Simplicial surfaces in GAP

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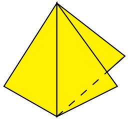

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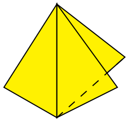

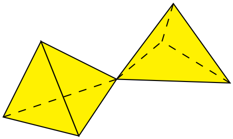

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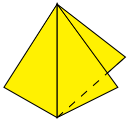

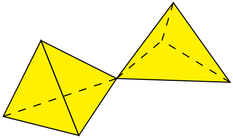

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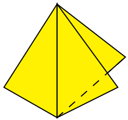

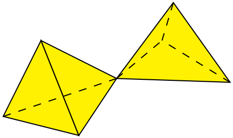

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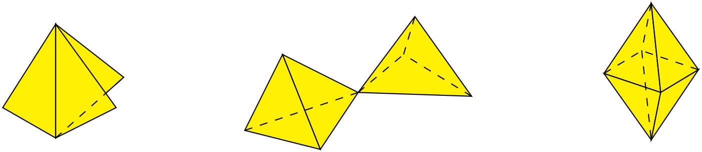

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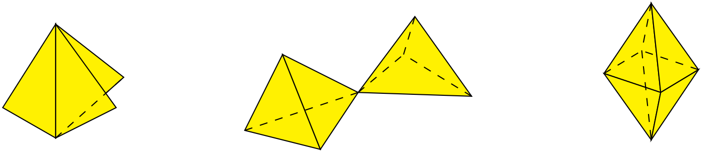

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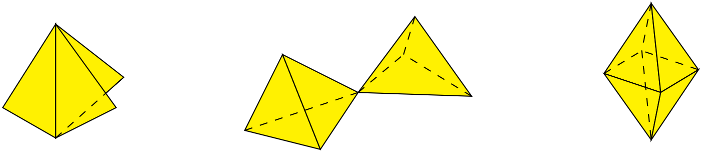

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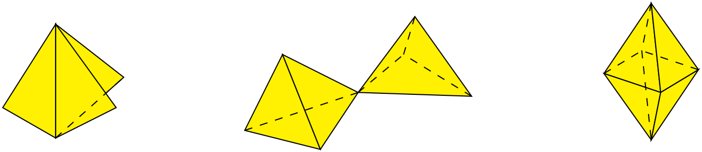

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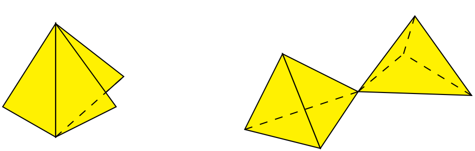

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- difference to `FinInG`–package by De Beule, Neunhöffer et al.
  - only two dimensions but it can work with colourings and foldings

1. General simplicial surfaces

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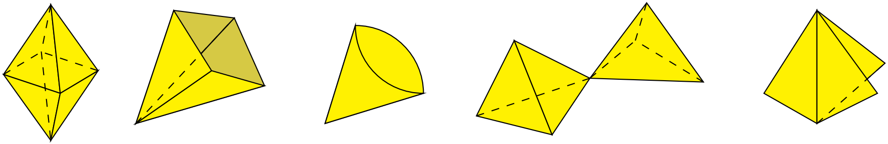

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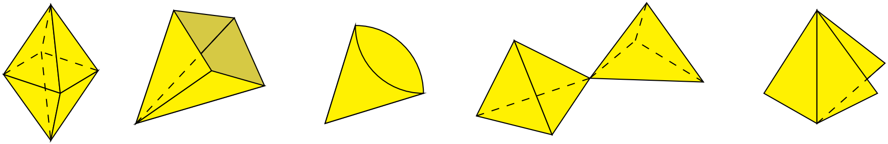

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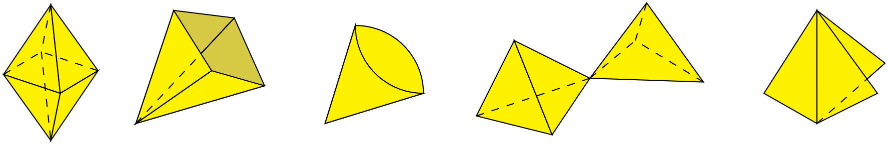

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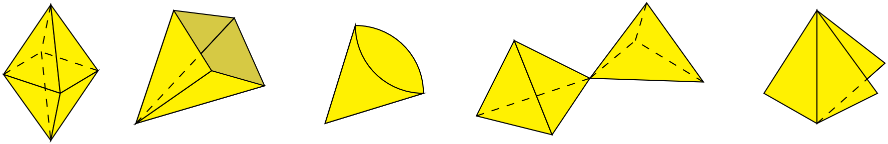

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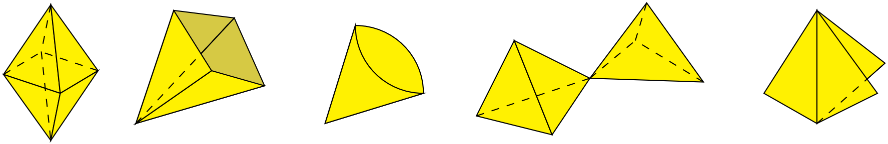

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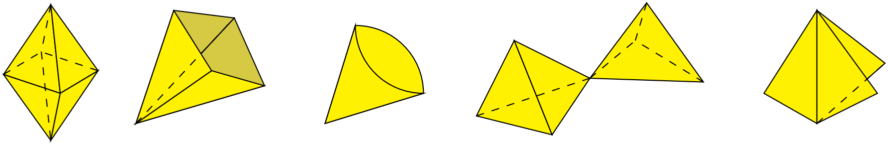

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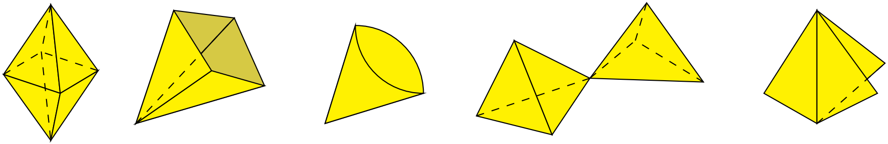

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- incidence relation between them
- every face is a triangle
- every vertex lies in an edge and every edge lies in a face

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Incidence structure can be interpreted as a coloured graph:

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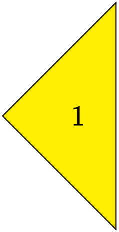

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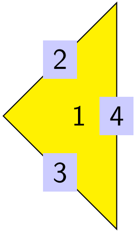

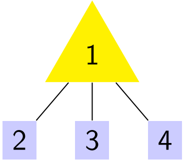

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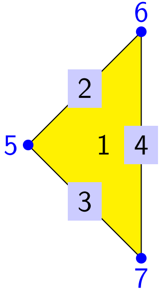

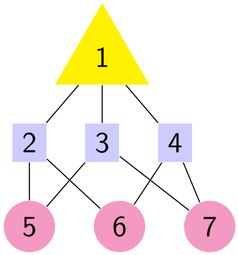

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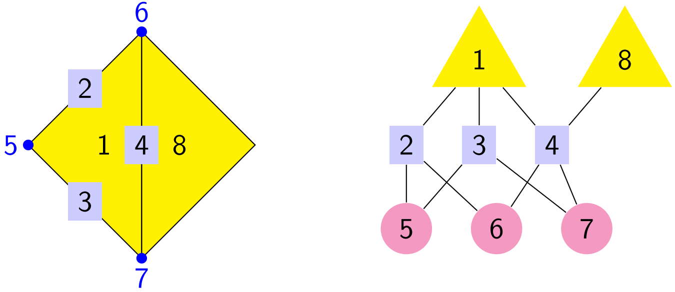

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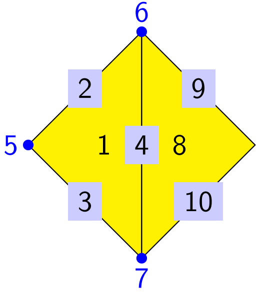

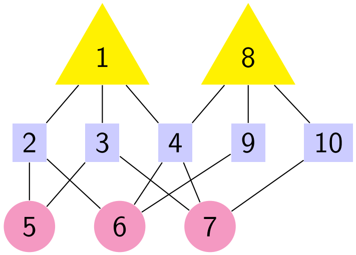

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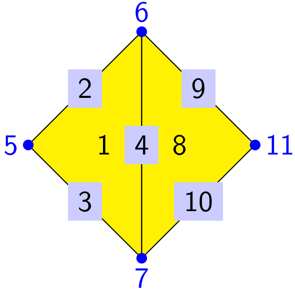

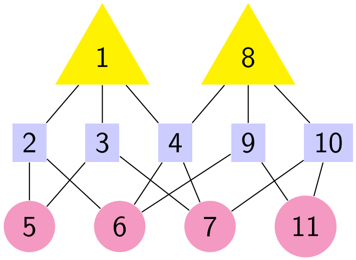

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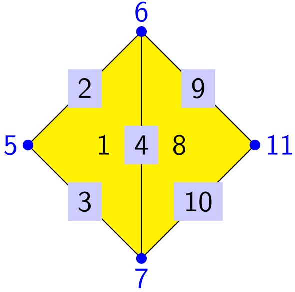

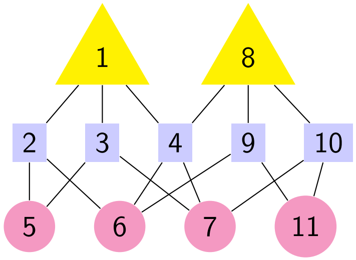

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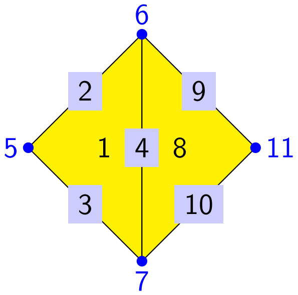

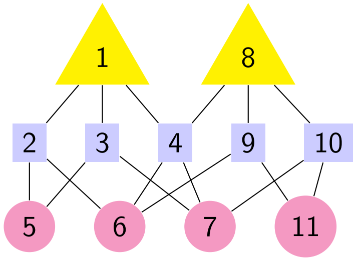

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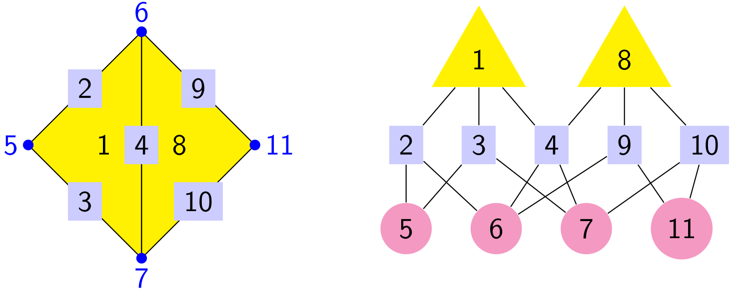

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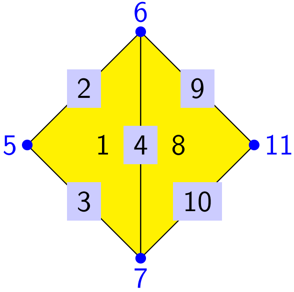

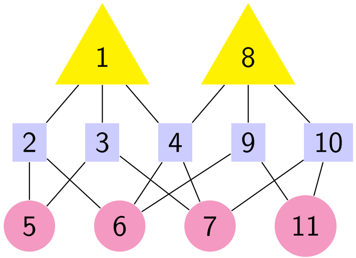

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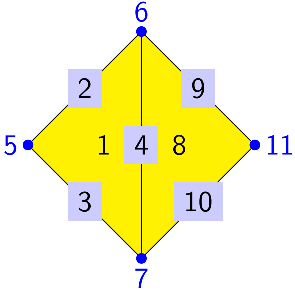

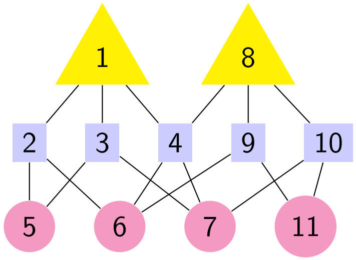

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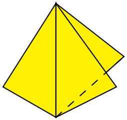

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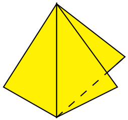

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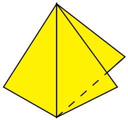

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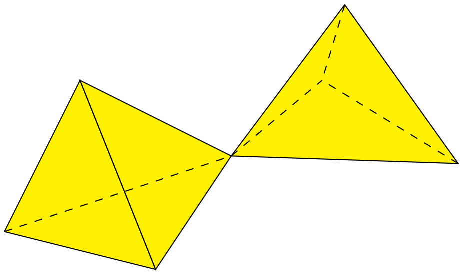

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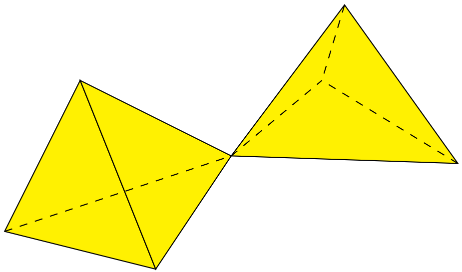

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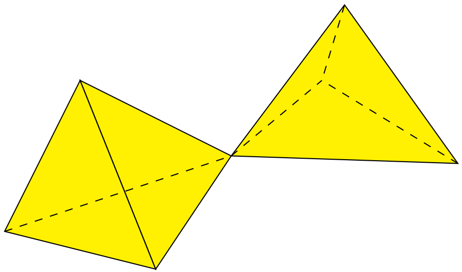

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A **simplicial surface** does not have these ramifications.

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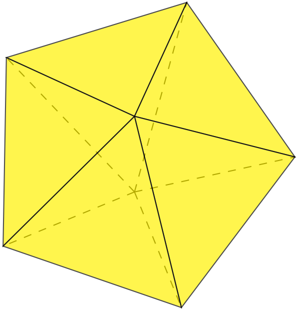

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Progress report of triangulated complexes

Already implemented:

- surface hierarchy
- elementary properties (e. g. connectivity, orientability)
- isomorphism testing
- classification of small surfaces (as data base)

Not yet implemented:

- automorphism group
- advanced properties (any wishes?)

Embedding questions

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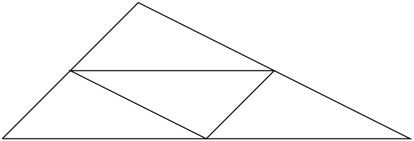

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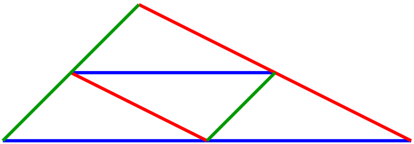
$\rightsquigarrow$ Edge–colouring encodes different lengths



Colouring as permutation

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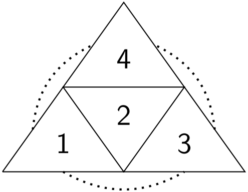
Consider tetrahedron



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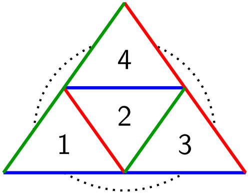

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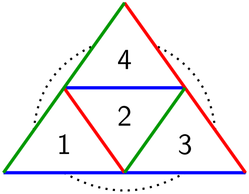

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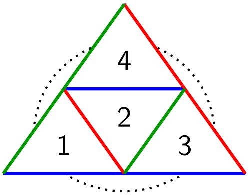

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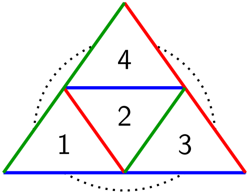

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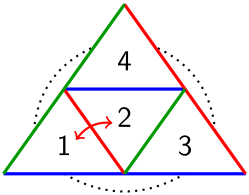

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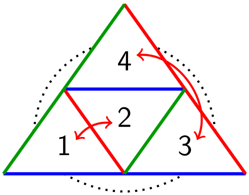

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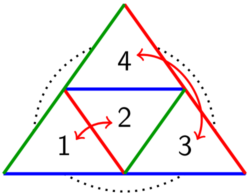

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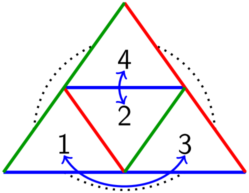

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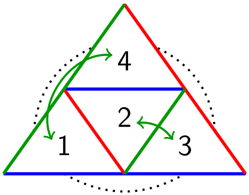

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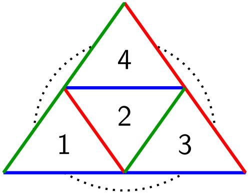

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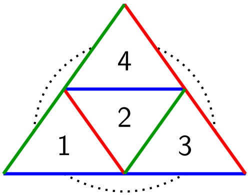

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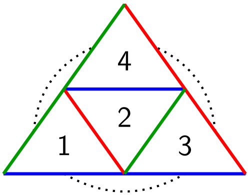

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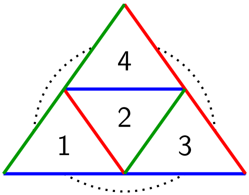

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  - The connected components of the surface correspond to the orbits of $\langle \sigma_a, \sigma_b, \sigma_c \rangle$ on the faces (fast computation for permutation groups)

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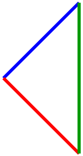
Consider a face of the surface



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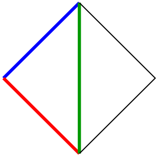

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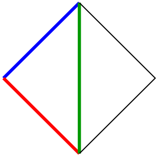

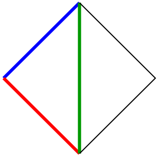

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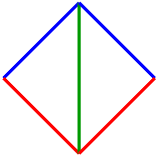

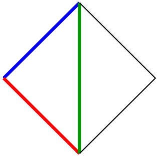

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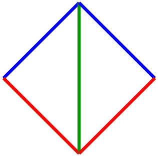

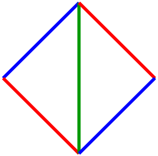

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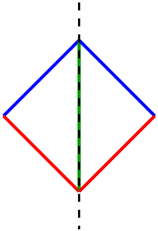

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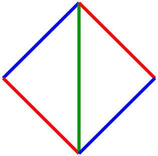

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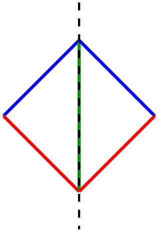

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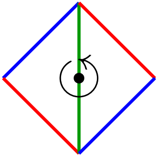

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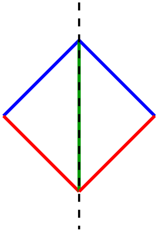

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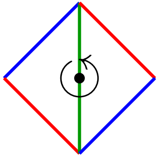

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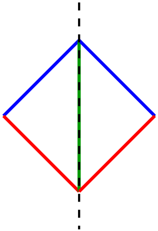

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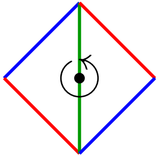

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Permutations and mr–assignment uniquely determine the surface.

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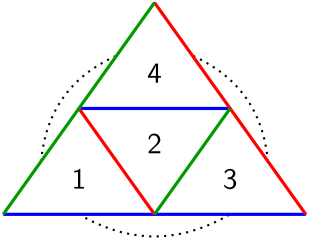

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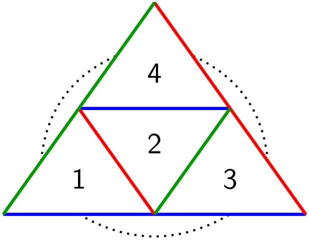

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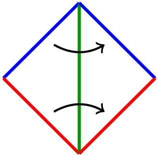

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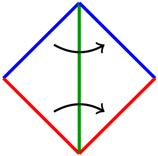

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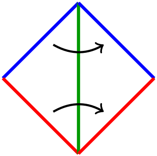

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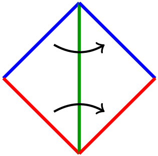

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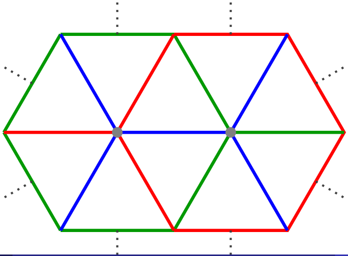

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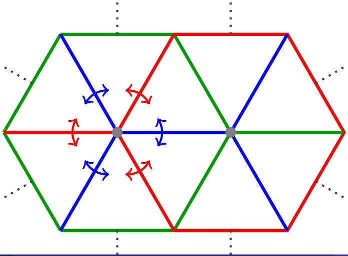

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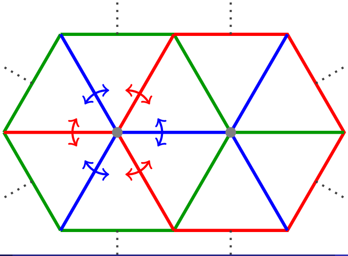

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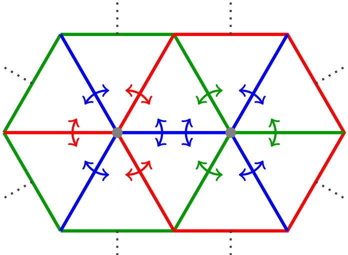

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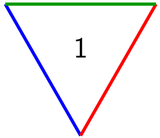

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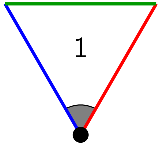

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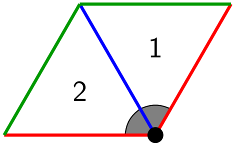

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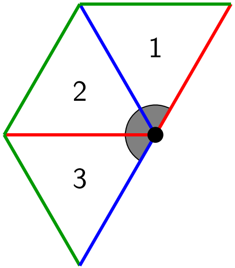

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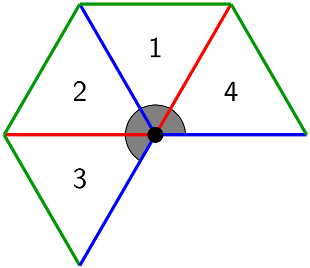

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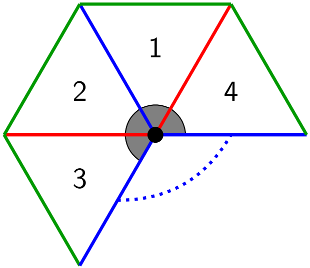

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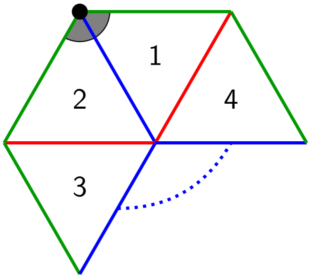

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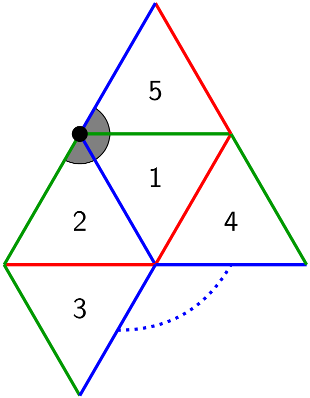

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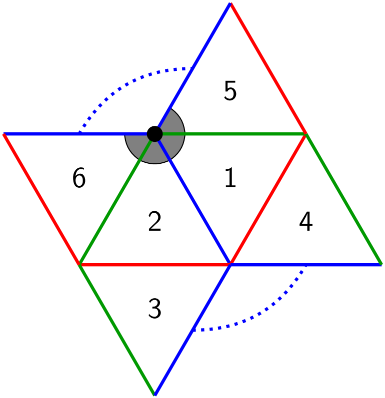

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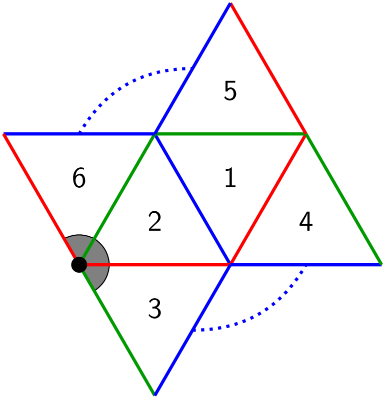

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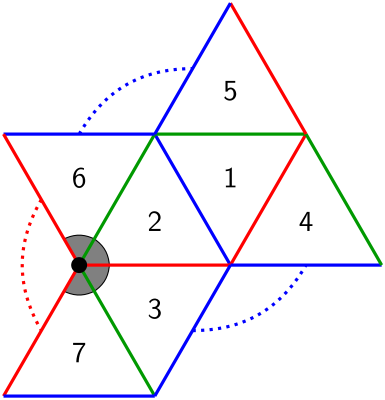

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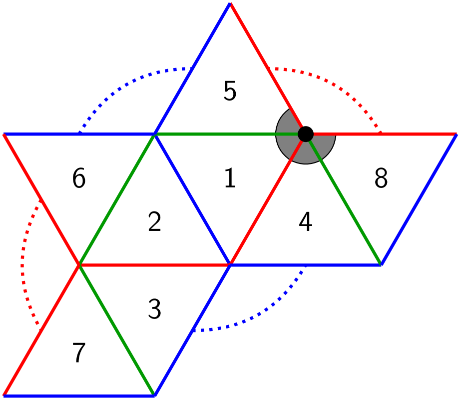

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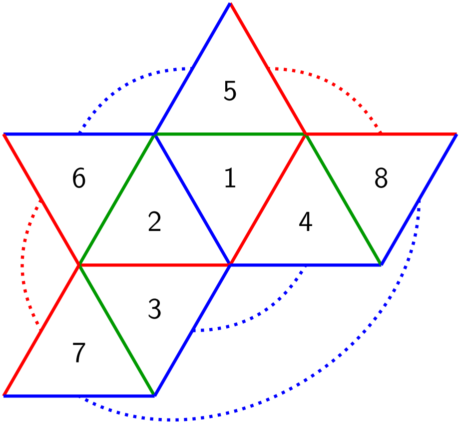

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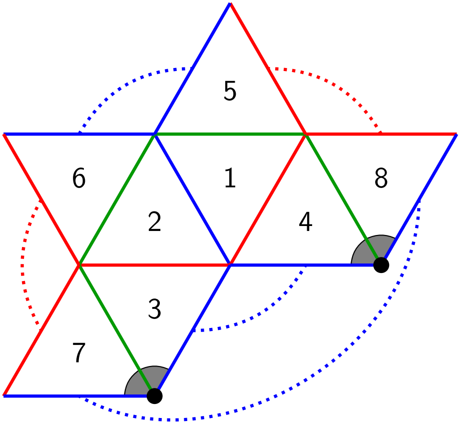

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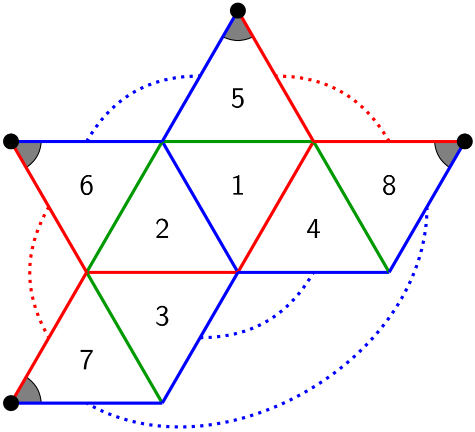

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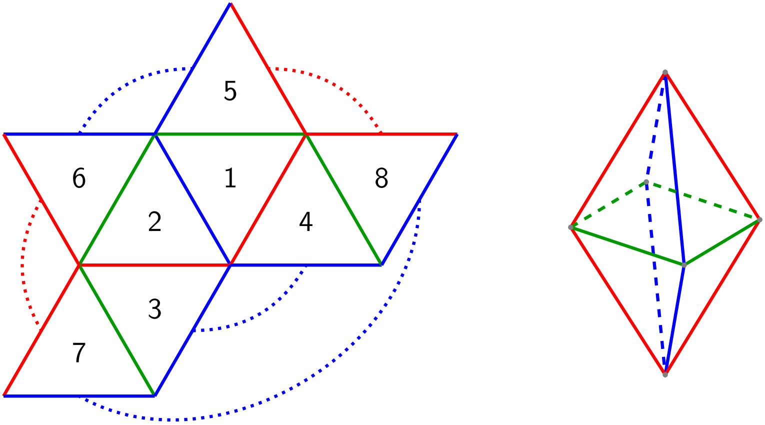

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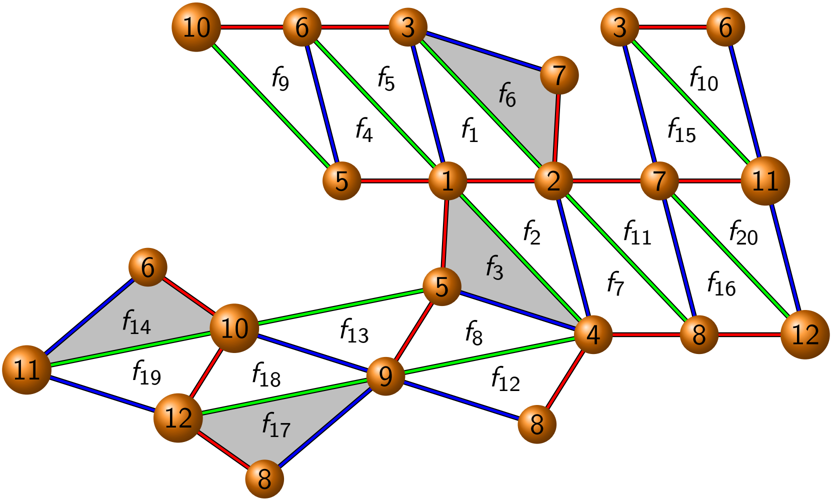

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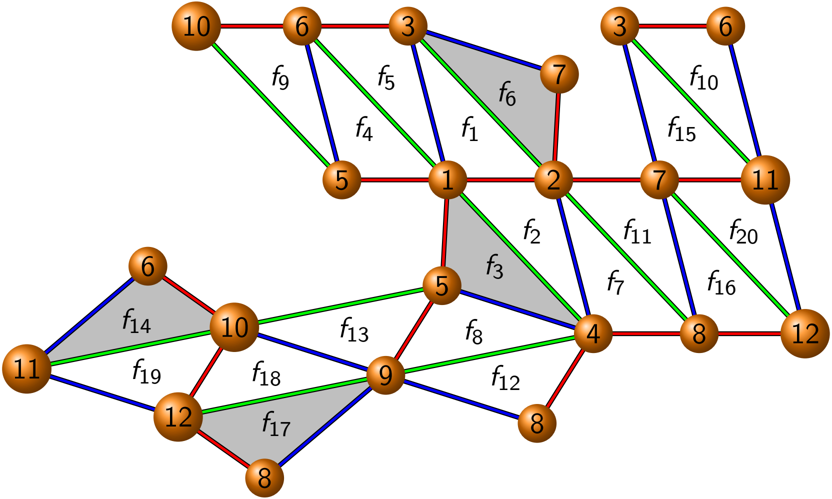

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Still missing:

- Research TODO?

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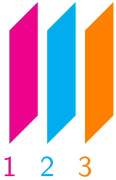

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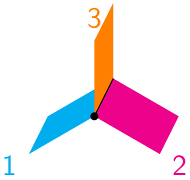

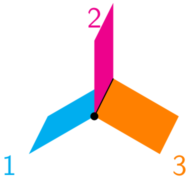

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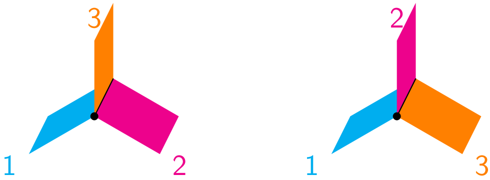

$\rightsquigarrow$ **folding complex**

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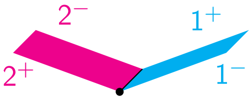

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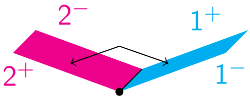

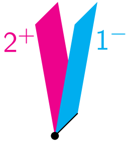

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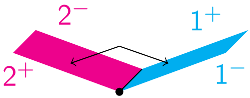

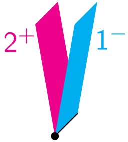

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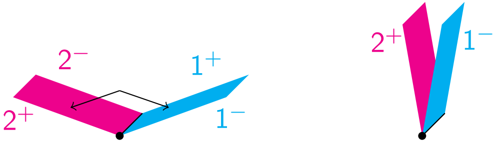

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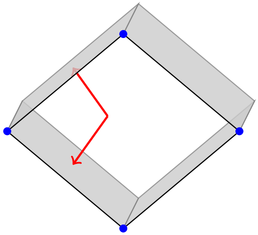

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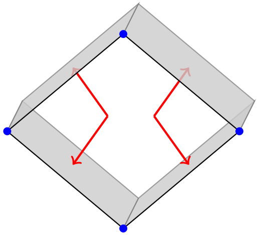

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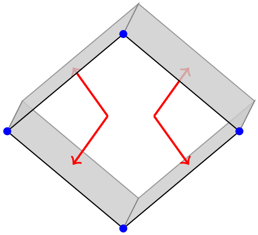

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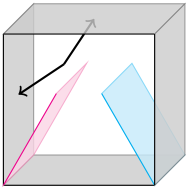

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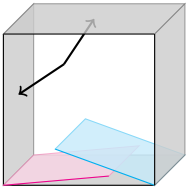

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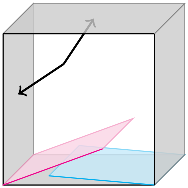

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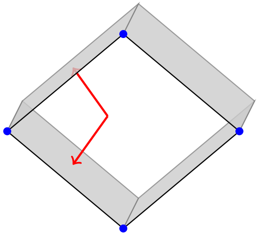

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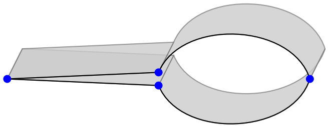

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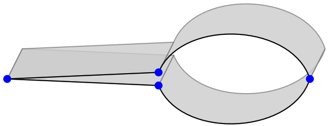

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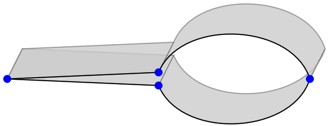

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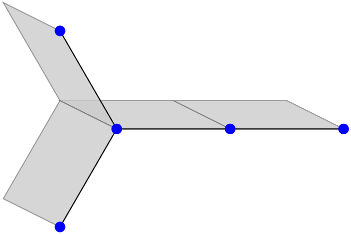

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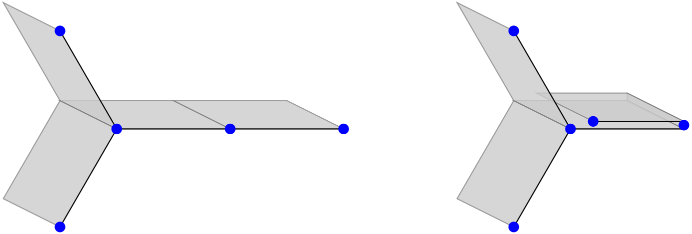

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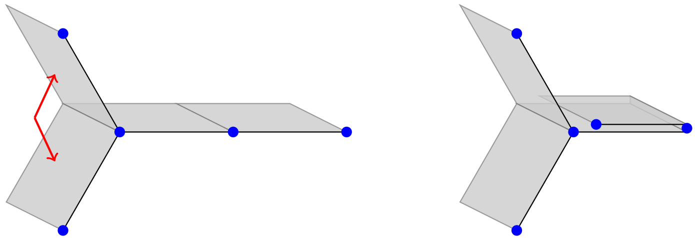

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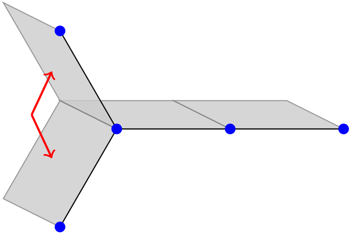

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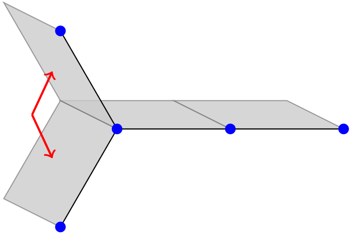

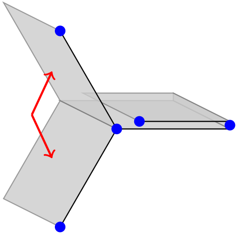

$\rightsquigarrow$ more structure on the set of possible foldings

Folding graph

Folding graph

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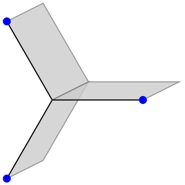

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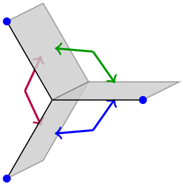

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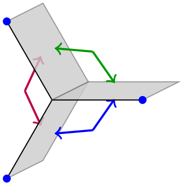

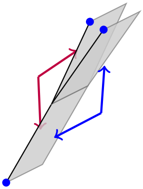

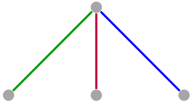

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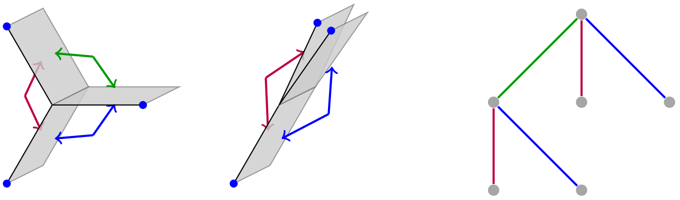

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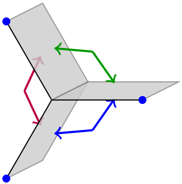

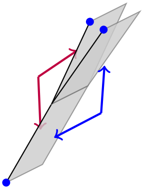

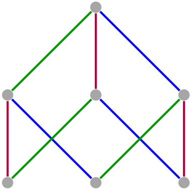

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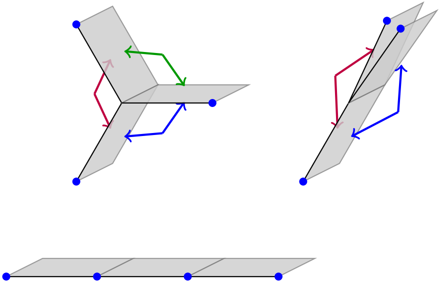

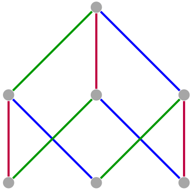

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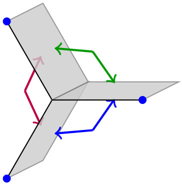

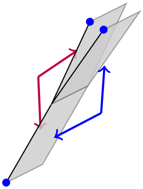

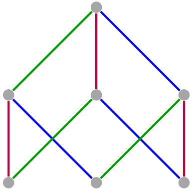

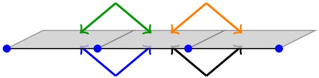

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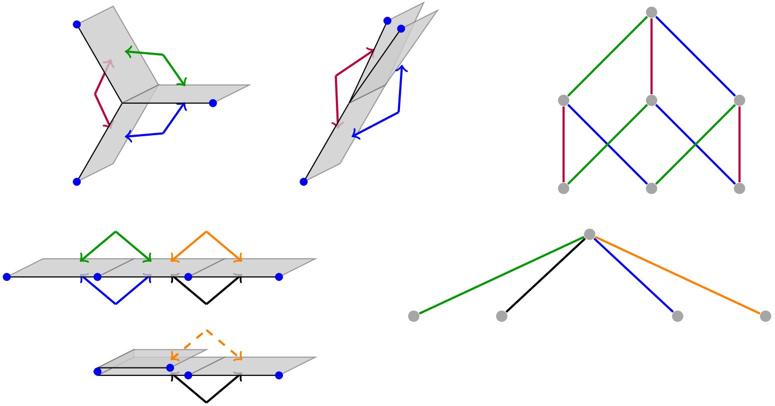

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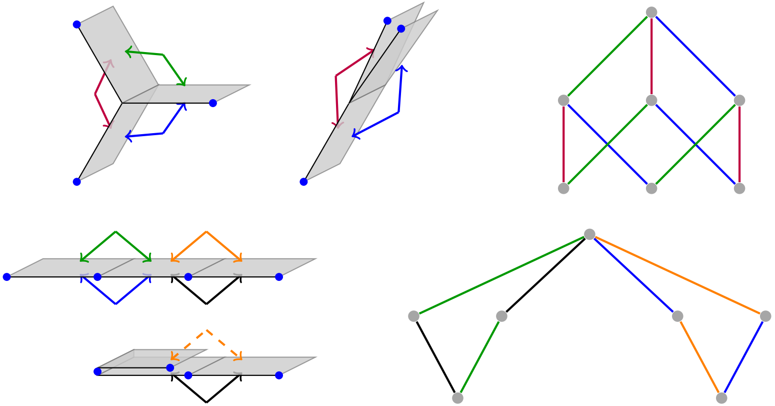

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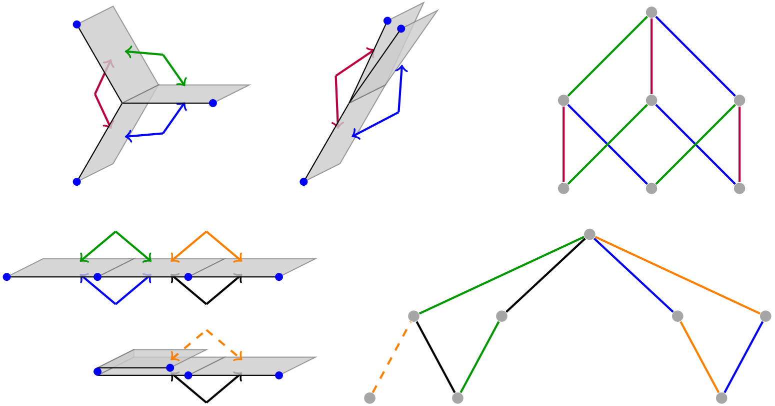

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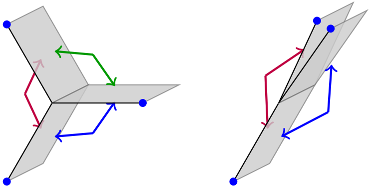

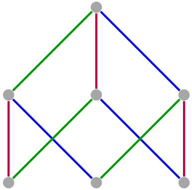

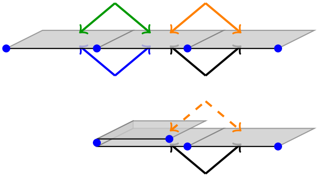

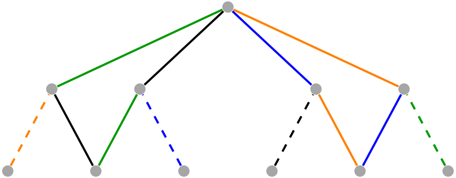

Larger graph

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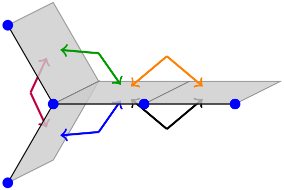

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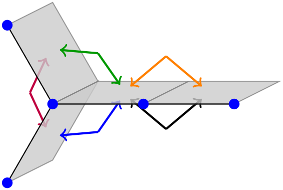

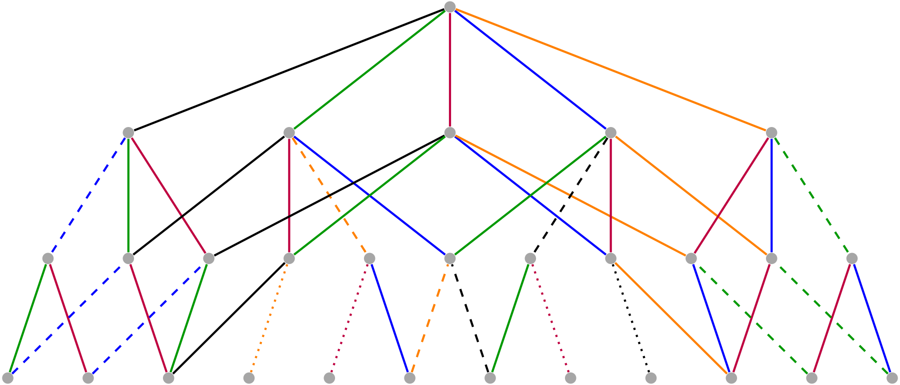

Drawback of folding plans

Drawback of folding plans

Some foldings that “should” be the same, aren’t:

Drawback of folding plans

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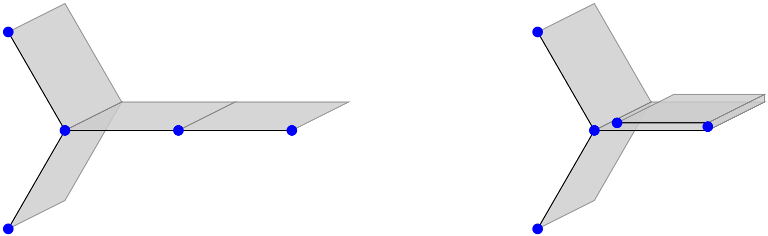

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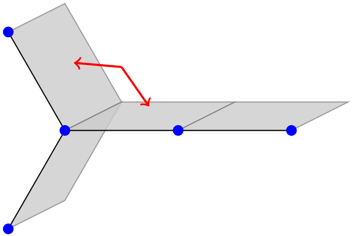

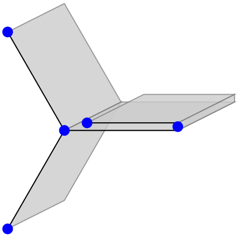

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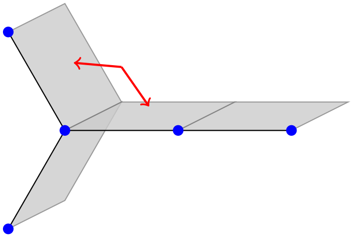

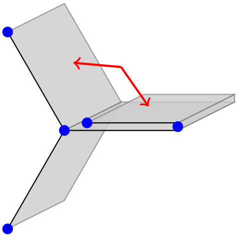

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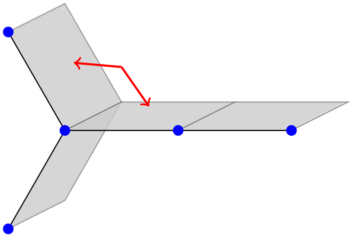

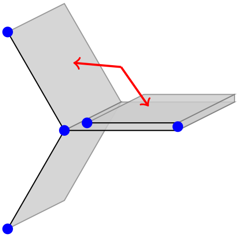

$\Rightarrow$ If you know the folding structure of a small complex,

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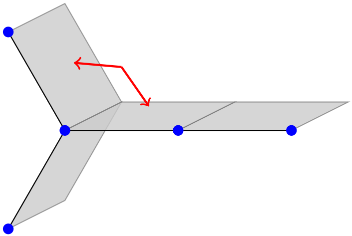

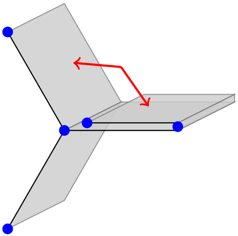

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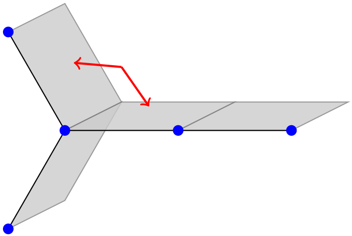

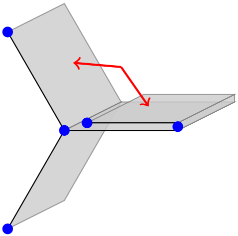

$\Rightarrow$ If you know the folding structure of a small complex, you can’t easily find the folding structure of an extended complex

$\rightsquigarrow$ Folding plans are not optimal to model folding

Progress report of abstract folding

Progress report of abstract folding

In development:

Progress report of abstract folding

In development:

- folding complex

Progress report of abstract folding

In development:

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- folding plans

Progress report of abstract folding

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Missing:

Progress report of abstract folding

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Missing:

- better folding description

Progress report of abstract folding

In development:

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Missing:

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- properties of folding graphs

Summary SimplicialSurfaces

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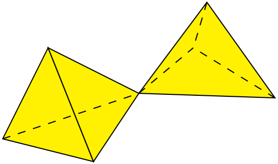
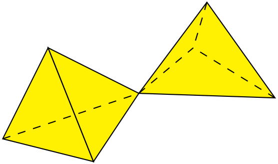
Triangulated complexes



Summary `SimplicialSurfaces`

Triangulated complexes

- mostly complete



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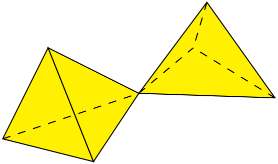
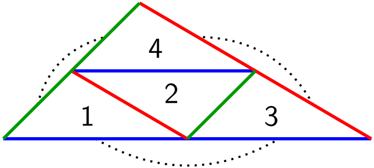
Edge colouring





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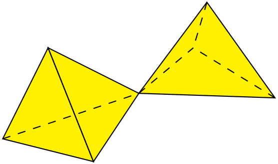

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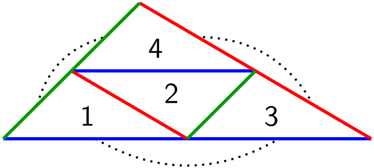

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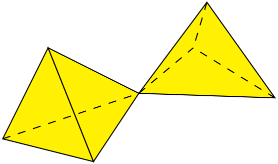

Edge colouring

- current theory implemented
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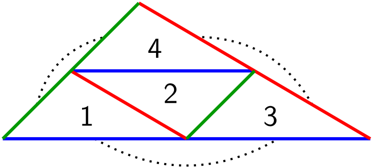

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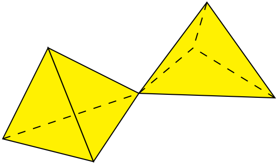

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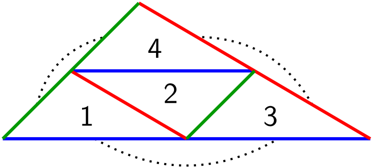
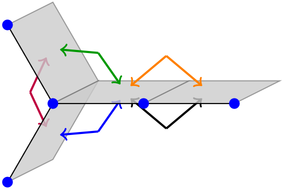


Abstract folding



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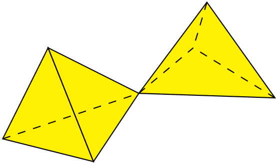

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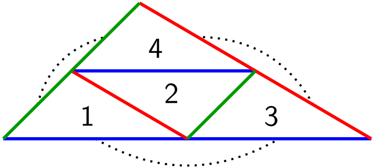
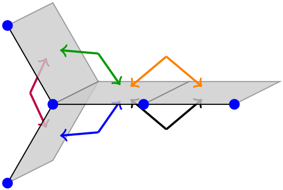


Abstract folding

- framework exists



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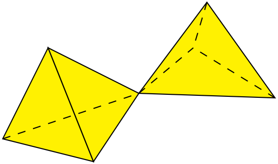

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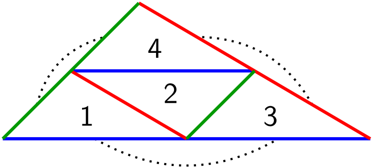

Abstract folding

- framework exists
- needs proper implementation