

Simplicial surfaces in GAP

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- 1 General polygonal complexes by incidence geometry
- 2 Edge colouring and group properties
- 3 Abstract folding

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Motivation

Goal: simplicial surfaces (and generalisations) in GAP



⇝ examples of **polygonal complexes**

No embedding

We do not work with embeddings (mostly)

- is very hard to compute
- if often unknown for an abstractly constructed surface
- is different from *intrinsic structure*

⇒ lengths and angles are not important

↪ incidence structure is intrinsic

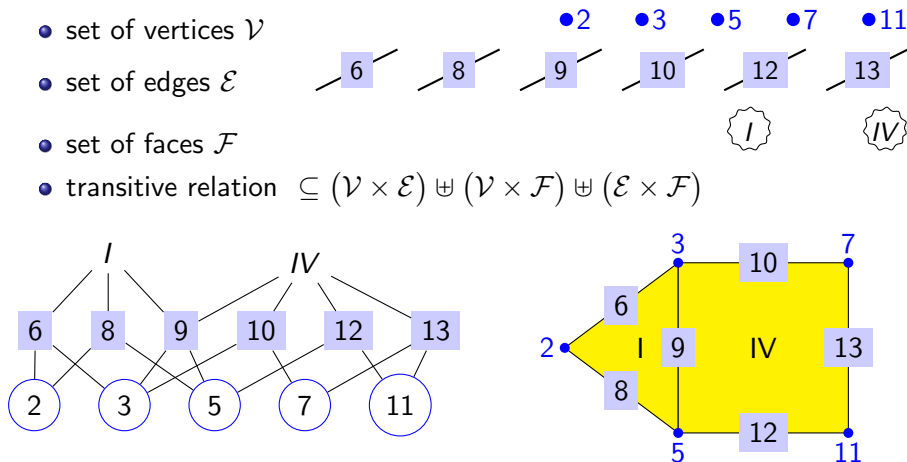
Incidence structure of polygonal complex

- set of vertices \mathcal{V}

- set of edges \mathcal{E}

- set of faces \mathcal{F}

- transitive relation $\subseteq (\mathcal{V} \times \mathcal{E}) \uplus (\mathcal{V} \times \mathcal{F}) \uplus (\mathcal{E} \times \mathcal{F})$



- 1 Every edge has exactly two vertices

- 2 Every face is a polygon

- 3 Every vertex lies in an edge and every edge lies in a face

Polygonal complexes

A **polygonal complex** is a two-dimensional incidence structure of vertices, edges and faces, such that:

① Every edge has exactly two vertices.



② Every face is a polygon.



③ Every vertex lies in an edge

④ Every edge lies in a face

Isomorphism testing

Incidence geometry allows "easy" isomorphism testing. Incidence structure can be interpreted as a coloured graph:



↪ reduce to graph isomorphism problem

Solved by NautyTracesInterface (by Gutsche, Niemeyer, Schweitzer)

General properties

Some properties can be computed for all polygonal complexes:

- Connectivity
- Euler–Characteristic

Orientability is **not** one of them. Counterexample:

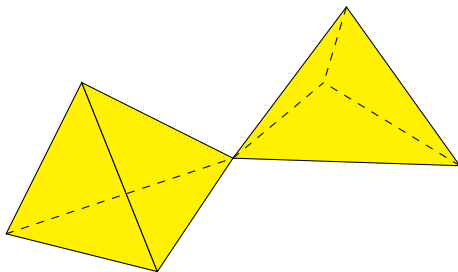


⇒ every edge lies in at most two faces (for well-definedness)

⇝ **ramified polygonal surfaces**

Why ramified?

Typical example of ramified polygonal surface:



⇒ It is not a surface – there is a *ramification* at the central vertex
A **polygonal surface** does not have these ramifications.

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Embedding question

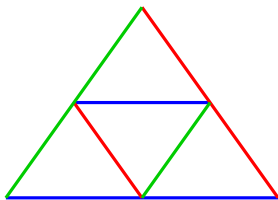
Given: A polygonal complex

- Can it be embedded?
- In how many ways?

Simplifications:

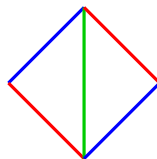
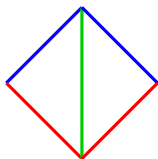
- 1 Only polygonal surfaces (surface that is build from polygons)
- 2 All polygons are triangles (**simplicial surfaces**)
- 3 All triangles are isometric

⇒ Edge-colouring encodes different lengths



How do faces fit together?

Consider a face of the surface and a neighbouring face
The neighbour can be coloured in two ways:



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