Simplicial surfaces in GAP

Markus Baumeister
(j/w Alice Niemeyer)

Lehrstuhl B für Mathematik
RWTH Aachen University

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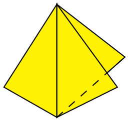

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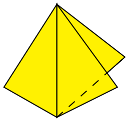

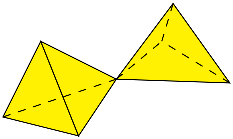

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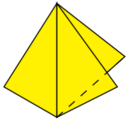

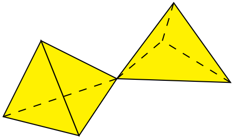

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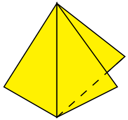

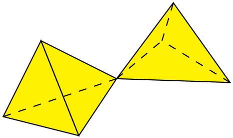

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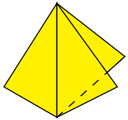

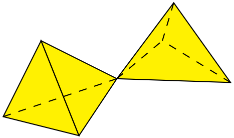

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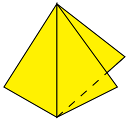

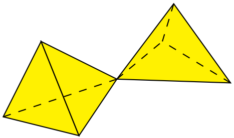

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$\rightsquigarrow$ incidence geometry

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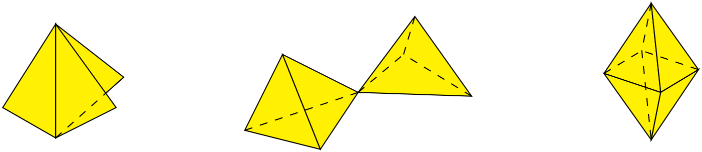

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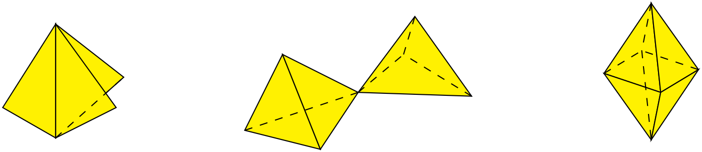

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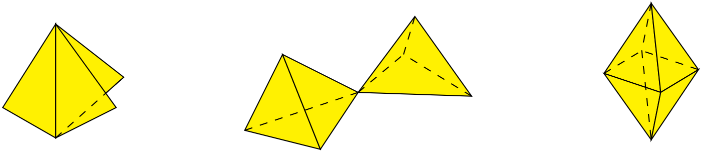

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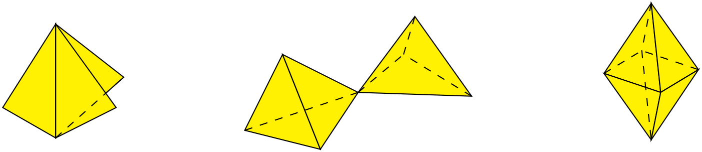

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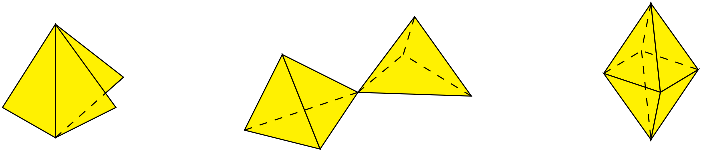

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  - we only have two dimensions but can work with colourings and foldings

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2. Edge colouring and group properties

3. Abstract folding

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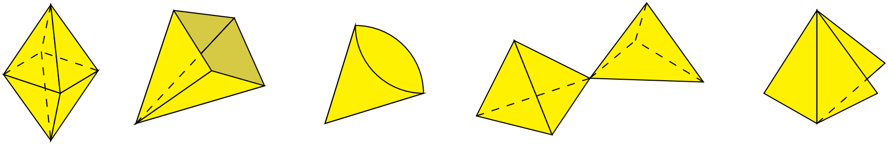

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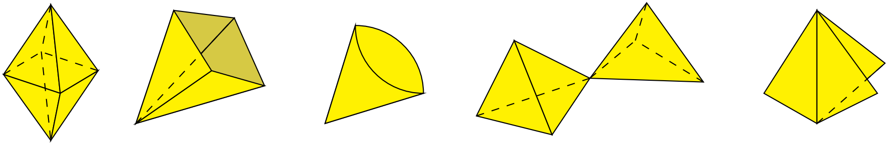

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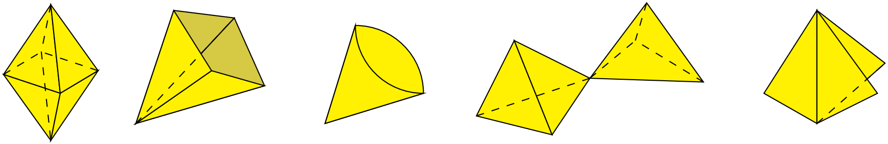

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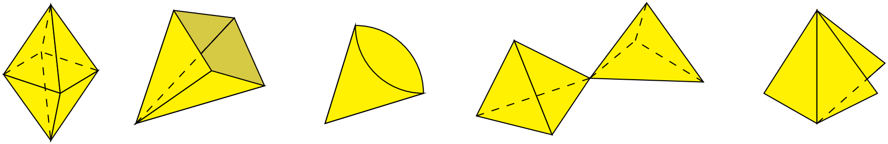

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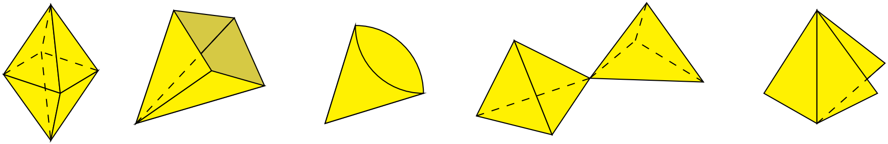

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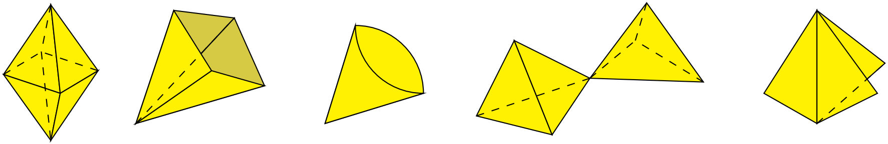

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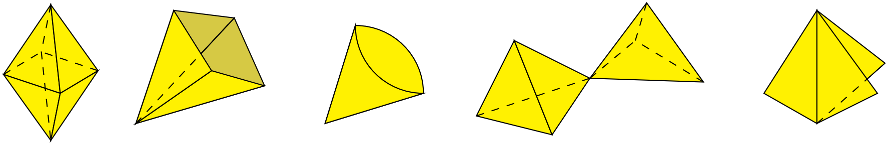

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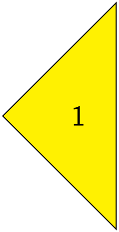

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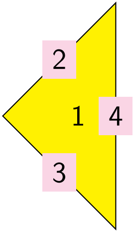

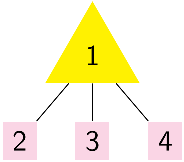

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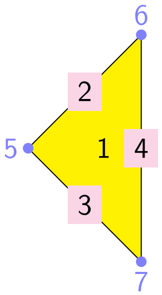

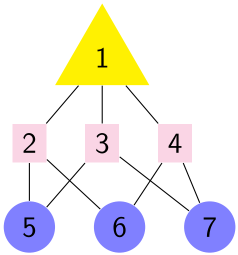

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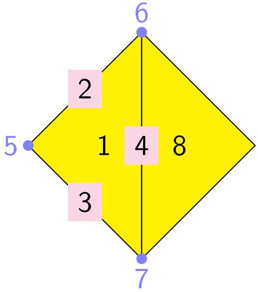

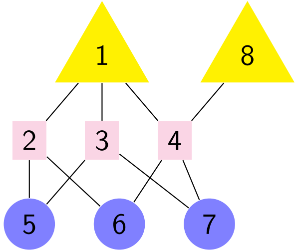

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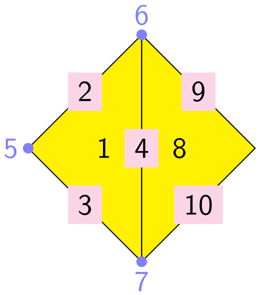

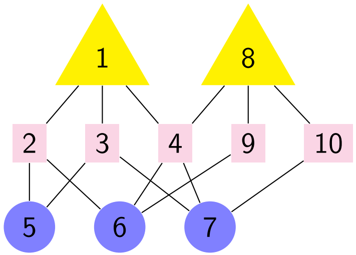

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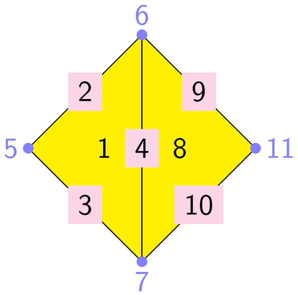

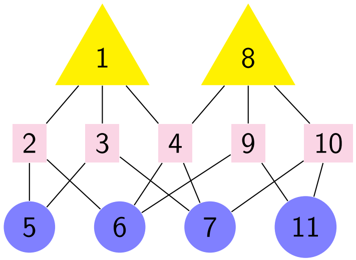

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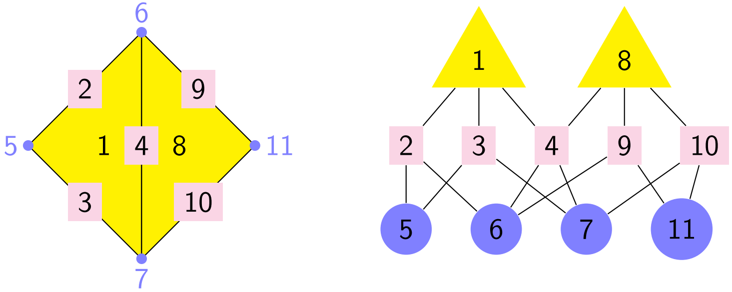

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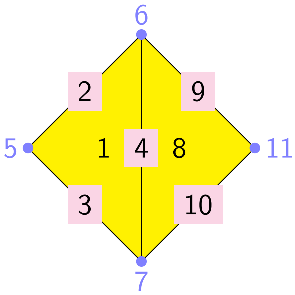

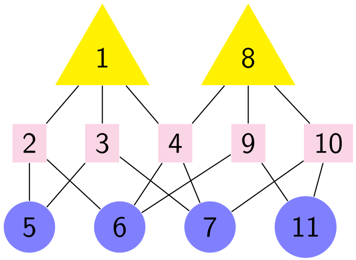

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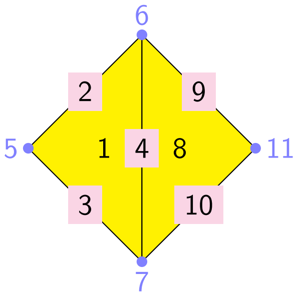

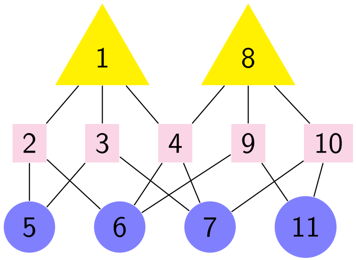

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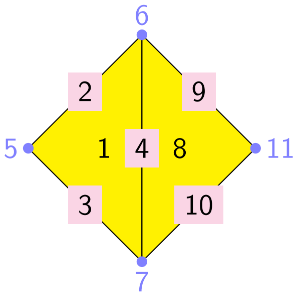

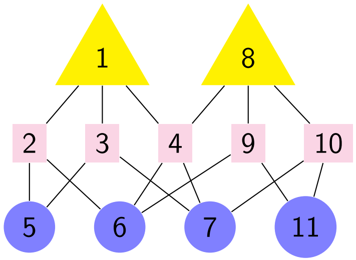

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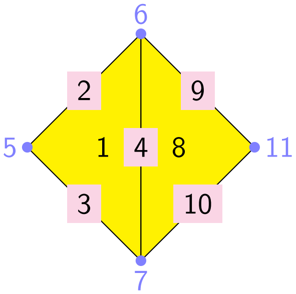

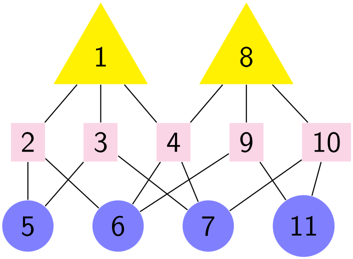

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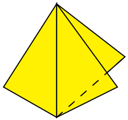

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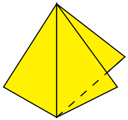

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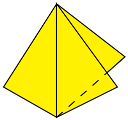

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$\rightsquigarrow$ **ramified simplicial surfaces**

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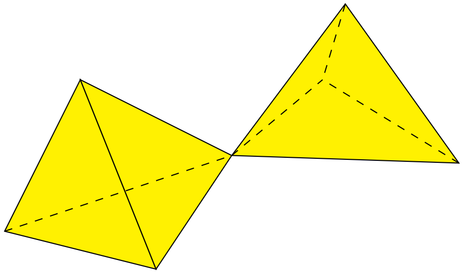

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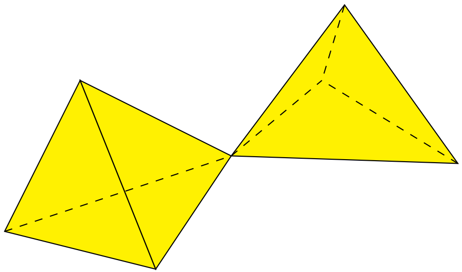

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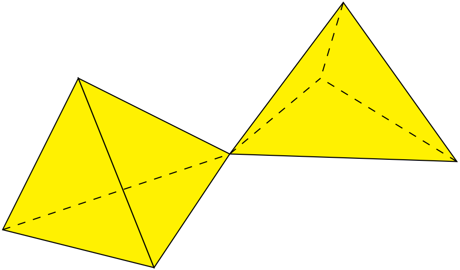

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A **simplicial surface** does not have these ramifications.

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- building blocks are those without a 3–cycle of edges (not around face)

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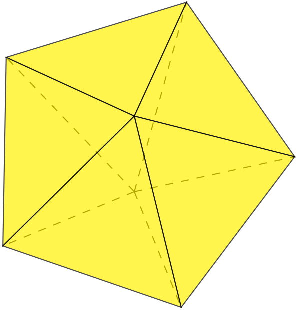

Progress report of triangular complexes

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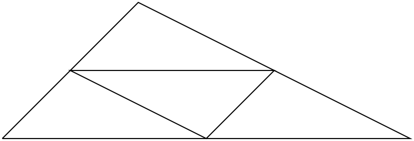

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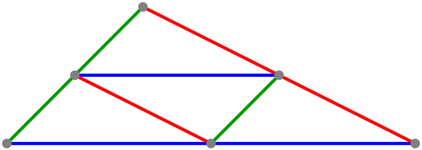

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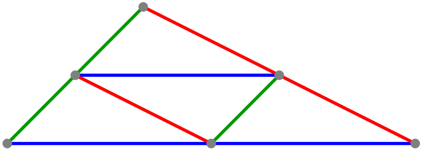

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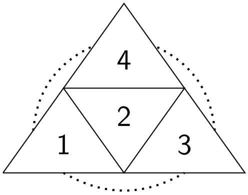

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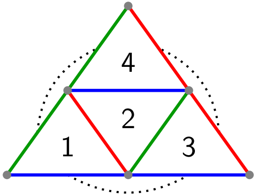

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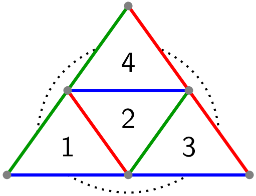

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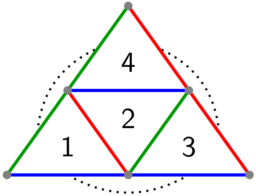

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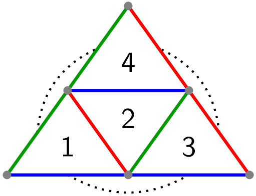

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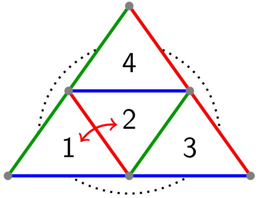

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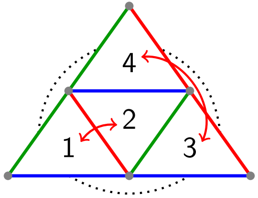

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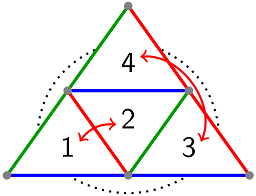

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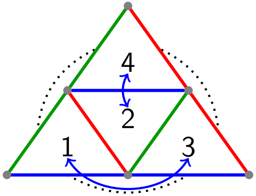

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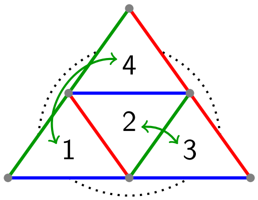

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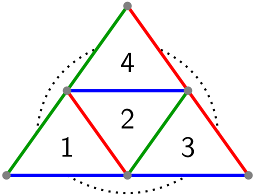

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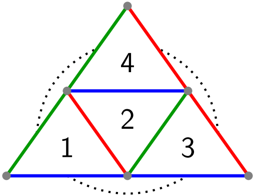

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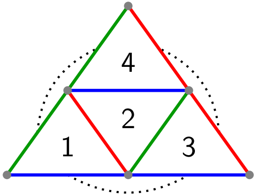

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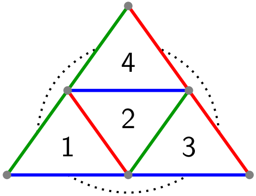

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  - The connected components of the surface correspond to the orbits of $\langle \sigma_a, \sigma_b, \sigma_c \rangle$ on the faces (fast computation for permutation groups)

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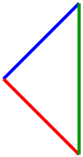
Consider a face of the surface



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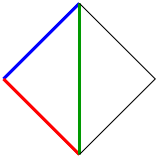

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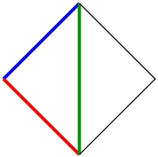

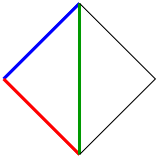

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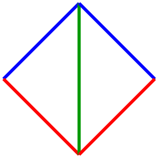

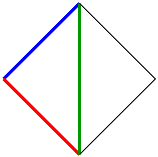

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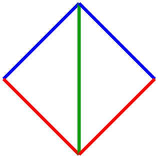

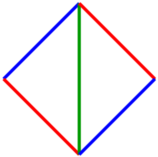

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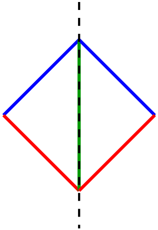
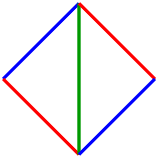


mirror (m)



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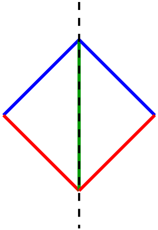

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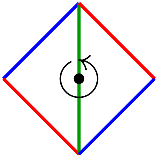

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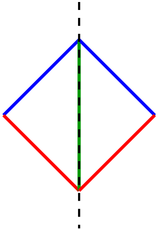

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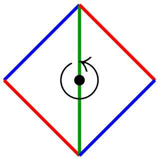

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Satz

Permutations and mr–assignment uniquely determine the surface.

Constructing surfaces from groups

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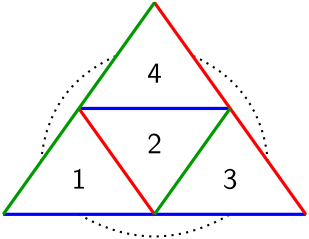

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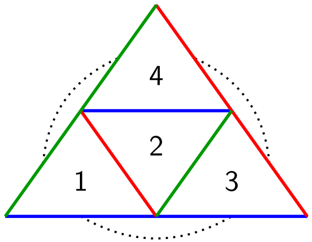

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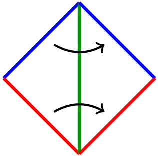

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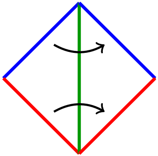

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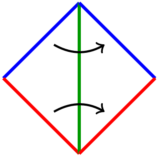

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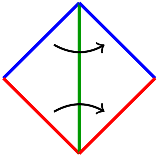

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- Mirror–colouring defines a map to the triangle

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Start with three involutions σ_a, σ_b, σ_c in permutation representation

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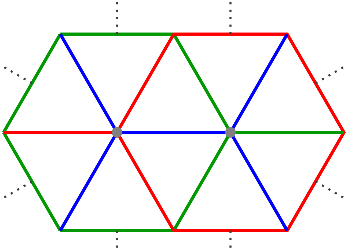

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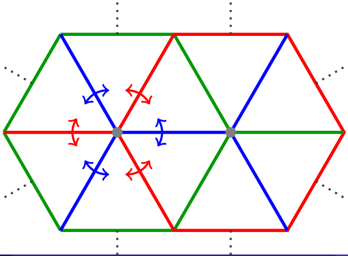

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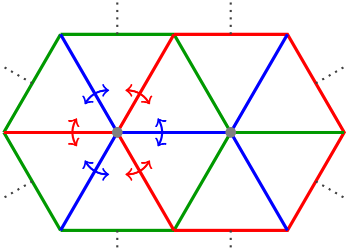


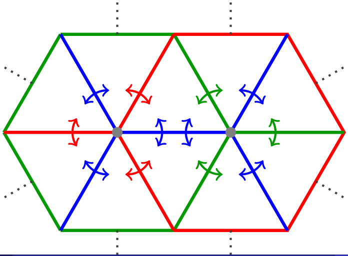
Construction from permutations

Start with three involutions σ_a, σ_b, σ_c in permutation representation (like generators of a finite group)

Satz

There exists a coloured surface with the given involutions where all edges are mirror edges.

- The faces are the points moved by the involution triple
- The edges are the cycles of the involutions
- The vertices are the orbits of $\langle \sigma_a, \sigma_b \rangle$ on the faces (for all pairs)



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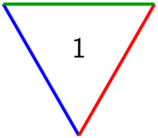

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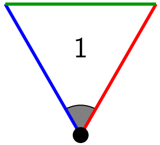

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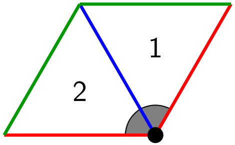

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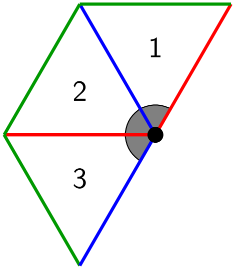

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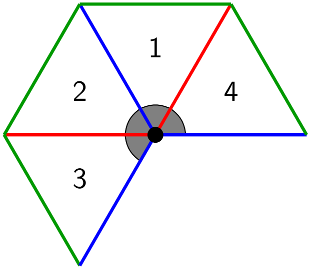

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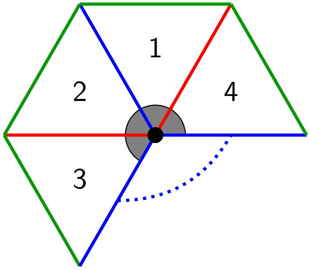

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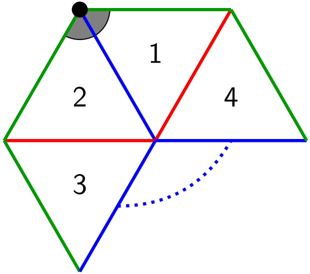

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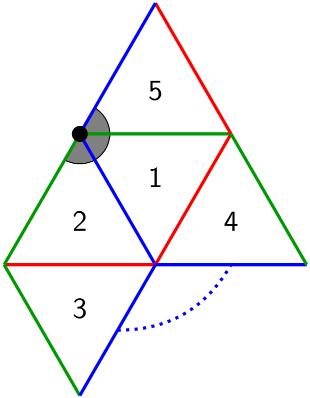

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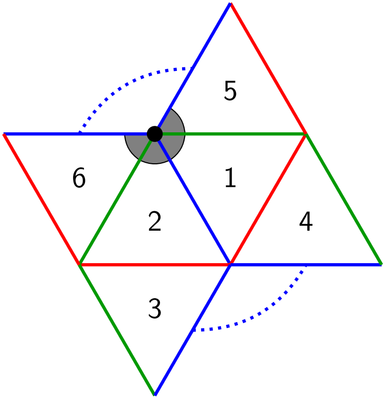

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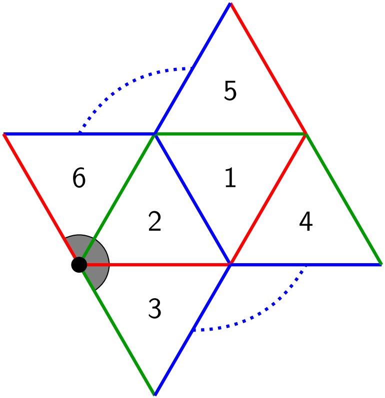

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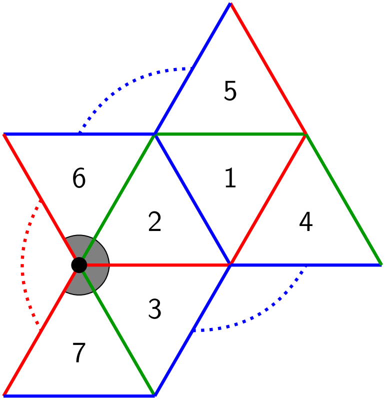

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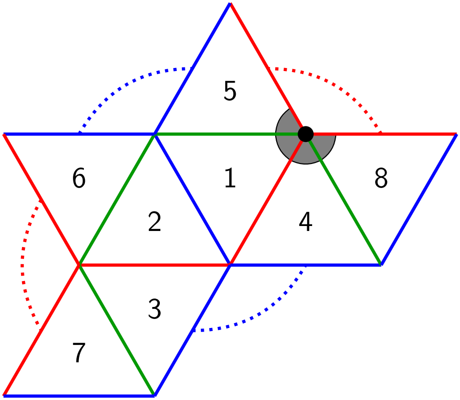

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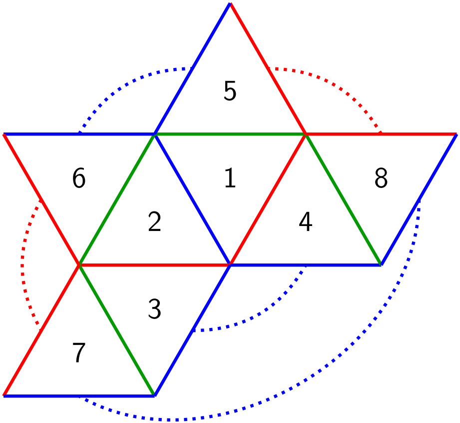

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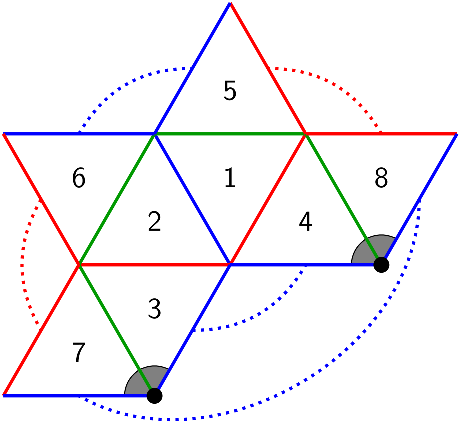

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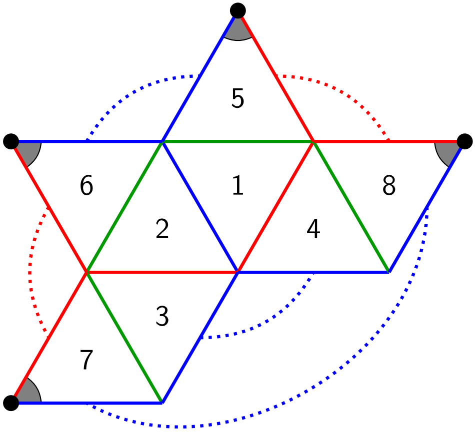

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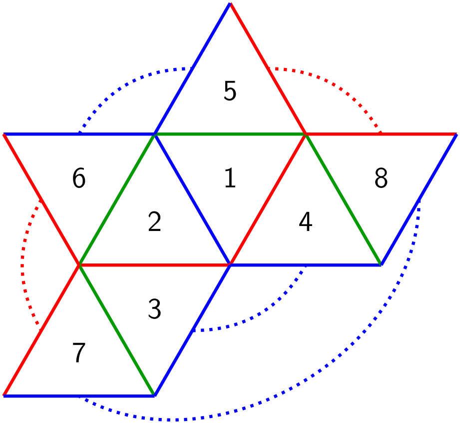

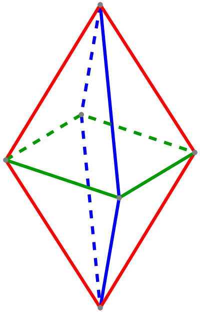

Net of an icosahedron

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`iko`: coloured icosahedron

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gap> DrawSurfaceToTikZ(iko,"NetIko.tex");
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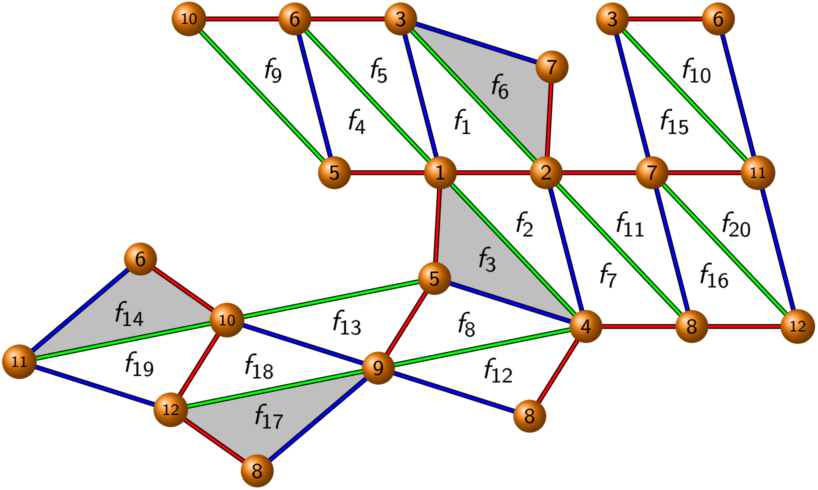

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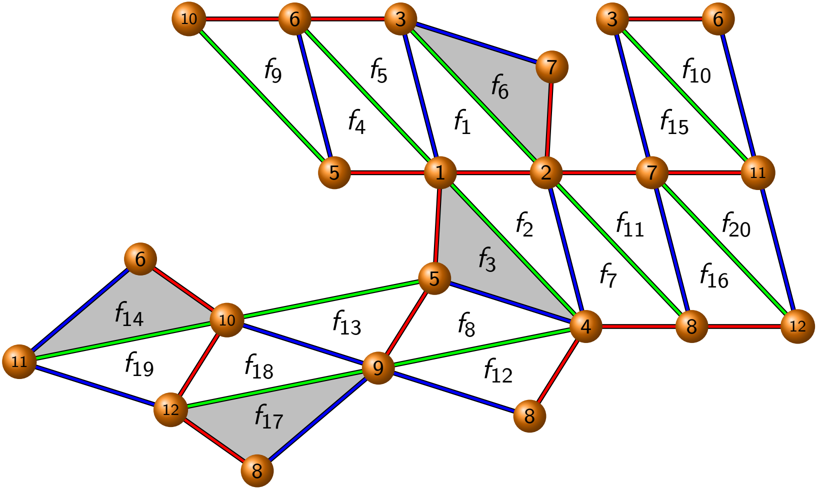

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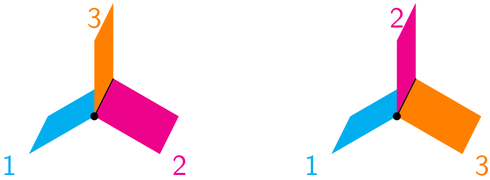

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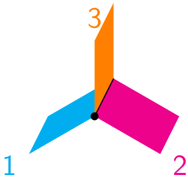

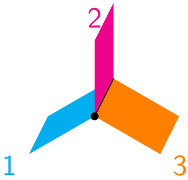

$\rightsquigarrow$ **folding complex**

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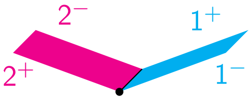

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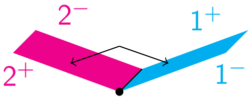

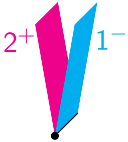

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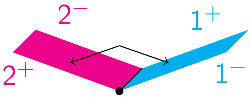

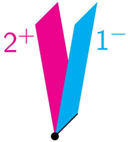

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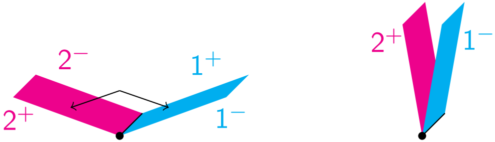

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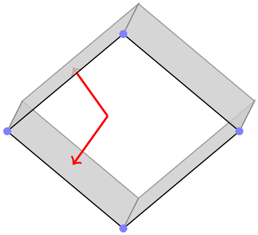

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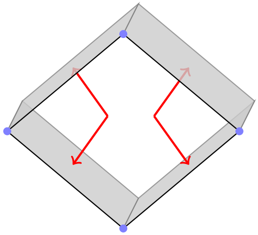

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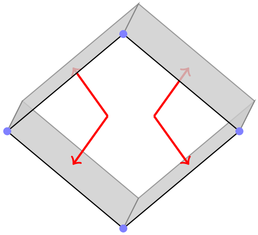

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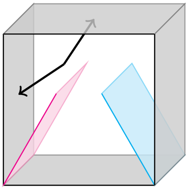

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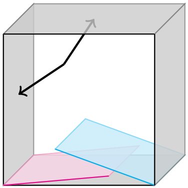

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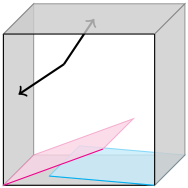

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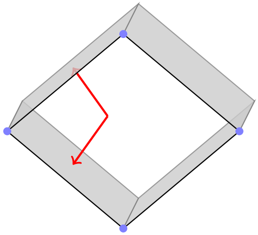

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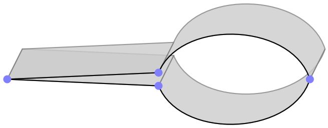

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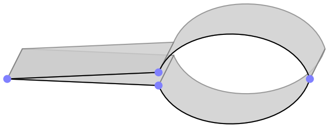
  $\rightsquigarrow$ Relax the rigidity–constraint:



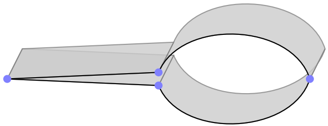
How does folding plan work?

Folding of two faces can induce folding of other faces:

- Can apply to arbitrarily many faces
- The induced folding is not unique

$\Rightarrow$ Identify only two faces at a time

  - $\rightsquigarrow$ Relax the rigidity–constraint:
  - Allow non–rigid configurations as transitional states



Structure of multiple foldings

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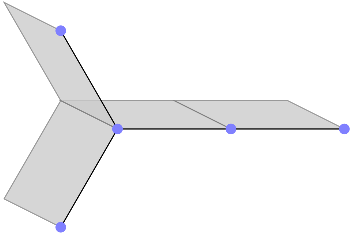

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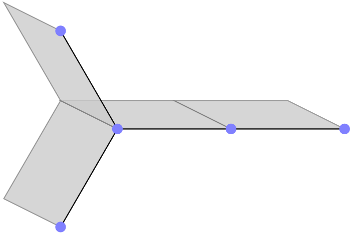

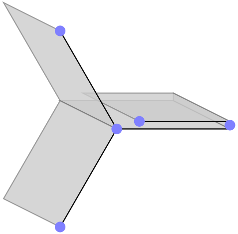

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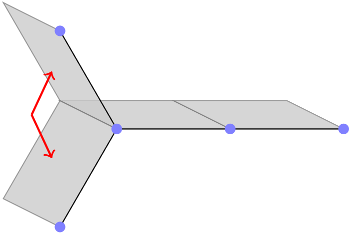

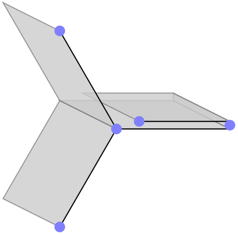

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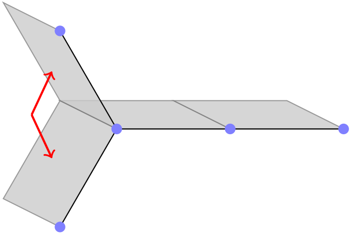

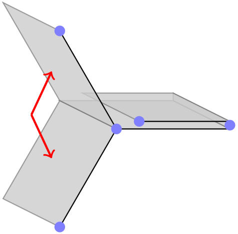

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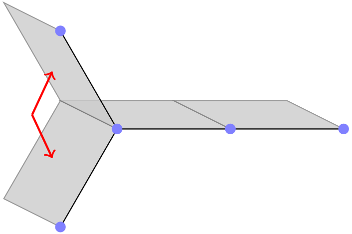

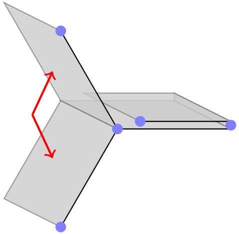

$\rightsquigarrow$ more structure on the set of possible foldings

Folding graph

Folding graph

- Vertices are folding complexes (modelling folding states)

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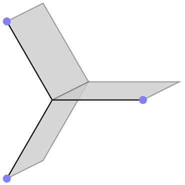

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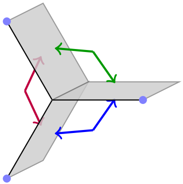

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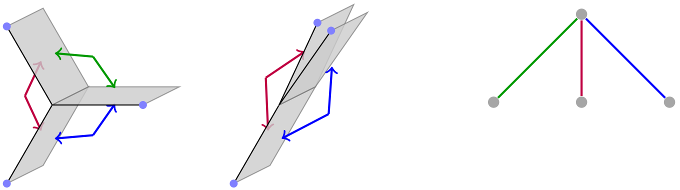

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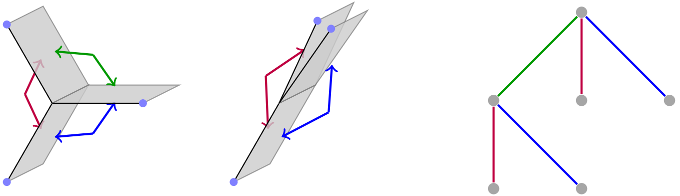

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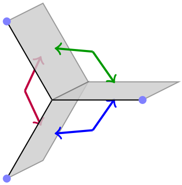

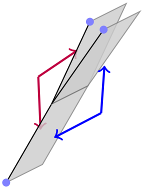

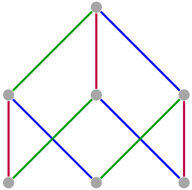

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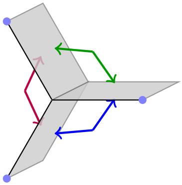

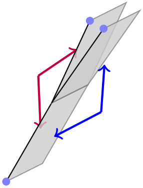

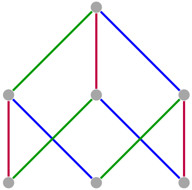

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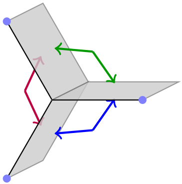

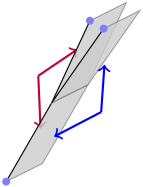

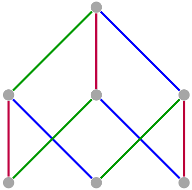

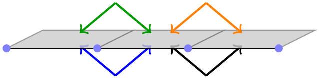

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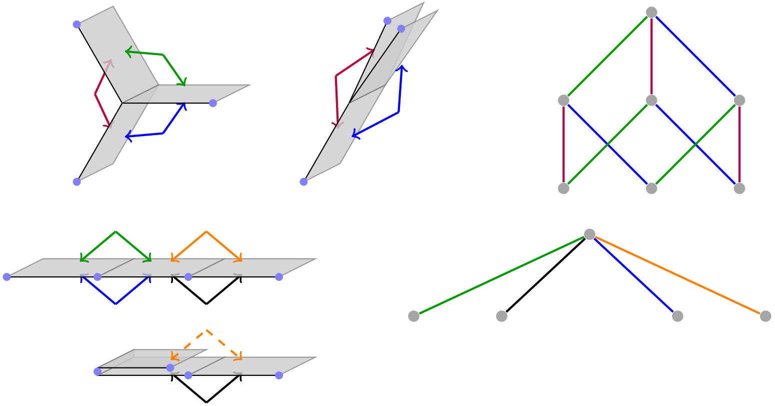

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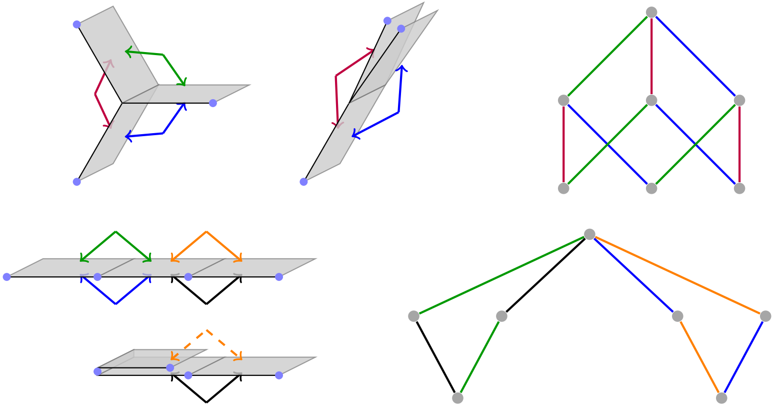

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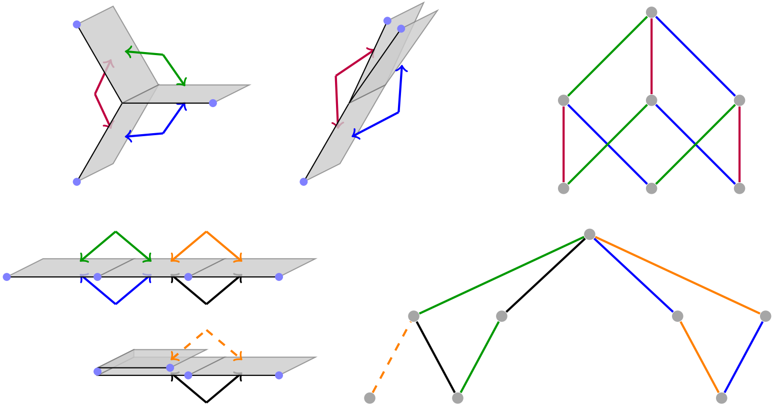

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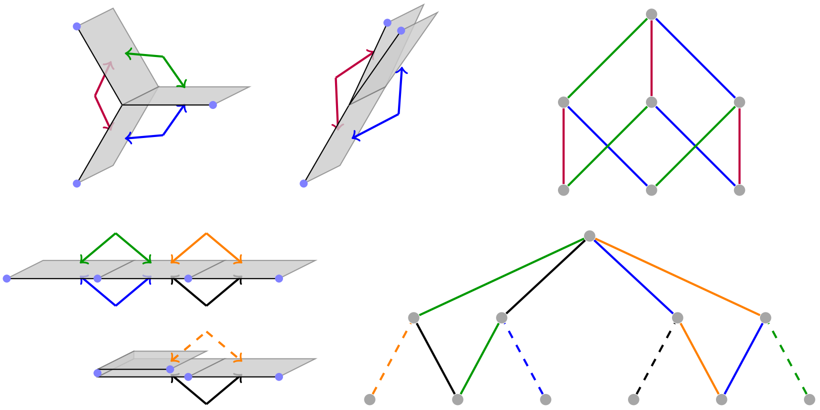

Larger graph

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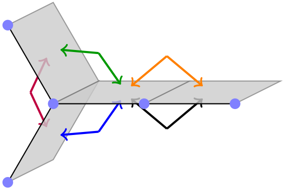

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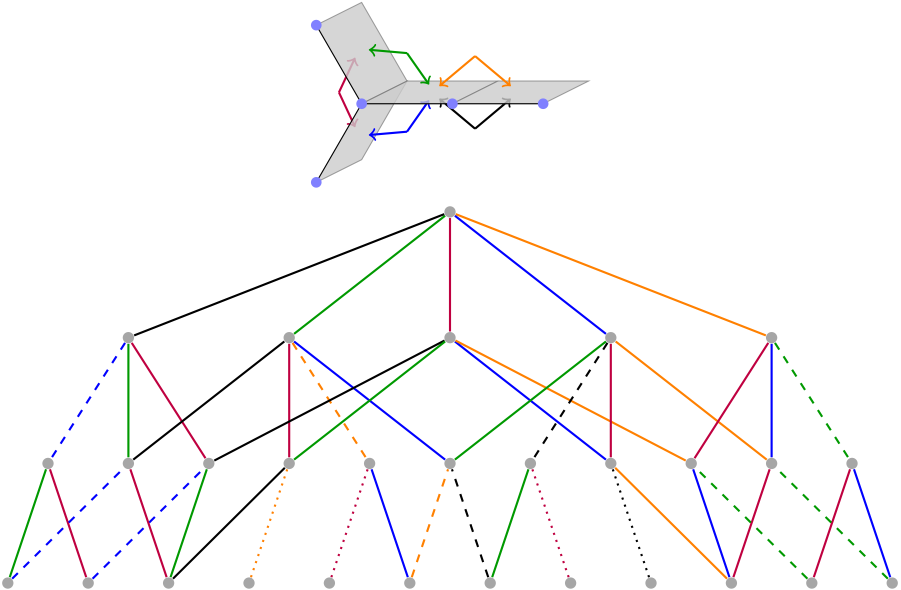

Drawback of folding plans

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Some foldings that "should" be the same, aren't:

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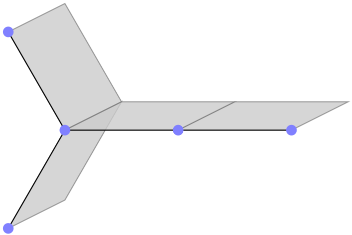

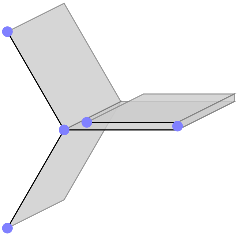

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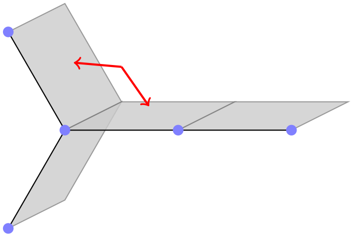

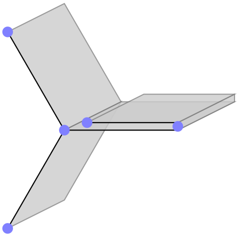

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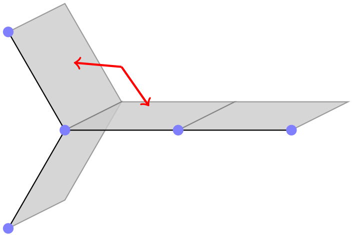

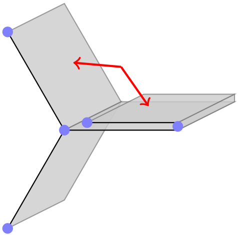

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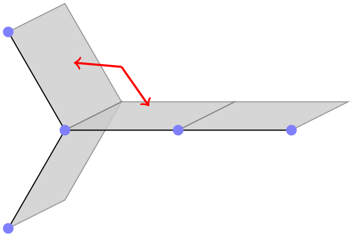

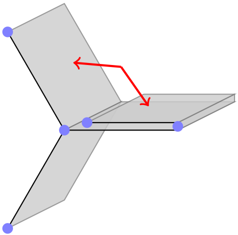

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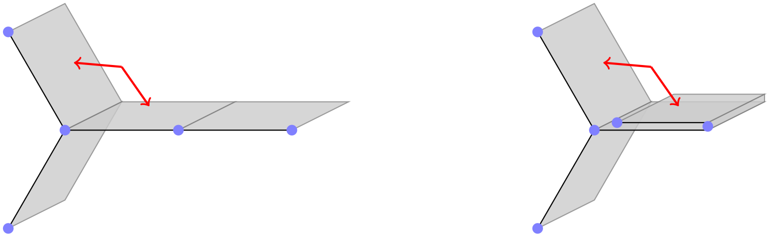

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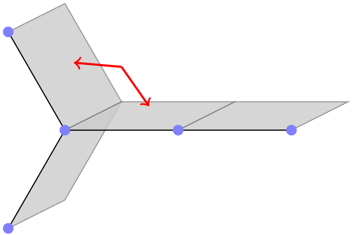

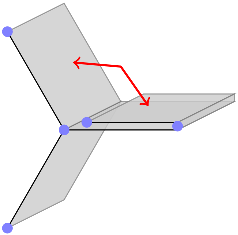

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$\rightsquigarrow$ Folding plans are not optimal to model folding

Progress report of abstract folding

Progress report of abstract folding

In development:

Progress report of abstract folding

In development:

- folding complex

Progress report of abstract folding

In development:

- folding complex
- folding plans

Progress report of abstract folding

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- better folding description

Progress report of abstract folding

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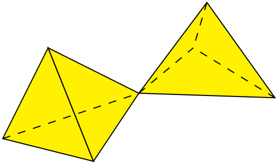
Summary: SimplicialSurfaces

Summary: `SimplicialSurfaces`

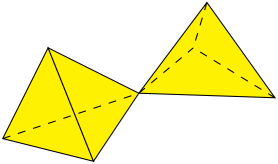
Triangular complexes



Summary: SimplicialSurfaces

Triangular complexes

- mostly complete



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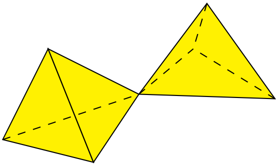
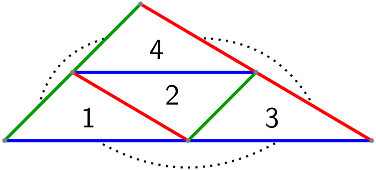


Edge colouring



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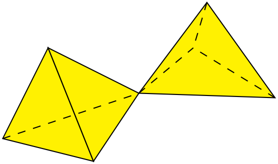

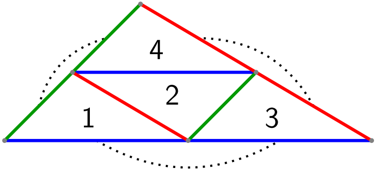

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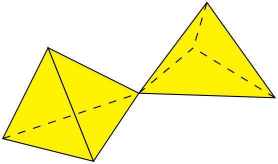

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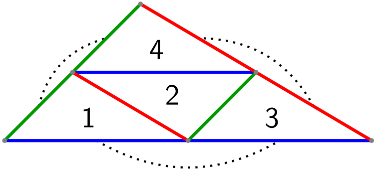

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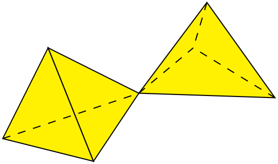

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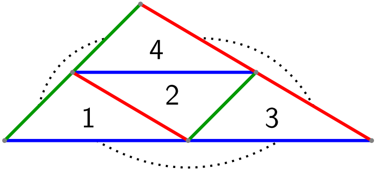
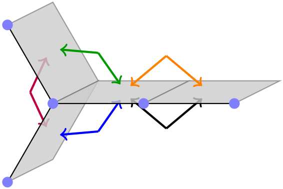


Abstract folding



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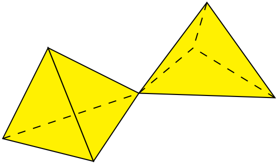

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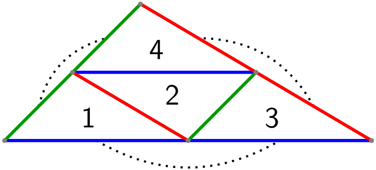
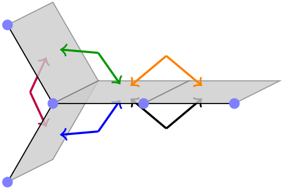


Abstract folding

- framework exists



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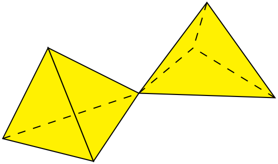

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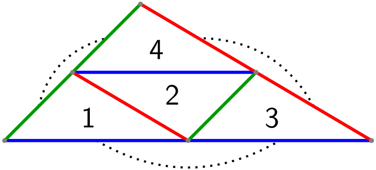

Abstract folding

- framework exists
- needs proper implementation