

A Proposal for a Parameterized Circulating Vector Field Guidance for Fixed Wing  
Unmanned Aerial Vehicles

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A Proposal for a Parameterized Circulating Vector Field Guidance for Fixed Wing  
Unmanned Aerial Vehicles

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## ABSTRACT

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A Proposal for a Parameterized Circulating Vector Field Guidance for Fixed Wing

Unmanned Aerial Vehicles (47 pp.)

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Unmanned Aerial Vehicles (UAVs) are guided to fly along straight line obstacle free paths that connect pre-planned waypoints. Initially undiscovered obstacles encountered during flight may require waypoints to be re-planned. Obstacles could be avoided without the need to re-plan mission waypoints by implementing vector field path following in conjunction with repulsive obstacle vector fields. Repulsive vector fields that combine weighted repulsive and attractive components to provide an optimal obstacle avoidance guidance will be investigated to avoid singularities and improve path tracking performance compared to waypoint guidance.

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## LIST OF SYMBOLS

$\vec{v}$	Vector field
$\vec{v}_{conv}$	Convergence component
$\vec{v}_{circ}$	Circulation component
$\vec{v}_{tv}$	Time-varying component
$G$	Convergence weight
$H$	Circulation weight
$L$	Time-varying weight
$q$	Spatial dimension set
$\alpha_i(x_1, x_2, \dots x_n, t)$	Implicit surface function
$n$	Number of spatial dimensions
$t$	Time
$i$	index
$\nabla_q$	Gradient with respect to spatial dimensions q
$M$	Gradient matrix
$a$	Velocity column vector
$\vec{V}$	Total field
$d$	Range
$P$	Decay weight
$R$	Radius
$\vec{v}_{repulsive}$	Repulsive vector field
$\vec{v}_{attractive}$	Attractive vector field
$u$	Speed

## LIST OF ACRONYMS

UAV	Unmanned Aerial Vehicles
VF	Vector Field
UAS	Unmanned Aerial System
VFF	Virtual Force Field
TPLVF	Tangent Plus Lyapunov Vector Field
RRT*	Optimal Rapid Radom Trees
DT	Delauny Triangulation
GVF	Gradient Vector Field

# 1 INTRODUCTION

## 1.1 Motivation and Problem Statement

Fixed wing Unmanned Aerial Vehicles (UAVs) are used for missions such as surveillance and reconnaissance that might put pilots in harms way [1]. Missions typically consist of sequential objectives represented as waypoints that the UAV follows. Waypoints may be pre-planned before flight where vehicle constraints and obstacles can be considered to prevent collisions or entry into no-fly zones. UAVs follow waypoints by implementing guidance algorithms that calculate headings to direct a UAV along a path connecting the waypoints. Obstacles may be discovered during flight that were unknown at initial planning and a new set of waypoints may have to be generated. Waypoints are typically computed at a ground station and are relayed to the UAV by radio, which may be problematic if communication with the UAV is lost. Guidance that accomplishes mission objectives while avoiding obstacles without the need for re-planning waypoints may be beneficial.

Vector Field (VF) guidance is a method that is mainly used for path following and can be useful for obstacle avoidance [2, 3]. VFs can produce continuous heading vectors that can be used to guide a UAV to coverage and follow a path. Vectors are calculated by summing together convergence and circulation terms that are weighted by static scalars. Obstacles can be represented as a path and given a negative convergence weight resulting in a repulsive field. Static repulsive VFs do not always route the UAV around an obstacle. Modifying repulsive VF parameters to be functions of common UAV states may be used to produce an optimal guidance. **The proposed research seeks to determine VF weighting functions that enable optimal obstacle avoidance.**

## 1.2 Methods Overview

The proposed research will be conducted in three phases where VF guidance singularities will be demonstrated, weighting functions will be investigated, and a developed GVF will be validated on a ground robot simulating a UAV. Phases I and II will be conducted in a simulation environment that combines mission paths and obstacles into a single GVF. Phase III will be conducted with a ground robot simulating a UAV guided by the modified GVF in real-time. Dubin's fixed wing constraints will be imposed in simulations and experiments.

## 1.3 Phase I

**Demonstrate GVF singularities for circular obstacles.** A simulation environment will be built that generates GVFs consisting of mission paths and obstacles. Circular and elliptical obstacles will be investigated and the resulting singularities will be characterized. Static weights will be used and the performance of the guidance measured in distance traveled and time of flight.

## 1.4 Phase II

**Investigate GVF weighting functions that influence obstacle avoidance.** UAV closing rate, position, and range will be used to develop dynamic GVF weights for convergence and circulation. The modified GVF will be compared against a static and strictly repulsive GVF. Distance traveled and time of flight will be used to as metrics to compare the modified GVF to the unmodified GVF.

## 1.5 Phase III

**Validate modified GVF model with ground robot experiments.** The modified GVF developed in Phase II will be implemented on a differential drive ground robot

simulating a fixed wing UAV. Guidance performance while avoiding static obstacles will be demonstrated.

## 1.6 Summary of Phases

Each phase consists of an **objective** that will be accomplished by executing *tasks*. Completion of all objectives and phases will result in the final deliverable.

**Phase I Objective:** Demonstrate Gradient Vector Field Singularities

*Tasks:*

1. *Build a GVF simulation environment*
2. *Evaluate scenarios where singularities are expected*
3. *Characterize location of singularities*

**Phase II Objective:** Investigate GVF weighting functions that influence obstacle avoidance

*Tasks:*

1. *Formulate circulation and convergence weights as functions of UAV state*
2. *Determine combination of GVF weights that produces optimal guidance in simulation*

**Phase III Objective:** Validate modified GVF model with ground robot experiments

*Tasks:*

1. *Build differential drive robot*
2. *Build robotic framework to take guidance commands*
3. *Repeat simulations performed in Phase II on ground robot*

**Deliverable:** Adaptive GVF parameterized weights optimal guidance for path following and static obstacle avoidance.

## 2 LITERATURE REVIEW

### 2.1 Introduction to Literature Review

### 2.2 Unmanned Aerial Vehicle

Unmanned Aerial Vehicles (UAVs) are pilotless aircraft used by military, police, and civilian communities for tasks such as reconnaissance, damage assessment, natural disaster surveying, and target tracking [4, 5]. Tasks may be carried out by a single UAV or in cooperation with another air or ground vehicle [6–8]. In general, UAVs are categorized into fixed wing and rotor craft varieties [9] that range in size, payload, and flight time capabilities. The aircrafts can be controlled remotely with a RC transmitter or fly on paths connecting pre-planned waypoints. Data can be collected with on-board sensors [10, 11], such as cameras, which can be stored or relayed to the ground. UAVs are part of an Unmanned Aerial System (UAS) which consists of the vehicle, autopilot, ground station, transmitter, and two way radio, depicted in Figure 2.1. Ground stations are responsible for monitoring the vehicle's status, planning missions, and generating obstacle free and flyable paths which are sent to the autopilot via two way radio. Pre-planed paths may be generated by connecting waypoints with cubic splines of minimum curvature [12].



Figure 2.1: Unmanned Aerial System (UAS)

The UAV autopilot is responsible for controlling a pre-planned path and maintaining vehicle stability while under the influence of external wind disturbances. Stable flight while path following is accomplished by implementing feed-back control, navigation, and guidance systems. A high level overview of the autopilots systems can be seen in Figure 2.2. Feed-back refers to the closure of an open-loop control system which allows a reference error to be calculated between the desired state of the UAV, the reference, and the current state of the UAV. Reference error is used to calculate the necessary actuator output required to modify the vehicles attitude and position while preventing unbounded oscillation. Attitude and position feed-back is provided by the navigation system by sampling on-board sensors. Filtering and fusing noisy data from multiple sources is often accomplished through estimation techniques such as the Kalman filter.

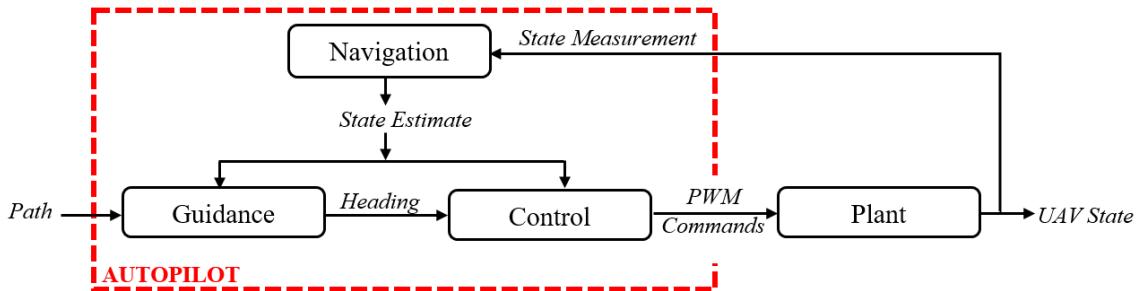


Figure 2.2: Autopilot's Navigation, Guidance, and Control Architecture

### 2.3 Path Planning

### 2.4 Guidance

#### 2.4.1 Waypoint Guidance

Waypoint guidance aligns the vehicle with the current active waypoint that lies along a pre-planned path. Paths are typically generated off-line and can be optimized for shortest distance traveled and further refined to be flyable for a particular vehicle. Paths

may also be optimized to produce flight patterns that increase sensor coverage of an area of interest [?].

If an obstacle lies along that sensor path, the UAV must avoid the obstacle but also return back to the sensor path such that a minimal length of the path is missed during data collection. The number of waypoints that divert around an obstacle effects how closely the UAV tracks the outside of the obstacle and how much of the original path can be traveled. Few obstacle diversion waypoints leads to excess path deviation. Increasing the number of diversion waypoints reduces path deviation, however has diminishing returns. A cost function  $\gamma$  can be used to measure the deviation from a planned path while avoiding obstacles with diversion waypoints, shown in Equation 2.1.

$$\gamma = \frac{1}{r_o} \int_0^{t_f} y dt \quad (2.1)$$

An example of a UAV following diversion waypoints is shown in Figure 2.3 and the cost associated with increasing number of waypoints in Figure 2.4.

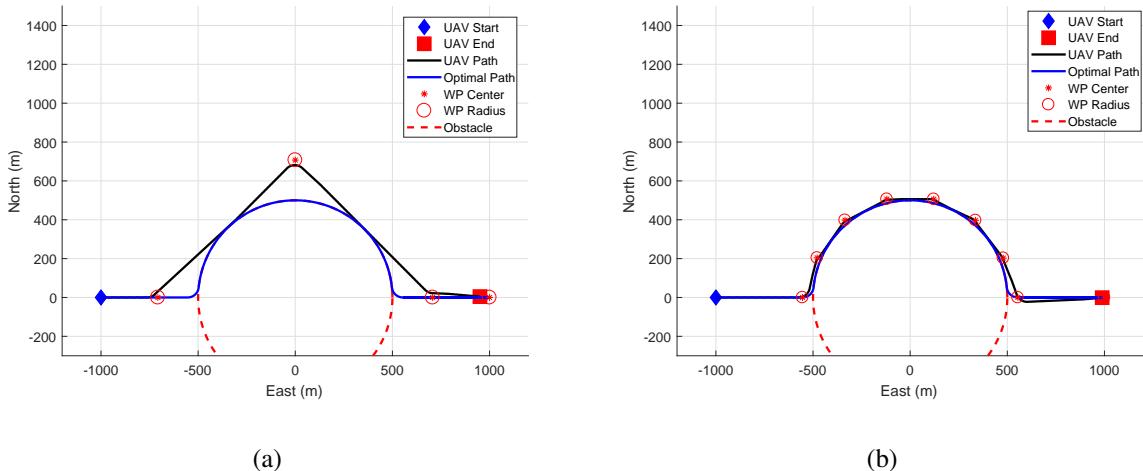


Figure 2.3: Obstacle Diversion Waypoints

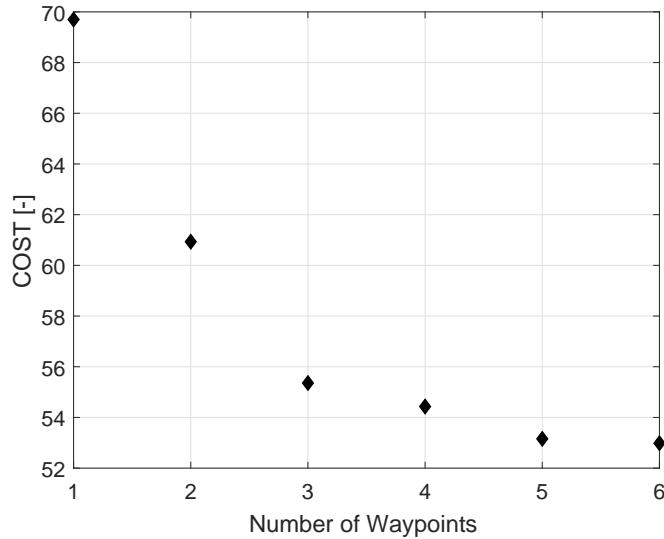


Figure 2.4: Cost impact versus number of waypoints

It would be beneficial to include obstacle avoidance into a UAVs guidance system to remove the need to communicate with the ground station or use an on-board path planner which may be accomplished with potential field or vector field.

#### 2.4.2 Potential Field

Potential field is based on the principle of artificial attractive and repulsive forces acting on a point mass that is guided to a desired goal while avoiding static and dynamic obstacles [13]. Goal states are represented as an attractive force that pulls a point mass in the direction of minimal energy while obstacles are represented as repulsive forces that act locally to push the point mass away. Potential field is also capable of acting as a path and trajectory planning algorithm [14], possibly eliminating the off-board path planner.

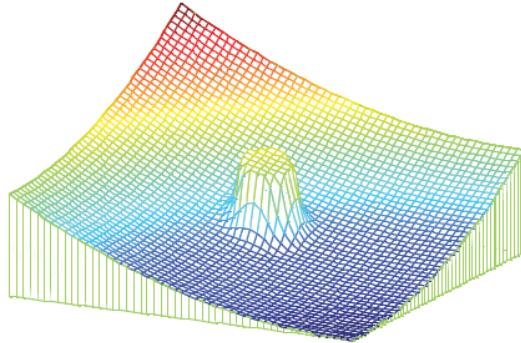


Figure 2.5: Single Obstacle Potential Field Gradient [18]

An example of potential field can be found in [15–17] which allowed for real time goal seeking with obstacle avoidance on a mobile ground robot equipped with ultrasonic sensors. The robot located at  $(x_0, y_0)$  is attracted towards a goal with constant magnitude force  $\vec{F}_t$  located at  $(x_t, y_t)$  and a distance  $d_t$  from the robot. In the immediate area of the robot, an active window exists which records integer certainty values inside discrete cells. Cells containing an obstacle provide a repulsive force  $\vec{F}_{i,j}$  opposite in direction to the line-of-sight from vehicle to cell location  $(x_i, y_j)$ , where  $(i, j)$  represents the cell index,  $F_{cr}$  is a constant repulsive force,  $W$  the vehicle's width,  $C_{i,j}$  a cell's certainty, and  $d_{i,j}$  the distance to the center of the cell with respect to robots center.

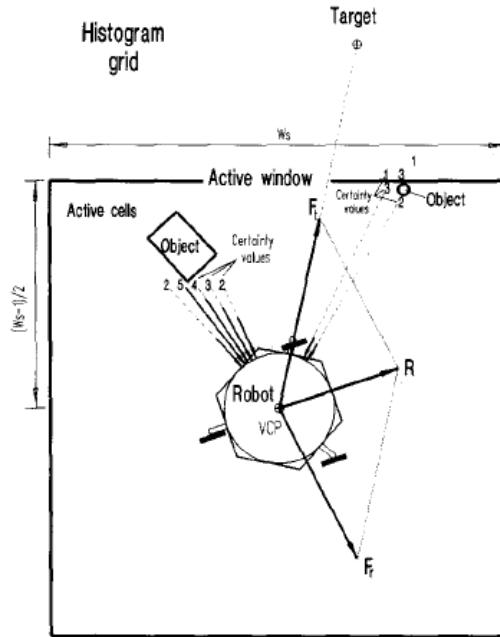


Figure 2.6: Virtual force field histogram acting on a mobile robot [16]

$$\vec{F}_{i,j} = \frac{F_{cr} W^n C_{i,j}}{d_{i,j}^n} \left( \frac{x_i - x_0}{d_{i,j}} \hat{x} + \frac{y_i - y_0}{d_{i,j}} \hat{y} \right) \quad (2.2)$$

The total repulsive force exerted on the robot is determined by summing the active cells, shown in Equation 2.3

$$\vec{F}_r = \sum_{i,j} \vec{F}_{i,j} \quad (2.3)$$

$$\vec{F}_t = F_{ct} \left( \frac{x_t - x_0}{d_t} \hat{x} + \frac{y_t - y_0}{d_t} \hat{y} \right) \quad (2.4)$$

Summing together attractive and repulsive forces produce a vector  $\vec{R}$  that can be used for heading guidance, shown in Equation 2.5.

$$\vec{R} = \vec{F}_r + \vec{F}_t \quad (2.5)$$

Major drawbacks to potential field were identified in [17] consisting of local minimum and oscillations in corridors. The local minimum problem occurs when closely spaced obstacle's potential combine to produce a well on the descent gradient where a pre-mature stable point is reached, shown in Figure 2.7. Additionally, closely spaced obstacles may also be difficult to pass between, shown in Figure 2.8a. Oscillations can also be experienced near obstacles or in narrow passages at high speeds, shown in Figure 2.8b.

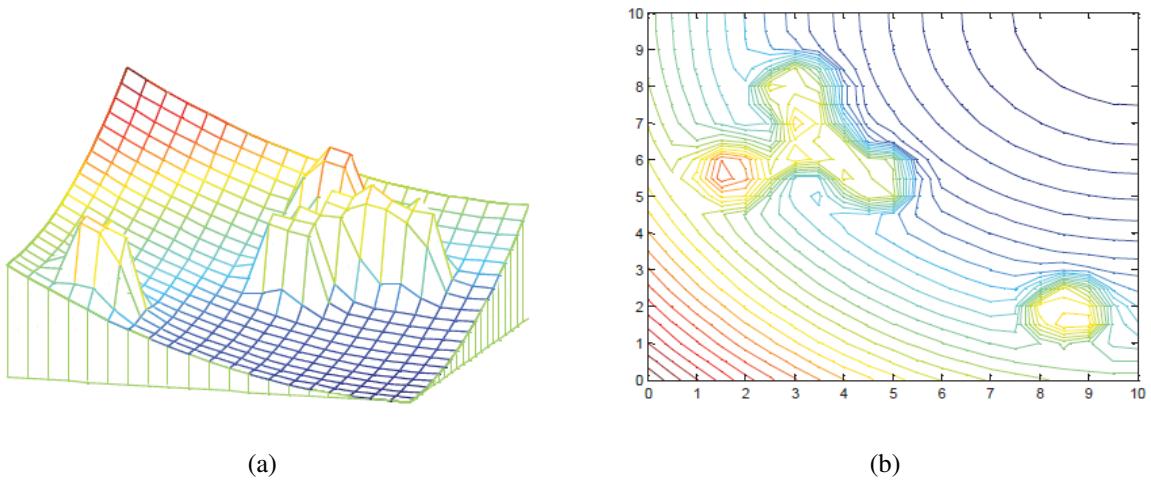


Figure 2.7: Potential Field Local Minimum [18]

Proposed solutions to local minimum include object clustering and virtual waypoint method [18], virtual escaping route [?], and use of navigation functions [19]. Oscillations in potential field were addressed in [?] and [?].

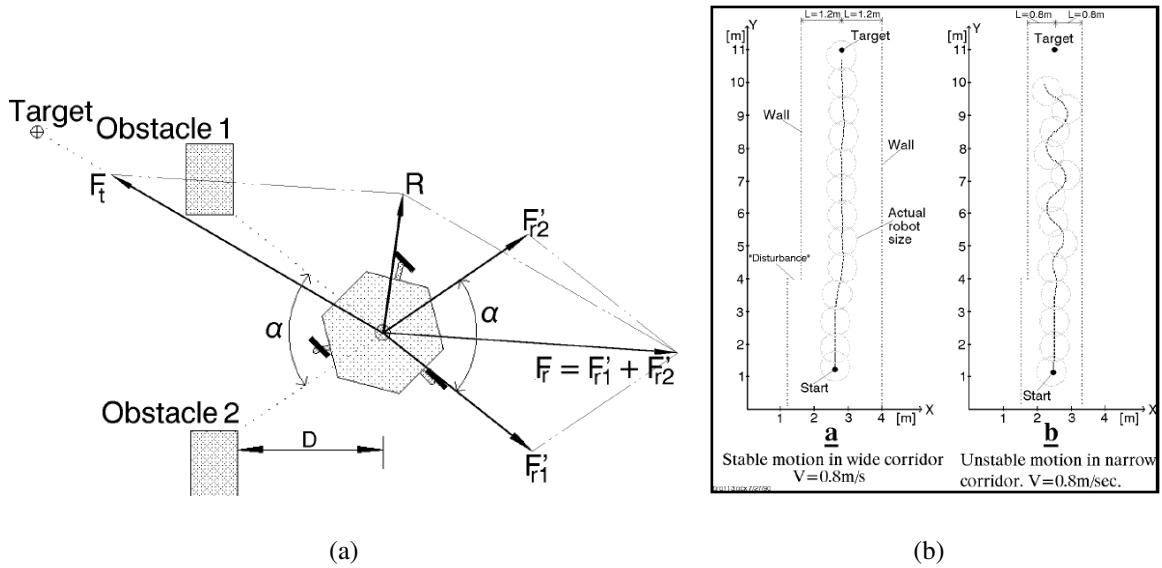


Figure 2.8: Potential Field Local Minimum [18]

Navigation functions [19] and obstacle clustering [18] have been used to prevent local minimums in potential field. Navigation functions relate kinematic constraints to the gradient potential to produce a bounded and local minimum free solution [14]. Clustering closely spaced obstacles into a single and equally repulsive obstacle prevents local minimum from forming, shown in Figure 2.9.

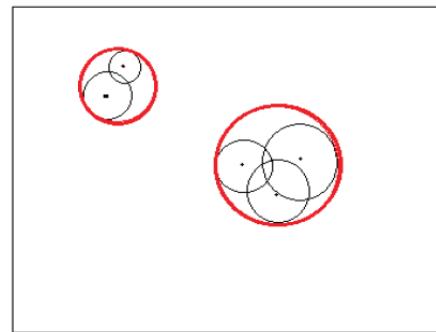


Figure 2.9: Obstacle Clustering [18]

Potential Field's ability to avoid obstacles and combine path planning, trajectory planning, and control into a single computationally inexpensive system makes it an attractive motion control system for robots seeking a singular point, even with the limitations discussed in [17].

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In addition to local minimum and oscillations, potential field may not be ideal for providing guidance to return to a sensor path after avoiding an obstacle. Unlike the mobile ground robots in [15], fixed wing UAVs must maintain a minimum forward velocity, have limited turning radius, and cannot converge to a single point. Vehicles with velocity and turn rate constraints may not return to a pre-planned path once the obstacle has been avoided. Vector fields that direct a UAV to paths connecting waypoints have been developed using Lyapunov and gradient vector field techniques.

#### **2.4.3 Path Following Vector Field Guidance**

The UAV guidance system is responsible for taking high level pre-planned paths from the ground station and providing a reference heading command to the control system. Several methods for path following guidance were investigated in [21] consisting of carrot chasing [22], non-linear guidance law, pure line-of-sight [23], linear quadratic regulator [24], and vector field method [25]. A Monte Carlo simulation with wind disturbances was conducted for the guidance methods above in [21] to determine each method's performance based on accuracy, robustness, and control effort. The vector field method followed the path with the least tracking error and control effort which is the primary goal of path following.

## 2.5 Vector Field Guidance

### 2.5.1 Introduction to Vector Field Guidance

Vector Field is a guidance and control approach that can be used to transition a robotic system from an initial state to a final state. Final states, or goals, act as artificial attractive forces that pull on the robotic system while obstacles act as artificial repulsive forces that push the robotic system away. Classes of vector fields can be categorized as point seeking or path following algorithms. Potential Field and Virtual Force Field (VFF) methods converge to a single point and avoid obstacles by applying artificial attractive and repulsive forces. Lyapunov and Gradient Vector Fields provide guidance that asymptotically converges and follows a path. Obstacle avoidance has been achieved with Gradient Vector Fields by assigning repulsive weights to a convergence term. Weights currently act as a high level specification of the desired guidance behavior and may be further optimized.

### 2.5.2 Lyapunov Vector Fields

Lyapunov vector fields produce heading guidance that asymptotically converges and circulates along a path passing through waypoints. Paths can be built from straight line and circular arc primitives taking UAV kinematic constraints into consideration. Vector Fields that guide to straight line and circular paths was introduced in [25]. Farther away from the path, vectors are constant and point in the direction perpendicular to the path. Within a transition region the vectors begin to rotate and point more parallel to the path. Vectors on the path point directly in the direction of the path. Lyapunov vector fields for straight line and circular arcs are shown in Figure 2.10.

Combining path primitives together can result in fairly complex paths such as that shown in Figure 2.11. Each path primitive has a vector field associated with it and determining which field to use can be approached in two different ways. Fields from all

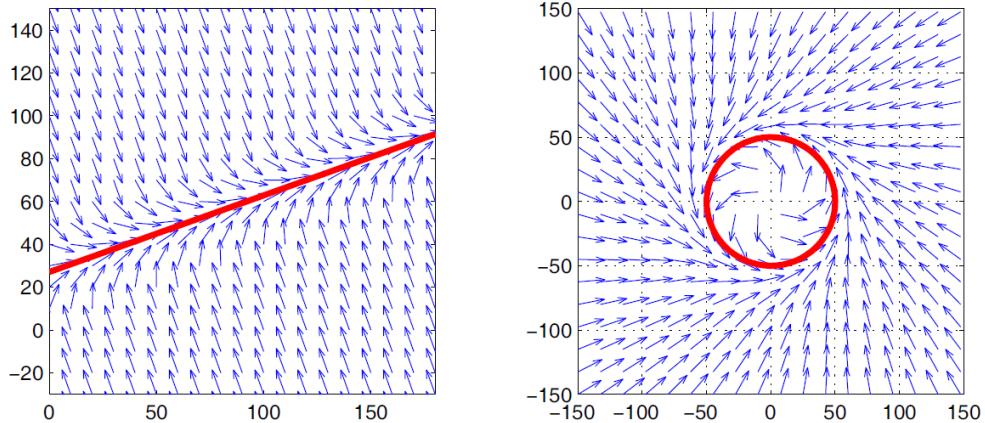


Figure 2.10: Lyapunov vector field for straight line and circular primitives [25]

of the primitives can be summed together similar to the attractive and repulsive forces in potential field. Second, fields can be selectively activated and deactivated based on the position of the UAV. Summing together vector fields, as pointed out in [25], can result in several problems including dead zones, sinks, and singularities. Selectively activating each vector field as a UAV nears waypoints was used in [25–28].

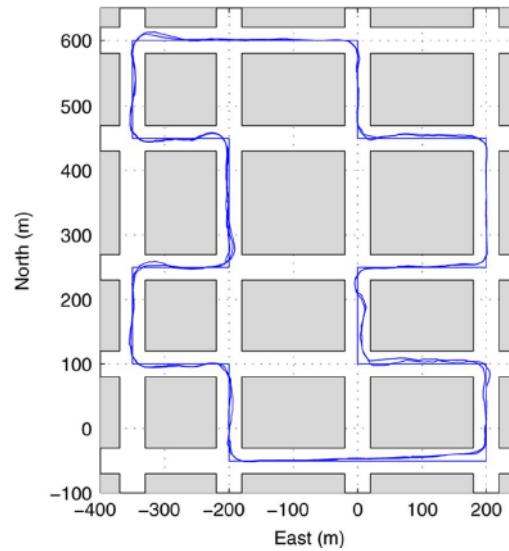


Figure 2.11: Straight path following in urban environment [25] using Lyapunov Vector Field

Lyapunov Vector field construction for curved paths was presented in [29] and is shown in Figure 2.12. Constructing a Vector Field for an arbitrary curve may allow for more complex paths and could eliminate the need for switching between primitives.

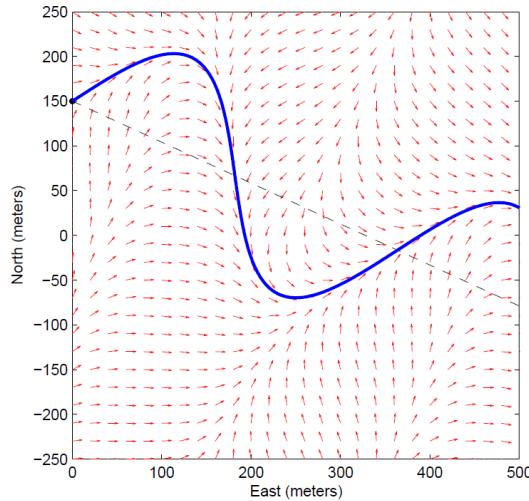


Figure 2.12: Lyapunov vector field approach curved path asymptotically [29]

Primitive circular vector fields were modified in [30, 31] via non-linear coordinate transformations to produce elliptical 2.13a, or racetrack 2.13b, fields. Transforming the circular field as a function of a Kalman filter's covariance matrix when sensing an unknown target was investigated in [31].

Target tracking Tangent Plus Lyapunov Vector Field (TPLVF) was introduced in [32] that produced shorter paths compared to Lyapunov alone. Outside of the standoff circle, tangent vectors provided the shortest distance to a standoff circle. Inside the standoff circle, no tangent lines exist and Lyapunov was used in its place. Figure 2.14 shows the difference in paths taken for Lyapunov and tangent vector fields outside the standoff circle. The TPLVF was later used for path planning to avoid obstacles in [33] while [34] constructed a tangent vector field for curved paths.

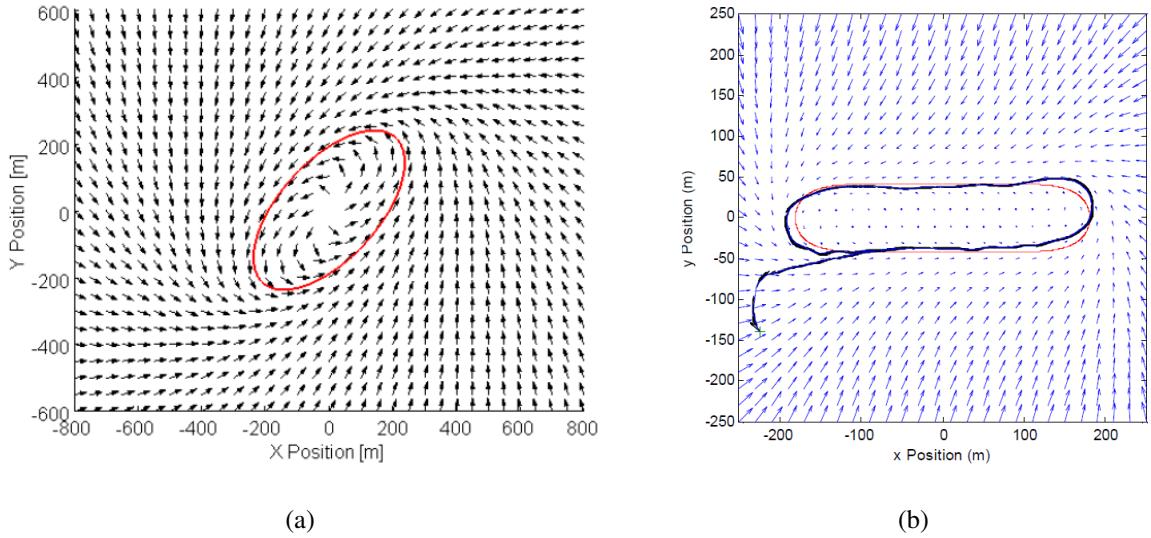


Figure 2.13: Elliptical VF produced by non-linear coordinate transformations a) [31] and b) [30]

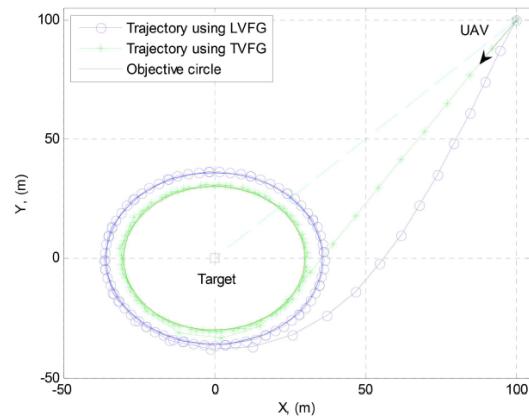


Figure 2.14: Tangent plus lyapunov vector fields for shortest path target tracking [33]

### 2.5.3 Non-Lyapunov Vector Fields

All methods that consider obstacles thus far built a vector field that guides the UAV to an obstacle free path. Another approach is to use vector fields as a high level specification for heuristic path planning algorithms [35]. An optimal Rapid Random Trees (RRT\*) algorithm used a vector field as a guide to explore the configuration space of the

UAV for an obstacle free path. Branches extend from the root, or initial location of the UAV, randomly throughout the map with a finite deviation from the initial vector field. When a branch encounters an obstacle it is trimmed and no longer explored. The path of minimum cost, or least distance, is selected for the UAV to use as a reference path. An example of the algorithm is shown in Figure 2.15.

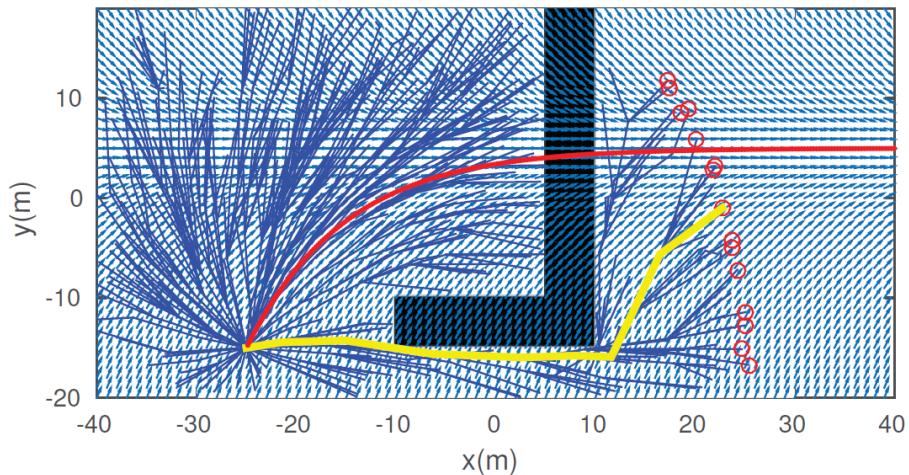


Figure 2.15: RRT\* path planner with a VF used as a task specification [35]

A Vector Field was constructed inside a configuration space with edges defined by Delaunay triangulation (DT) in [36]. A simulation of a robot traversing a vector field inside a set of DTs can be seen in Figure 2.16. Vector fields designed to stay inside a region of DTs may be used with optimal path planning algorithms for navigating urban environments [37].

So far all of the vector field methods discussed have avoided obstacles by planning paths around them. Paths are typically calculated at the ground station and if communication is lost a new path may not be relayed to a UAV encountering a new obstacle. A possible solution is using vector fields to provide a repulsive force such as that seen in [2, 3, 38].

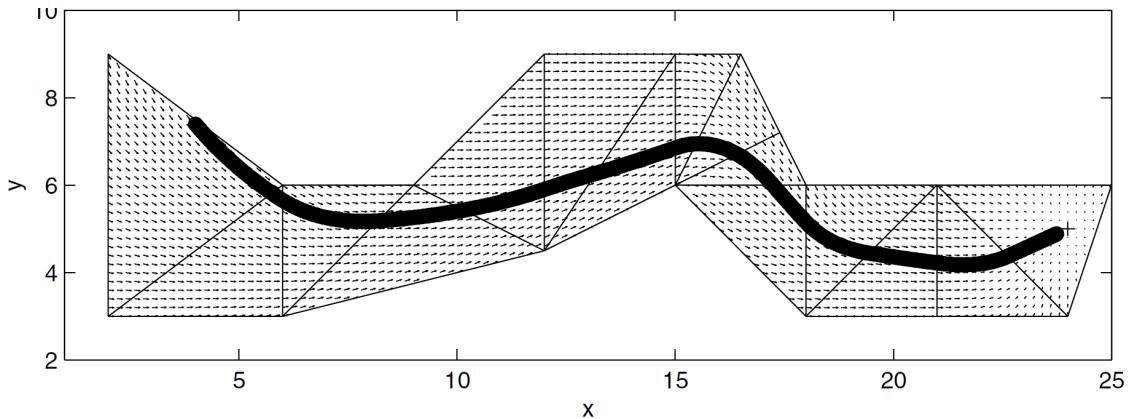


Figure 2.16: Vector field within a set of delaunay triangles [36]

#### 2.5.4 Gradient Vector Field

Gradient vector field (GVF) is a method that produces an  $n$ -dimensional vector field that converges and circulates a target path [39]. Paths can be static or time-varying and consist of points that lie at the intersection of surfaces defined by implicit functions. The surface functions are used to calculate a total vector field that is a sum of a convergence, circulation, and time-varying term. Convergence effectively attracts a robot to the target curve while circulation guides the robot to traverse the target curve. Time-varying is a feed-forward term that accounts for changes in the target path as a function of time  $t$ . The total field  $\vec{v}$  is summarized in Equation 2.6.

$$\vec{v} = \vec{v}_{conv} + \vec{v}_{circ} + \vec{v}_{tv} \quad (2.6)$$

Multiplicative scalar weights ( $\mathbf{G}, \mathbf{H}, \mathbf{L}$ ) have been used to modify the strength of each term in [40], shown in Equation 2.7

$$\vec{v} = \mathbf{G}\vec{v}_{conv} + \mathbf{H}\vec{v}_{circ} + \mathbf{L}\vec{v}_{tv} \quad (2.7)$$

Producing a GVF that converges and circulates a target path consisting of points  $q = [x_1(t), x_2(t) \dots x_n(t)] \in \mathbb{R}^n$ , first requires ( $n-1$ ) implicit surface functions  $\alpha_i(x_1, x_2, \dots x_n, t)$ ,  $i = 1, 2, \dots n - 1$  to be defined. UAVs are restricted to  $\mathbb{R}^3$  spatial configuration space, therefore two surface functions must be defined to calculate a GVF. Surface functions must have 1) *continuous first order partial derivatives* and 2) *bounded second order partial derivatives* to guarantee convergence to the curve [40]. The total gradient vector field converges and circulates the intersection, or level sets, of the two surfaces. For a UAV traveling at constant altitude, the first surface can be represented as a flat plane in Equation 2.8. Intersecting the flat plane with a static vertical plane, Equation 2.9, produces a gradient vector field that converges and follows a straight line, Figure 2.17.

$$\alpha_1 = z \quad (2.8)$$

$$\alpha_2 = x \quad (2.9)$$

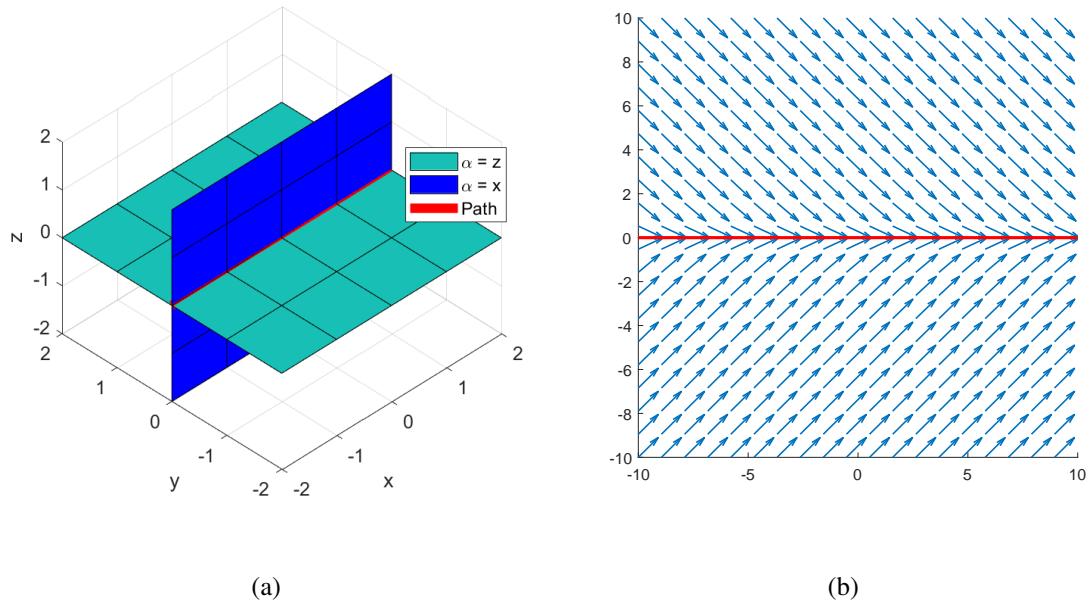


Figure 2.17: Intersection of planes (a) and resultant Gradient Vector Field (b)

Similarly, intersecting the flat plane with a static cylinder, Equation 2.10, results in a vector field that converges and follows a circular path shown in Figure 2.18.

$$\alpha_2 = x^2 + y^2 - r^2 \quad (2.10)$$

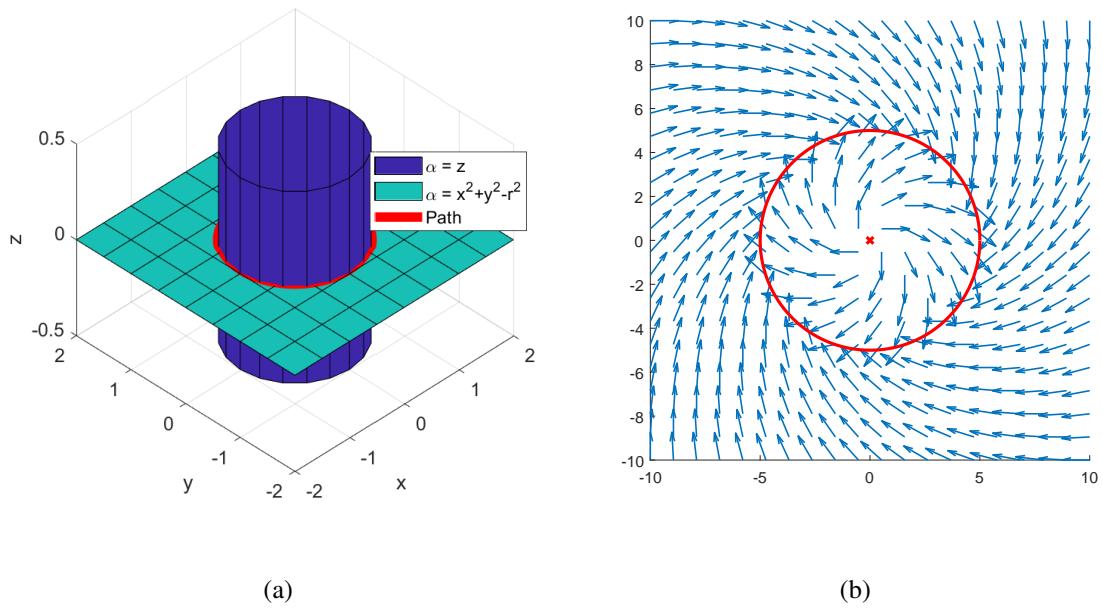


Figure 2.18: Intersection of plane / cylinder (a) and resultant Gradient Vector Field (b)

Static paths do not require the time-varying term of the vector field calculation and can be considered zero in such cases. The convergence term of the vector field is calculated by summing the product of each implicit function with its own gradient, which is shown in Equation 2.11.

$$\vec{v}_{conv} = \sum_{i=1}^{n-1} \alpha_i \nabla_q \alpha_i \quad (2.11)$$

Evaluating equation 2.11 for a cylinder intersecting a plane results in a vector field that converges to a circular path shown in Figure 2.19a. Note that the vectors are not of equal length for the entire configuration space, decaying in strength when approaching the

curve. Normalizing the field was done in [2, 40, 41] to allow for convenient weighting of each term at a later process. The normalized convergence field for a circular path is shown in Figure 2.19b.

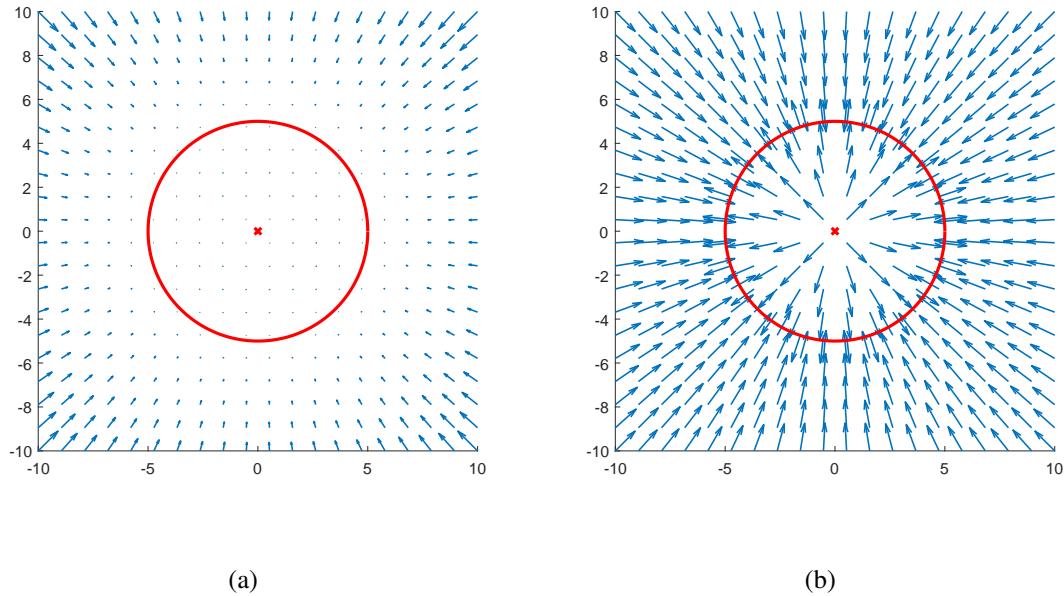


Figure 2.19: GVF circular attractive field without normalization (a) and with normalization (b)

The circulation term is calculated by taking the wedge product of each surface's gradient, shown in Equation 2.12. For surfaces in  $\mathbb{R}^3$ , the wedge product can be simplified to the cross product shown in Equation 2.13.

$$\vec{v}_{circ} = \wedge_{i=1}^{n-1} \nabla_q \alpha_i \quad (2.12)$$

$$\vec{v}_{circ} = \nabla_q \alpha_1 \times \nabla_q \alpha_2 \quad (2.13)$$

Evaluating the circulation term results in a vector field that is parallel to a circular path, shown in Figure 2.20a. The field is normalized to produce a field with equal length vectors for the configuration space which is shown in Figure 2.20b.

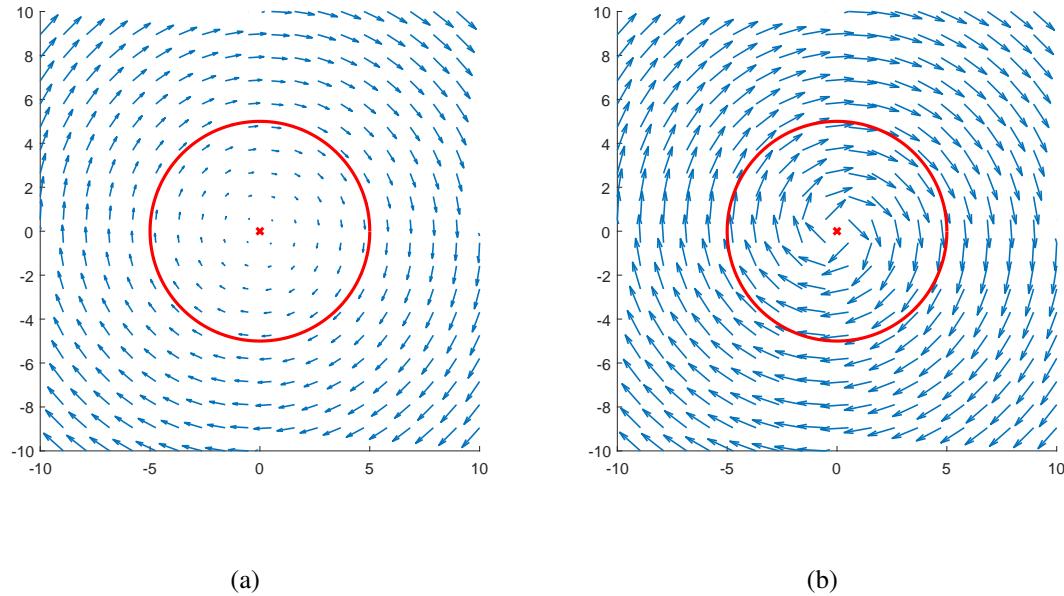


Figure 2.20: Circular GVF without normalization (a) and with normalization (b)

For dynamic paths that vary in time  $t$ , feedforward compensation is accomplished by calculating the time-varying term. The time-varying field  $\vec{v}_{tv}$  is calculated by multiplying the inverse of a gradient matrix  $M$  with the column vector  $a$  shown in Equations 2.15 and 2.16 respectively. The resulting vector field for a circular path moving in the positive x direction can be shown in Figure 2.21a and the normalized version in Figure 2.21b.

$$\vec{v}_{tv} = M^{-1}a \quad (2.14)$$

$$M = \begin{bmatrix} \nabla \alpha_1^T \\ \nabla \alpha_2^T \\ (\nabla \alpha_1 \times \nabla \alpha_2)^T \end{bmatrix} \quad (2.15)$$

$$a = \begin{bmatrix} \frac{\partial \alpha_1}{\partial t} & \frac{\partial \alpha_2}{\partial t} & 0 \end{bmatrix}^T \quad (2.16)$$

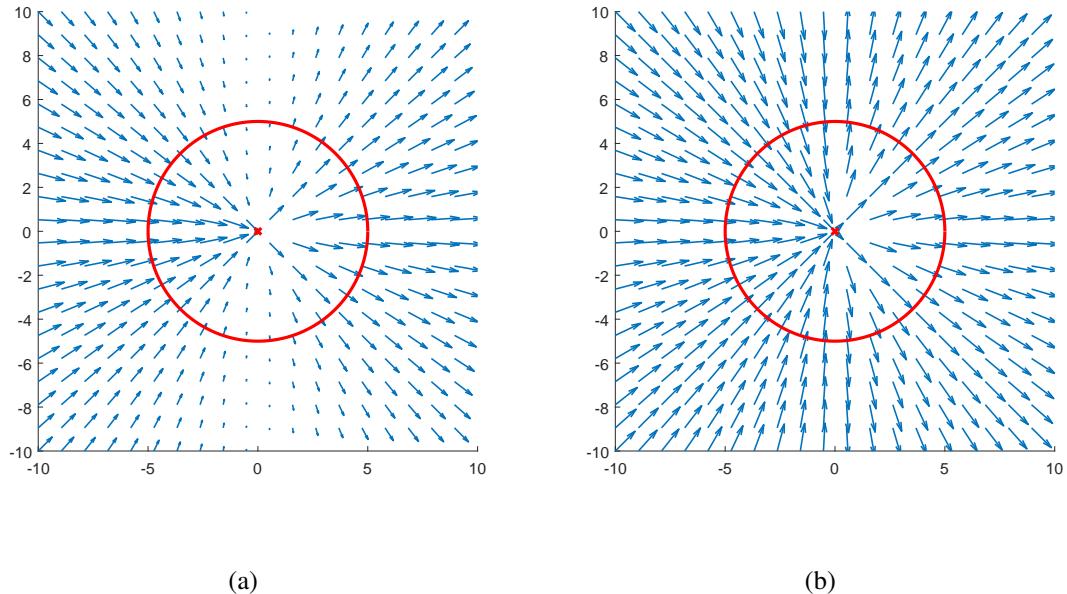


Figure 2.21: Circular time-varying GVF without normalization (a) and with normalization (b)

Summing together the normalized convergence, circulation, and time-varying terms results in the total normalized and weighted field field  $\vec{V}$  shown defined in Equation 2.17. Scalar weights,  $(G, H, L)$ , are added after normalization to increase or decrease the contribution of convergence, circulation, and time-varying respectively.

$$\vec{V} = G\|\vec{v}_{conv}\| + H\|\vec{v}_{circ}\| + L\|\vec{v}_{tv}\| \quad (2.17)$$

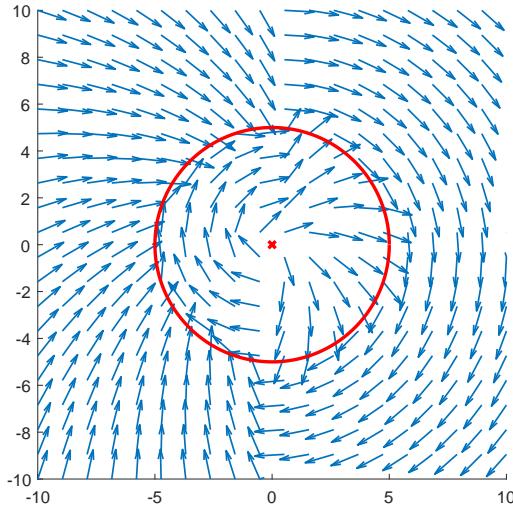


Figure 2.22: GVF with unity ( $\mathbf{G}=\mathbf{H}=\mathbf{L}=\mathbf{I}$ ) weights for a moving circular path

The GVF weights are decoupled from each term calculation, therefore modifications to the GVF weights do not require a new derivation of the resulting field. Magnitude of the weights effectively scales the contribution of each term. Negating the GVF weights has been used to provide a repulsive field for obstacle avoidance in [2] for a UAV tracking a moving loiter circle. Assigning weights  $\mathbf{G}=-\mathbf{1}$  and  $\mathbf{H}=\mathbf{L}=\mathbf{0}$  rotates the attractive field vectors in Figure 2.19b  $180^\circ$  about the tail of each vector, which is shown in Figure 2.23a. Note how the repulsive field vectors point away from the circular curve for the entire configuration space. Encircling an obstacle by a path produces vectors that may provide guidance into the obstacle, which is not desired. Reducing the radius to be smaller than the radius of the obstacle produces a GVF with nearly all vectors directing away.

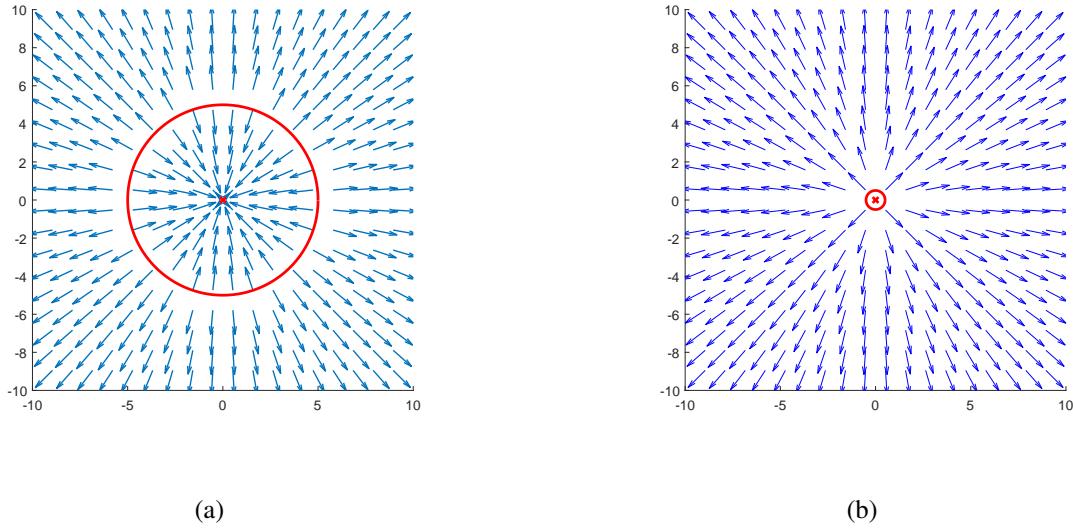


Figure 2.23: Repulsive field with large path radius (a) and small radius (b)

The strength of the repulsive field was varied as a function of proximity  $d$  by multiplying the repulsive field by a decay function  $P$  bounded on the interval  $[-1,0]$ , shown in Equation 2.18 and Figure 2.24. The radius  $R$  is the distance from the center of the field where vectors have effectively zero strength.

$$P = R \frac{\tanh(2\pi d - \pi) + 1}{2} \quad (2.18)$$

$$\vec{v}_{repulsive} = -G \|v_{conv}\| P \quad (2.19)$$

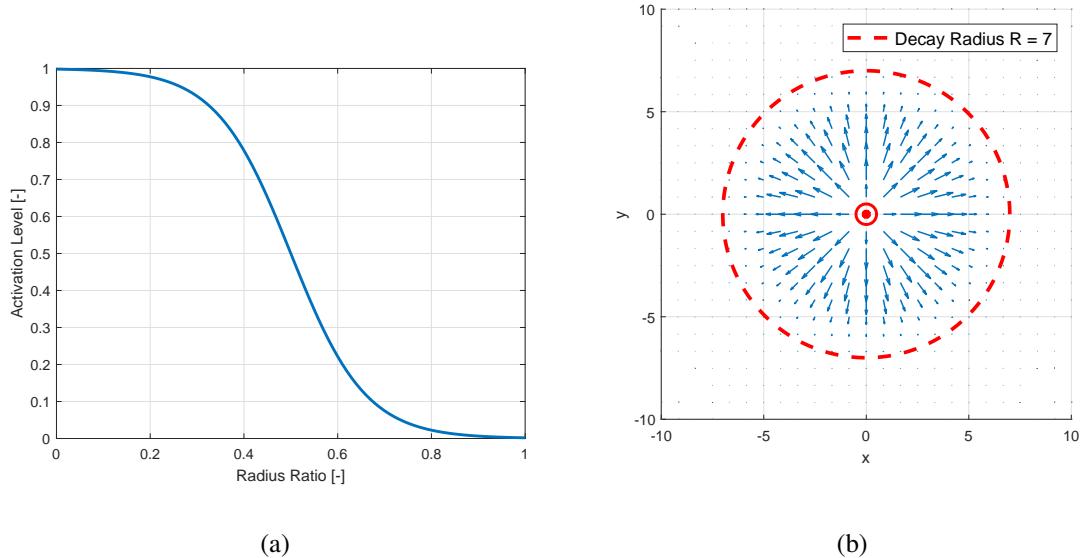


Figure 2.24: Repulsive field activation [2] (a) resulting repulsive field (b)

Attractive and repulsive fields were added together similar to that in [3] and the resulting vector shown in Equation 2.20. The resultant heading vector was used to guide a UAV to track a moving loiter circle while avoiding static obstacles, shown in Figure 2.25.

$$\vec{V} = \vec{v}_{attractive} + \vec{v}_{repulsive} \quad (2.20)$$

GVF weights have been used for high level specification of the desired behavior for a UAV, whether it be for convergence, circulation, or avoidance. Repulsive GVF have considered obstacle fields that strictly repel. The strictly repulsive guidance is effective at directing a UAV away from an obstacle, but provides no information on how to circumnavigate one. Additionally, further specification on the minimum radius and strength of the repulsive field may be useful in preventing a UAV from violating the obstacle circle. A preliminary simulation shows that a UAV encountering an obstacle while tracking a path may result in violation of the obstacle space, shown in Figure 2.26. UAVs traveling at lower

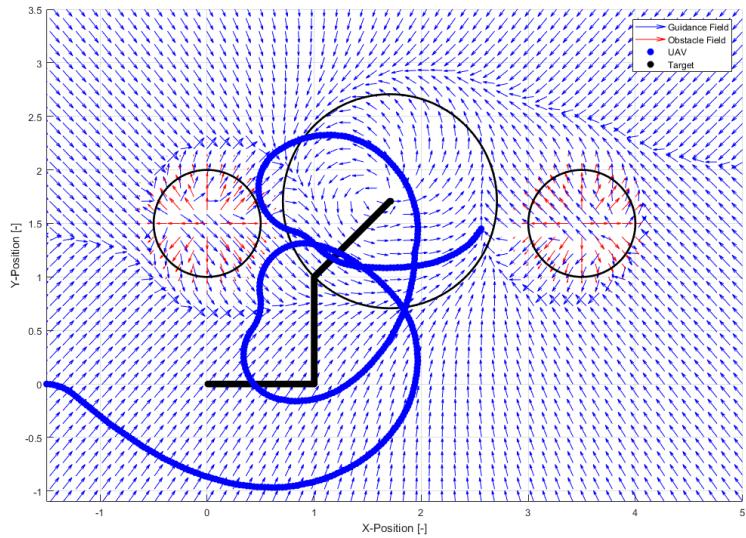


Figure 2.25: Place holder image of UAV following ground target [2]

velocities may also enter a trap situation where the guidance is confined to an infinite loop.

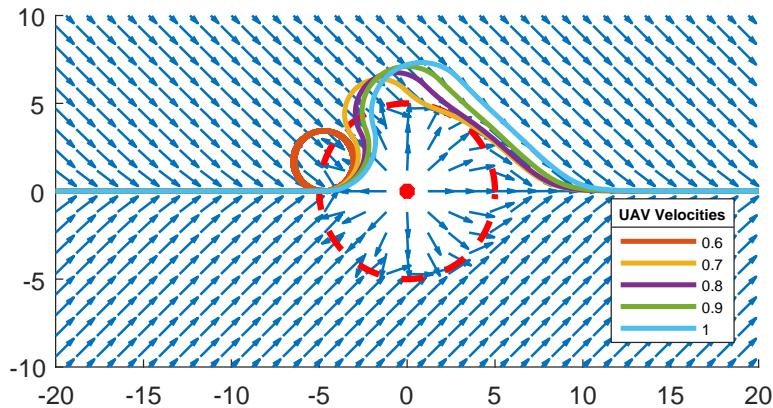


Figure 2.26: Strictly repulsive obstacle on straight path

Adding circulation  $H=I$  to the GVF in Figure 2.26 prevented the trap situation for a velocity  $u = 0.6$  and reduced the circumnavigation distance, which is shown in Figure 2.27. Specifying vector field weights as functions of a UAV's state may enable an optimal guidance for obstacle avoidance.

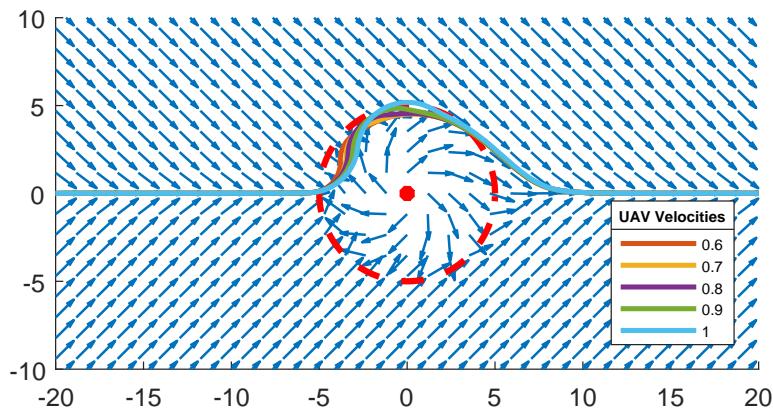


Figure 2.27: Circulating repulsive obstacle on straight path

## 2.6 Unmanned Aerial Vehicle Simulation

Testing new guidance, navigation, and control algorithms on flight hardware can be costly, require significant time, and requires an adequately large airspace. Before spending the time to reserve airspace and allocate man hours for flight tests it is important to test algorithms in a controlled environment. One way to accomplish testing without actual flight is through validation through mobile robots simulating fixed wing constraints [42], [43], [44]. Vector fields have been used in robot platforms other than UAVs, including ground [45] and marine [46].

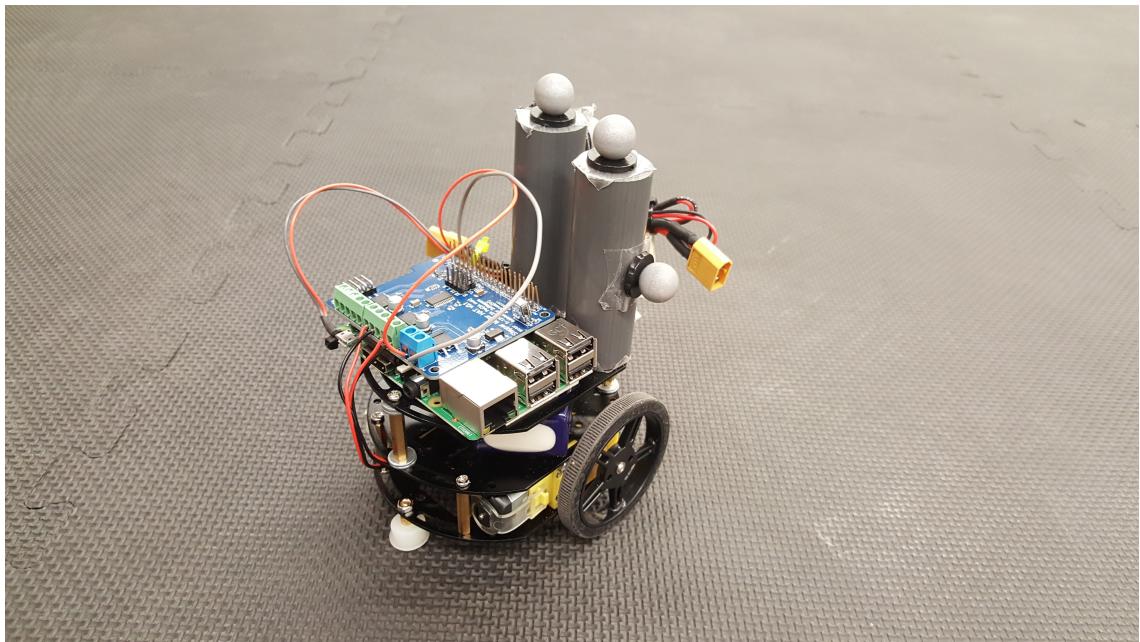


Figure 2.28: Differential drive mobile robot simulating fixed wing UAV Dubins constraints

## 2.7 Literature Review Summary

Vector fields can provide guidance and control for robots through the use of artificial attractive and repulsive forces. Converging to a singular point while avoiding obstacles can be achieved with Potential Field or Virtual Force Field methods. Fixed wing UAVs

cannot converge to a singular point therefore vector fields that asymptotically converge and follow a path is beneficial. Lyapunov and Gradient Vector Fields have been used for path following, standoff tracking, and obstacle avoidance. Gradient Vector Field provides convenient and decoupled access to scalar multiplicative weights for the convergence, circulation, and time-varying terms. Negative weights can be used for obstacle avoidance, however have so far been used only as high level specification of guidance behavior. Specifying vector field weights as functions of a UAV's state may enable an optimal guidance for obstacle avoidance. Validation of a modified GVF guidance can be performed on mobile ground robots simulating UAV dynamics.

## 3 METHODOLOGY

### 3.1 Introduction to Methodology

### 3.2 Phase I

Phase I. Demonstration of vector field trap situations and vector field singularities

Consists of path following vector field guidance summed with obstacle field

- Field Construction
- Straight Path
- Circular path
- obstacle definition
- Obstacle Field
- Field decay radius definition
- tanh function
- repulsive and repulsive+circulation side-by-side

### 3.3 Phase II

### 3.4 Phase III

## **4 RESULTS**

### **4.1 Introduction to Methodology**

### **4.2 Phase I**

### **4.3 Phase II**

### **4.4 Phase III**

## **5 CONCLUSIONS**

## **6 FUTURE WORK**

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