Simplicial surfaces in GAP

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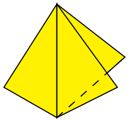

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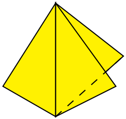

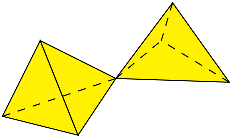

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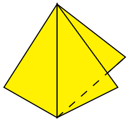

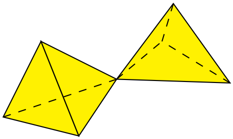

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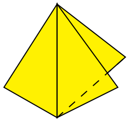

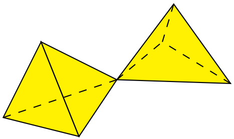

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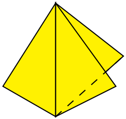

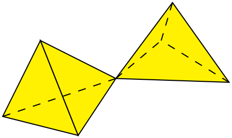

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$\rightsquigarrow$ incidence geometry

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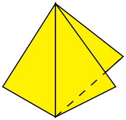

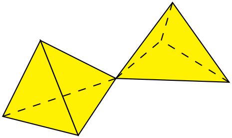

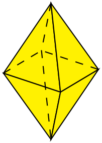

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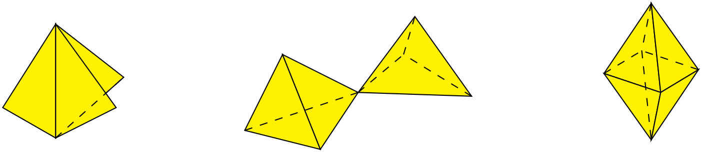

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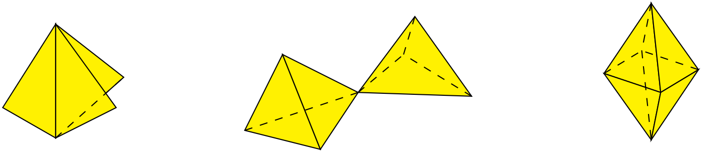

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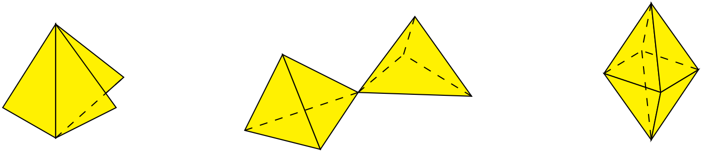

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  - only two dimensions but it can work with colourings and foldings

1. General simplicial surfaces

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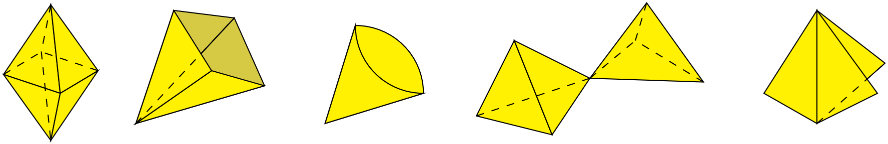

Triangular complexes

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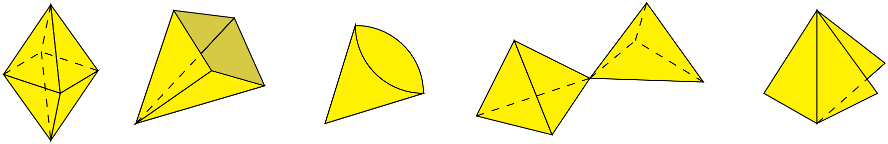

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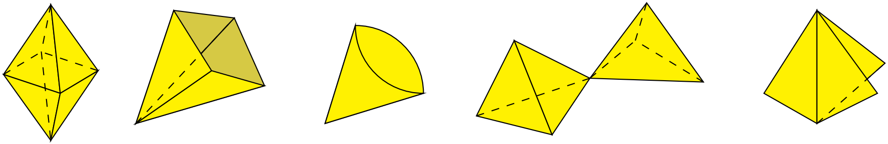

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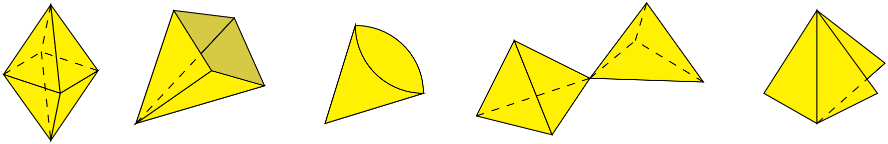

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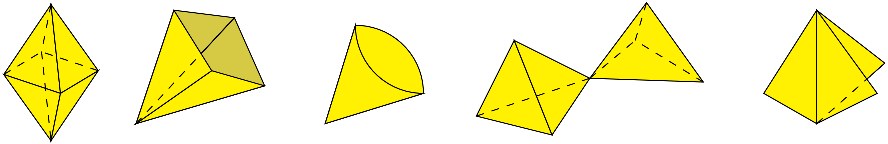

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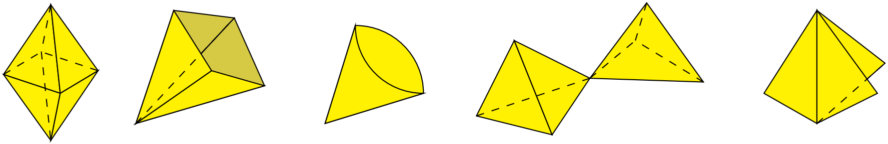

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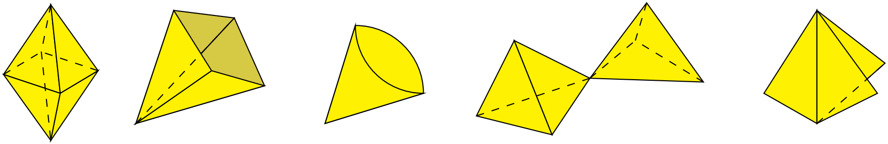

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- incidence relation between them
- every face is a triangle
- every vertex lies in an edge and every edge lies in a face

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Incidence structure can be interpreted as a coloured graph:

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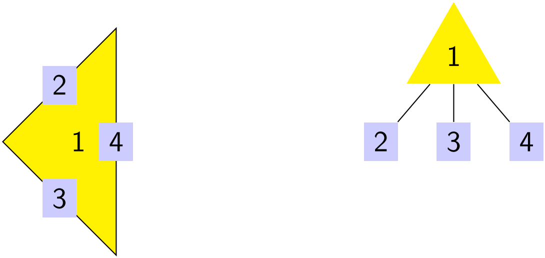

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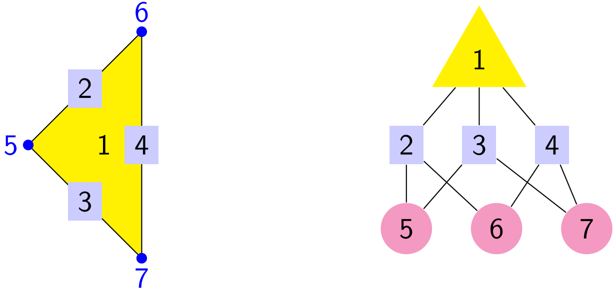

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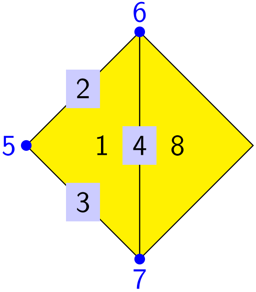

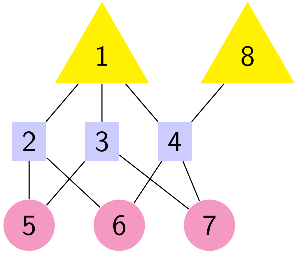

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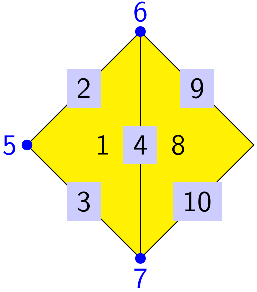

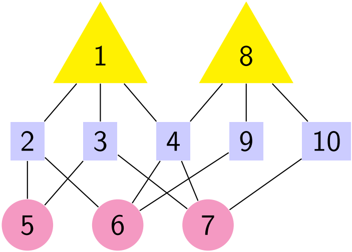

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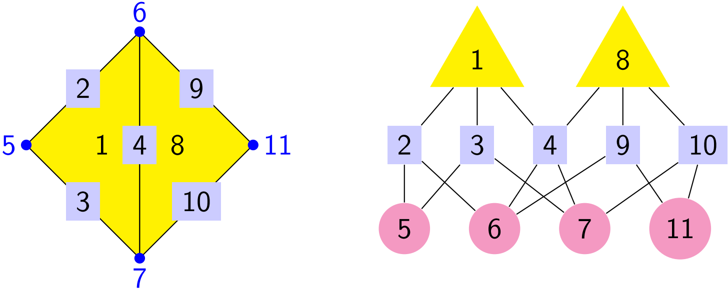

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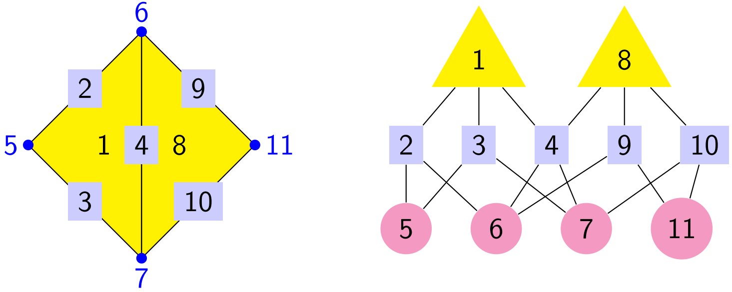

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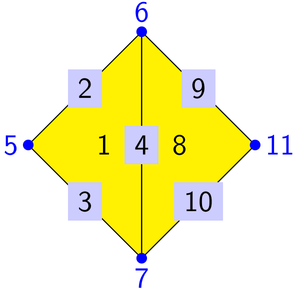

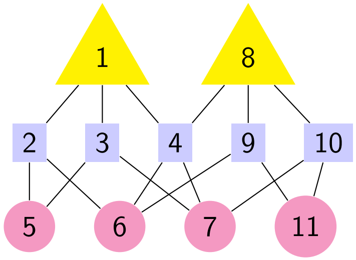

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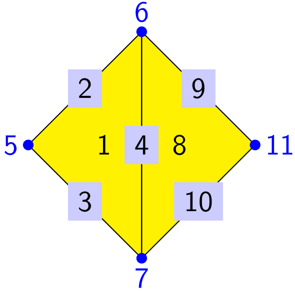

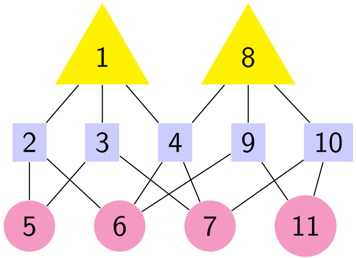

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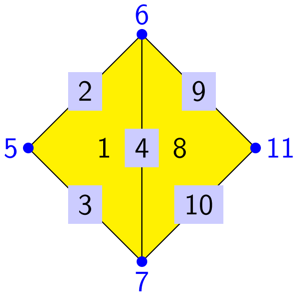

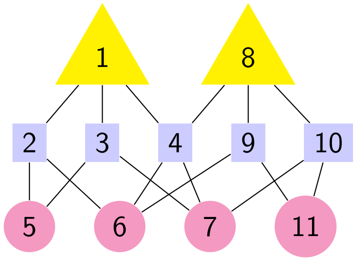

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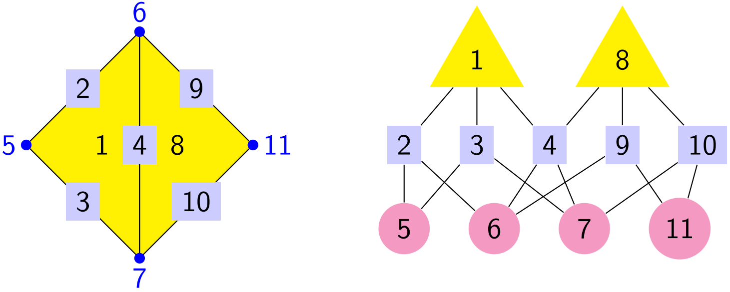

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- also returns automorphism group

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Some properties can be computed for all triangular complexes:

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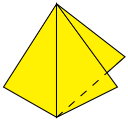

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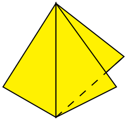

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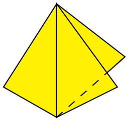

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$\rightsquigarrow$ **ramified simplicial surfaces**

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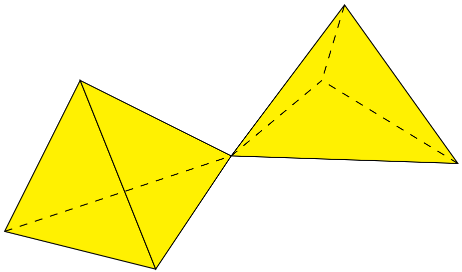

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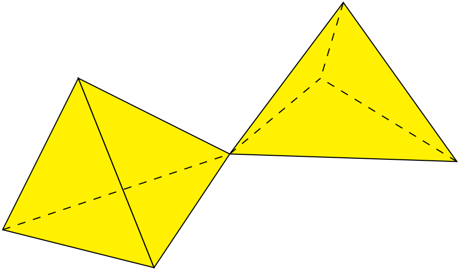

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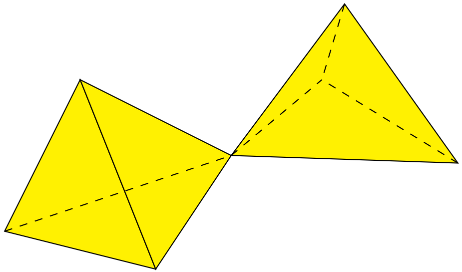

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A **simplicial surface** does not have these ramifications.

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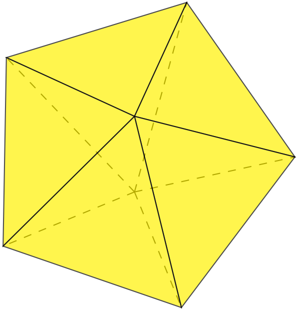

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Embedding questions

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- Can it be embedded?

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- Can it be embedded?
- In how many ways?

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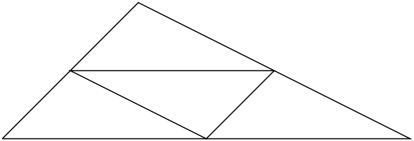

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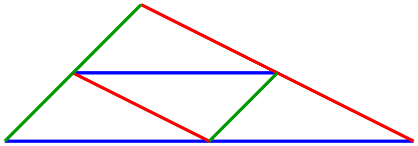
$\rightsquigarrow$ Edge–colouring encodes different lengths



Colouring as permutation

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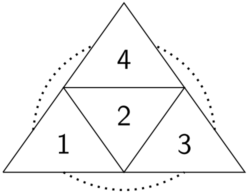
Consider tetrahedron



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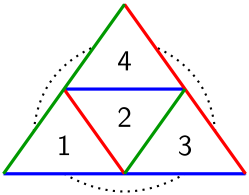

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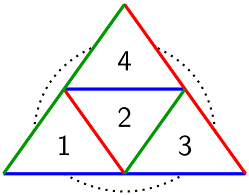

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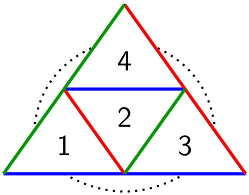

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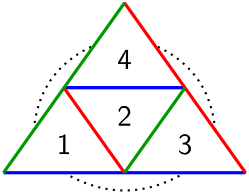

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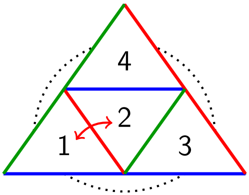

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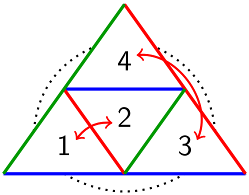

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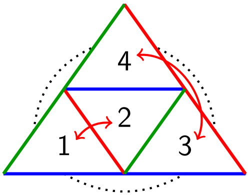

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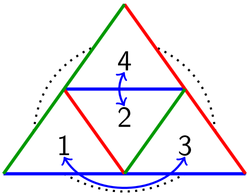

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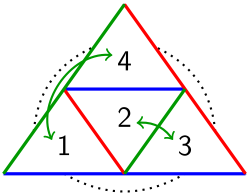

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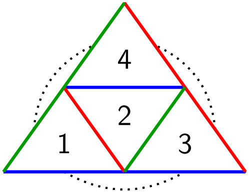

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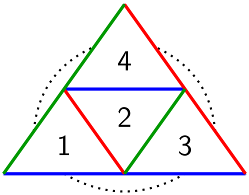

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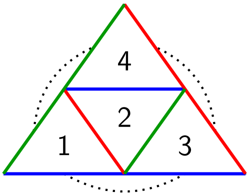

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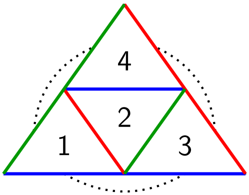

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- The connected components of the surface correspond to the orbits of $\langle \sigma_a, \sigma_b, \sigma_c \rangle$ on the faces (fast computation for permutation groups)

How do faces fit together?

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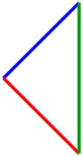
Consider a face of the surface



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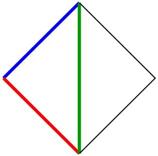

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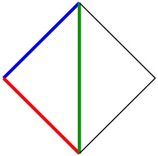

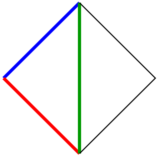

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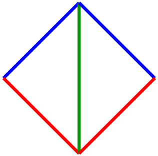

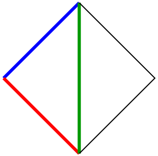

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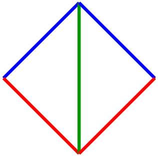

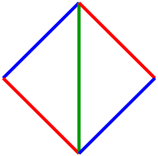

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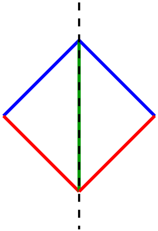
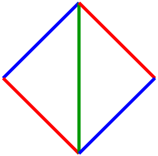


mirror (m)



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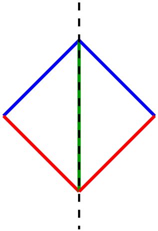

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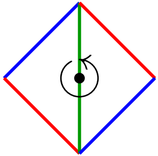

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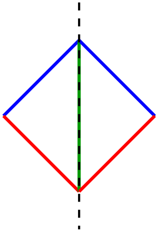

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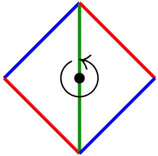

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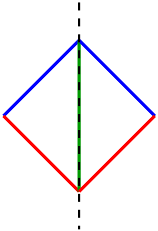

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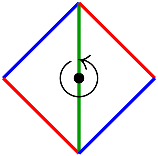

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Permutations and mr–assignment uniquely determine the surface.

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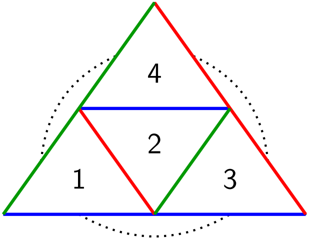

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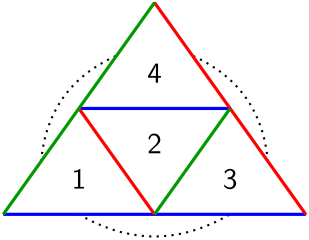

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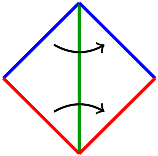

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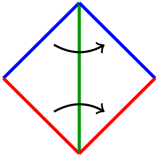

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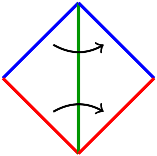

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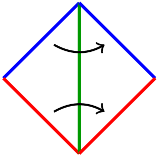

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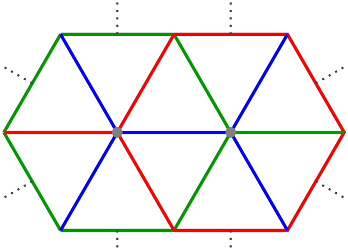

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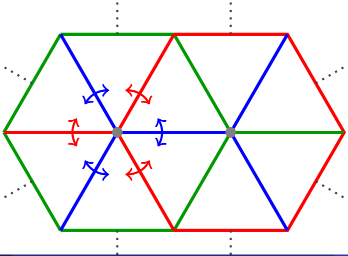

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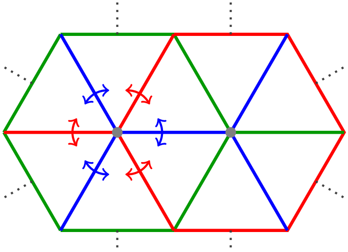

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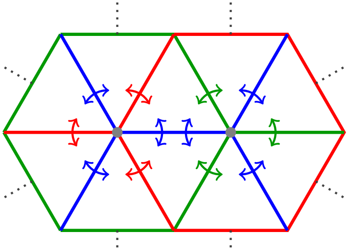

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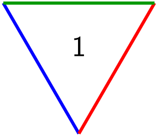

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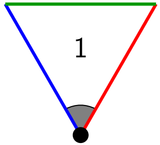

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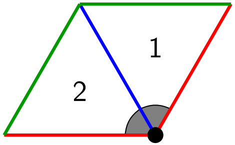

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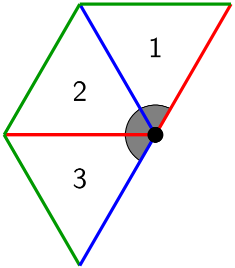

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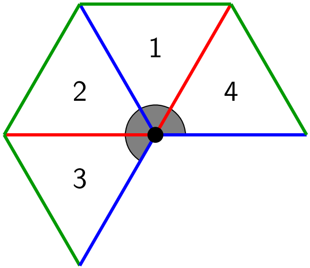

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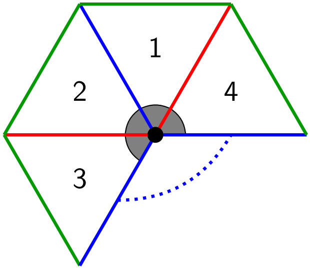

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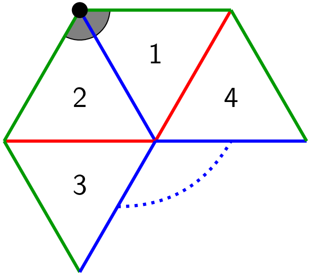

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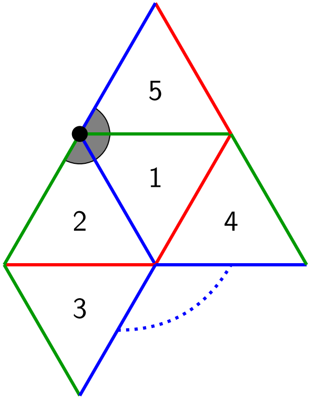

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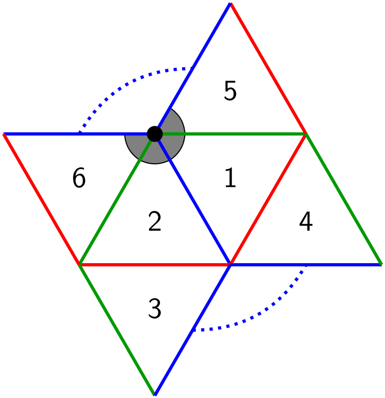

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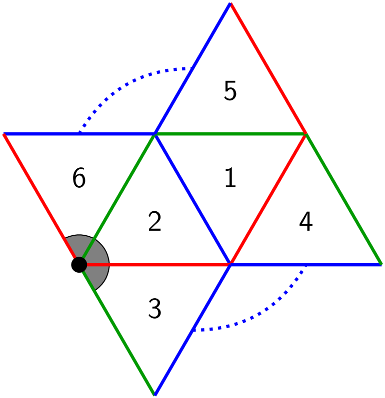

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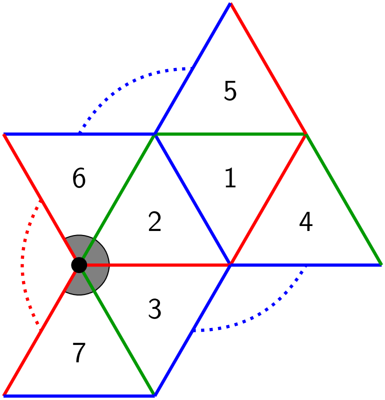

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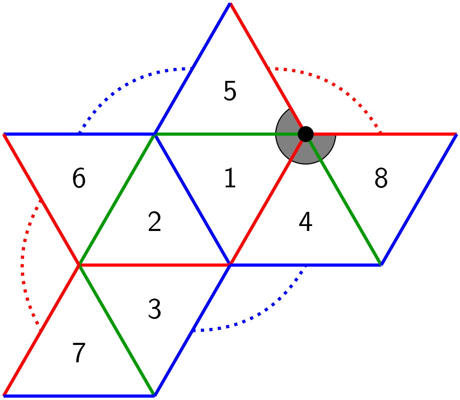

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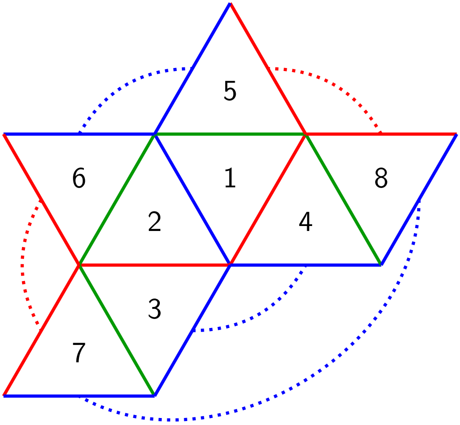

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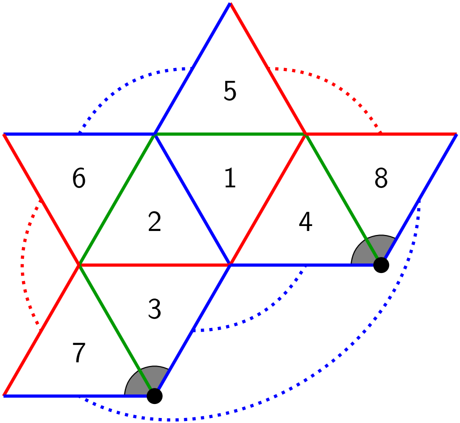

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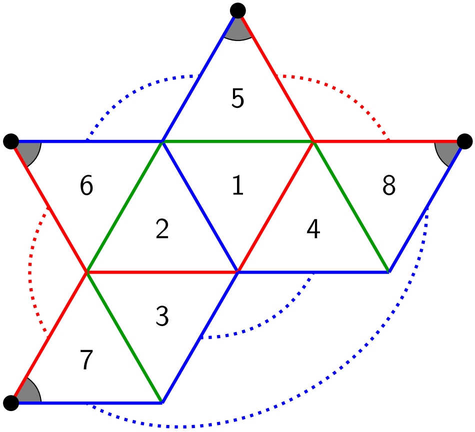

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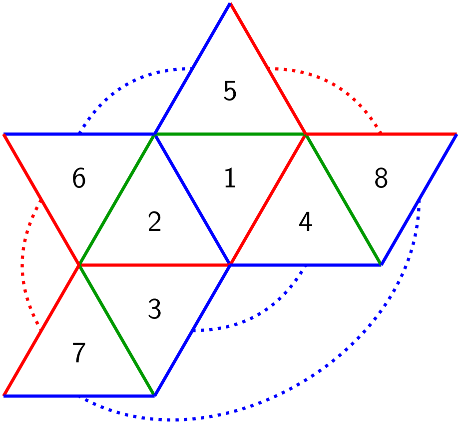

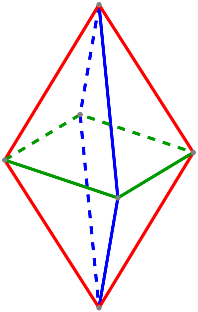

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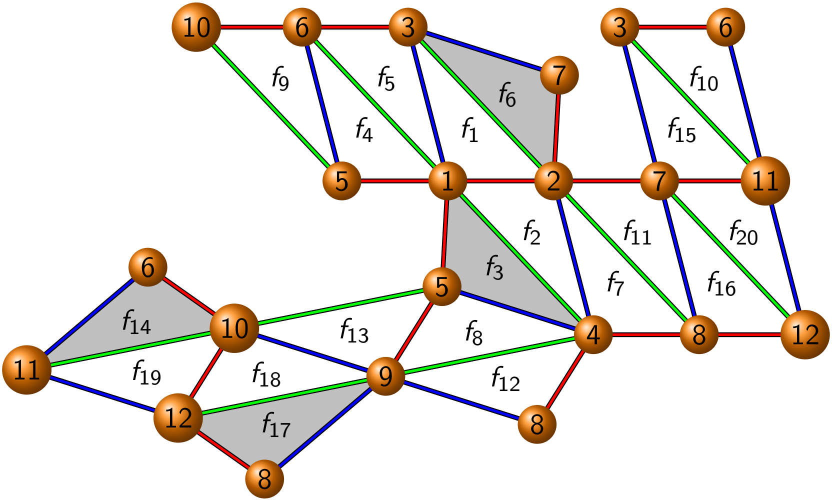

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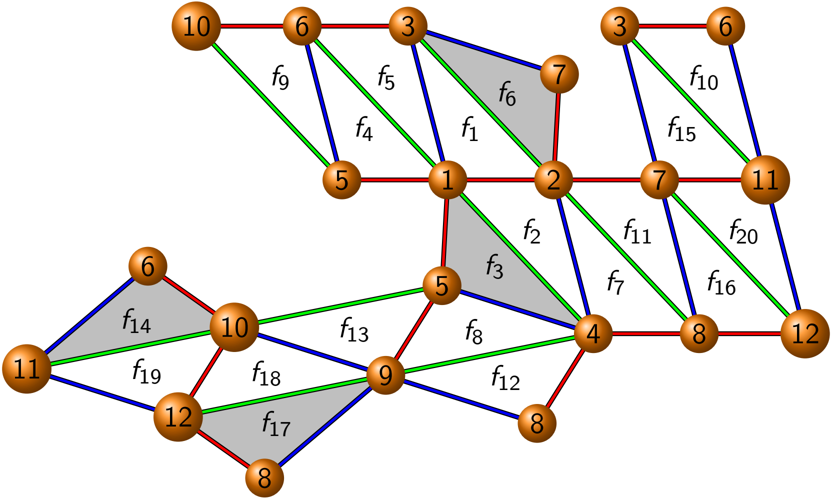

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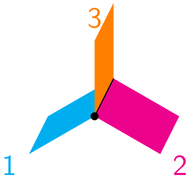

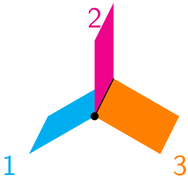

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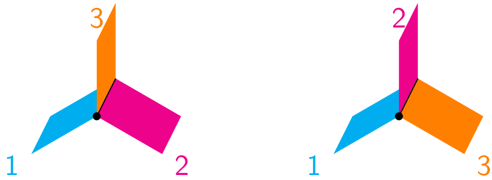

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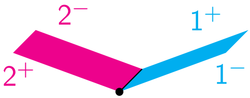

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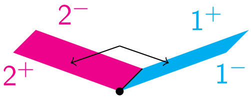

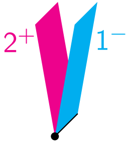

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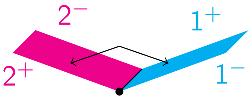

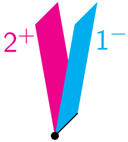

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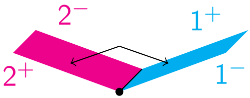

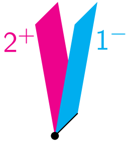

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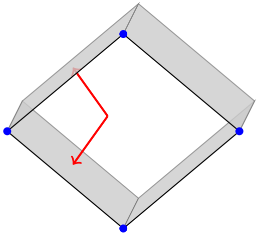

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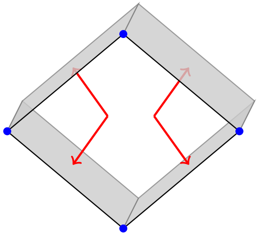

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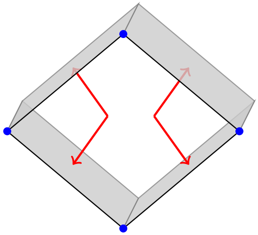

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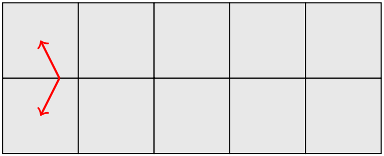

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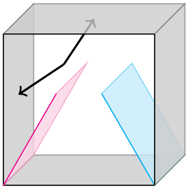

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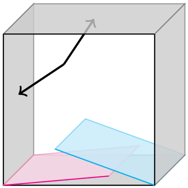

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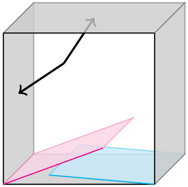

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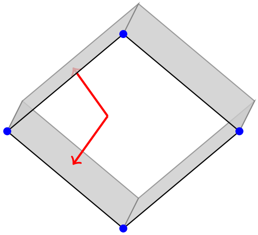

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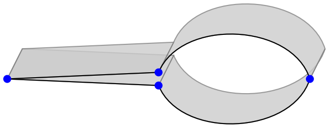

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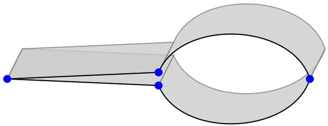

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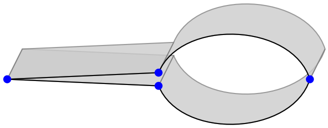

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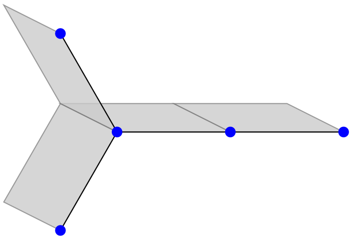

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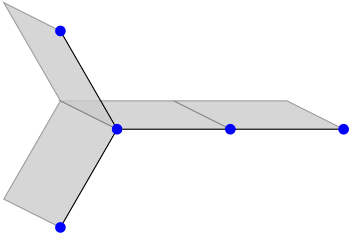

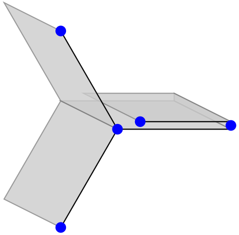

Structure of multiple foldings

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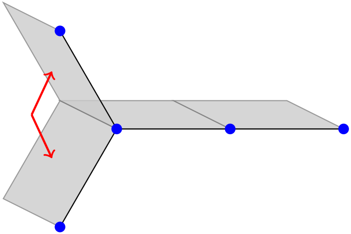

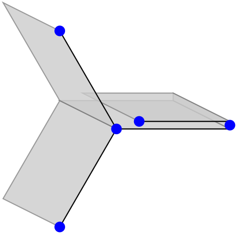

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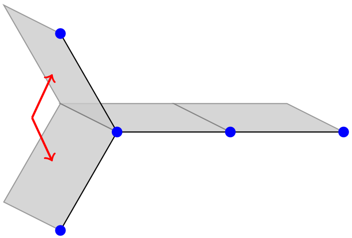

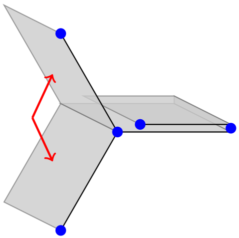

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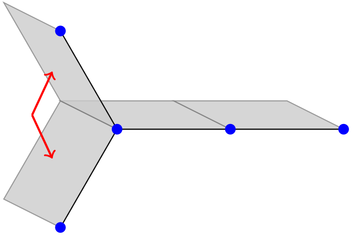

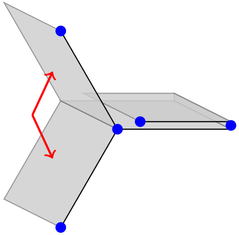

$\rightsquigarrow$ more structure on the set of possible foldings

Folding graph

Folding graph

- Vertices are folding complexes (modelling folding states)

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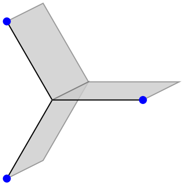

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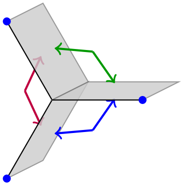

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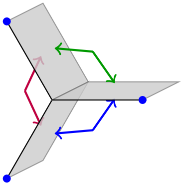

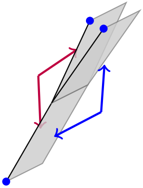

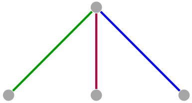

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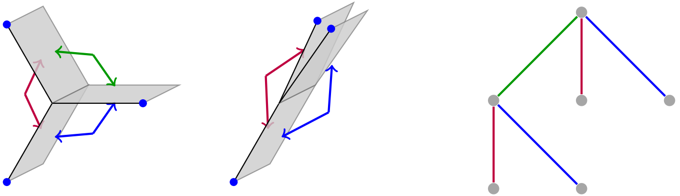

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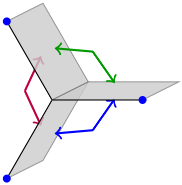

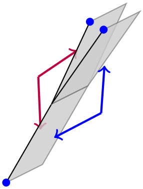

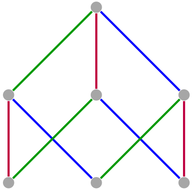

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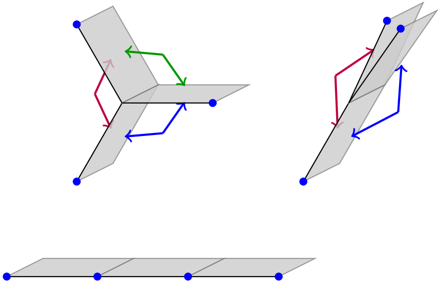

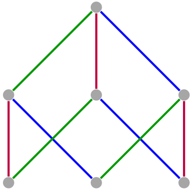

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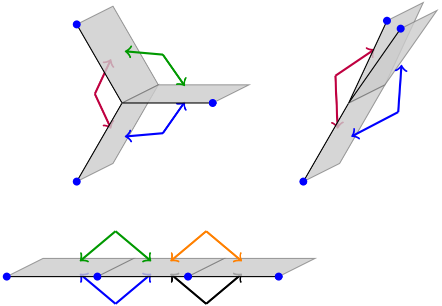

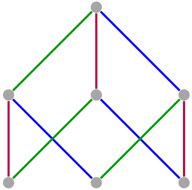

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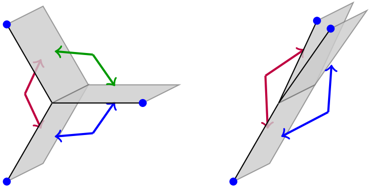

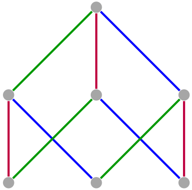

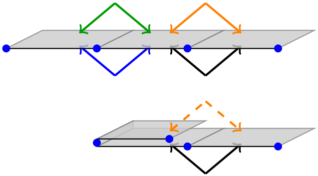

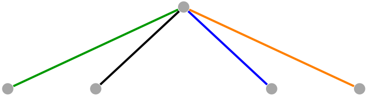

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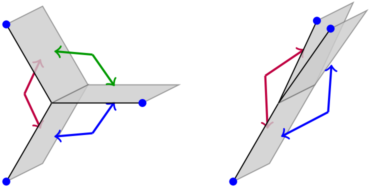

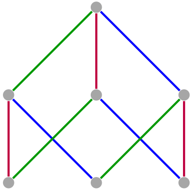

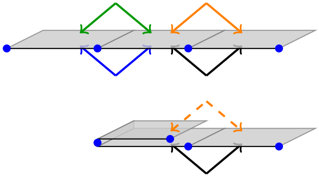

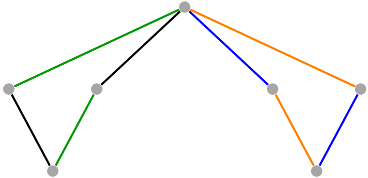

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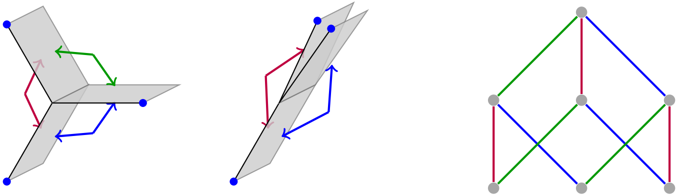

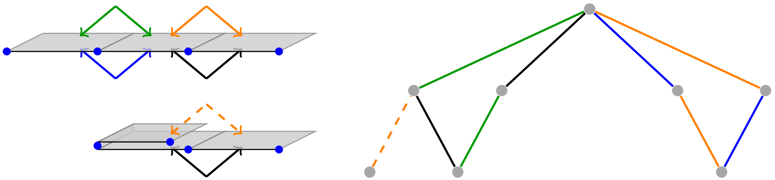

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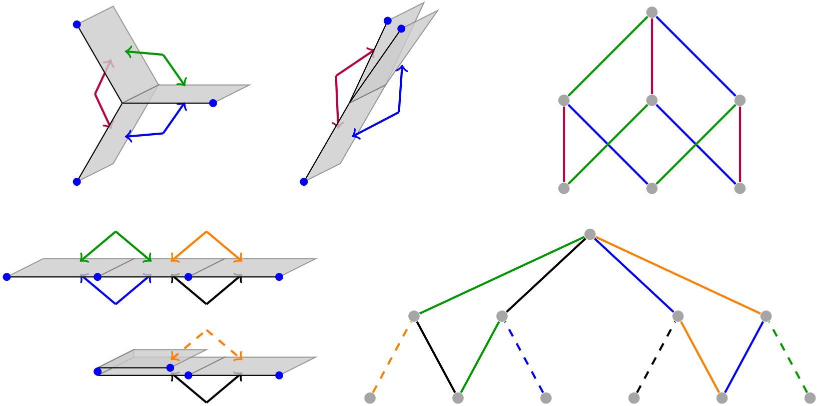

Larger graph

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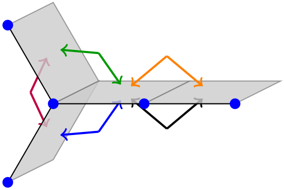

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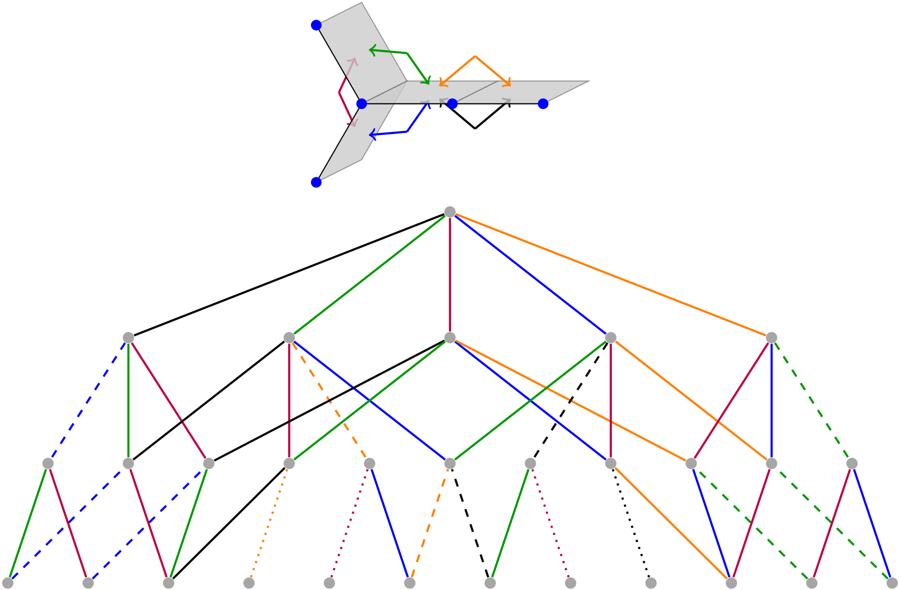

Drawback of folding plans

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Some foldings that “should” be the same, aren’t:

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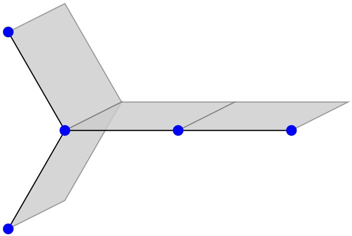

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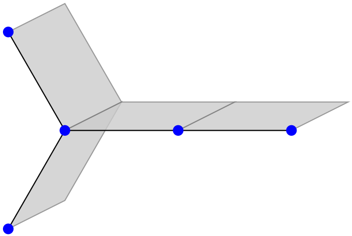

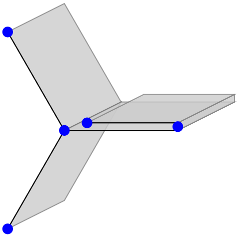

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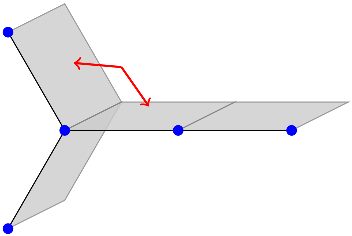

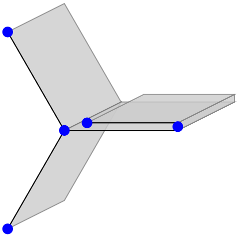

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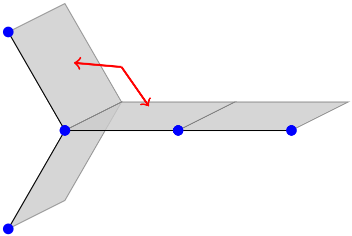

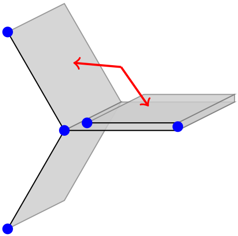

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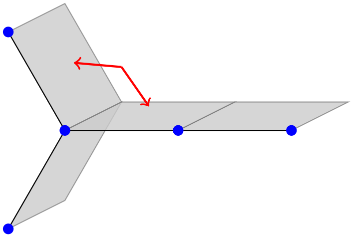

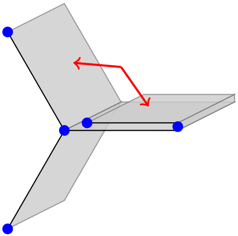

$\Rightarrow$ If you know the folding structure of a small complex,

Drawback of folding plans

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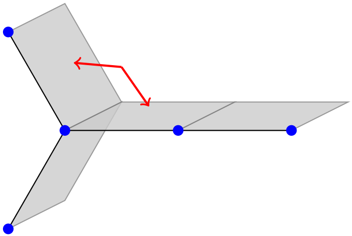

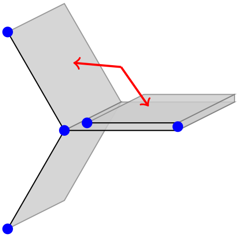

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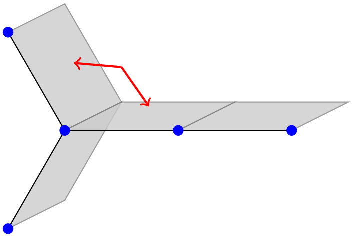

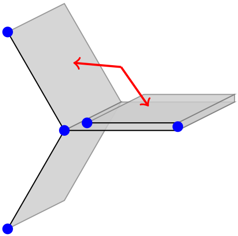

$\Rightarrow$ If you know the folding structure of a small complex, you can’t easily find the folding structure of an extended complex

$\rightsquigarrow$ Folding plans are not optimal to model folding

Progress of abstract folding

Progress of abstract folding

In development:

Progress of abstract folding

In development:

- folding complex

Progress of abstract folding

In development:

- folding complex
- folding plans

Progress of abstract folding

In development:

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- folding graph

Progress of abstract folding

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Missing:

Progress of abstract folding

In development:

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Missing:

- better folding description

Progress of abstract folding

In development:

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Missing:

- better folding description
- properties of folding graphs

Summary SimplicialSurfaces

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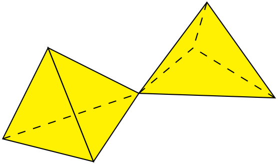
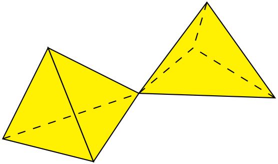
Triangulated complexes



Summary SimplicialSurfaces

Triangulated complexes

- mostly complete



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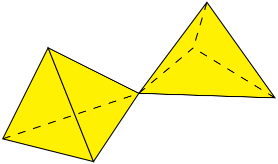
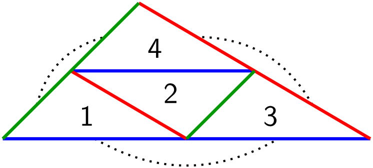


Edge colouring



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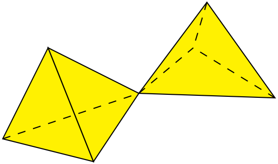
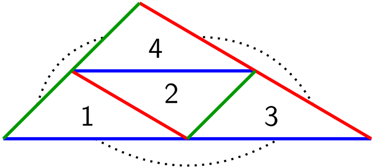


Edge colouring

- current theory implemented



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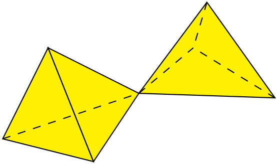
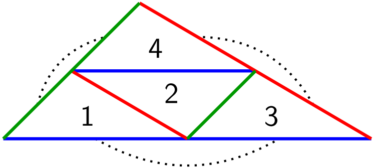


Edge colouring

- current theory implemented
- a lot of theory missing



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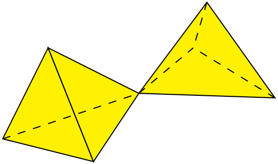

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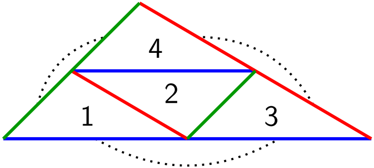
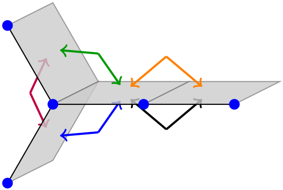


Abstract folding



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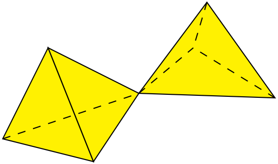

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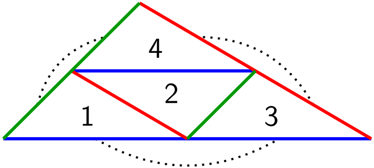
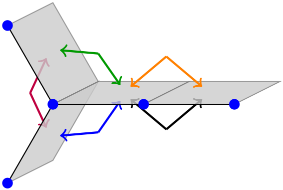


Abstract folding

- framework exists



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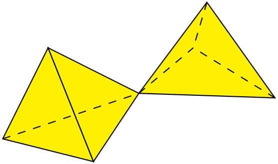

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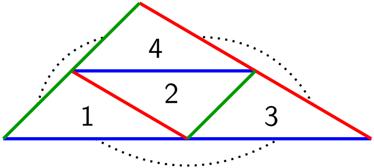

Abstract folding

- framework exists
- needs proper implementation