A Proposal for a Parameterized Circulating Vector Field Guidance for Fixed Wing Unmanned Aerial Vehicles

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the faculty of

the Russ College of Engineering and Technology of Ohio University

In partial fulfillment
of the requirements for the degree
Master of Science

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ABSTRACT

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LIST OF SYMBOLS

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LIST OF ACRONYMS

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1 Introduction

1.1 Motivation and Problem Statement

Unmanned Aerial Vehicles (UAVs) are powerful robotics tools used by both military and civilian communities alike. Advances in lightweight materials and commercial electronics have increase the range, payload, and reliability of the crafts. Reduced costs have brought the technology to a wide range of communities which have found uses such as surveillance, reconnaissance, and aerial photography to name a few. As the complexity of the tasks increase, the complexity of the guidance and control systems increase as well. Traditionally, guidance and control laws have been regarded as separate systems responsible for independent tasks. Potential field and vector field methods have blurred the line between guidance and control systems, whereas now they have shared the burden of complex tasks such as obstacle avoidance and path following. Potential field has been effective for obstacle avoidance and goal seeking for a singular discrete point, however is not initially intended to be used for tasks such as path following. Vector field methods have been effective at providing guidance and control for fixed wing UAVs for converging to and following a path. Vector fields can be constructed in a number of ways, however a convenient method first introduced in [GPMP09] constructs a field by summing together three components consisting of convergence, circulation, and time varying. The filed is generated by calculating the integral lines converging to and following the level sets of intersecting arbitrary surfaces in n-dimensions. Unlike potential field, vector fields do not take into account the dynamics of a vehicle being provided the guidance which provides no guarantee that the UAV will avoid obstacles. Improving the guidance provided by the vector field for obstacle avoidance may be possible by modifying the circulation term of a vector field as a function of a vehicles state with respect to the obstacle.

1.2 Methods Overview

The Gonsalves vector field method will be applied to generate attractive and repulsive vector fields for the purposes of converging to and following a path while avoiding obstacles. The circulation term of the vector field can be modified to increase the contribution of circulation around the target. Developing the modified vector field guidance will be done in three distinct phases consisting of function development, function evaluation through simulation, and finally validation through simulating a fixed wing UAV on a non-holonomic ground robot.

1.3 Phases

1.3.1 Phase I

Construct time parametrized functions for the circulation term of the vector field dependent on common flight measurements. Common states measured by a UAV include position, closing velocity and heading. Varying the circulation contribution to the total vector field as a function of the vehicles state will be useful for reducing the risk of the UAVs controls being saturated and entering the region the obstacle is occupying.

1.3.2 Phase II

Simulate and compare the parametrized circulation functions with a non-parametrized vector field guidance for circular obstacles. The performance of the parametrized circulation functions developed in Phase I will be evaluated for a range of vehicle turn rate constraints and obstacle radii. Distance traveled, control effort exerted, and time taken will be used to compare the performance of each parameterized function of circulation to determine which function is the most effective.

1.3.3 Phase III

Simulate a fixed wing UAV with a ground robot guided by the modified vector field guidance to validate simulation results and demonstrate real time guidance is achievable with parametrized circulation modification. The parameterized circulation function vector field will be implemented on a mobile robot simulating the constraints of a fixed wing aircraft for the purposes of validating the use of the algorithm for real time robotic guidance and control.

1.4 Summary of Objectives

- Construct time parametrized functions for the circulation term of the vector field dependent on common flight measurements.
- Simulate and compare the parametrized circulation functions with a non-parametrized vector field guidance for circular obstacles.
- Simulate a fixed wing UAV with a ground robot guided by the modified vector field guidance to validate simulation results and demonstrate real time guidance is achievable with parametrized circulation modification

2 LITERATURE REVIEW

2.1 Introduction to Literature Review

The following sections provide a discussion on the literature in regards to the modeling of fixed wing unamnned aerial vehicles and the navigation, guidance, and control systems that govern the behavior of the crafts. First, the fixed wing is introduced and a brief discussion on the modeling techniques commonly used for navigation, guidance, and control design. An overview of the structure of autopilots that execute the models is presented. Then, an overview of navigation, guidance, and control is given along with the cutting edge techniques for guidance. (Revisit this section when a rough draft is complete)

2.2 Fixed Wing Unmanned Aerial Vehicle

2.2.1 Introduction to Fixed Wing UAV

Fixed wing unmanned aerial vehicles (UAVs) have seen an increase in use performing tasks such as surveillance, reconnaissance, environmental surveying, aerial photography, and competitive racing. A large spectrum of fixed wing UAVs are in operation today being used by the military and civilians. Larger UAVs may require long stretches of runway to takeoff, while smaller hand launched UAVs do not. Fixed wing UAVs are ideal for tasks that require endurance, high altitude flight, or a large payload. A brief overview of the complex non-linear models used to describe fixed wing UAVs and the simplifications made in literature is discussed, followed by a discussion on the autopilot systems commonly used to control the crafts.

2.2.2 Modeling

The fixed wing UAV is a 6-DOF system with four degrees of control consisting of pitch, roll, yaw, and thrust. Due to the under-actuation of the craft, several states of the



Figure 2.1: 3DR Aero

craft are coupled. Lift, pitch, thrust, all of these properties are interconnected and coupled in such a way to produce non-linear high order differential equations which can be difficult to model. Models that represent the position and attitude of the craft as well as the Cartesian velocities and rate of attitude change result in a 12 state variable system of high order non-linear differential equations.

High order non-linear models can be linearized and simplified to be used in low level control systems to maintain vehicle stability. Beyond the control aspect of UAVs, the high order non-linear models are not always necessary. It is common in literature to simplify the motion of fixed wing UAVs in terms of simple Dubins vehicle kinematic equations [CCA09] [LJ17] [Nel05] [Gri06] [JLLB16]. When simplified kinematics are used when developing navigation and guidance systems, it is assumed that there already exists a low level control loop that maintains vehicle stability and performs according to the kinematic model in Equations 2.1-2.2.

$$\dot{x} = V\cos(\theta) \tag{2.1}$$

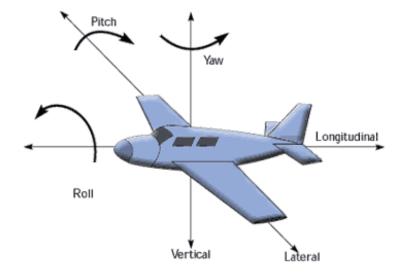


Figure 2.2: Pitch Roll and Yaw

$$\dot{y} = V \sin(\theta) \tag{2.2}$$

The position of the vehicle at any point in 2D Cartesian space is defined as (x, y) and the velocity of the vehicle in each axis is defined as (\dot{x}, \dot{y}) . The heading of the vehicle θ is most frequently used as the input $u = \theta$. To more closely represent the dynamics of a fixed wing, constraints are given to the velocity V and turning rate $\dot{\theta}$.

$$V \ge V_{stall} \tag{2.3}$$

$$-\dot{\theta}_{max} \le \dot{\theta} \ge \dot{\theta}_{max} \tag{2.4}$$

The devices that are assumed to be in control of the UAV when designing guidance and navigation systems are called autopilots or flight controllers, which will be discussed briefly in the next section.

2.2.3 Autopilot

The autopilot is a software and hardware package that brings navigation, guidance, and control techniques realization. A typical autopilot system is depicted in Figure 2.3, consisting of a hard shell protecting sensors with numerous input and output connectors for power, communication, and servo actuation. The flight controller is responsible for several major tasks including maintaining vehicle stability, turning high level commands into low level control effort, and recording and transmitting data to ground stations.

Autopilots accomplish these tasks by implementing several layers of algorithms consisting of navigation, guidance, and control which will be discussed next.



Figure 2.3: Pixhawk Flight Controller

2.3 Navigation, Guidance and Control

2.3.1 Introduction to NGC

Nearly all of the systems that go into an autopilot system for a fixed wing UAV can be put under the category of Navigation, Guidance, or Control (NGC). Traditionally, NGC could be thought of separate but equally important algorithm layers that aid an autopilot in accomplishing a given task. Navigation is the study of measuring and filtering the state of the vehicle. Guidance produces a commanded state based on high level requests from a path planner or user. Control maintains vehicle stability while attempting to achieve the state commanded by the high level guidance system. Lately, the lines between guidance and control have become less clear as the systems become more integrated with each other, resulting in more complex capabilities. Continuing with the convention in literature, navigation will be discussed as a separate unit while guidance and control will be discussed simultaneously.

2.3.2 Navigation

Navigation is the study of measuring the position and attitude of the UAV for the purposes of building guidance and providing feedback for the low level control system. Modern flight controllers contain a package of sensors called an Inertial Measurement Unit (IMU) which consists of a 3-axis accelerometer, gyroscope, compass, and barometer. The accelerometer and gyroscope are measured to determine the translational and angular acceleration of the craft. The compass measures the heading, and the barometer measures air pressure which is correlated to altitude. GPS is often used to determine the local position of the UAV. One of the major challenges in navigation is the noise and uncertainty in sensor measurements. Filters are used to fuse information and provide more accurate estimates of the vehicles position and rate of change.

2.3.3 Guidance and Control

Guidance and control have traditionally been separate systems, but as the need for UAVs to perform complex tasks increases, so does the complexity of the control systems. A common high level request provided to a UAV is to follow an arbitrary path which can be done with traditional controls [CITATION]. A control system for following a moving path was developed in [OAE16], which was successful at reducing the tracking error in simulation and real flight tests. To accomplish path following of a mobile path, the control law becomes complex and less intuitive.

Instead of relying on complex control laws, there has been much research on methods that combine guidance and control. Two categories will be discussed consisting of potential field and vector field methods. In literature, the two methods have been seen used interchangeably. In the work presented, potential field is in reference to a gradient potential converging to a local minimum while vector field is in reference to a space of vectors whose integral lines converge and follow a path. It can be argued that vector fields are essentially a potential field, but for organizational purposes, they will be referred to as completely different methods.

2.3.4 Potential Field

- Introduction to potential field

Potential field is a real-time robotic manipulator algorithm that distributes the task of goal seeking and obstacle avoidance among multiple layers of control [Kha86]. The robot's workspace is represented as a gradient potential of attractive and repulsive artificial forces that drive the robot to a desired state. Goals are represented as an attractive force while obstacles provide a repulsive force. The potential field is constructed by modeling the robots motion in terms of Lagrangian mechanics shown in Equation 2.5. The Lagrangian is defined as the difference in kinetic energy $T(x, \dot{x})$ and potential energy

U(x) in a system. The goal of the system imparts a potential U_{xd} while obstacles impart a repulsive potential U_o .

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = F \tag{2.5}$$

$$L(x, \dot{x}) = T(x, \dot{x}) - U(x)$$
 (2.6)

$$U_{art}(x) = U_{xd} + U_o(x) \tag{2.7}$$

Potential field has been shown to be successful at driving a robot from an initial state to a goal state while avoiding obstacles by taking the path of least action formed by the gradient. One of the weaknesses of potential field identified early on is the methods susceptibility to local minima, preventing the robot from reaching the desired global minima, or goal state [KB91]. Local minima can be avoided in the potential field method if navigation functions are used [GKM10].

Potential field is useful for point-to-point guidance and control which is often the task of UAVs traveling to waypoints, however it is often desired for a UAV to converge to and follow a path.

2.3.5 Vector Field

Introduction to Vector Field

Several methods have been developed to generate vector fields that converge to and follow paths. Histogram virtual force field (VFF) breaks the workspace into discrete cells that contain information in regards to the certainty of the presence of an obstacle [BK90] [BK91]. Cells containing an obstacle apply an artificial guidance force away from the cell. Goals apply a global attractive force that drive the robot towards the desired state. The resultant guidance vector for the VFF method is the sum of the contributions of obstacle

repulsive cells and the globally attractive goal. Lyapunov and navigation functions have been successfully used to provide guidance for converging to and following a path. Nelson et al. developed a method for generating a vector field that converges to and follows a path for line and circular primitives [Nel05]. Sinks, dead zones, and singularities were avoided when constructing more complex flight paths from the primitives by only allowing a single field to be active at any time. A vector field construction method for curved paths was presented in [Gri06] by extending Nelson's method. Elliptical paths have been generated by from primitives by applying coordinate transformations to the field [Fre].

Vector fields have many advantages to traditional guidance. UAVs often encounter disturbances in the form of wind which can be difficult to plan for. In the event a UAV encounters wind disturbances and is pushed off course, the field will drive the vehicle back on course [dMKB+17]. In addition to path following, vector field has been used to track a moving target in 3D [MTE+16]. When the location of the target is unknown, loitering about an uncertain target can be achieved by building a circular vector field and applying a linear coordinate transformation to form an elliptical loiter based on the uncertainty in targets position estimate [Fre07].

- Following curved paths in a constant wind [Gri06]

Another method for constructing a vector field is by forming integral curves that converge at the intersection of surfaces [GPMP09].

 construct an n-dimensional vector field by forming integral curves that converge at the intersection of surfaces - Field is a result of the sum of 3 terms - Convergence -Circulation - Time varying

$$\boldsymbol{u} = G\nabla_q V + H \wedge_{i=1} \nabla_q \alpha_i - M(\alpha)^{-1} \boldsymbol{a}(\alpha)$$
 (2.8)

- Define all variables - u is the resulting 2x1 vector - Gradient - $alpha_i$ is a surface function - Wedge product simplifies to cross product in 2d - G, H, L scalar weighting

quantities - The intersection of the surfaces represents a path to converge and follow - Cylinder and plane example

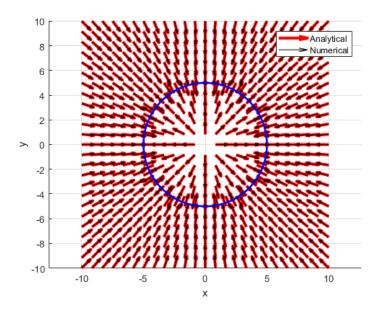


Figure 2.4: Convergence

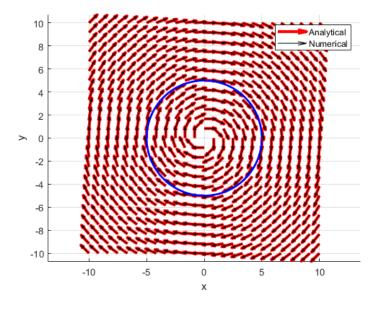


Figure 2.5: Circulation

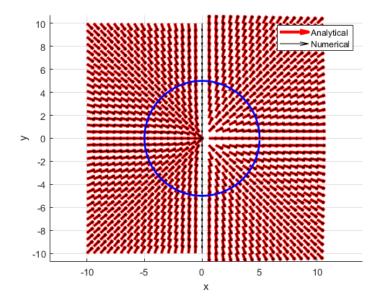


Figure 2.6: Time Varying

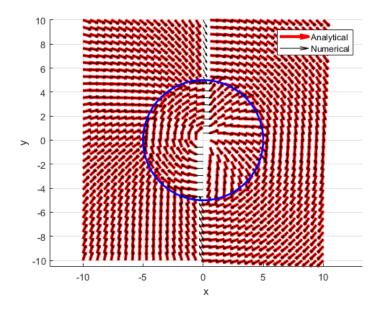


Figure 2.7: Total Field

• Cooperative Standoff Tracking of Uncertain moving targets (2007, Frew)

- VF usefulness extended to loitering about an uncertain target
- Lyapunov vector field generation for a circular loiter
- Linear transformation applied to stretch the field into an ellipse shape

2.4 Literature Review Summary

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APPENDIX: AN APPENDIX

A.1 A Section in the Appendix