Simplicial surfaces in GAP

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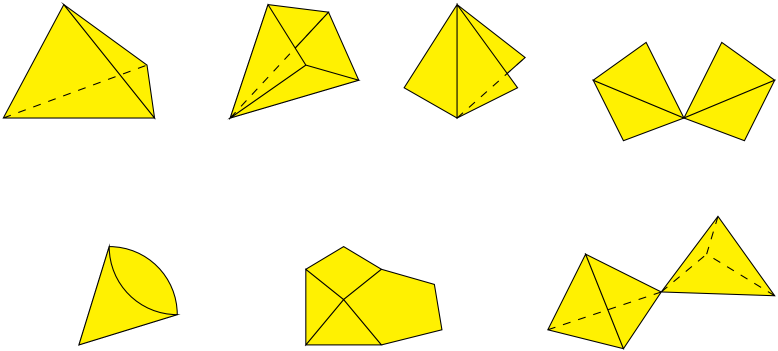
Motivation

Goal: simplicial surfaces (and generalisations) in GAP



$\rightsquigarrow$ examples of **polygonal complexes**

No embedding

We do not work with embeddings (mostly)

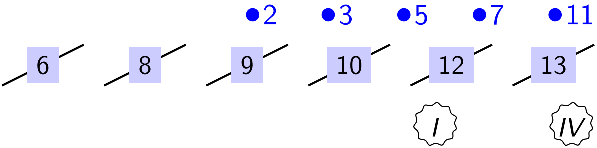
- is very hard to compute
- if often unknown for an abstractly constructed surface
- is different from *intrinsic structure*

$\Rightarrow$ lengths and angles are not important

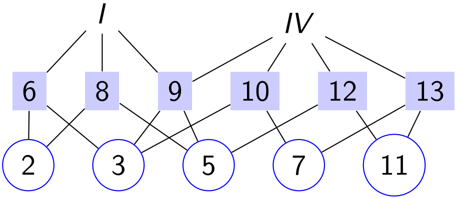
$\rightsquigarrow$ incidence structure is intrinsic

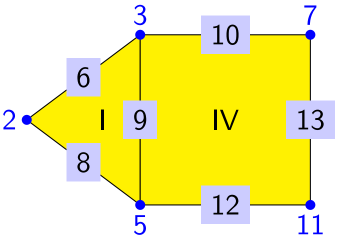
Incidence structure of polygonal complex

- set of vertices $\mathcal{V}$
- set of edges $\mathcal{E}$
- set of faces $\mathcal{F}$



- transitive relation $\subseteq \left(\mathcal{V} \times \mathcal{E}\right) \uplus \left(\mathcal{V} \times \mathcal{F}\right) \uplus \left(\mathcal{E} \times \mathcal{F}\right)$





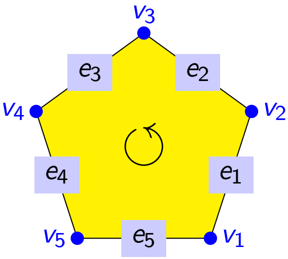
1. Every edge has exactly two vertices
2. Every face is a polygon
3. Every vertex lies in an edge and every edge lies in a face

Polygonal complexes

A **polygonal complex** is a two–dimensional incidence structure of vertices, edges and faces, such that:

1. Every edge has exactly two vertices.

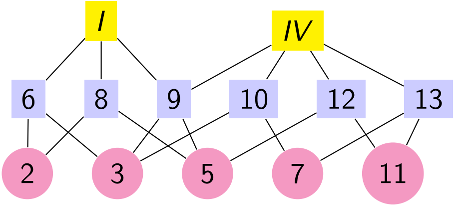
2. Every face is a polygon.



3. Every vertex lies in an edge
4. Every edge lies in a face

Isomorphism testing

Incidence geometry allows "easy" isomorphism testing. Incidence structure can be interpreted as a coloured graph:



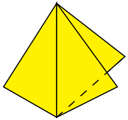
$\rightsquigarrow$ reduce to graph isomorphism problem
Solved by NautyTracesInterface (by Gutsche, Niemeyer, Schweitzer)

General properties

Some properties can be computed for all polygonal complexes:

- Connectivity
- Euler–Characteristic

Orientability is **not** one of them. Counterexample:

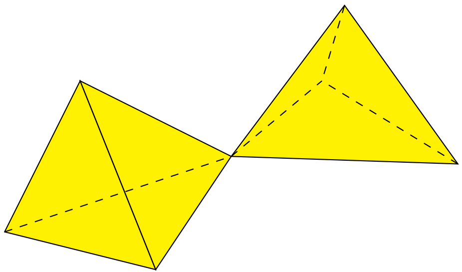


$\Rightarrow$ every edge lies in at most two faces (for well–definedness)

$\rightsquigarrow$ **ramified polygonal surfaces**

Why ramified?

Typical example of ramified polygonal surface:



$\Rightarrow$ It is not a surface – there is a *ramification* at the central vertex
A **polygonal surface** does not have these ramifications.

Embedding question

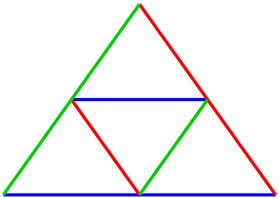
Given: A polygonal complex

- Can it be embedded?
- In how many ways?

Simplifications:

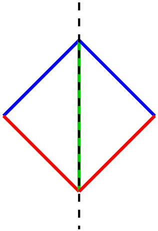
1. Only polygonal surfaces (surface that is build from polygons)
2. All polygons are triangles (**simplicial surfaces**)
3. All triangles are isometric

$\rightsquigarrow$ Edge–colouring encodes different lengths

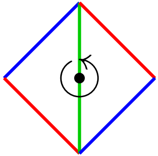


How do faces fit together?

Consider a face of the surface and a neighbouring face
The neighbour can be coloured in two ways:



mirror (m)



rotation (r)

This gives an **mr–assignment** for the edges.