Simplicial surfaces in GAP

Markus Baumeister

??.08.2017

1. General polygonal complexes by incidence geometry
2. Edge colouring and group properties
3. Abstract folding

Motivation

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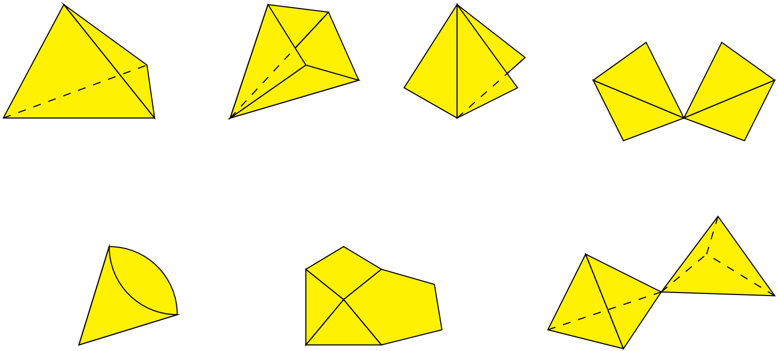

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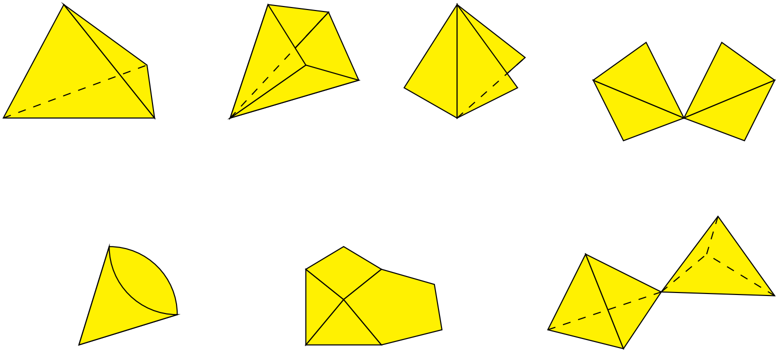

$\rightsquigarrow$ examples of **polygonal complexes**

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$\rightsquigarrow$ incidence structure is intrinsic

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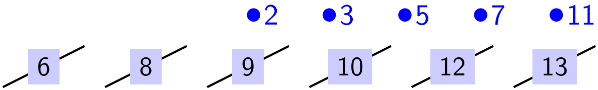

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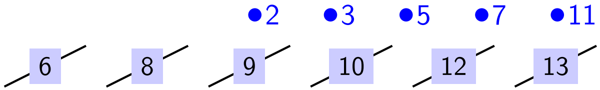

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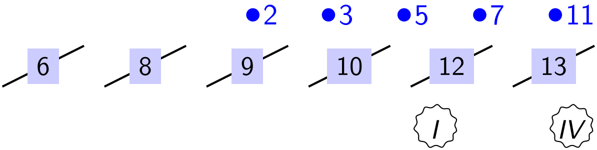

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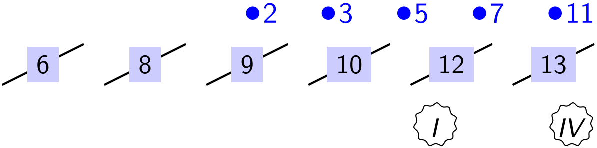

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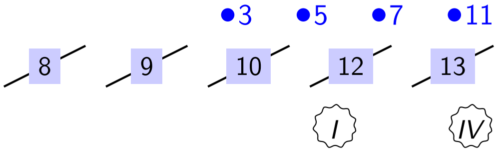

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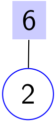

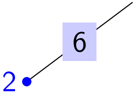

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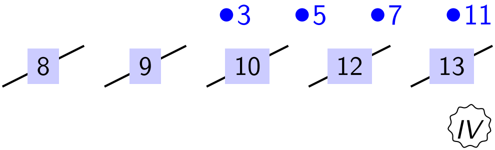

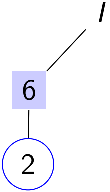

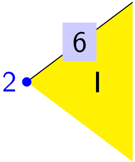

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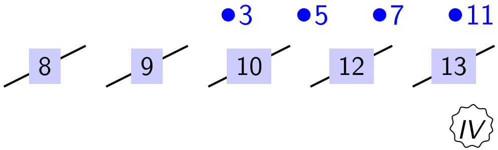

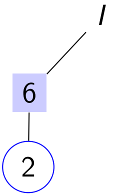

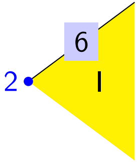

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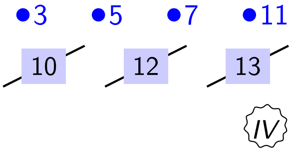

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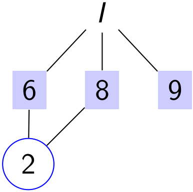

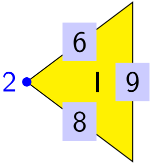

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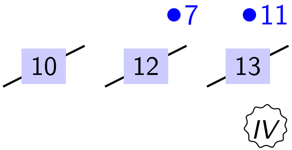

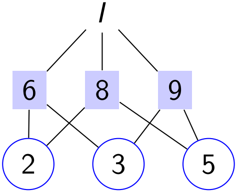

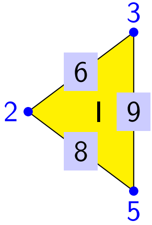

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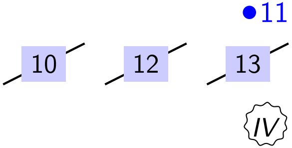

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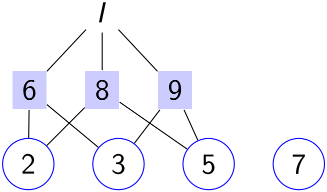

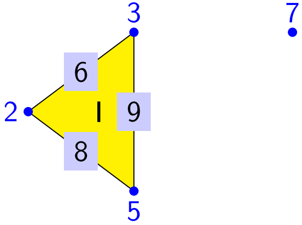

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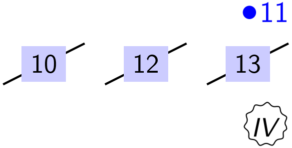

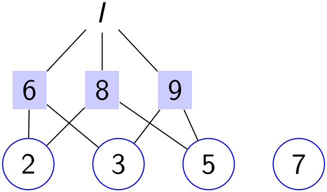

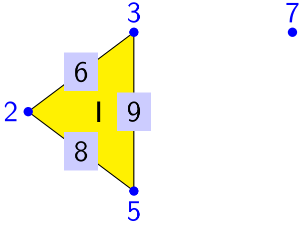

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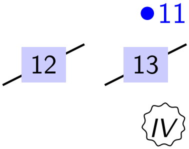

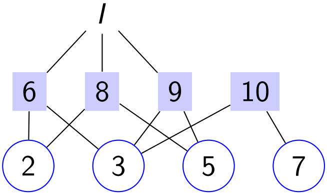

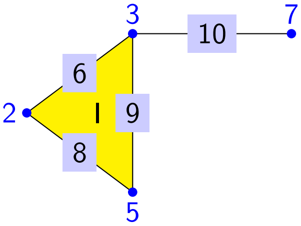

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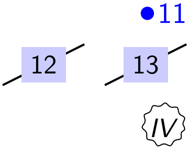

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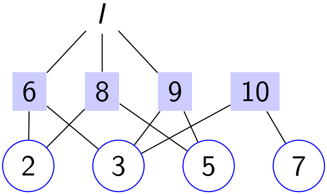

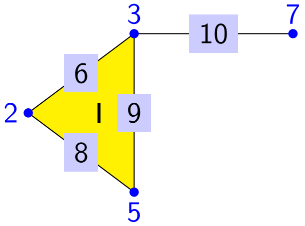

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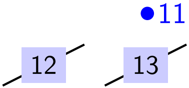

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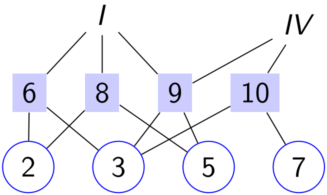

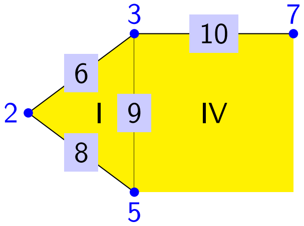

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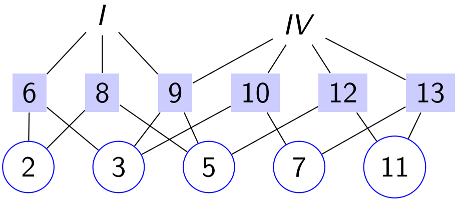

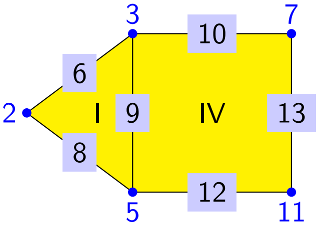

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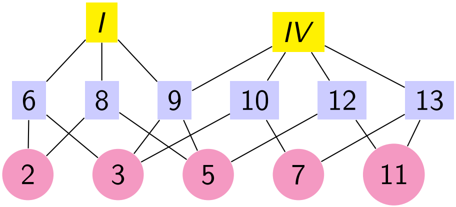

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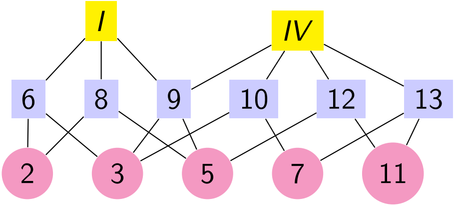

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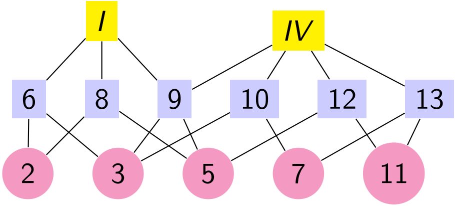

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Solved by NautyTracesInterface (by Gutsche, Niemeyer, Schweitzer)

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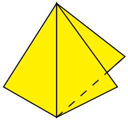

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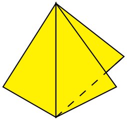

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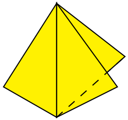

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$\rightsquigarrow$ **ramified polygonal surfaces**

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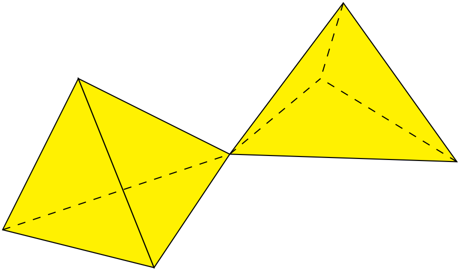

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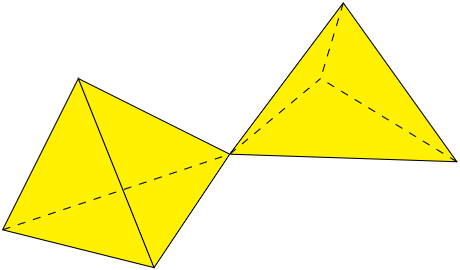

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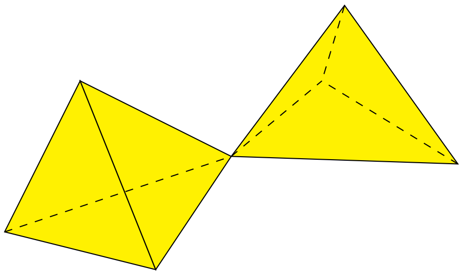

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A **polygonal surface** does not have these ramifications.

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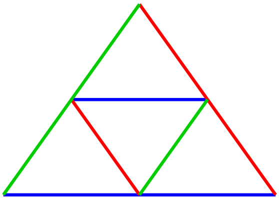
$\rightsquigarrow$ Edge–colouring encodes different lengths



Colouring as permutation

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Consider tetrahedron

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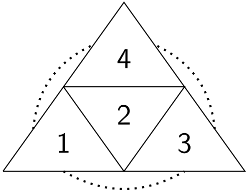

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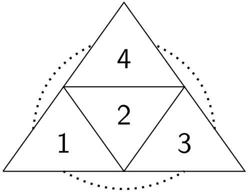

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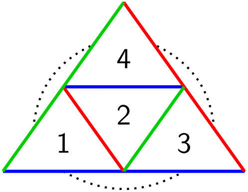

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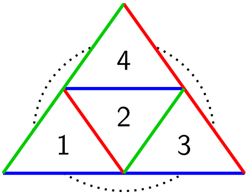

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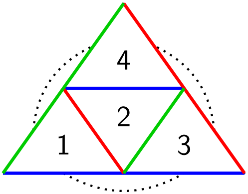

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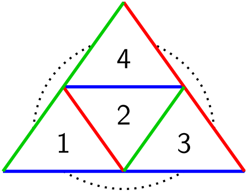

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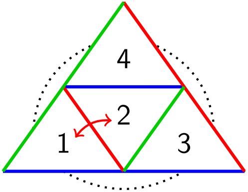

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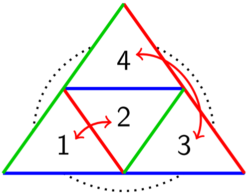

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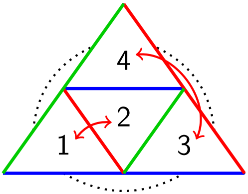

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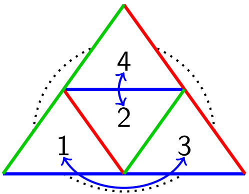

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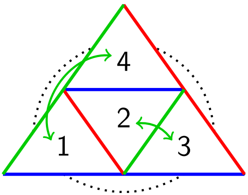

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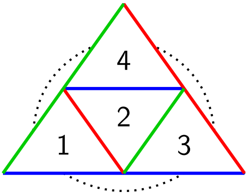

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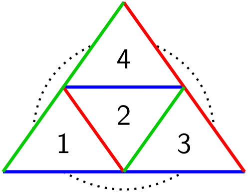

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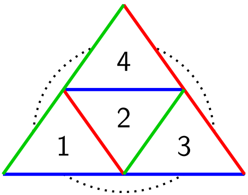

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  - The connected components of the surface correspond to the orbits of $\langle \sigma_a, \sigma_b, \sigma_c \rangle$ on the faces

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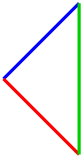

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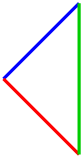

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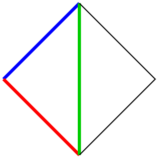

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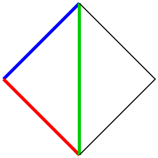

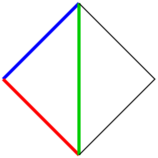

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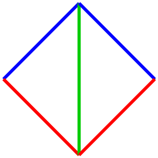

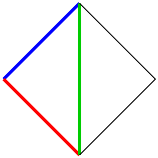

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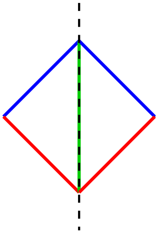
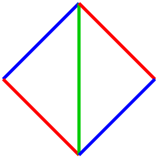


mirror (m)



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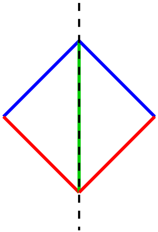

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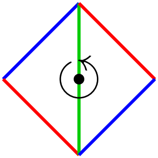

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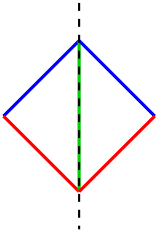

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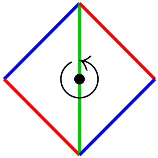

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Permutations and mr–assignment uniquely determine the surface.

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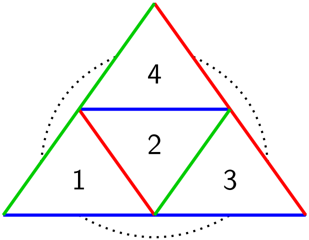

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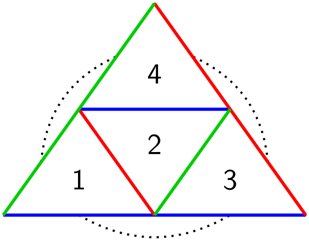

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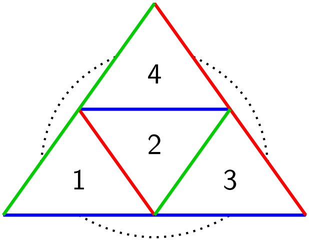

has an rrr–structure
The easiest structure is an mmm–structure.

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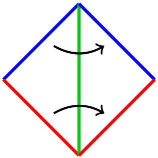

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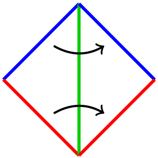

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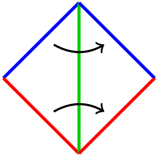

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- Colouring defines a map to the triangle.

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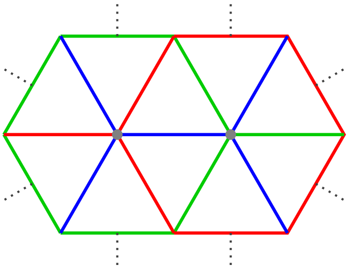

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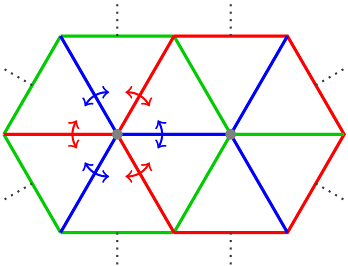

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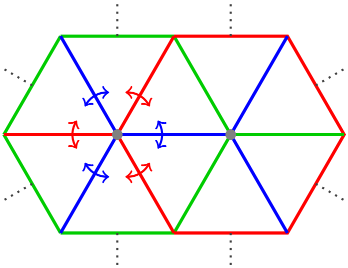

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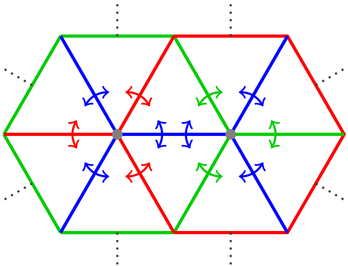

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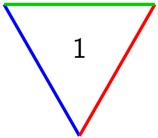

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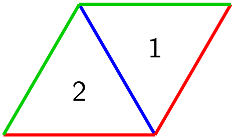

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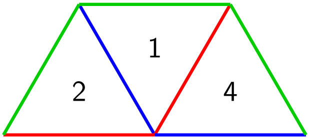

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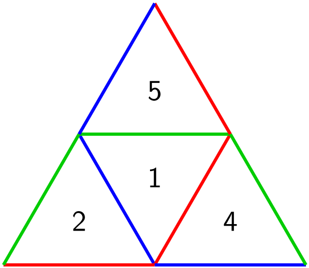

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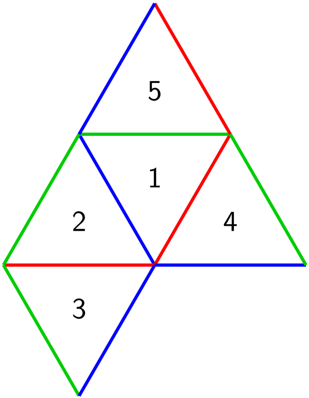

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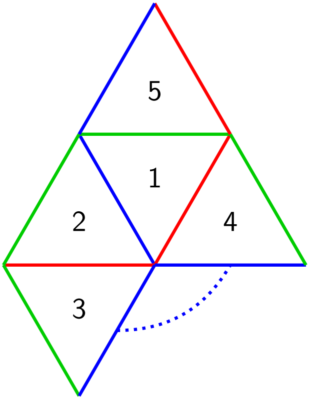

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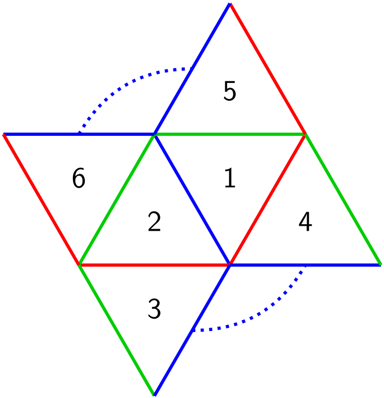

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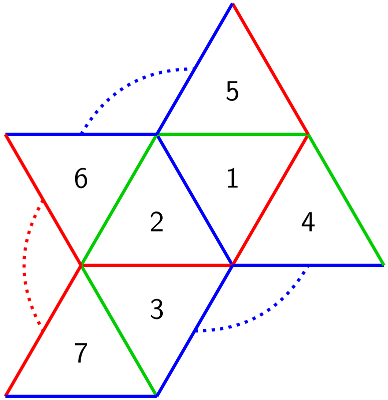

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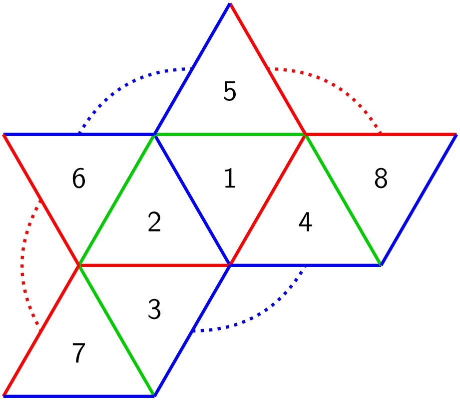

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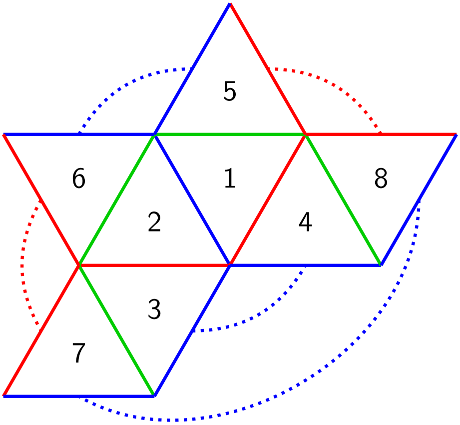

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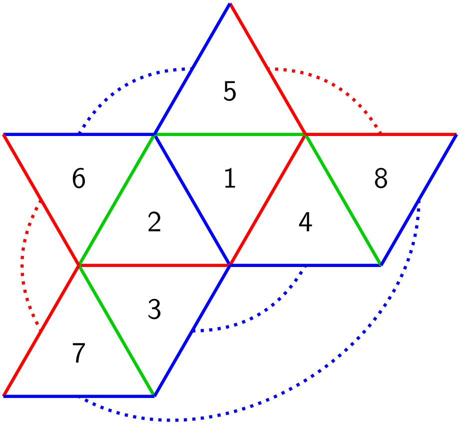

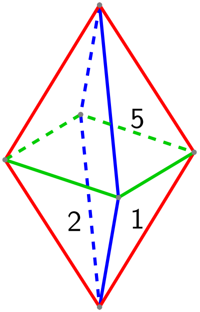

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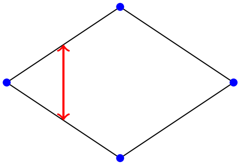

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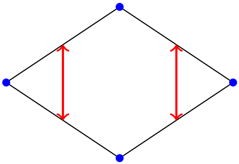

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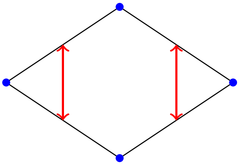

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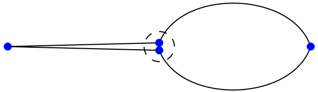

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Unordered Folding (Covering)

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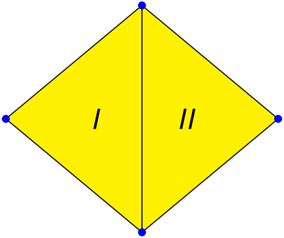
Why do we need more than a polygonal complex?

Unordered Folding (Covering)



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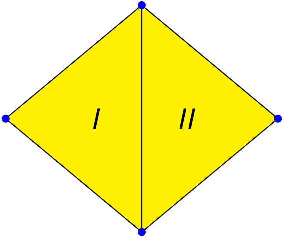

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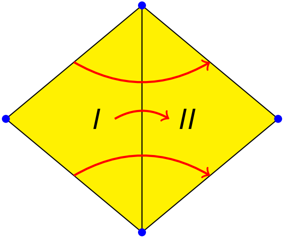

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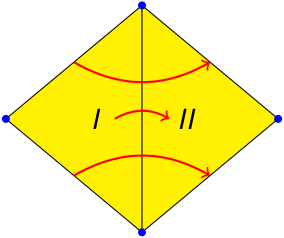

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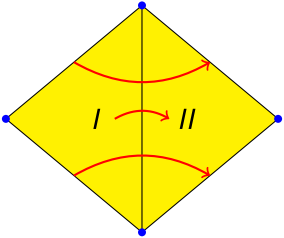

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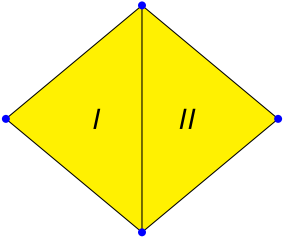

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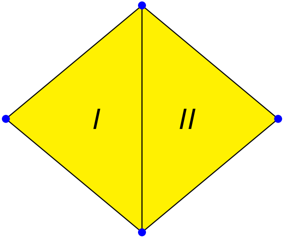

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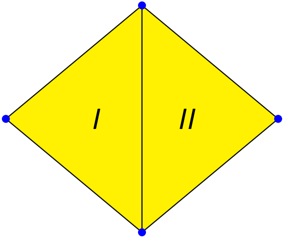

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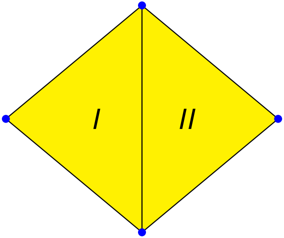

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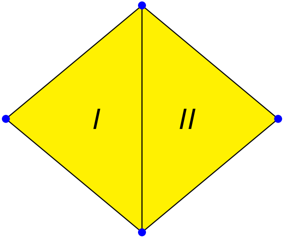

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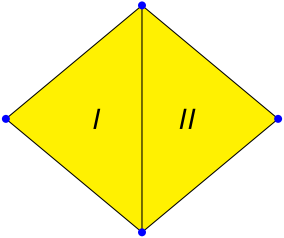

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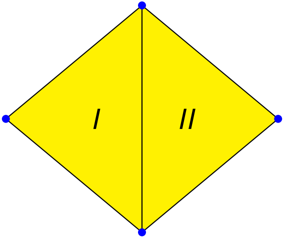

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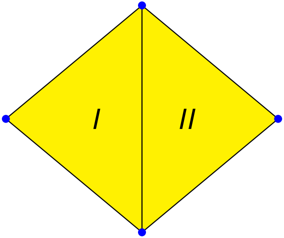

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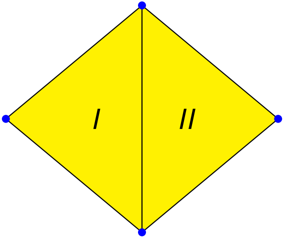

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$\Rightarrow$ Unordered folding is coarsening of equivalence relation

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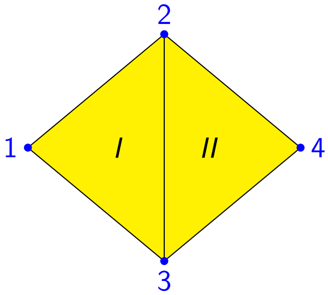

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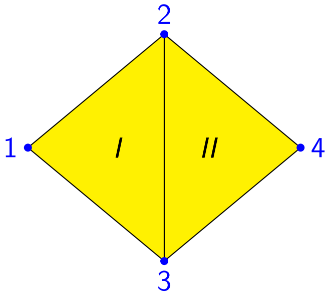

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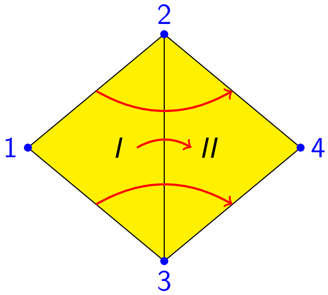

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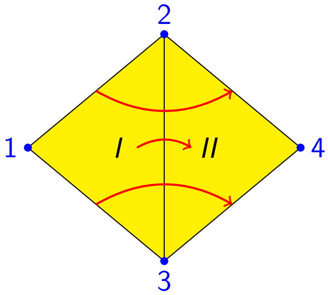

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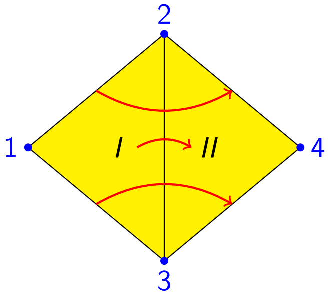

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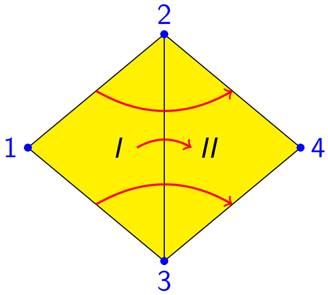

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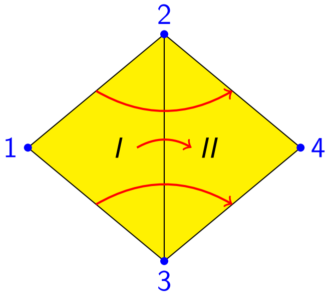

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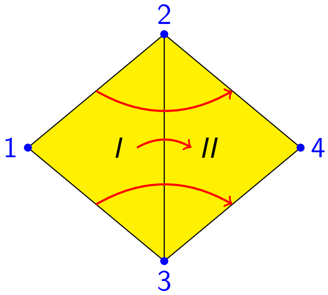

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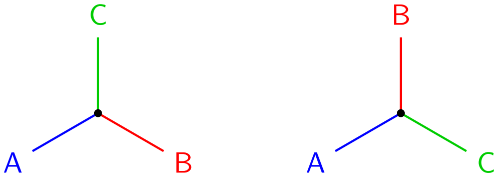

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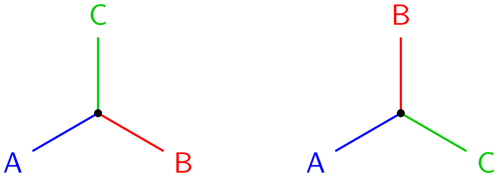

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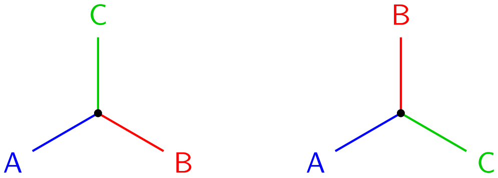

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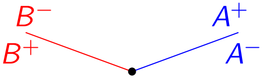

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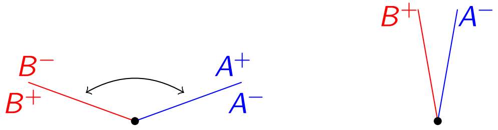

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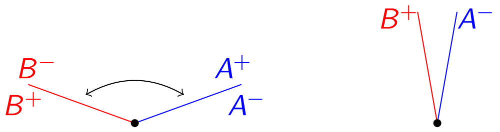

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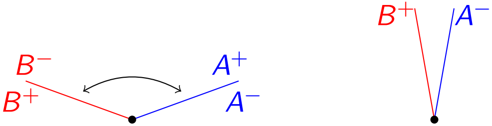

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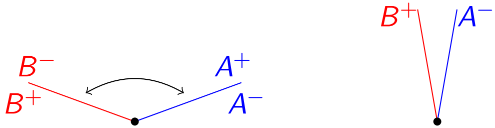

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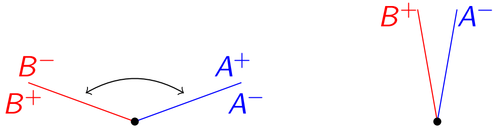

$\Rightarrow$ Define folding by two face sides (**folding plan**)
$\rightsquigarrow$ Allows reversible (un)folding

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With folding plans we can perform the same folding in different folding complexes

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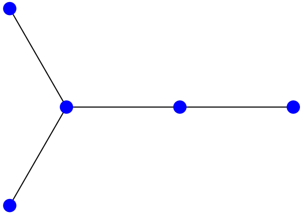

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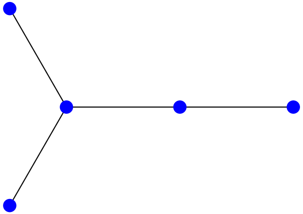

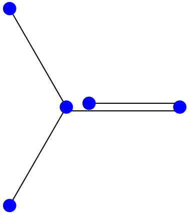

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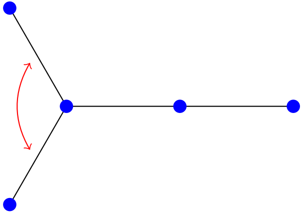

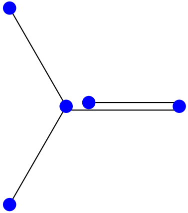

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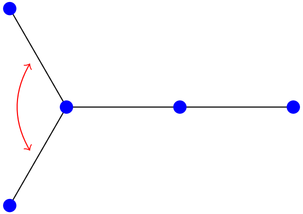

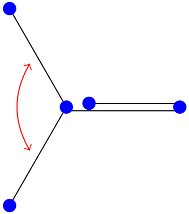

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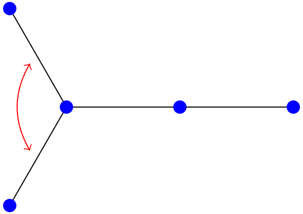

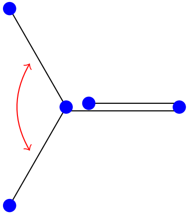

$\rightsquigarrow$ more structure on the set of possible foldings

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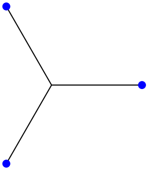

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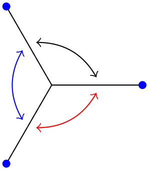

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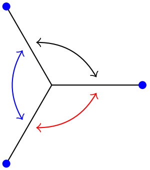

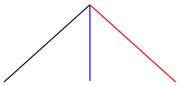

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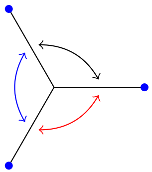

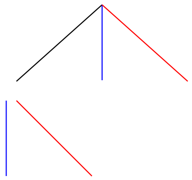

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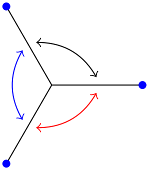

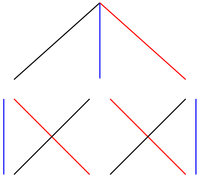

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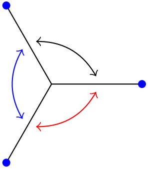

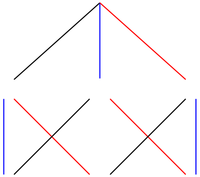

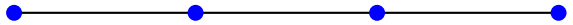

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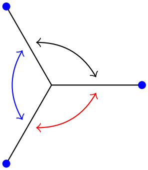

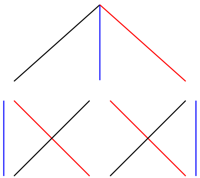

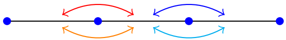

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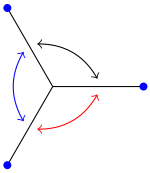

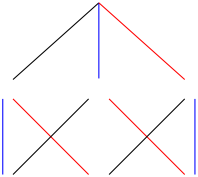

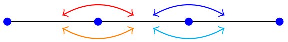

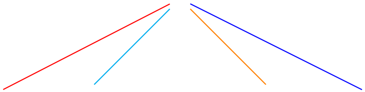

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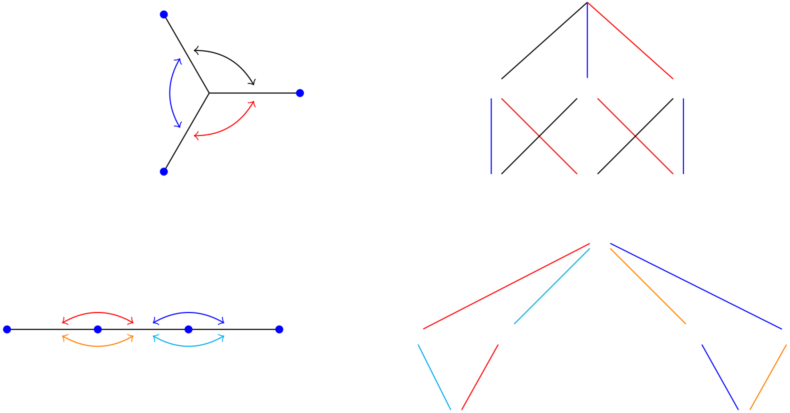

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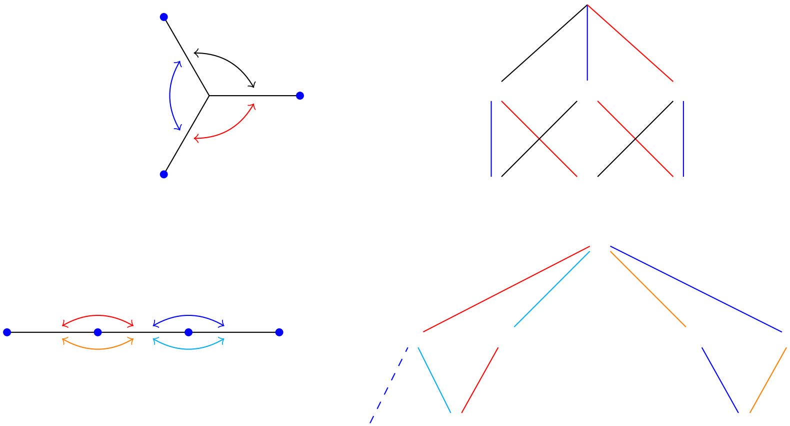

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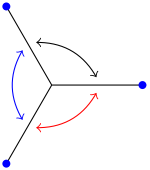
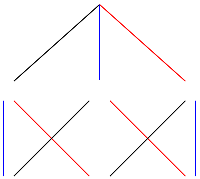
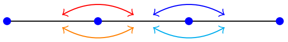
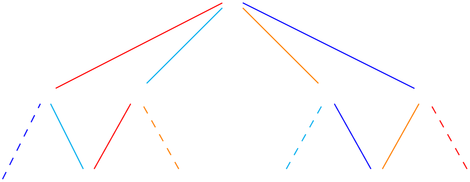

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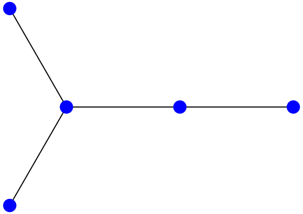

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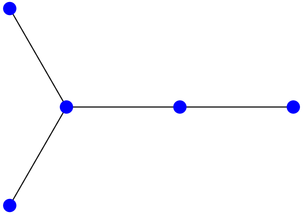

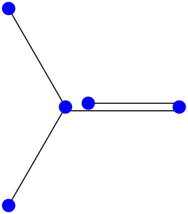

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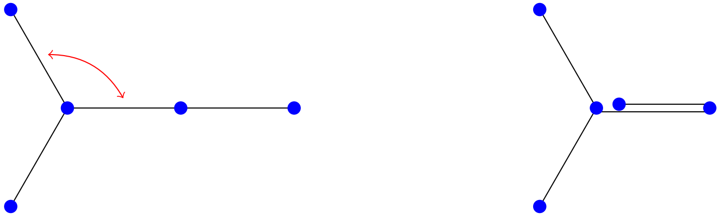

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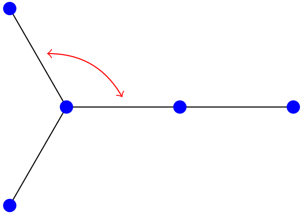

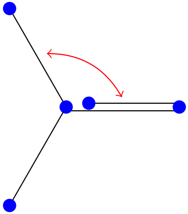

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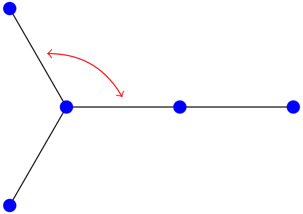

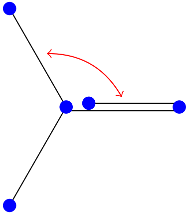

$\Rightarrow$ If you know the folding structure of a small complex,

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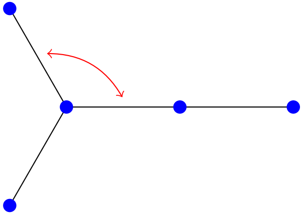

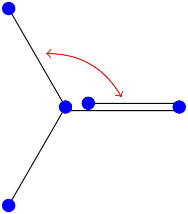

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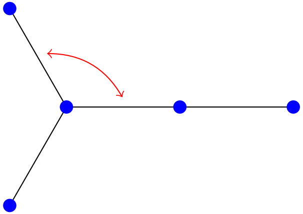

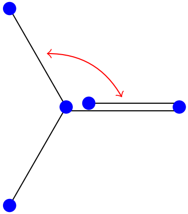

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$\rightsquigarrow$ Folding plans are not optimal to model folding.

Questions?