications. The SPS directly depends on the MR's and obstacles' physical constraints such that as the MR approaches to an obstacles, the potential value rises exponentially to inform the existence of a nearby obstacle. It is noted that the proposed method can be directly applied to the MR autonomous navigation problem in the static environments which has many practical applications such as consignments discharging to a warehouse in an industrial site and organizing them, purging the places which are perilous for human being to stay, exploring planets, guiding and helping elderly people, etc. Furthermore, since the proposed method relies on the fact that the visual information of the environment is available, then it can also be implemented in the dynamic environments and then used for path planning applications of multi-robotic systems (like swarm robotic) and robot manipulators. Finally, it is emphasized that, although the proposed method has demonstrated its capability of escaping the local minima traps in all performed simulations, but as a future work it is of high interest to proof this feature of the proposed algorithm theoretically.

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