Simplicial surfaces in GAP

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Lehrstuhl B für Mathematik
RWTH Aachen University

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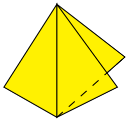

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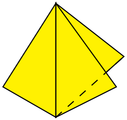

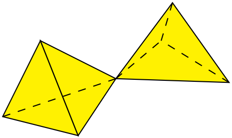

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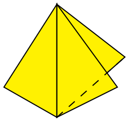

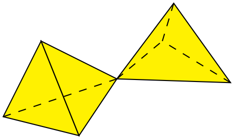

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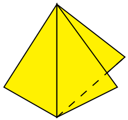

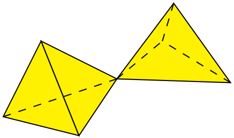

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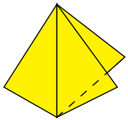

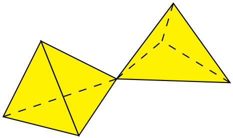

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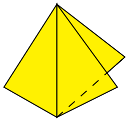

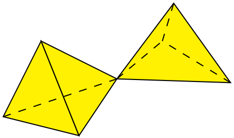

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$\rightsquigarrow$ incidence geometry

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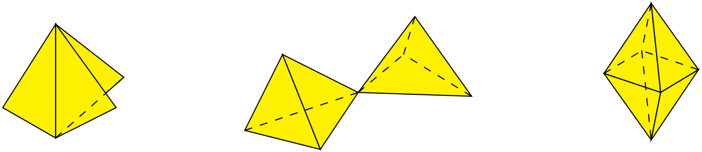

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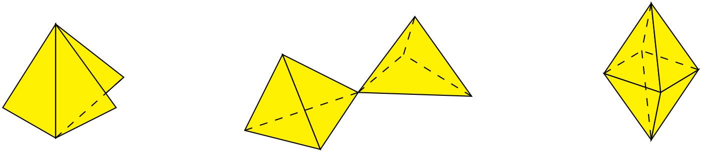

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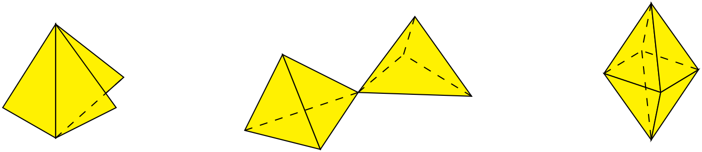

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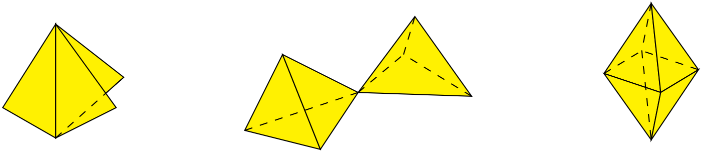

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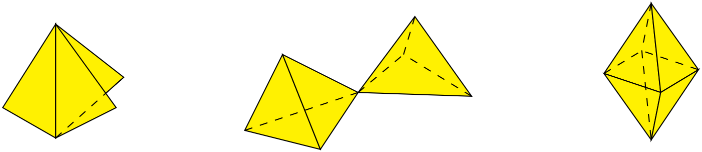

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  - only two dimensions but it can work with colourings and foldings

1 General simplicial surfaces

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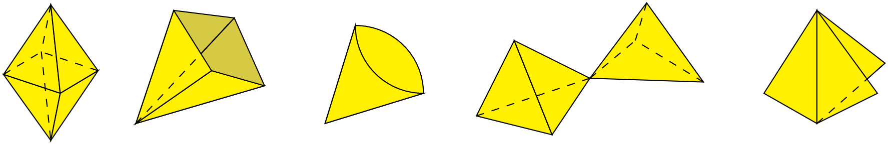

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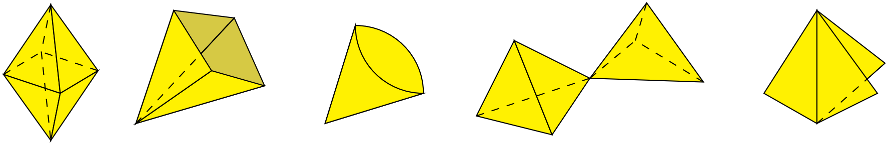

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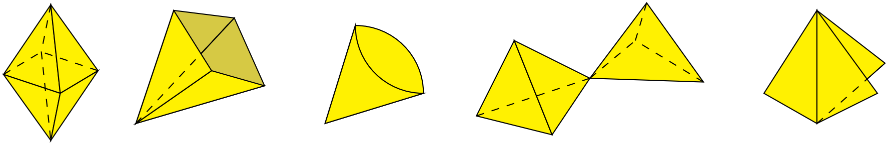

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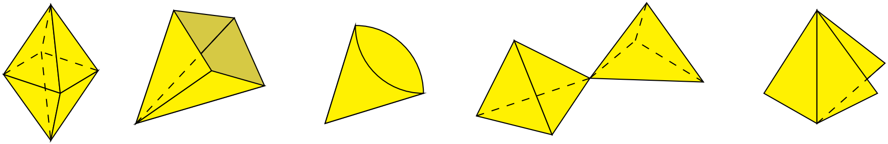

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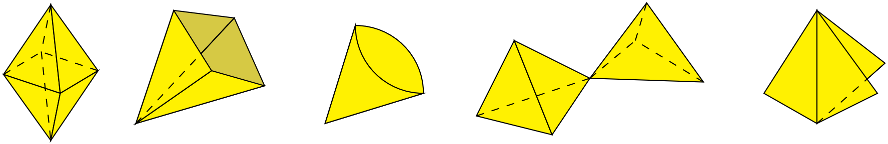

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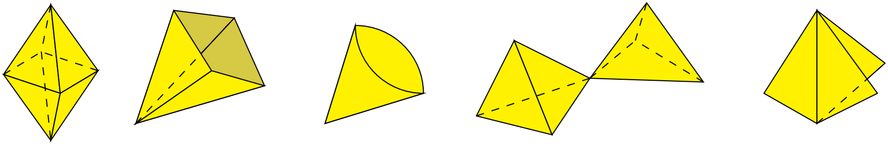

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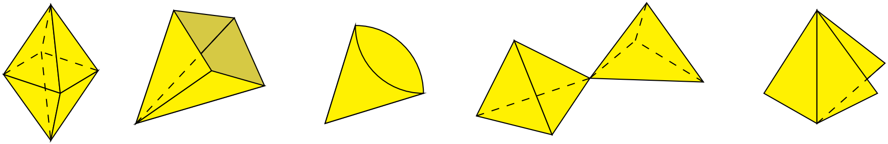

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- every vertex lies in an edge and every edge lies in a face

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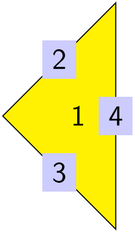

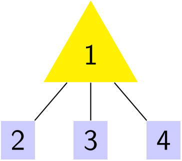

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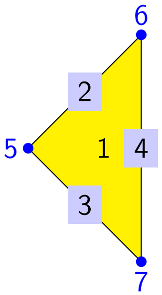

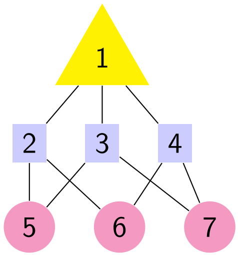

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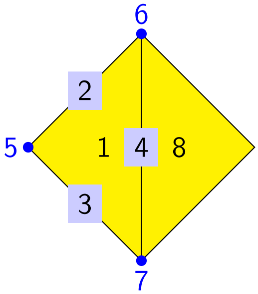

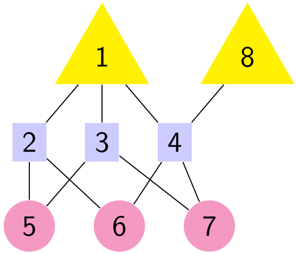

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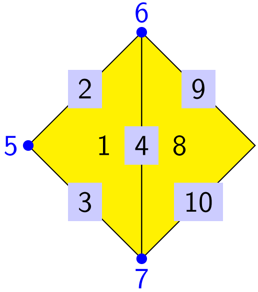

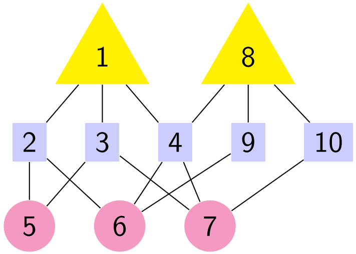

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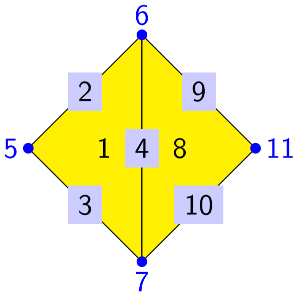

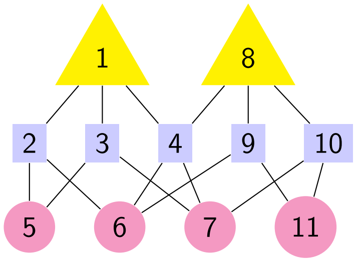

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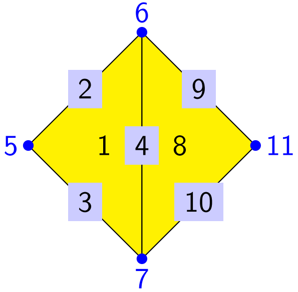

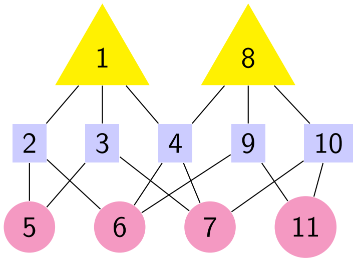

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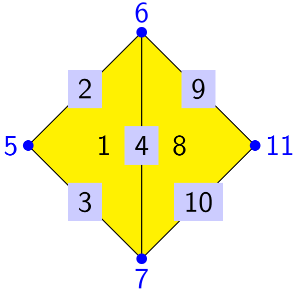

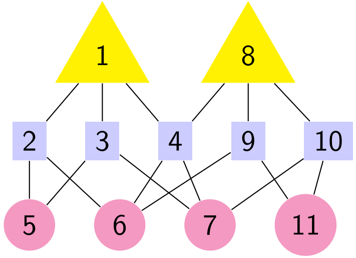

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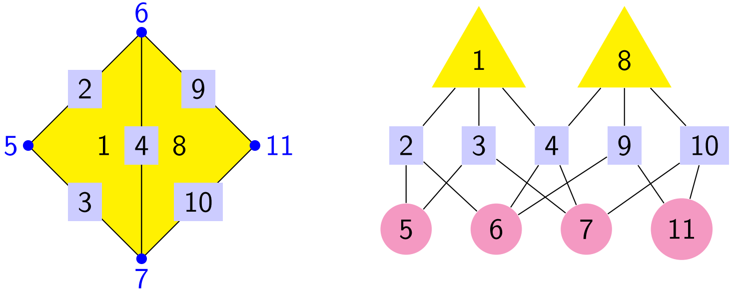

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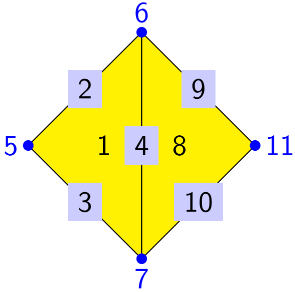

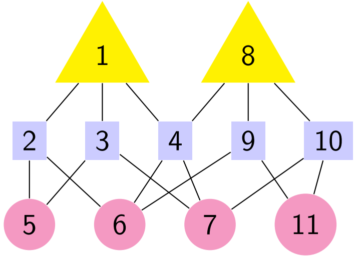

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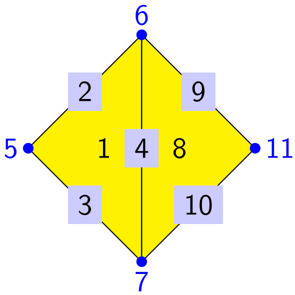

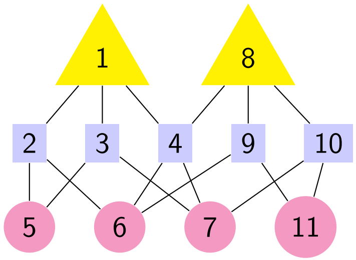

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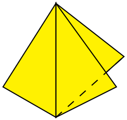

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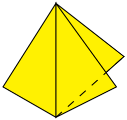

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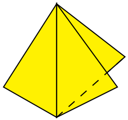

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$\rightsquigarrow$ **ramified simplicial surfaces**

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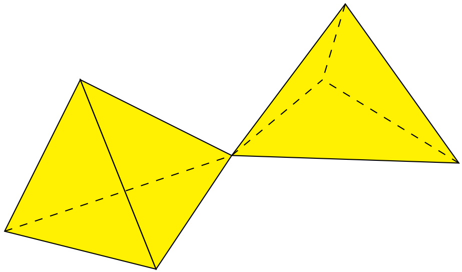

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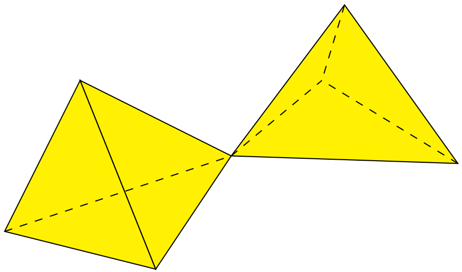

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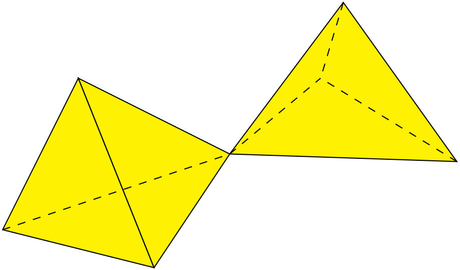

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A **simplicial surface** does not have these ramifications.

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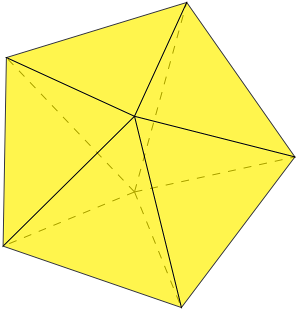

Progress report of triangulated complexes

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Embedding questions

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Given: A triangular complex

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- Can it be embedded into $\mathbb{R}^3$?

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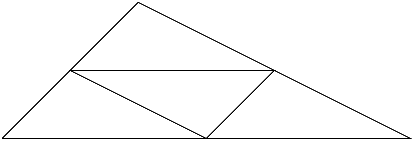

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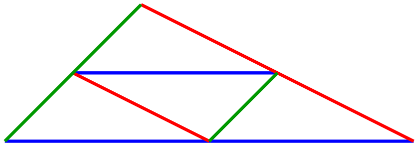
$\rightsquigarrow$ Edge–colouring encodes different lengths



Colouring as permutation

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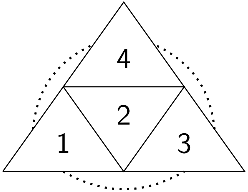
Consider tetrahedron



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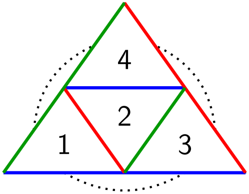

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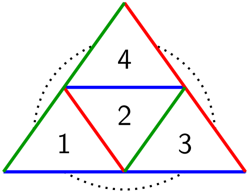

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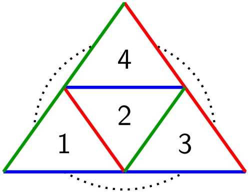

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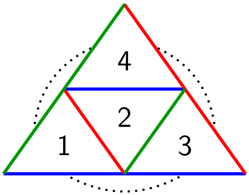

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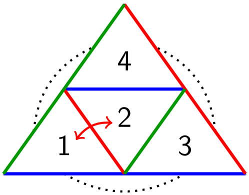

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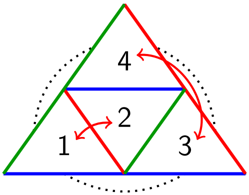

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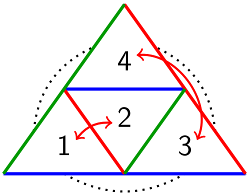

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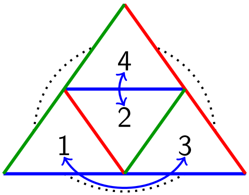

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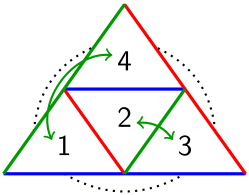

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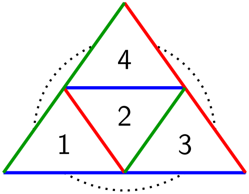

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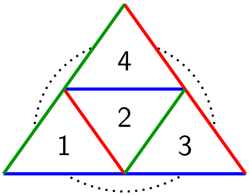

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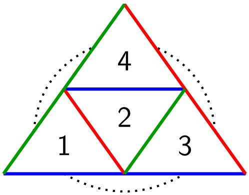

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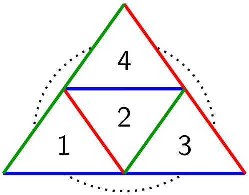

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How do faces fit together?

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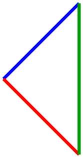
Consider a face of the surface



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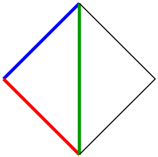

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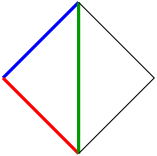

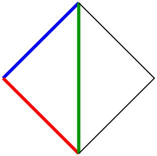

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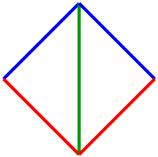

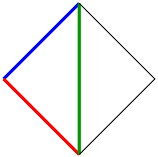

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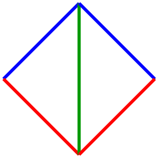

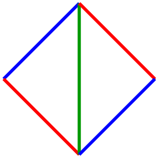

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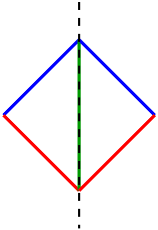
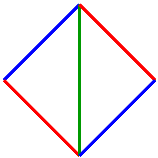


mirror (m)



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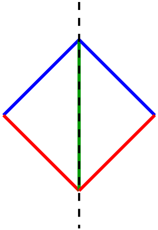

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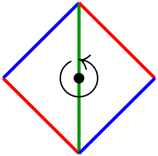

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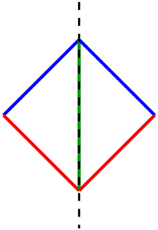

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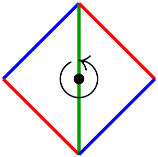

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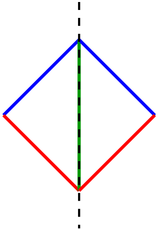

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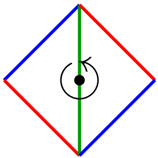

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Lemma

Permutations and mr–assignment uniquely determine the surface.

Constructing surfaces from groups

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A general mr–assignment leads to complicated surfaces.

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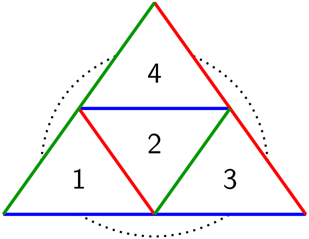

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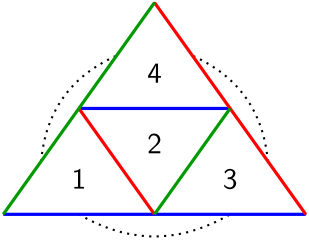

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The mirror–case

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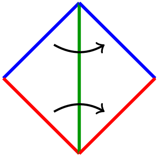

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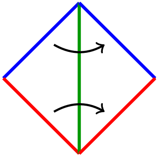

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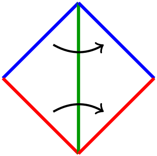

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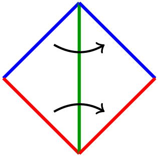

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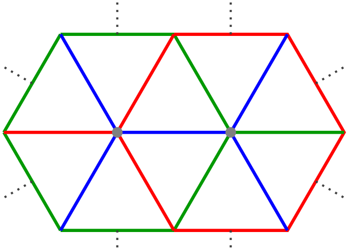

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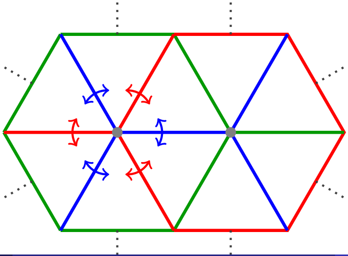

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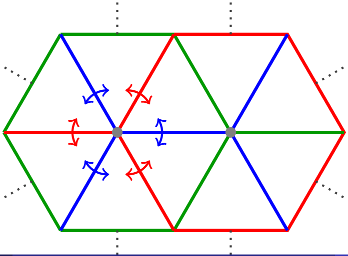

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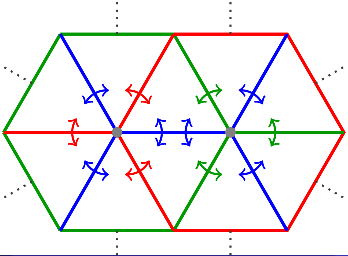

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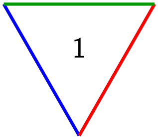

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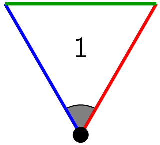

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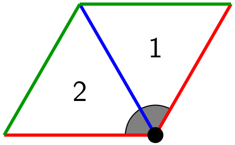

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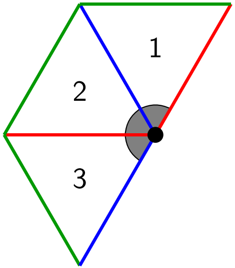

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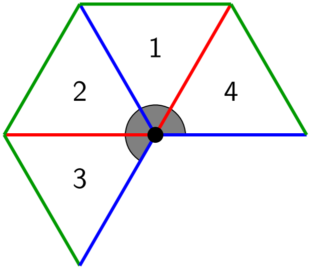

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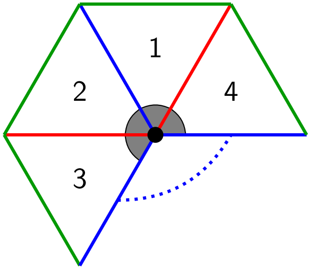

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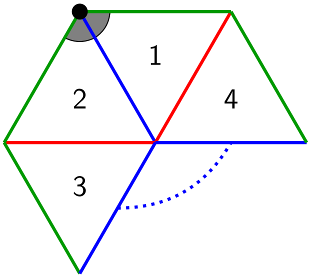

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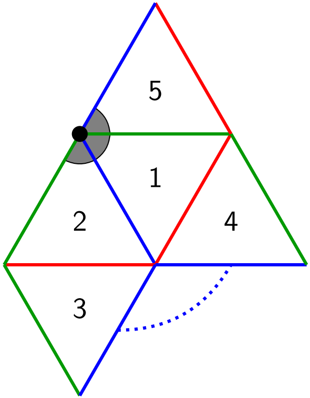

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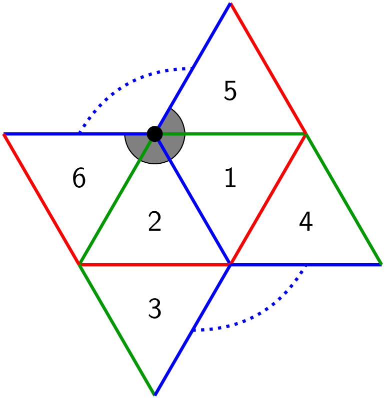

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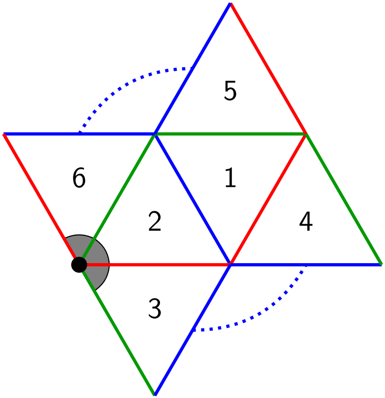

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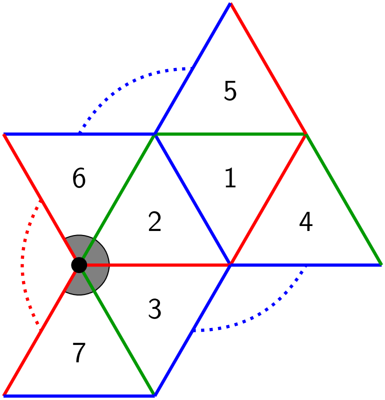

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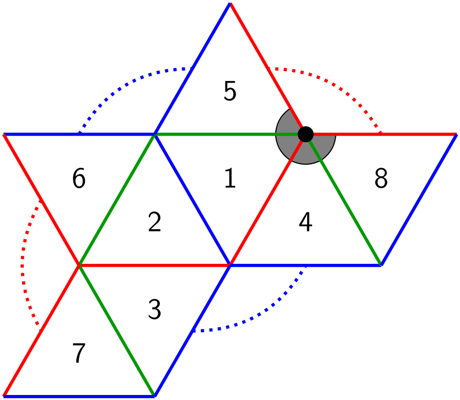

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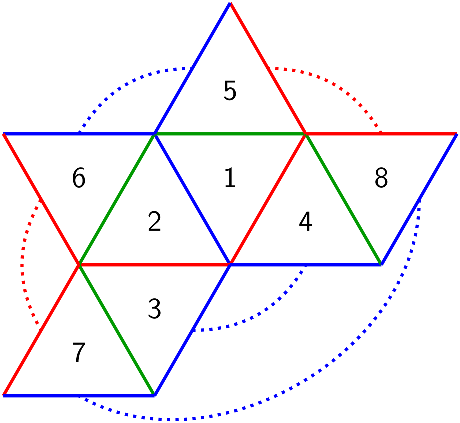

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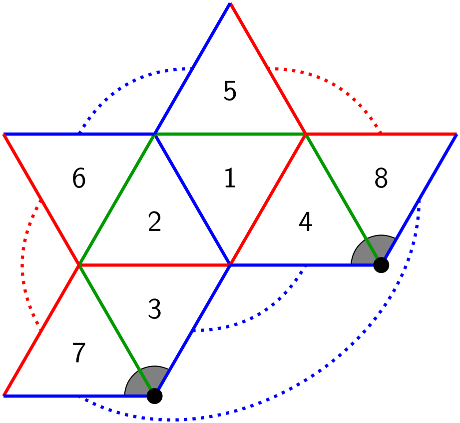

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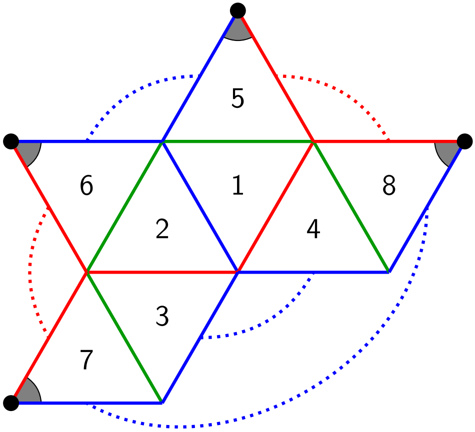

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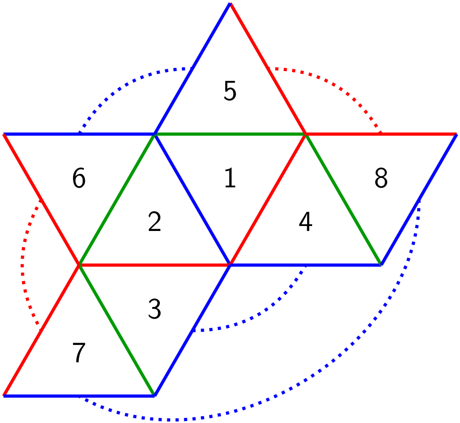

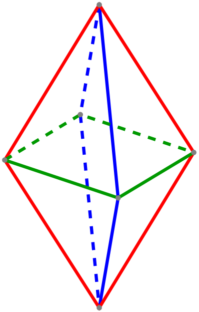

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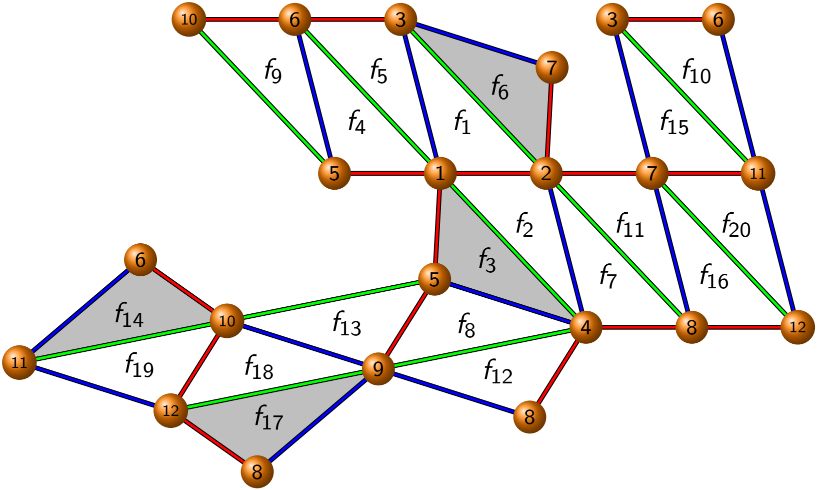

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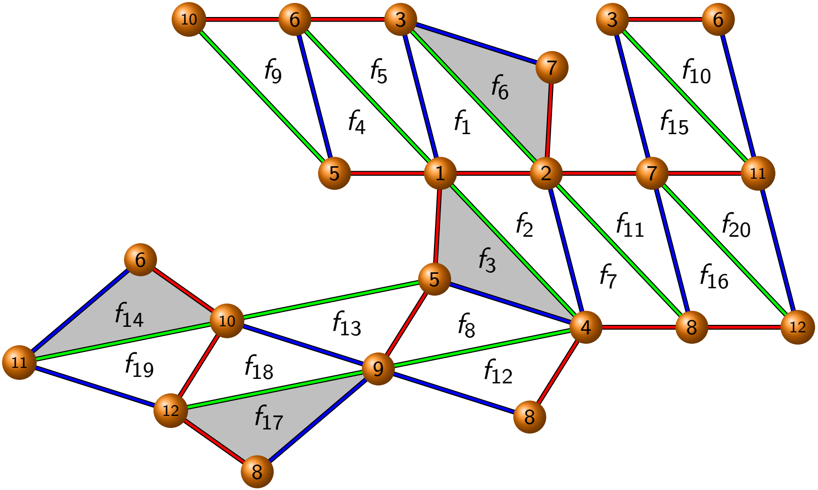

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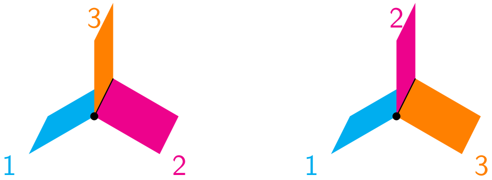

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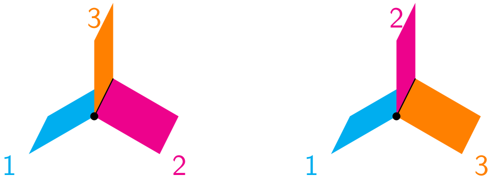

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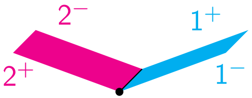

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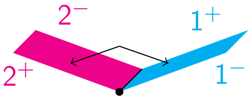

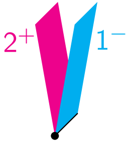

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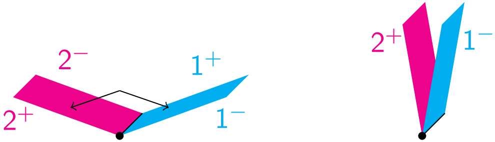

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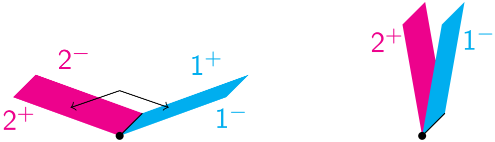

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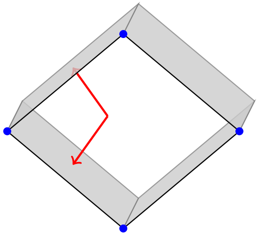

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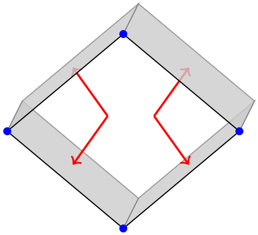

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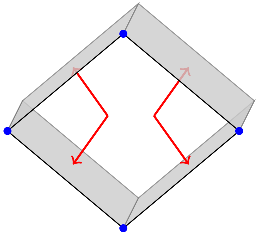

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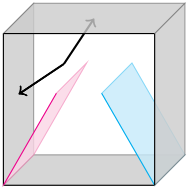

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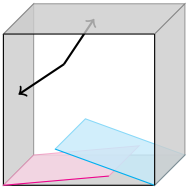

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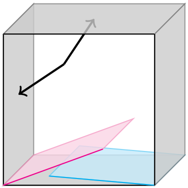

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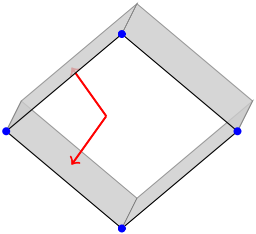

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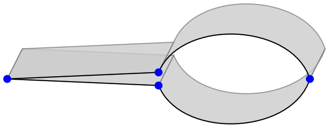

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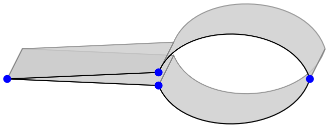

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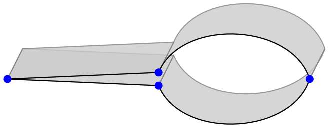

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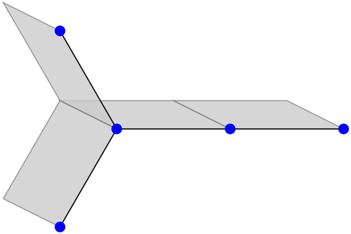

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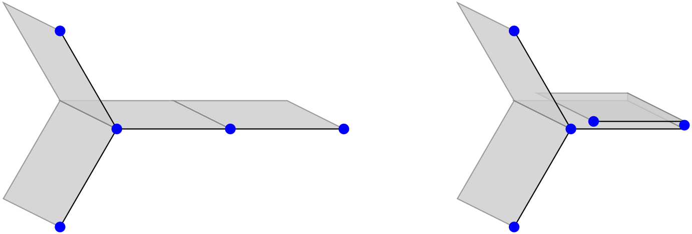

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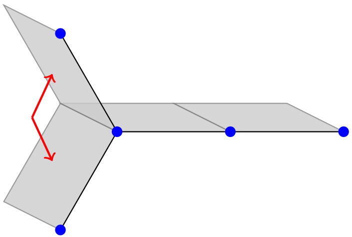 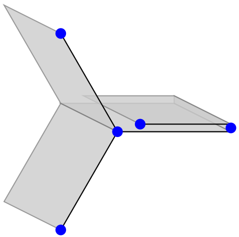

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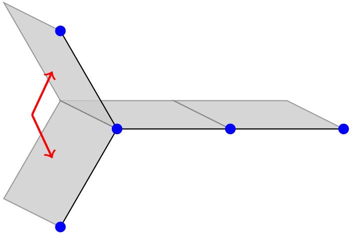

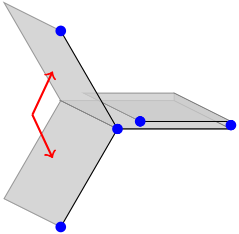

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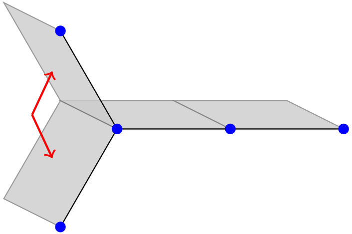

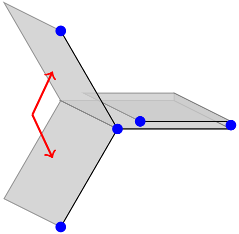

$\rightsquigarrow$ more structure on the set of possible foldings

Folding graph

Folding graph

- Vertices are folding complexes (modelling folding states)

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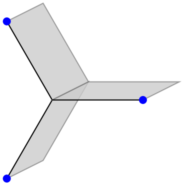

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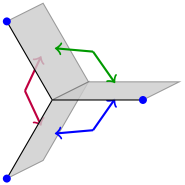

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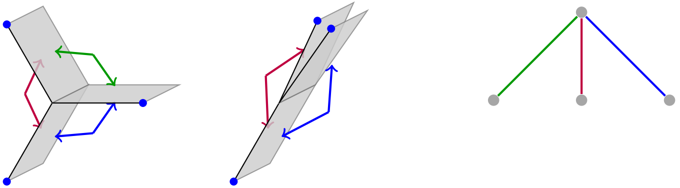

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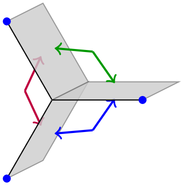

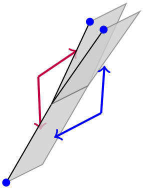

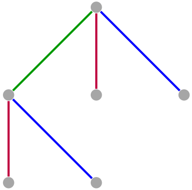

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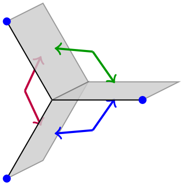

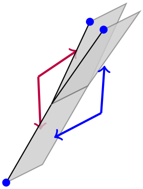

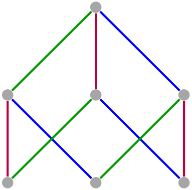

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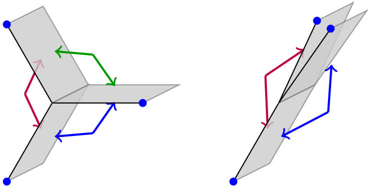

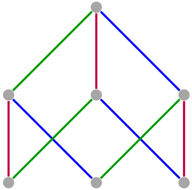

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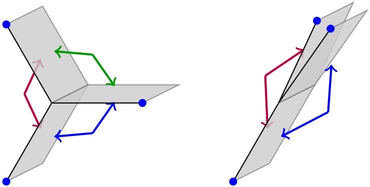

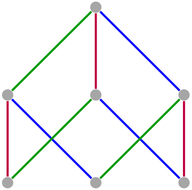

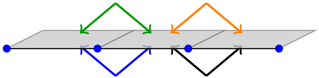

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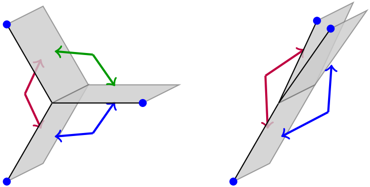

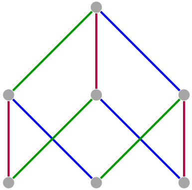

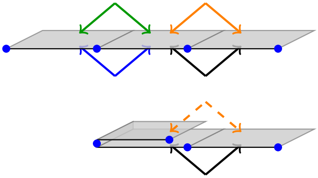

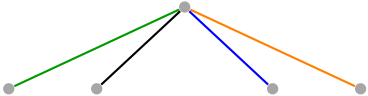

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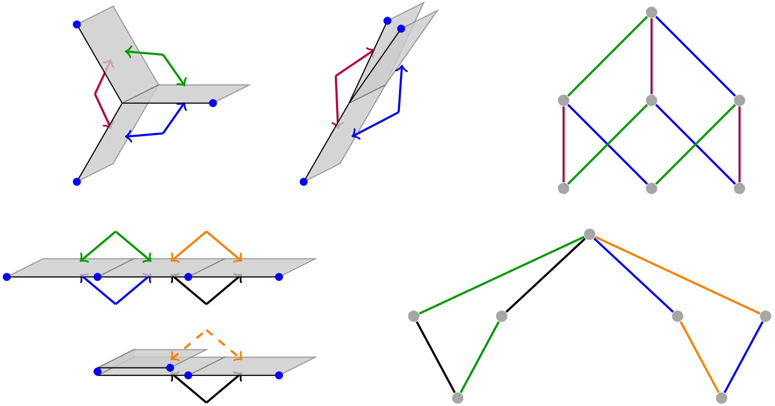

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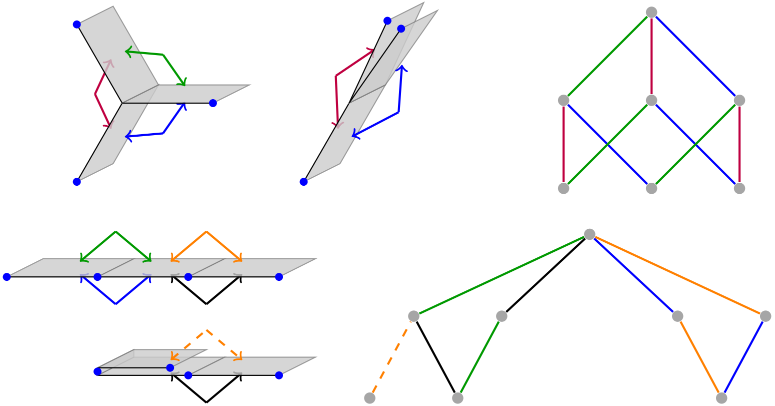

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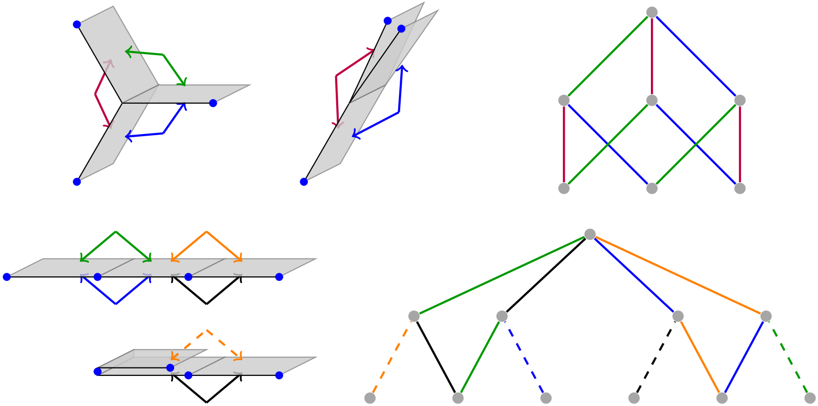

Larger graph

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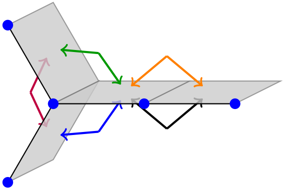

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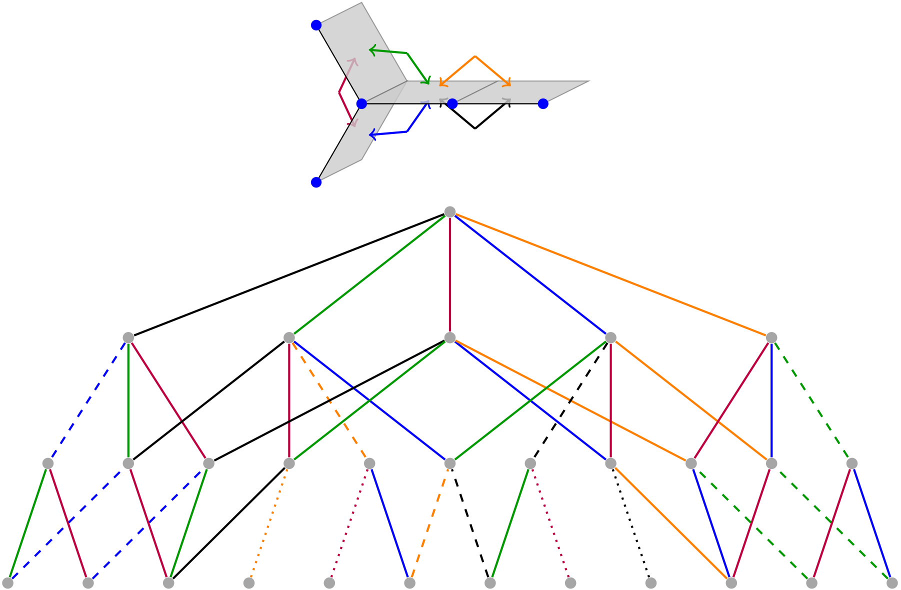

Drawback of folding plans

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Some foldings that “should” be the same, aren’t:

Drawback of folding plans

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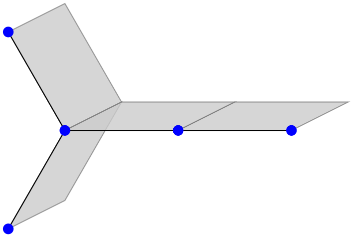

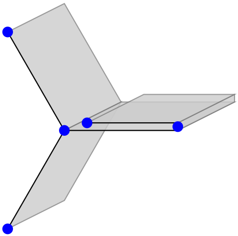

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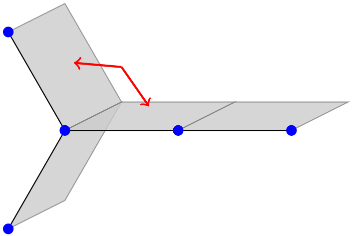

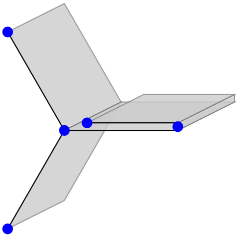

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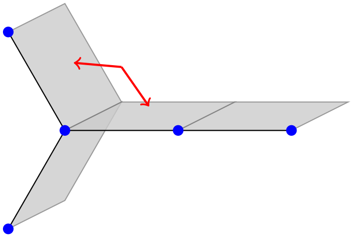

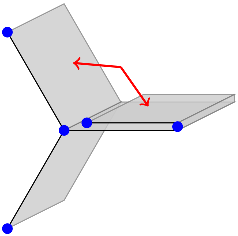

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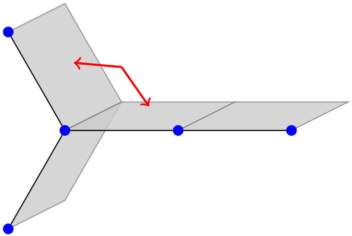

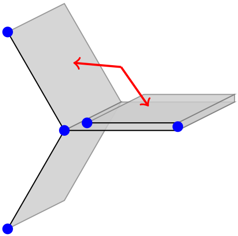

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Drawback of folding plans

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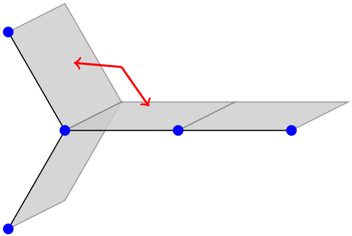

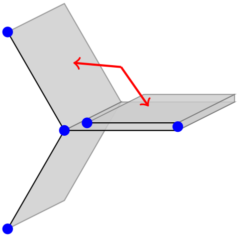

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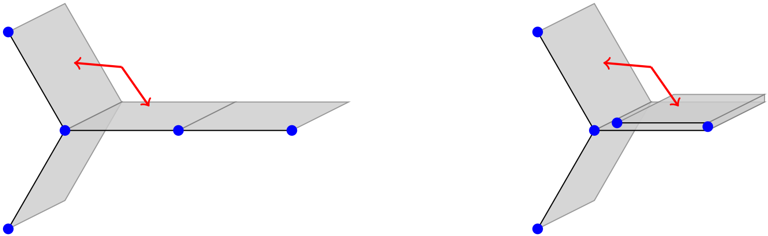

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$\rightsquigarrow$ Folding plans are not optimal to model folding

Progress report of abstract folding

Progress report of abstract folding

In development:

Progress report of abstract folding

In development:

- folding complex

Progress report of abstract folding

In development:

- folding complex
- folding plans

Progress report of abstract folding

In development:

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- folding graph

Progress report of abstract folding

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Missing:

Progress report of abstract folding

In development:

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Missing:

- better folding description

Progress report of abstract folding

In development:

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Missing:

- better folding description
- properties of folding graphs

Summary `SimplicialSurfaces`

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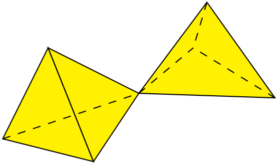
Triangulated complexes



Summary `SimplicialSurfaces`

Triangulated complexes

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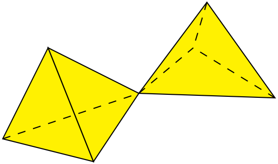

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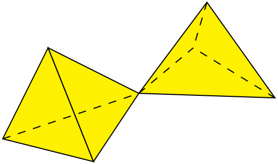
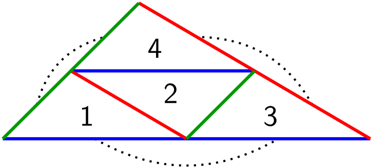


Edge colouring



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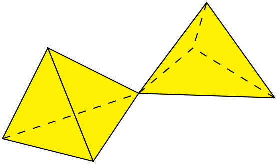

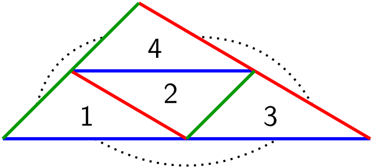

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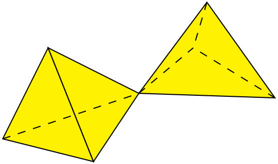

Edge colouring

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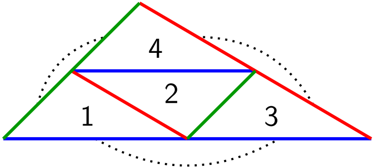

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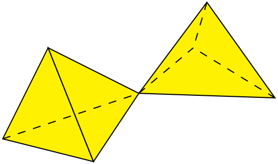

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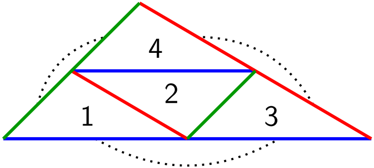
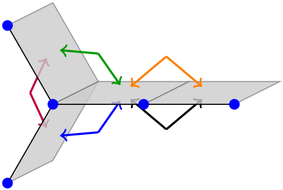


Abstract folding



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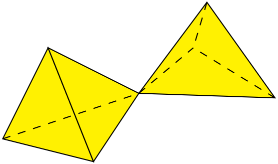

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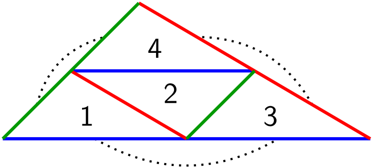
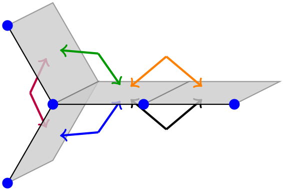


Abstract folding

- framework exists



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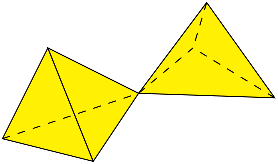

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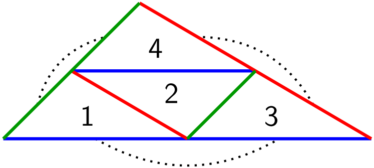

Abstract folding

- framework exists
- needs proper implementation