Simplicial surfaces in GAP

Markus Baumeister

30.08.2017

Motivation

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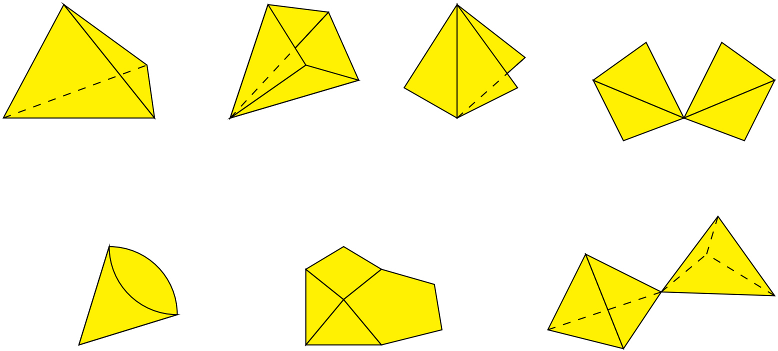

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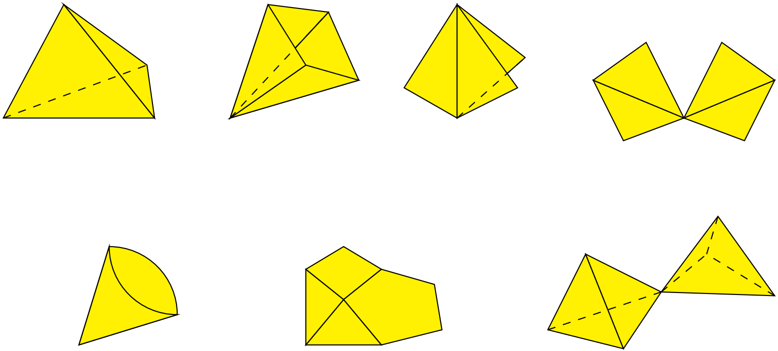

$\rightsquigarrow$ examples of **polygonal complexes**

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$\rightsquigarrow$ incidence structure is intrinsic

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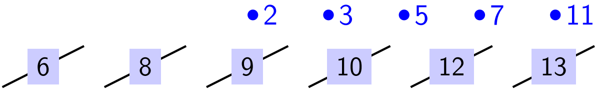

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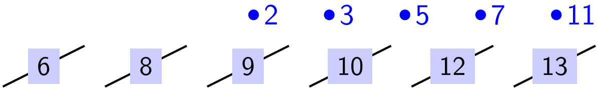

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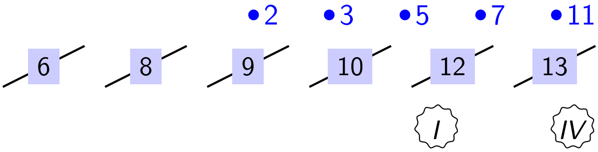

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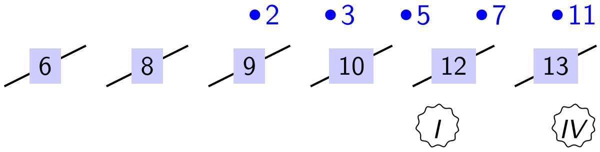

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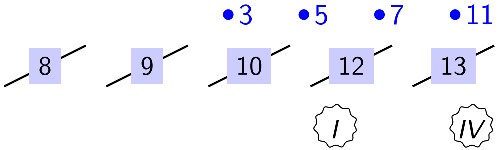

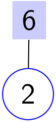

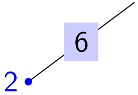

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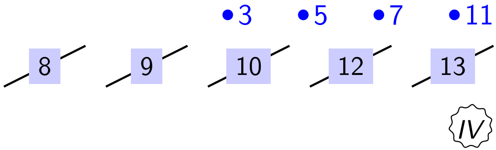

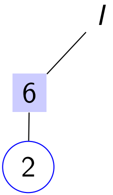

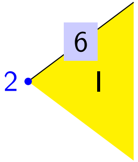

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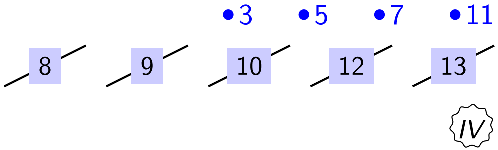

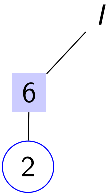

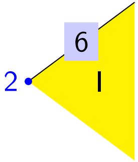

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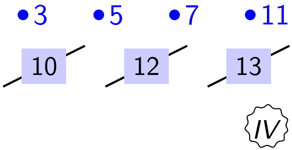

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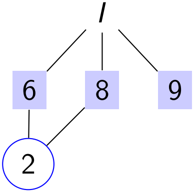

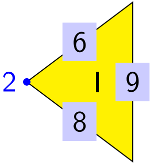

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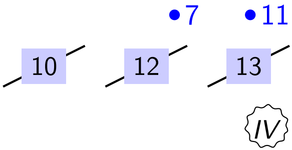

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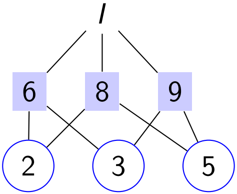

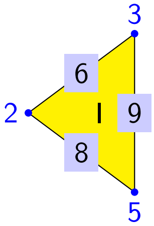

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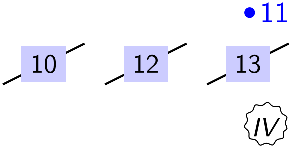

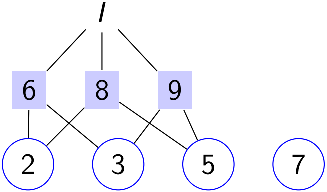

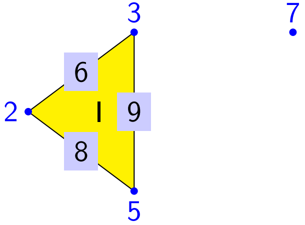

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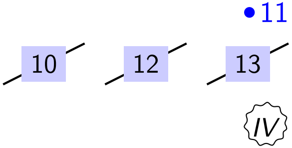

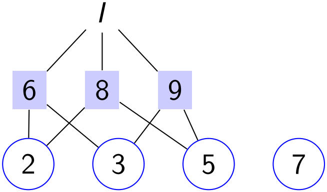

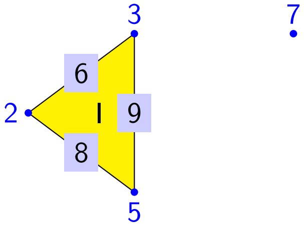

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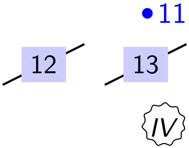

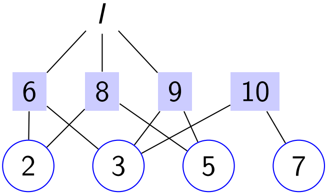

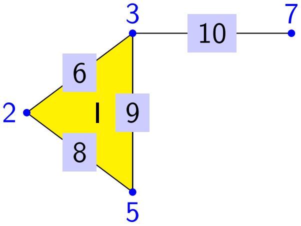

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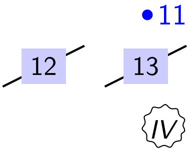

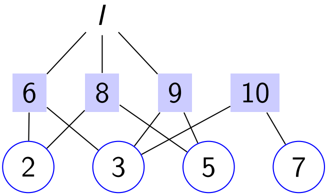

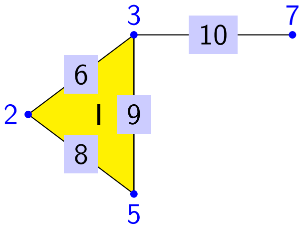

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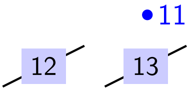

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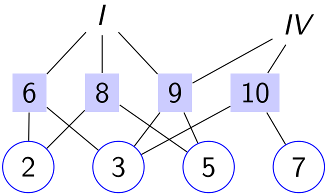

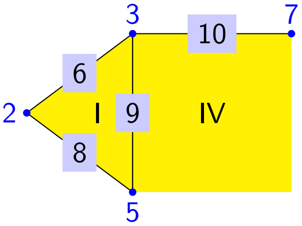

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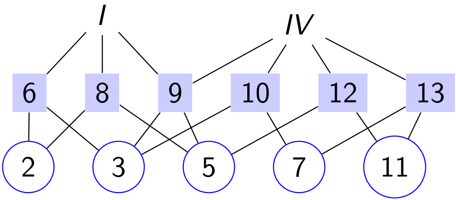

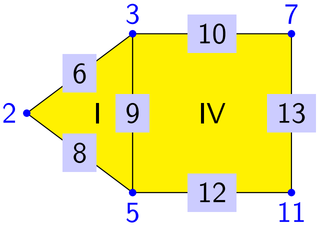

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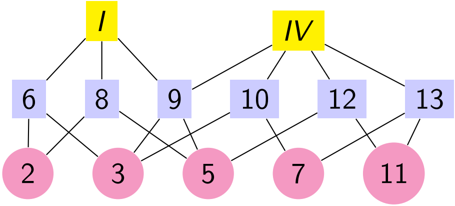

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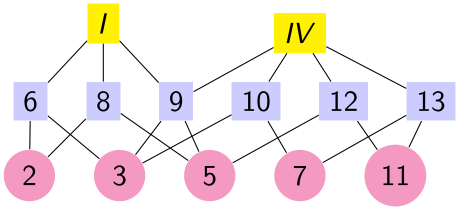

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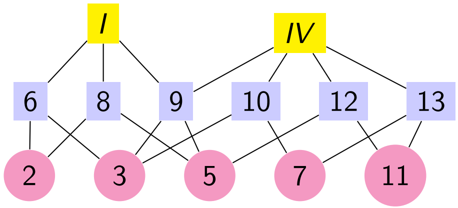

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Solved by NautyTracesInterface (by Gutsche, Niemeyer, Schweitzer)

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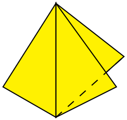

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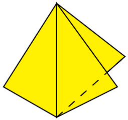

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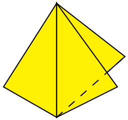

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$\rightsquigarrow$ **ramified polygonal surfaces**

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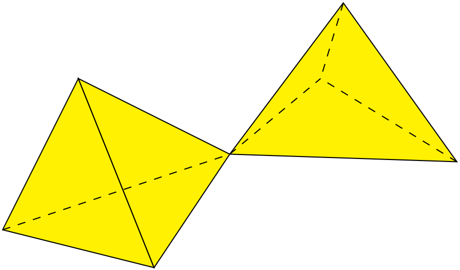

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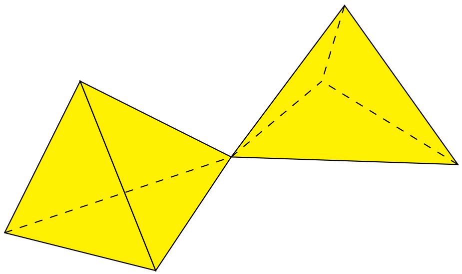

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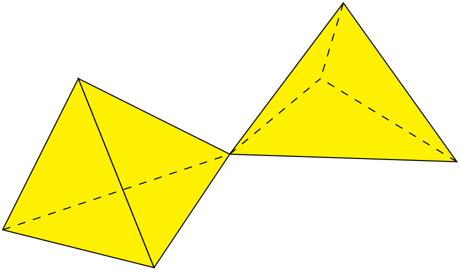

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A **polygonal surface** does not have these ramifications.

1. General polygonal complexes by incidence geometry
2. Edge colouring and group properties
3. Abstract folding

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Simplifications:

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2. All polygons are triangles (**simplicial surfaces**)
3. All triangles are isometric

Embedding question

Given: A polygonal complex

- Can it be embedded?
- In how many ways?

Simplifications:

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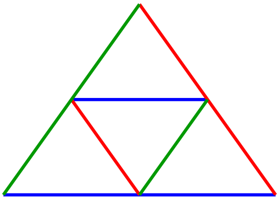
$\rightsquigarrow$ Edge–colouring encodes different lengths



Colouring as permutation

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Consider tetrahedron

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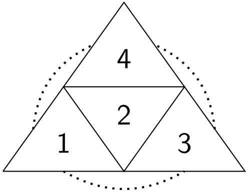

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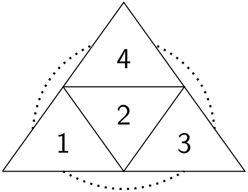

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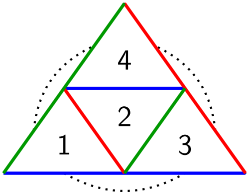

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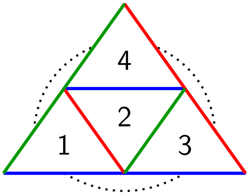

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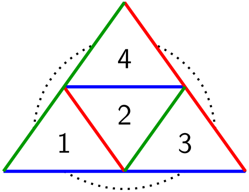

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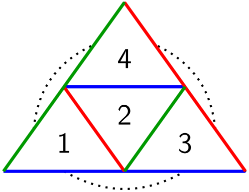

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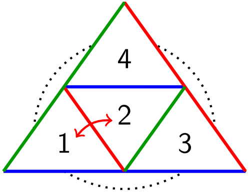

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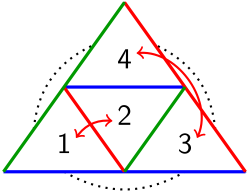

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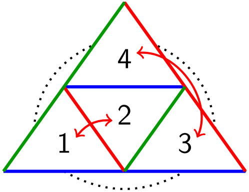

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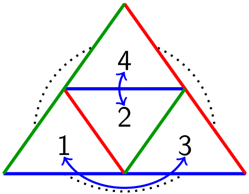

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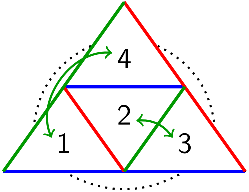

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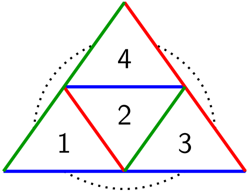

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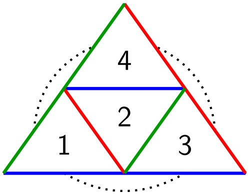

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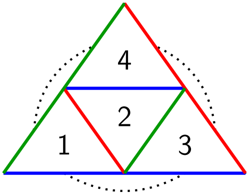

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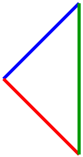

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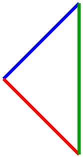

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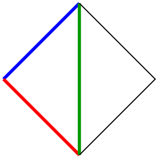

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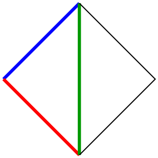

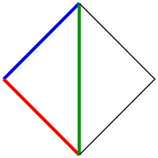

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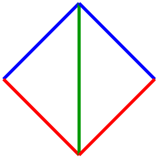

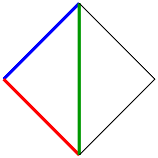

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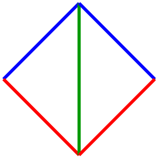

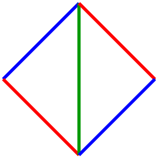

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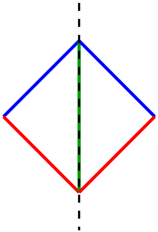
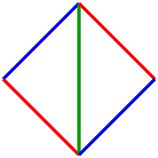


mirror (m)



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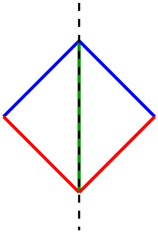

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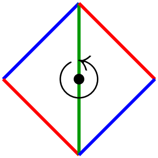

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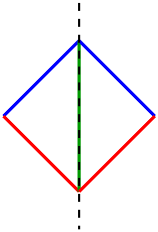

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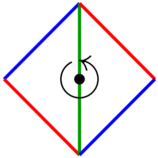

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This gives an **mr–assignment** for the edges.

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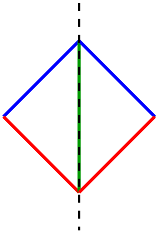

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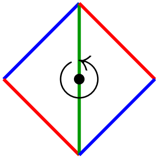

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Permutations and mr–assignment uniquely determine the surface.

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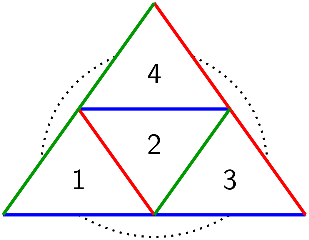

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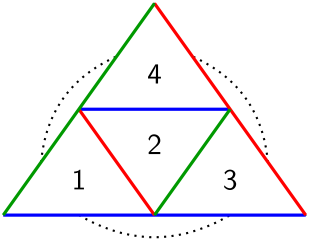

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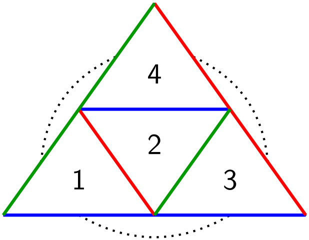

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The easiest structure is an mmm–structure.

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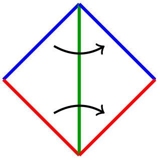

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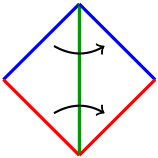

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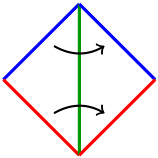

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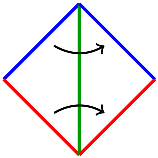

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- Colouring defines a map to the triangle.

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Start with three involutions σ_a, σ_b, σ_c

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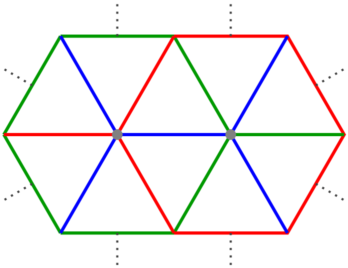

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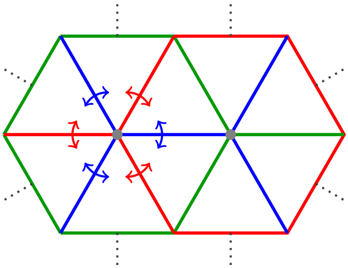

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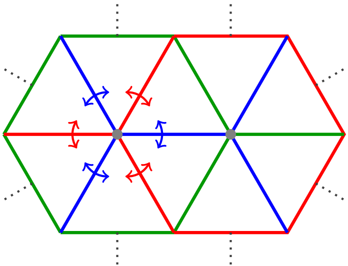

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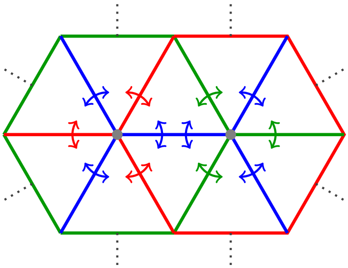

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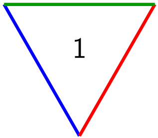

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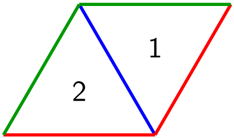

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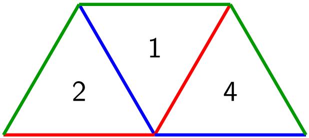

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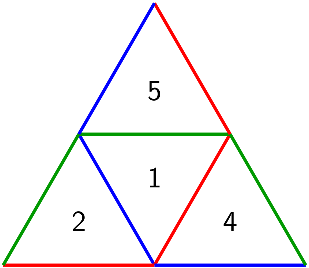

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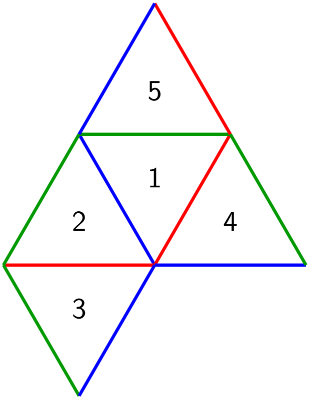

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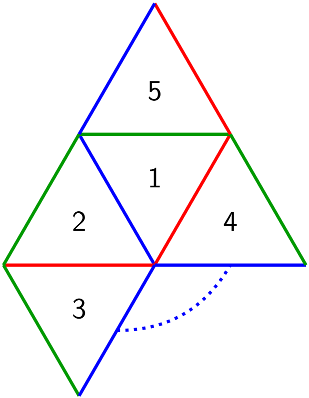

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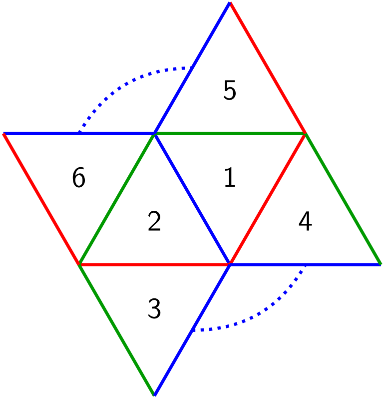

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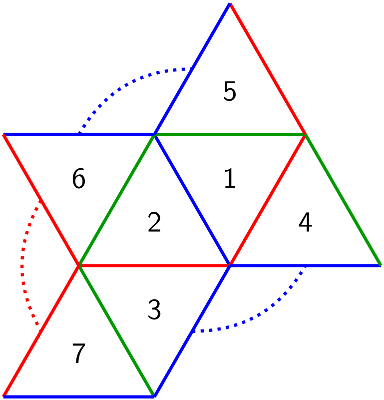

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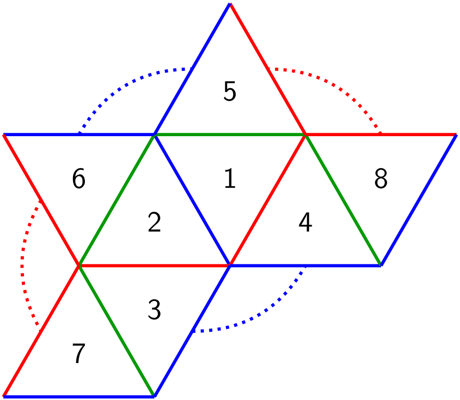

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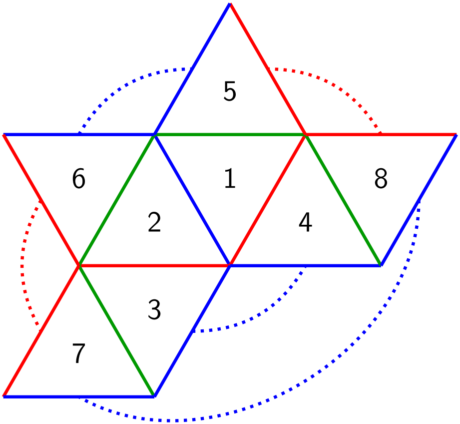

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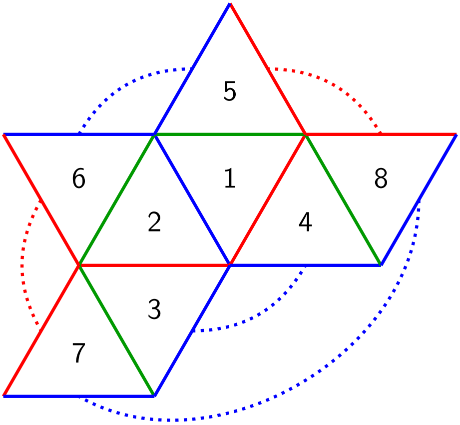

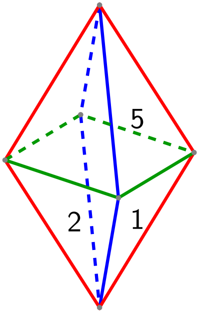

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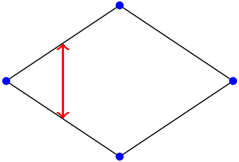

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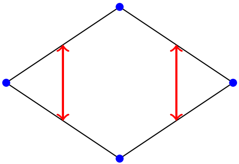

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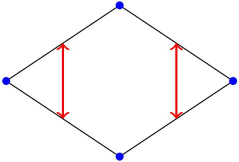

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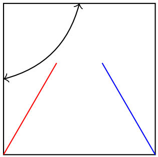

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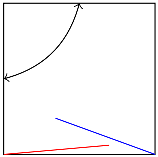

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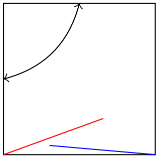

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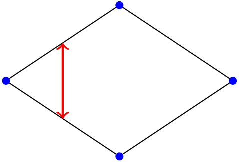

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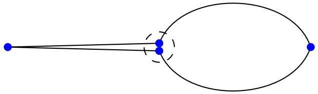

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Unordered Folding (Covering)

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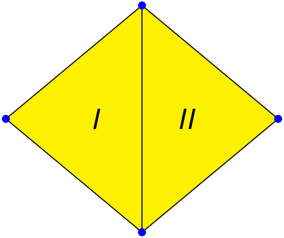

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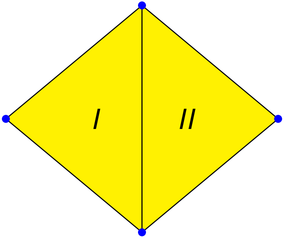

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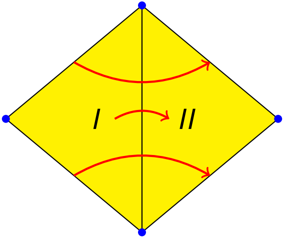

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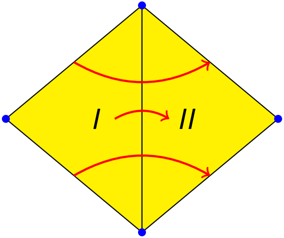

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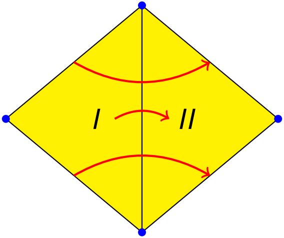

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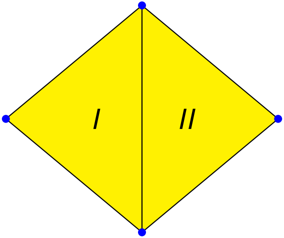

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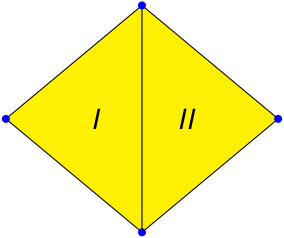

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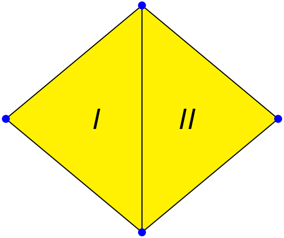

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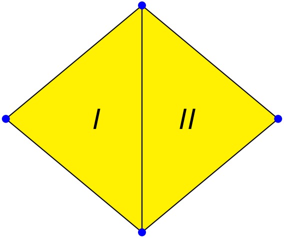

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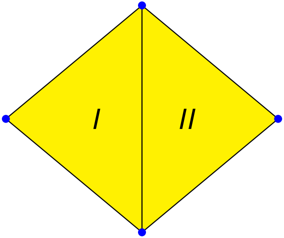

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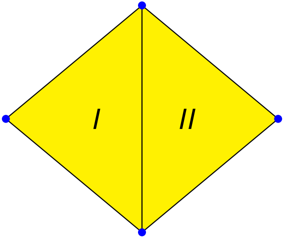

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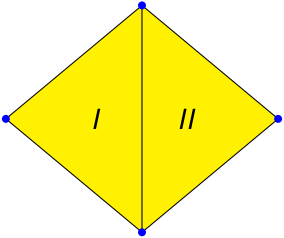

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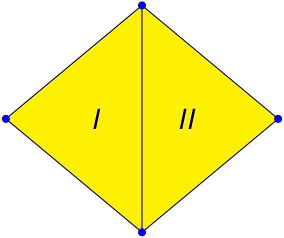

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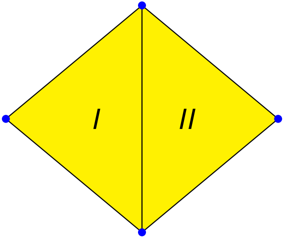

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$\Rightarrow$ Unordered folding is coarsening of equivalence relation

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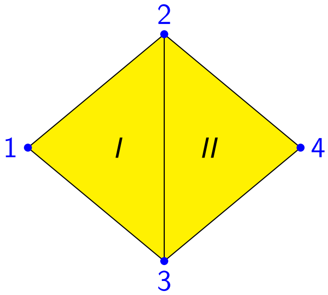

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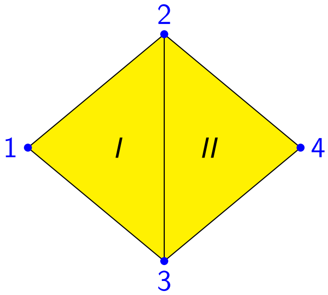

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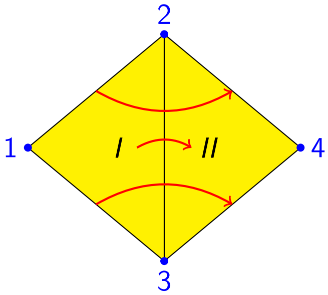

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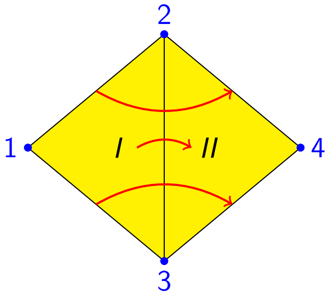

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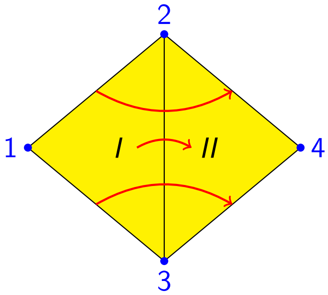
3. Add those pairs to the equivalence relation



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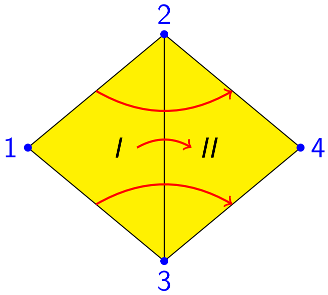

Only restriction:

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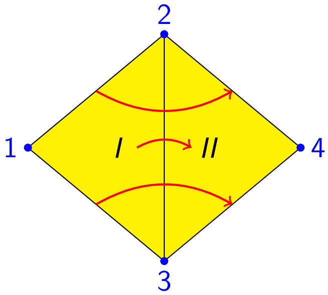

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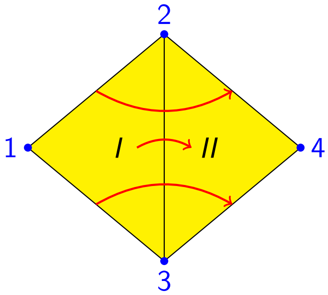

Only restriction:
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Limitation of unordered folding

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We can't work with ordering of faces:

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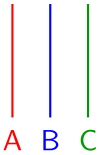

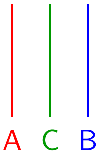

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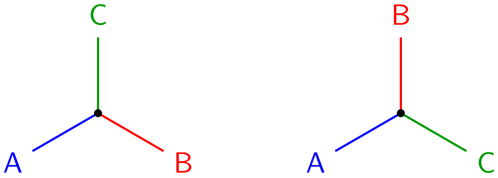

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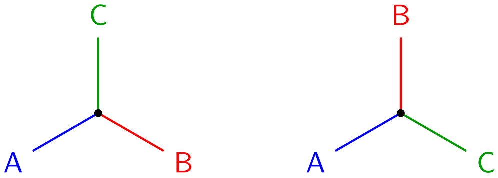

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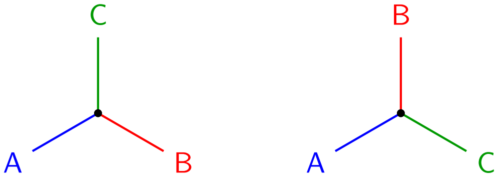

$\rightsquigarrow$ define order of faces around edges (we will skip the details)

Folding complex

Folding complex

Definition

A **folding complex**

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Before (no ordering):

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Changed definition of folding

Folding with ordering:

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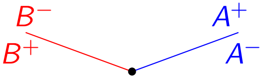

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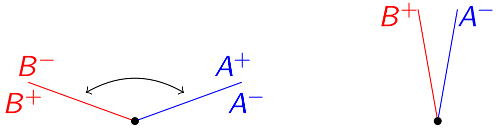
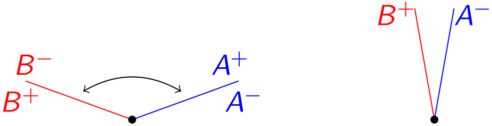


Changed definition of folding

Folding with ordering:

1. Choose two faces that are not folded together
2. Choose how to identify them and extend the equivalence relation
3. Choose the sides of the faces that will meet

$\rightsquigarrow$ Each face has two sides



Changed definition of folding

Folding with ordering:

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3. Choose the sides of the faces that will meet and modify the orderings

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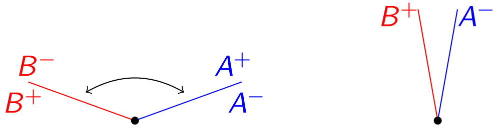

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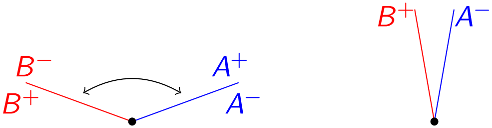

$\Rightarrow$ Define folding by two face sides (**folding plan**)

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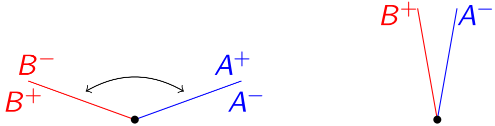

$\Rightarrow$ Define folding by two face sides (**folding plan**)
$\rightsquigarrow$ Allows reversible (un)folding

Structure of multiple foldings

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With folding plans we can perform the same folding in different folding complexes

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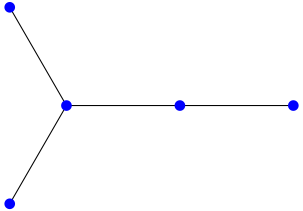

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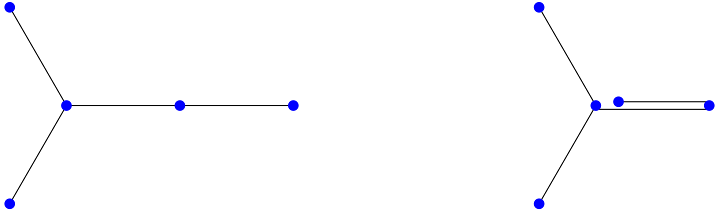

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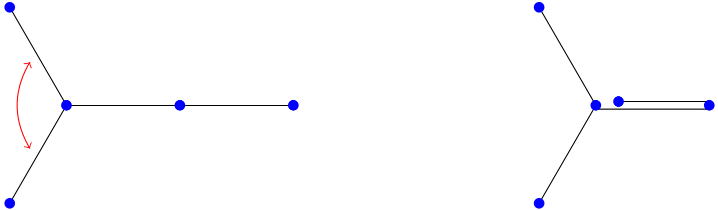

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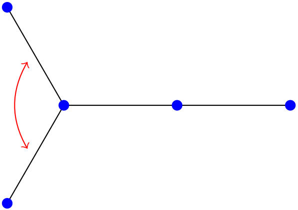

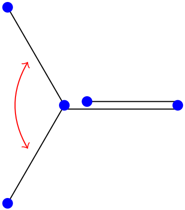

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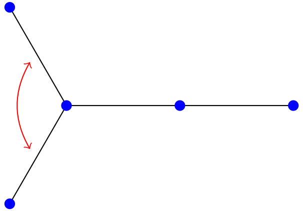

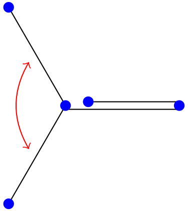

$\rightsquigarrow$ more structure on the set of possible foldings

Folding graph

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- Vertices are folding complexes (modelling folding states)

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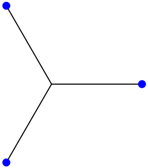

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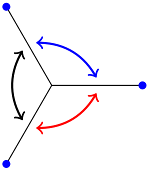

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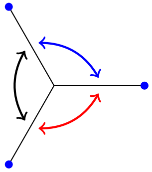

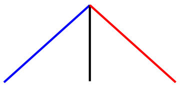

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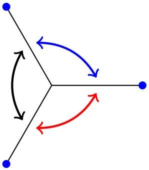

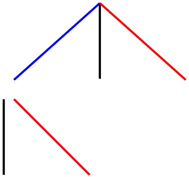

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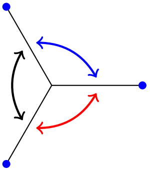

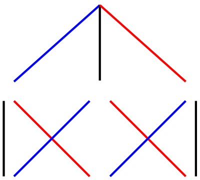

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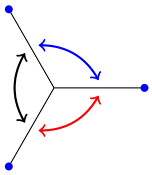

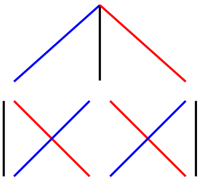

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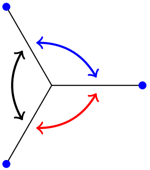

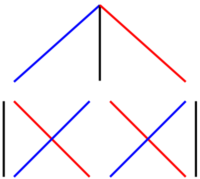

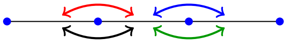

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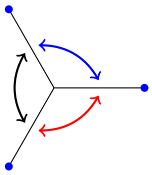

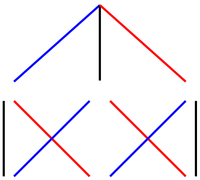

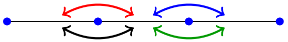

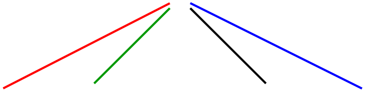

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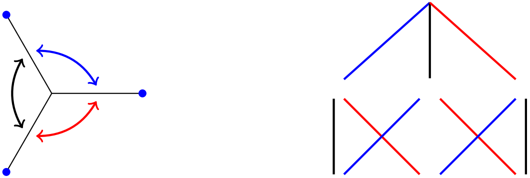

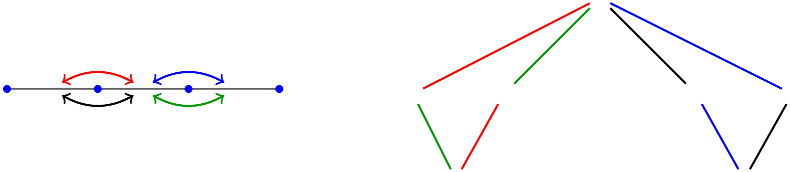

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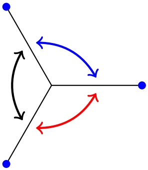

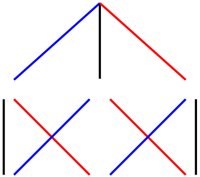

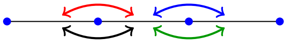

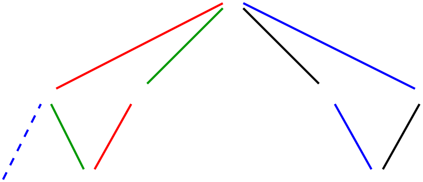

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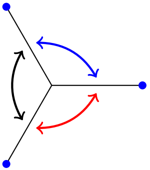

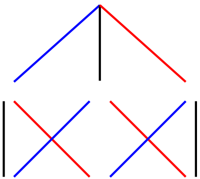

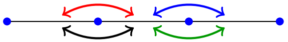

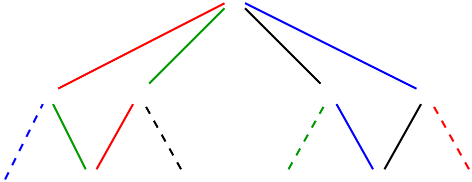

Drawback of folding plans

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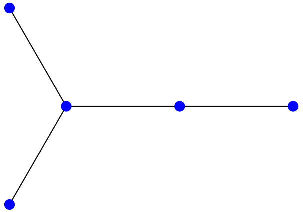

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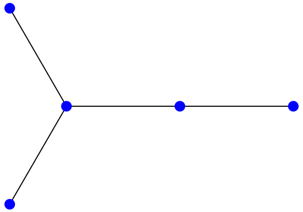

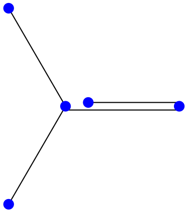

Drawback of folding plans

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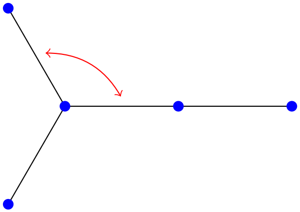

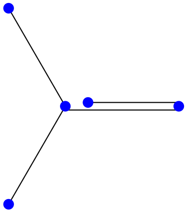

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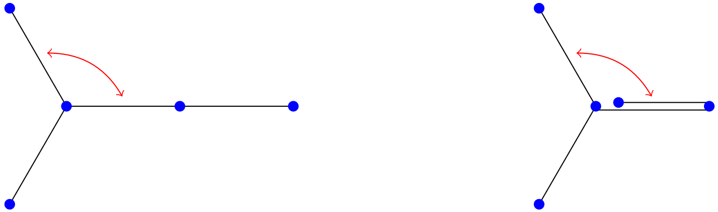

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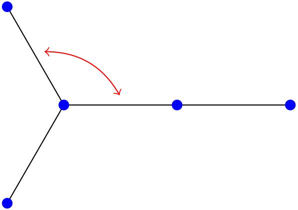

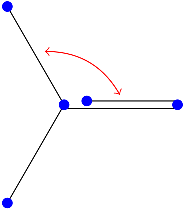

$\Rightarrow$ If you know the folding structure of a small complex,

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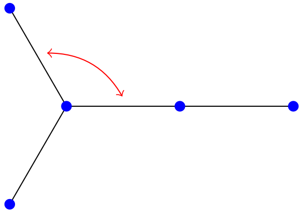

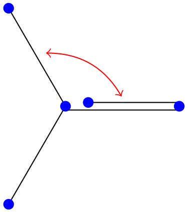

$\Rightarrow$ If you know the folding structure of a small complex, you can't easily find the folding structure of an extended complex

Drawback of folding plans

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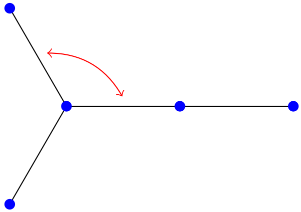

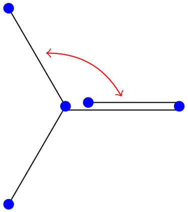

$\Rightarrow$ If you know the folding structure of a small complex, you can’t easily find the folding structure of an extended complex

$\rightsquigarrow$ Folding plans are not optimal to model folding.

Questions?