Simplicial surfaces in GAP

Markus Baumeister

??.08.2017

1. General polygonal complexes by incidence geometry

2. Edge colouring and group properties

3. Abstract folding

Motivation

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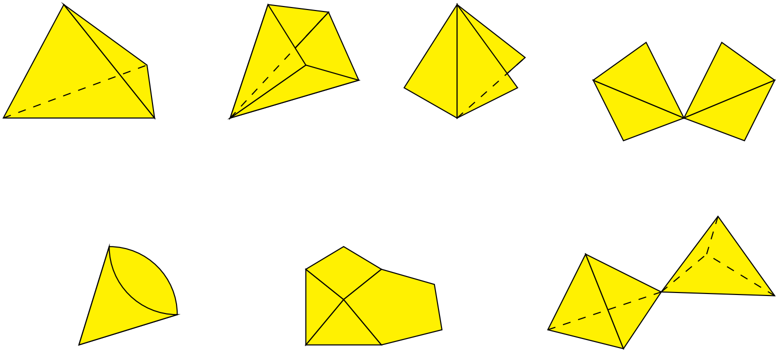

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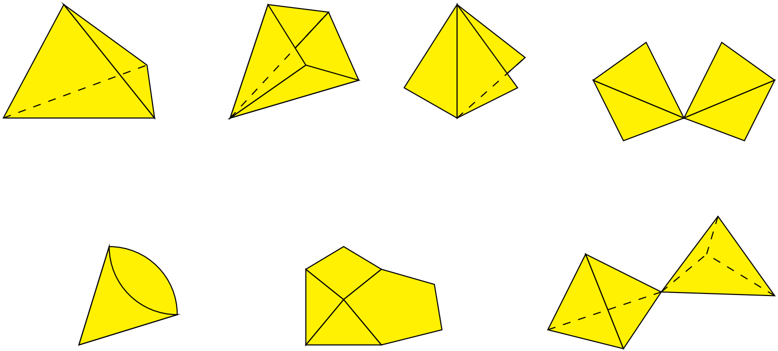

$\rightsquigarrow$ examples of **polygonal complexes**

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$\rightsquigarrow$ incidence structure is intrinsic

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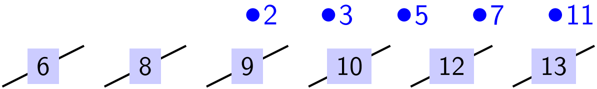

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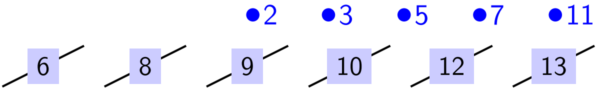

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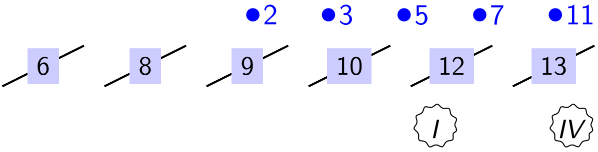

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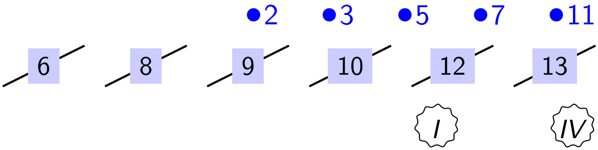

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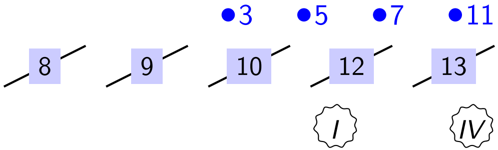

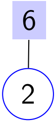

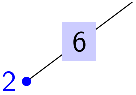

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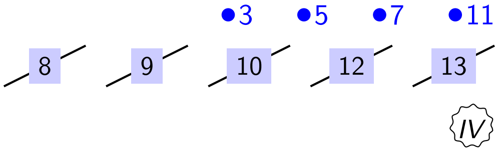

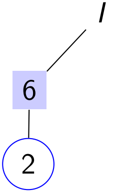

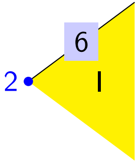

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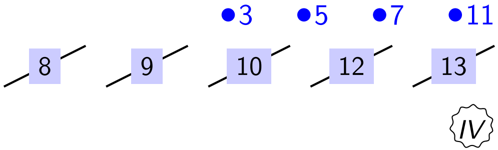

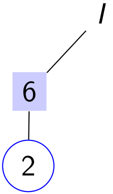

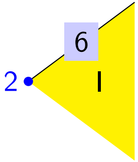

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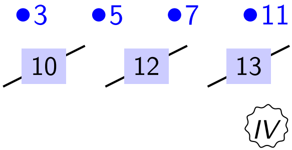

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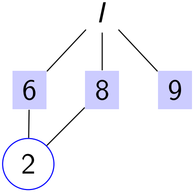

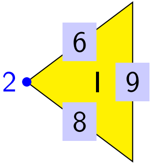

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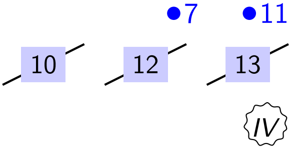

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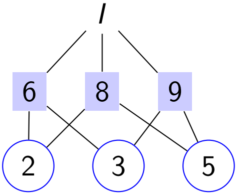

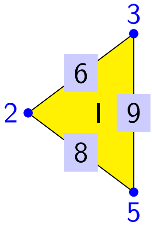

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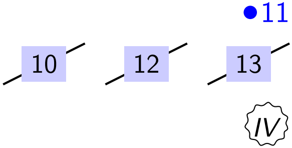

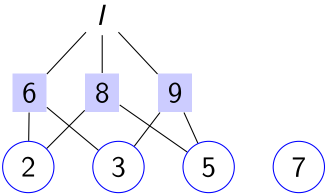

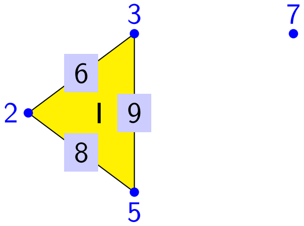

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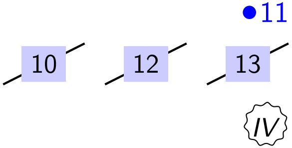

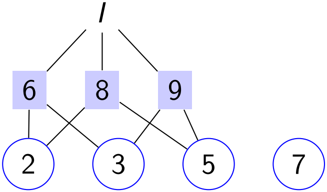

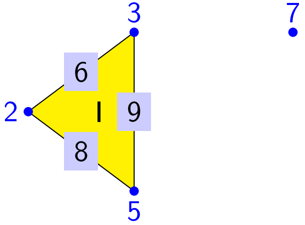

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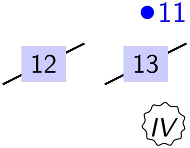

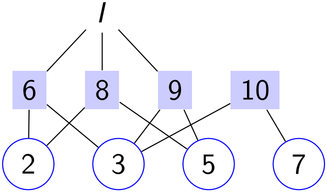

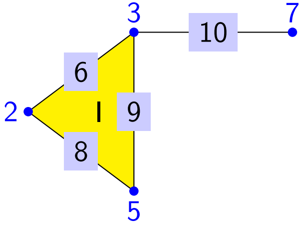

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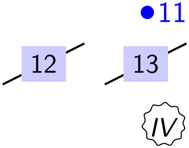

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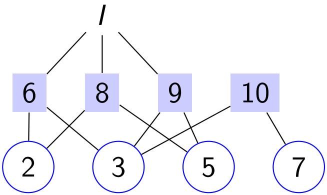

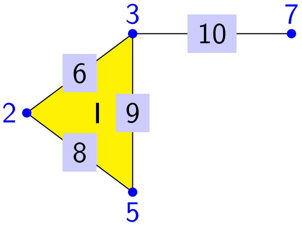

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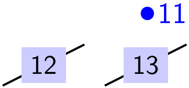

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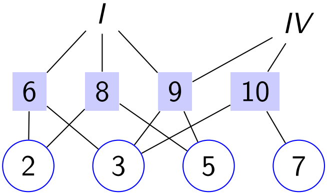

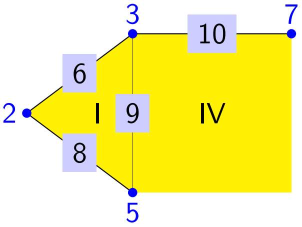

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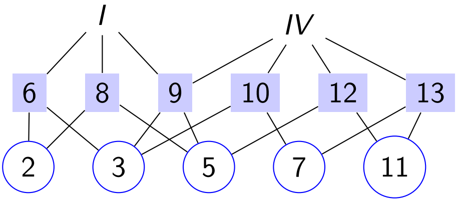

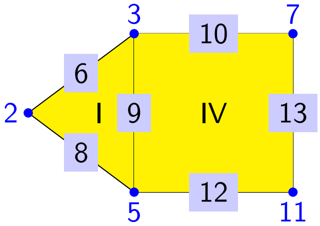

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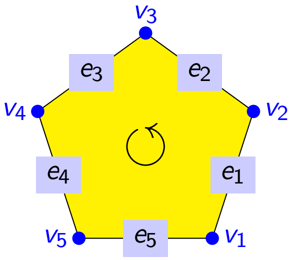

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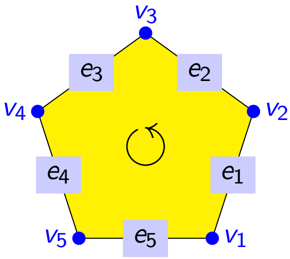

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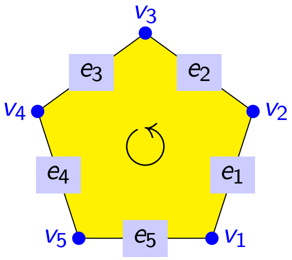

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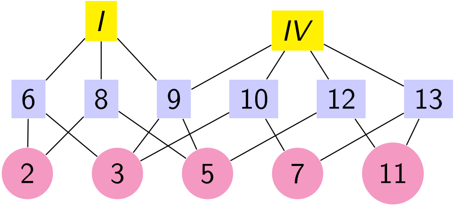

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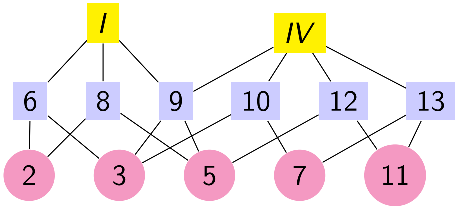

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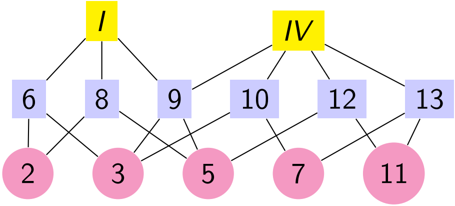

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Solved by NautyTracesInterface (by Gutsche, Niemeyer, Schweitzer)

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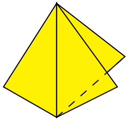

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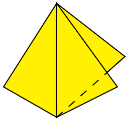

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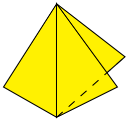

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$\rightsquigarrow$ **ramified polygonal surfaces**

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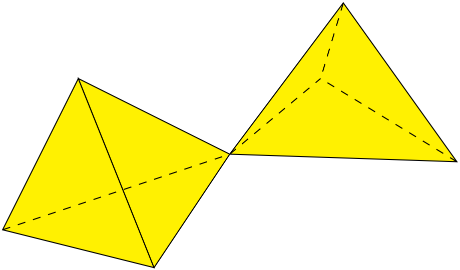

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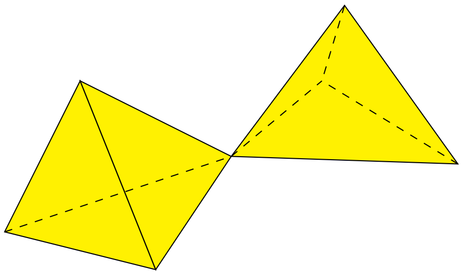

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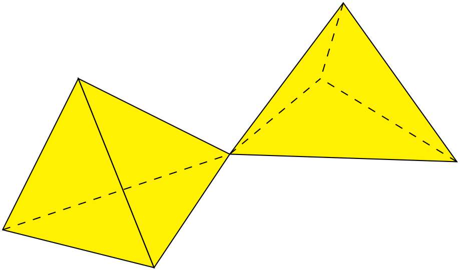

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A **polygonal surface** does not have these ramifications.

Embedding question

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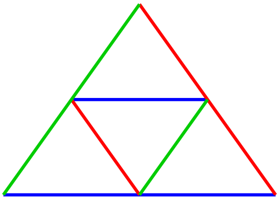
$\rightsquigarrow$ Edge–colouring encodes different lengths



Colouring as permutation

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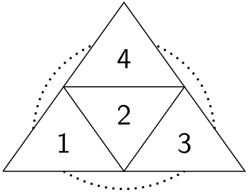

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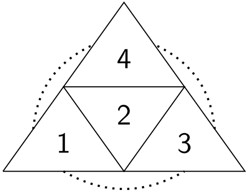
Consider tetrahedron with edge colouring



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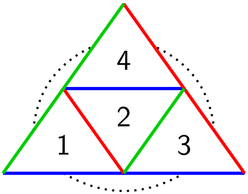

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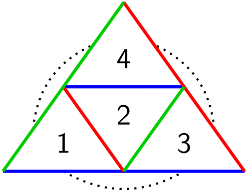

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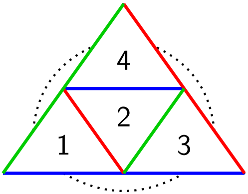

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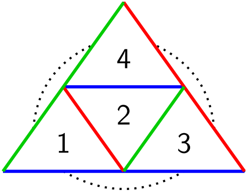

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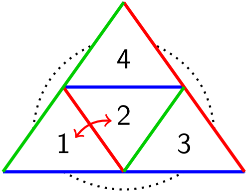

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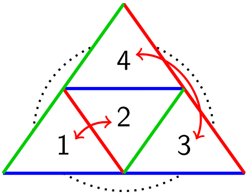

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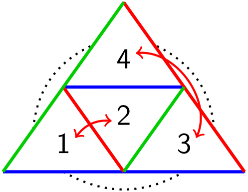

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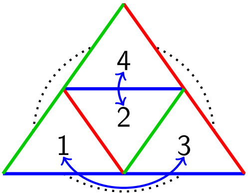

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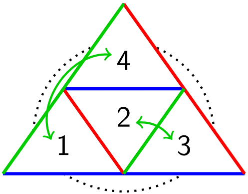

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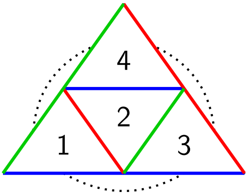

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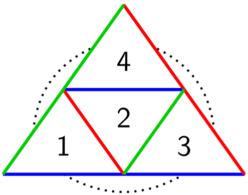

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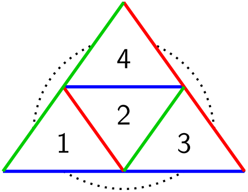

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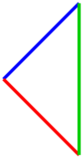

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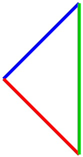

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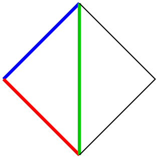

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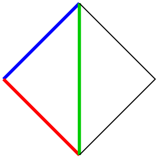

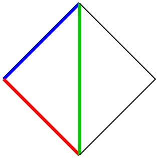

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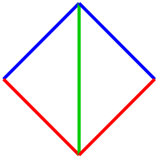

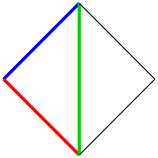

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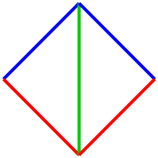

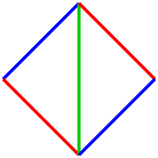

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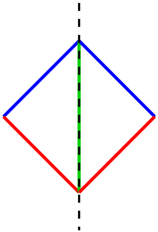

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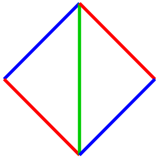

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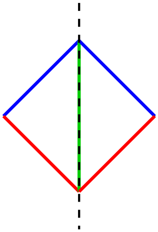

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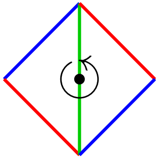

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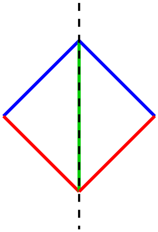

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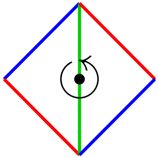

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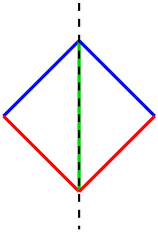

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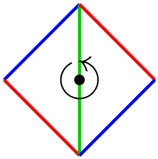

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Permutations and mr–assignment uniquely determine the surface.

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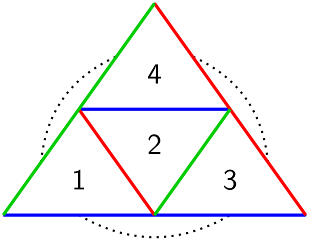

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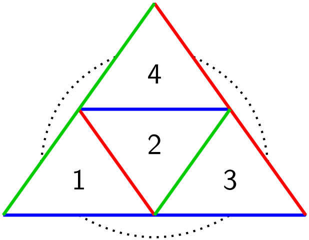

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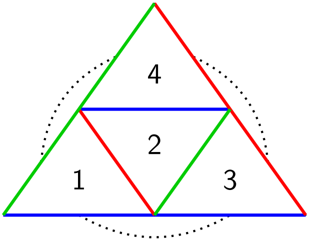

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The easiest structure is an mmm–structure.

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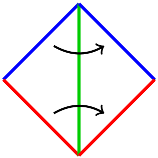

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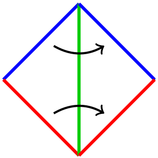

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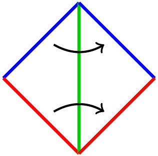

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- Colouring defines a map to the triangle.

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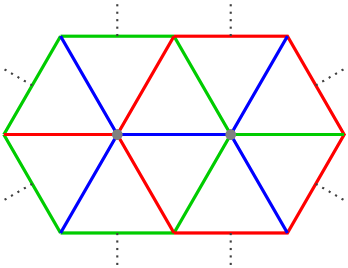

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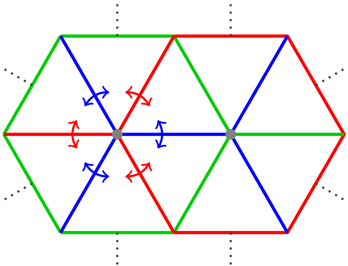

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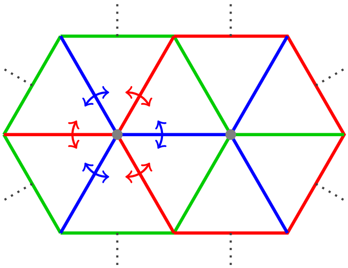

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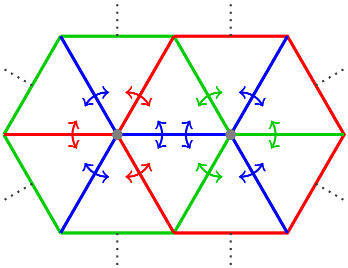

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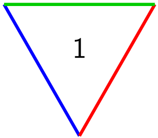

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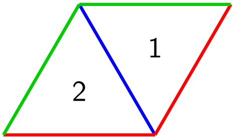

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$\sigma_b = (1,4)(2,3)(5,8)(6,7)$
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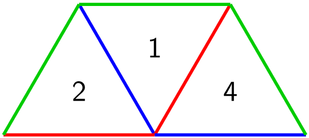

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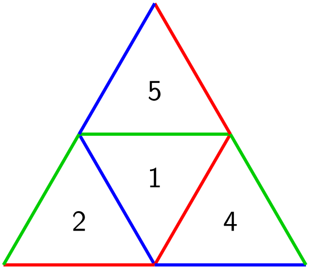

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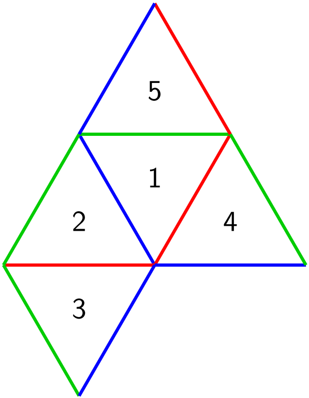

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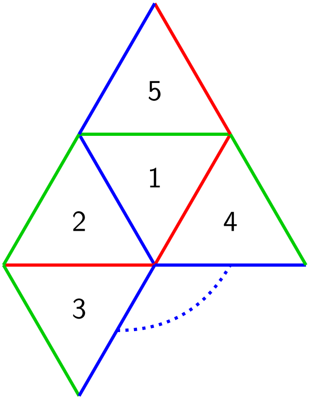

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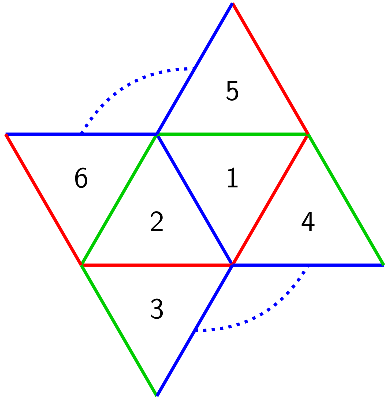

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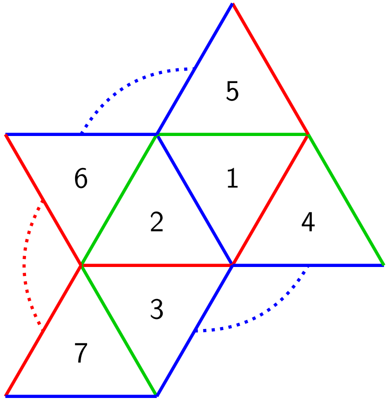

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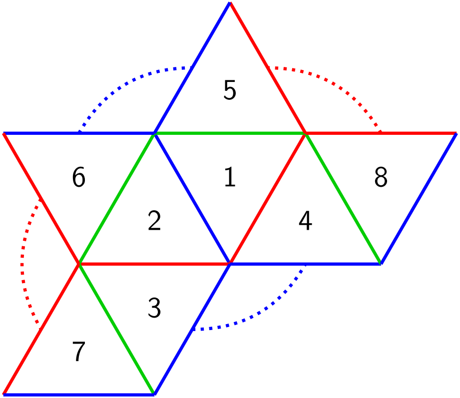

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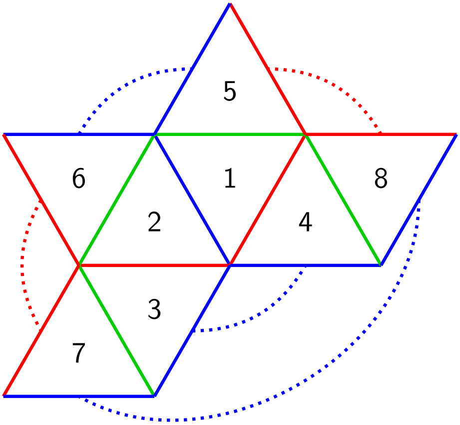

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