Vector field path following and obstacle avoidance singularity mitigation via look-ahead flight envelope

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Unmanned Aerial Vehicles conventionally navigate by following a series of pre-planned waypoints that may have to be re-planned when flying in a dynamic environment or encountering previously unknown obstacles. Waypoints are generally planned off-line and relayed to the UAV, taking up time and autopilot communication resources. Attractive path following and repulsive obstacle avoidance vector fields have been summed together to produce UAV guidance that follows pre-planned paths and avoids obstacles without the need to re-plan. Summing attractive and repulsive vector fields may produce small regions of null guidance, called singularities, which could potentially lead to trap situations. An investigation into singularity mitigation by vector field weight parameterization is presented.

I. Nomenclature

UAV	=	Unmanned Aerial Vehicle
VF	=	Vector Field
VFF	=	Virtual Force Field
LVF	=	Lyapunov Vector Field
GVF	=	Goncalves Vector Field

II. Introduction

Unmanned Aerial Vehicles are pilotless aircraft used by military, police, and civilian communities for tasks such as reconnaissance, damage assessment, natural disaster surveying, and target tracking [1, 2]. Tasks can be performed by a single UAV or with a team of other air, ground, or marine vehicles [3–5]. Autonomous vehicle missions are typically accomplished by navigating a series of waypoints [1] or path following [6]. Waypoints are conventionally generated off-line at a ground station and relayed over radio to the UAVs autopilot. Obstacles such as buildings, terrain, and other vehicles can be avoided by planning waypoints around obstacles. The on-board guidance directs the UAV towards the

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current active waypoint, once the UAV has reached a pre-defined distance from the waypoint the UAV is directed to the next waypoint. An example of a UAV following waypoint guidance while avoiding an obstacle is shown in Figure 1

Fig. 1 UAV path from waypoint guidance

During waypoint navigation the UAV may encounter obstacles or environmental changes that would require a new set of obstacle free waypoints to be generated. For highly uncertain or dynamic environments, there may have to be frequent updates which increases the communication overhead of the autopilot. Additionally, if communication is delayed or lost, waypoints may not be updated rapidly enough and the UAV may fail to avoid the obstacles.

Obstacle free paths in static and dynamic environments have been generated with the potential field method, which models a robot's workspace as a gradient of artificial attractive and repulsive forces [7]. Potential field combines path planning, trajectory planning, and control into a single system [8]. Paths can be generated by placing a point mass at an initially high potential and allowing it to descend a gradient until the point reaches the goal, located at a global minimum potential. Obstacles provide a limited repulsive force, pushing the mass away from the obstacle.

A histogram based potential field method can be found in [9–11] which allowed for real time goal seeking with obstacle avoidance. Sensors on-board a ground robot located at (x_0, y_0) detect obstacles within a pre-defined window containing a fixed number of cells. Cells containing an obstacle provide a repulsive force $\overrightarrow{F_{i,j}}$ opposite in direction to the line-of-sight from vehicle to cell location (x_i, y_j), where (i, j) represents the cell index, F_{cr} is a constant repulsive force, W the vehicle's width, $C_{i,j}$ a cell's certainty, and $d_{i,j}$ the distance to the center of the cell with respect to robots center.

$$\overrightarrow{F_{i,j}} = \frac{F_{cr} W^n C_{i,j}}{d_{i,j}^n} \left(\frac{x_i - x_0}{d_{i,j}} \hat{x} + \frac{y_i - y_0}{d_{i,j}} \hat{y} \right) \tag{1}$$

The total repulsive force exerted on the robot is determined by summing the active cells, shown in Equation 2

$$\overrightarrow{F_r} = \sum_{i,j} \overrightarrow{F_{i,j}} \tag{2}$$

The robot is attracted to the goal by force $\overrightarrow{F_t}$ with constant magnitude F_{ct} and along the LOS from robot center to goal, located at (x_t, y_t) and a distance d_t, shown in Equation 3

$$\overrightarrow{F_t} = F_{ct}\left(\frac{x_t - x_0}{d_t}\hat{x} + \frac{y_t - y_0}{d_t}\hat{y}\right) \tag{3}$$

Summing together attractive and repulsive forces produce a vector that can be used for heading guidance, shown in Equation 4

$$\overrightarrow{R} = \overrightarrow{F_r} + \overrightarrow{F_t} \tag{4}$$

Major drawbacks to potential field were identified in [11] consisting of local minimum and oscillations in corridors. The local minimum problem occurs when closely spaced obstacle's potential combine to produce a well on the descent gradient where a pre-mature stable point is found. Proposed solutions to local minimum include object clustering and virtual waypoint method [12], virtual escaping route [13], and use of navigation functions [14]. Oscillations in potential field were studied in [15] and [16].

In addition to local minimum and oscillations, potential field converges to a singular point which is not possible for fix wing aircraft. Similar to conventional waypoint guidance, the active goal point would change as a function of proximity. Simulating a UAV using VFF as guidance for a Dubins vehicle was performed and is shown in Figure 2, where a single obstacle cell located at the origin. The UAV initially travels directly toward the goal located at $(20, 0)$ until the obstacle is encountered, at which point a repulsive force is applied. The UAV avoids the obstacle, however significantly deviates and fails to get back on the path between waypoints. For certain applications, such as following a curved ground track, surveying, or target following, it may be beneficial to follow an explicit path.

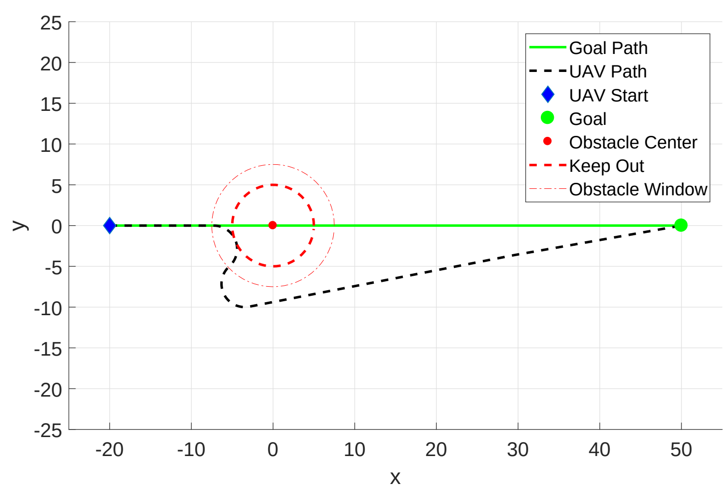


Fig. 2 Dubins vehicle encountering an obstacle while navigating to a waypoint

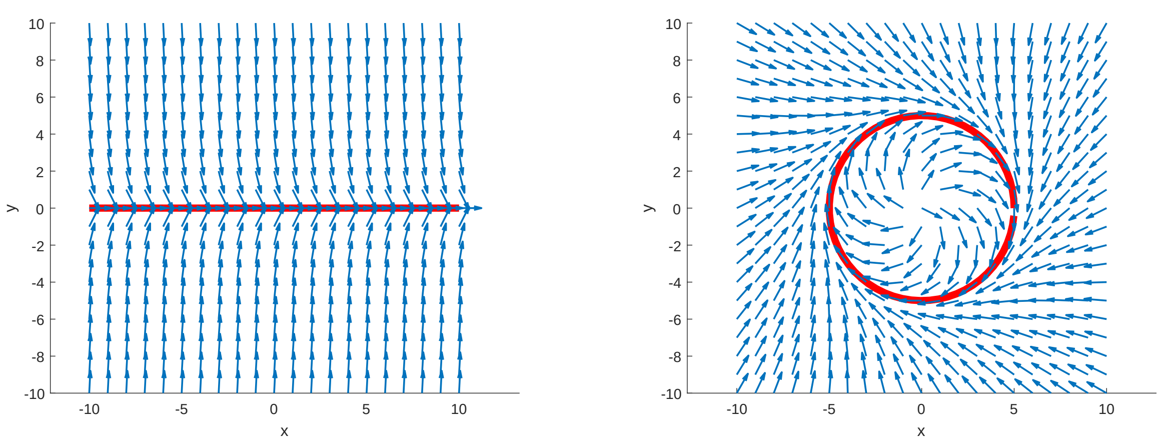
Path following can be accomplished with vector fields which produce a heading guidance that asymptotically converges and circulates a path. A comparison between vector field and waypoint guidance techniques was presented in [17] where each method was evaluated based on its complexity, robustness, and accuracy. Vector field produced guidance that was both robust to external wind disturbances while maintaining a low cross track error. The two most prominent methods for generating vector fields in literature consist of the Lyapunov [18–23] and Goncalves [24–27] method.

Lyapunov vector fields for converging and following straight and circular paths were described in [18]. For converging and following a straight path, a guidance vector χ^d is determined in Equation 5, where χ^∞ is the course approach angle, y is the lateral distance to the path, and k is a positive constant that determines the rate of transition between convergence and following. An example of a Lyapunov vector field converging and following a straight line is shown in Figure 3a.

$$\chi^d(y) = -\chi^\infty \frac{2}{\pi} \tan^{-1}(ky) \tag{5}$$

For converging and following a circular path, a guidance vector χ^d is determined in Equation 6, where γ is the UAVs angular position with respect to the circle, r is the paths radius, d is the distance from the circles center, and k is a positive constant that determines the transition behavior. An example of a Lyapunov vector field for converging and following a circular path is shown in Figure 3b.

$$\chi^d(d) = \gamma - \frac{\pi}{2} - \tan^{-1}\left(k\frac{d-r}{r}\right) \tag{6}$$



(a) (b)

Fig. 3 Lyapunov vector field converging and following a) straight path b) circular path

Straight and circular path vector fields can be selectively activated throughout flight to form more complex paths, shown in [18–20, 28]. Lyapunov vector field for curved path following was presented in [23] which may allow for more complex paths and eliminates the need to switch between vector fields.

The Gonvalves Vector Field (GVF) method produces a similar field, however has several advantages over LVFs. GVF produces an n-dimensional vector field that converges and circulates to both static and time varying paths. Additionally, convergence, circulation, and time-varying terms that make up the GVF are decoupled from each other allowing for easy weighting of the total field. GVFs converge and circulate at the intersection, or level set, of $n-1$ dimensional implicit surfaces ($\alpha_i : \mathbb{R}^n \rightarrow \mathbb{R} | i = 1, ..., n-1$). The integral lines of the field are guaranteed to converge and circulate the level set when two conditions are met: 1) the implicit surface functions are positive definite and 2) have bounded derivatives. Consider the space with dimensions in set $\mathbf{q}$:

$$\mathbf{q} = \begin{bmatrix} x_1, x_2, ..., x_n \end{bmatrix} \tag{7}$$

The total vector field $\overrightarrow{V}$ is calculated by:

$$\overrightarrow{V} = G\nabla V + H \wedge_{i=1}^{n-1} \nabla_q \alpha_i - LM(\alpha)^{-1} a(\alpha) \tag{8}$$

or in component form:

$$\vec{V} = \vec{V}_{conv} + \vec{V}_{circ} + \vec{V}_{tv} \tag{9}$$

where $\vec{V}_{conv}$ produces vectors perpendicular to the path, $\vec{V}_{circ}$ produces vectors parallel to the path, and $\vec{V}_{tv}$ is a feed-forward term that produces vectors accounting for a time varying path.

Convergence is calculated by:

$$\vec{V}_{conv} = G\nabla V \tag{10}$$

where scalar G is multiplied by the gradient of the definite potential function V:

$$V = -\sqrt{{\alpha_1}^2 + {\alpha_2}^2} \tag{11}$$

Circulation is calculated by taking the wedge product of the gradient:

$$\vec{V}_{circ} = \wedge_{i=1}^{n-1} \nabla_q \alpha_i \tag{12}$$

In the case of ($n = 3$) the wedge product simplifies as the cross product:

$$\vec{V}_{circ} = \nabla_q \alpha_1 \times \nabla_q \alpha_2 \tag{13}$$

The feed-forward time-varying component is calculated by:

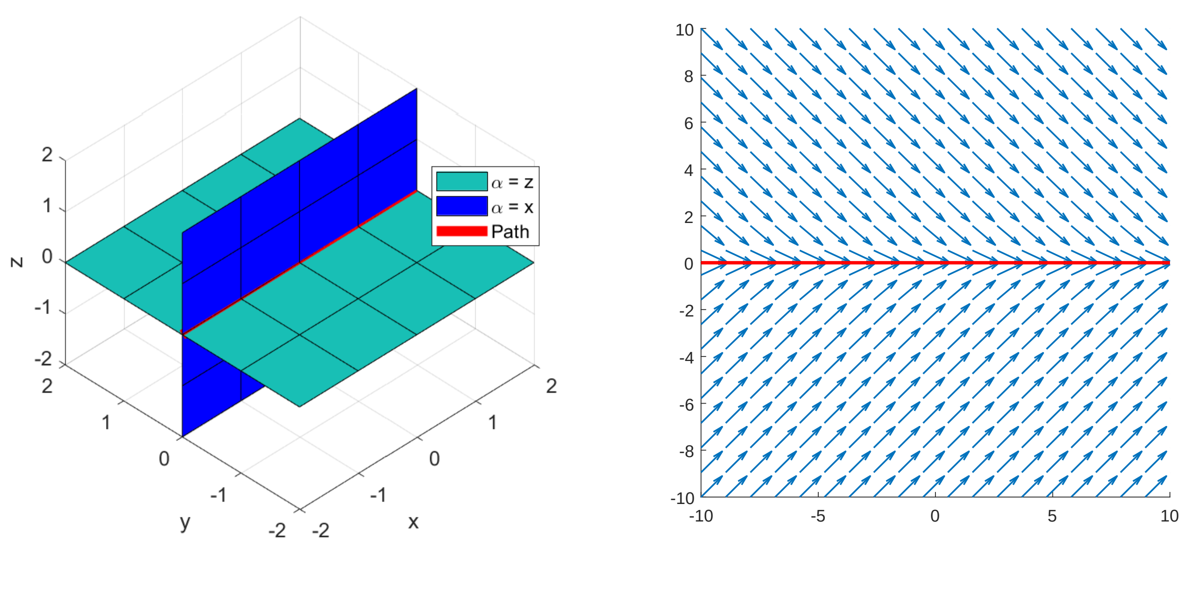
$$\vec{V}_{tv} = M^{-1} a \tag{14}$$

where,

$$M = \begin{bmatrix} \nabla \alpha_1^T \\ \nabla \alpha_2^T \\ (\nabla \alpha_1 \times \nabla \alpha_2)^T \end{bmatrix} \tag{15}$$

$$a = \begin{bmatrix} \frac{\partial \alpha_1}{\partial t} & \frac{\partial \alpha_2}{\partial t} & 0 \end{bmatrix}^T \tag{16}$$

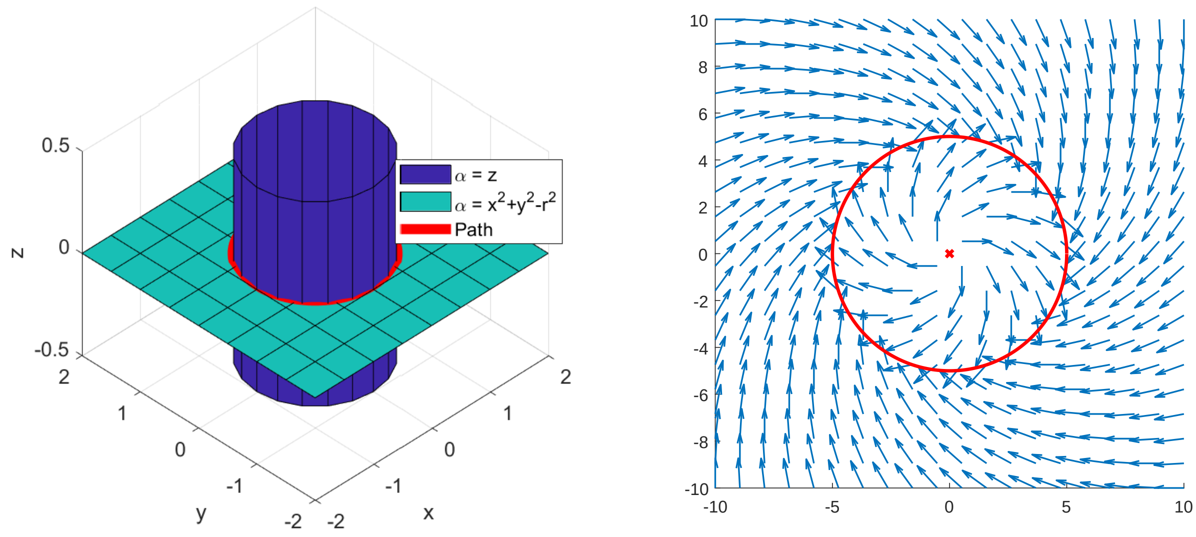
Intersecting two flat planes ($\alpha_1 = z, \alpha_2 = x$) produces a GVF that converges and circulates a straight path, shown in Figure 5. A circular path can be produced by intersecting a plane and a cylinder ($\alpha_1 = z, \alpha_2 = x^2 + y^2 - r^2$).



(a)

(b)

Fig. 4 GVF converging and circulating straight path



(a)

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Fig. 5 GVF converging and circulating circular path

The standoff tracking scenario presented in [Wilhelm] tasked a fixed wing UAV with loitering around a moving ground target while adding obstacle avoidance constraints. A circular attractive vector field was attached to a moving ground target. Repulsive vector fields centered at the obstacles and weighted by hyperbolic tangent decay functions were summed with the attractive circular field to produce a target loitering and obstacle avoidance guidance. The performance of Lyapunov [21] and gradient vector field [24–26] were compared for their cross track error with respect to the loiter circle. Gradient vector field had favorable performance due to compensation for a time-varying vector field. The gradient vector field technique also has the benefit of decoupled weighting parameters for convergence, circulation, and time-varying terms, allowing for easy modification of field behavior.

Decay functions for avoidance fields were investigated in [Zhu] for obstacles present on a straight path. When summing attractive and repulsive vector fields there is the possibility of guidance singularities, where magnitude and direction are equal and opposite. The presence of singularities were not addressed in [Wilhelm] and [Zhu], mentioned briefly in [18] and observed in [29]. For fixed wing UAVs the lack of guidance may prevent the UAV from avoiding an obstacle, while multi-rotor UAVs may end up in a trap situation. Singularities may be present at any location where a goal field and obstacle field are of equal strength.

Dubin's vehicle's position $\overrightarrow{X}$ at time t is calculated from the integral of the velocity vector $\overrightarrow{U}$. The vehicle has a constant velocity magnitude u_{uav} at a heading θ. The rate at which θ changes with respect to time is based on limitations of the craft itself.

$$\overrightarrow{U}(t) = u_{uav} \begin{bmatrix} cos(\theta(t)) \\ sin(\theta(t)) \end{bmatrix} \tag{17}$$

$$\overrightarrow{X}(t) = \overrightarrow{U} dt + \overrightarrow{X}(t-1) \tag{18}$$

$$\dot{\theta} \leq 20 deg/s \tag{19}$$

III. Conclusion

Appendix

Acknowledgments

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