

Simplicial surfaces in GAP

Markus Baumeister

?? .08.2017

- 1 General polygonal complexes by incidence geometry
- 2 Edge colouring and group properties
- 3 Abstract folding

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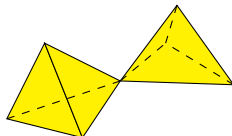
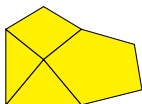
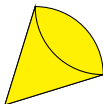
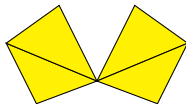
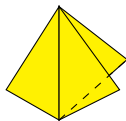
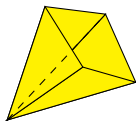
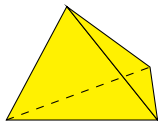
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Goal: simplicial surfaces (and generalisations) in GAP

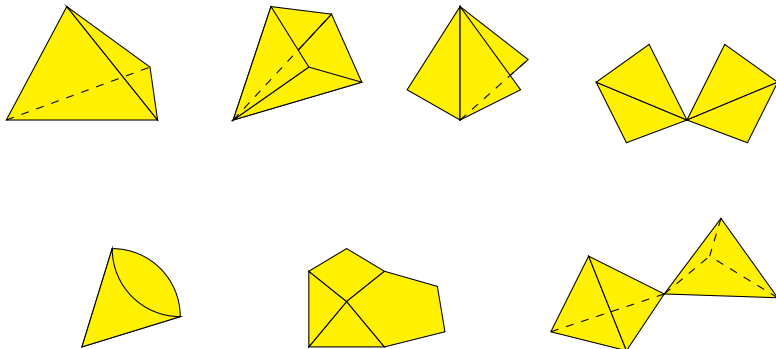
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⇝ examples of **polygonal complexes**

No embedding

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↪ incidence structure is intrinsic

Incidence structure of a polygonal complex

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- set of vertices \mathcal{V}

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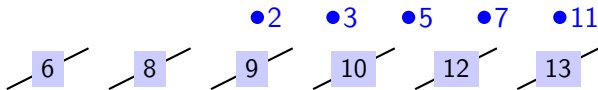
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- set of edges \mathcal{E}

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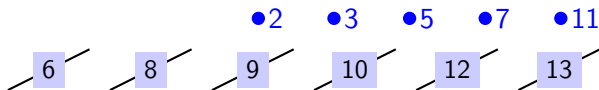
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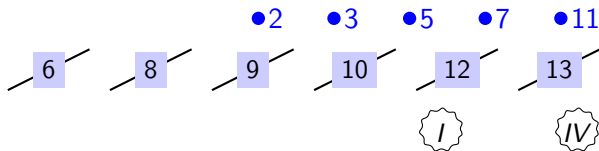
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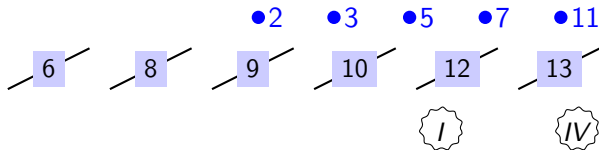
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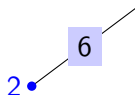
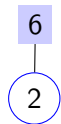
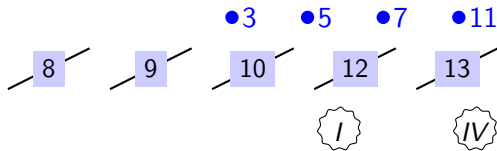
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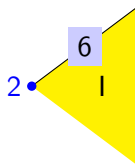
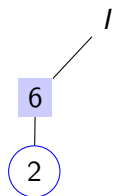
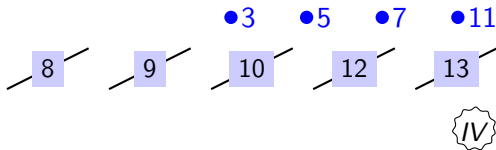
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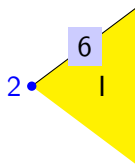
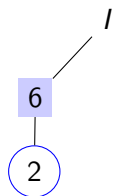
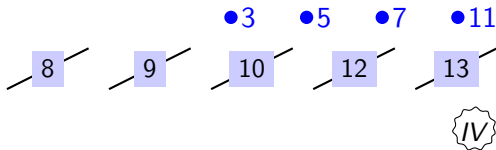
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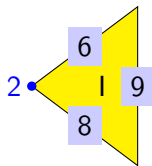
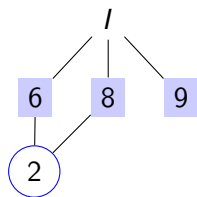
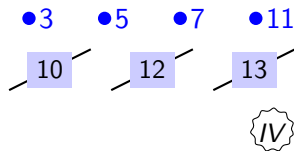


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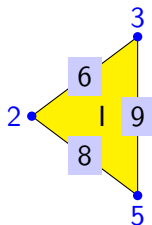
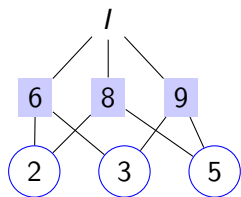
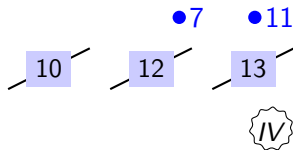


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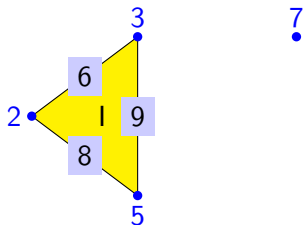
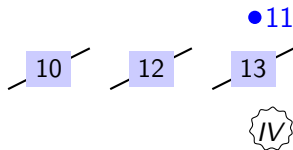
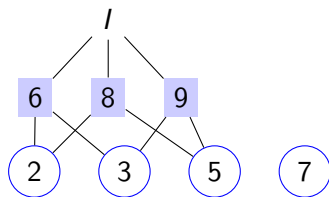


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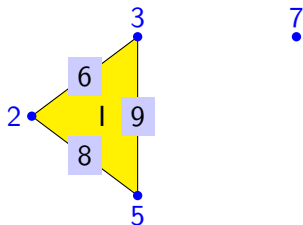
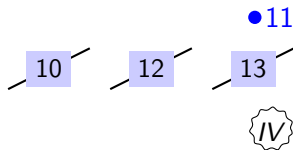
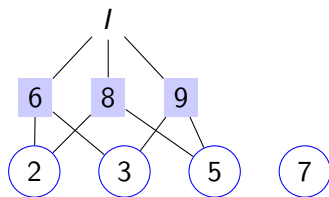


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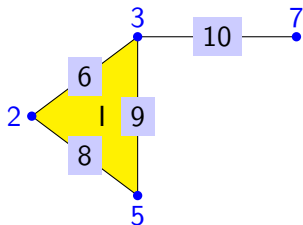
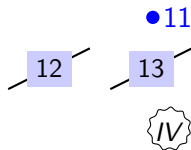
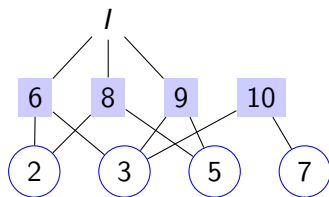


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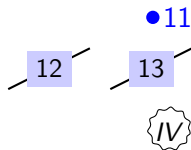
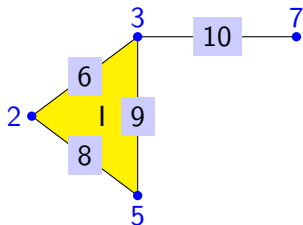
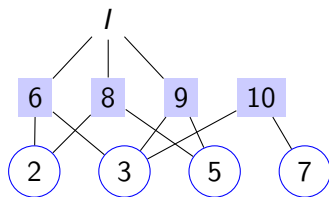


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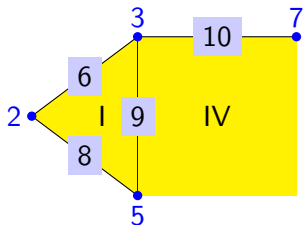
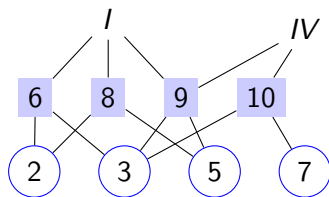
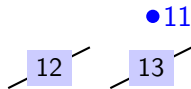


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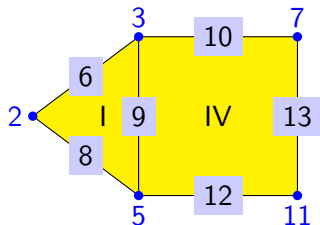
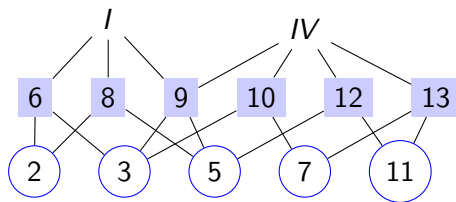


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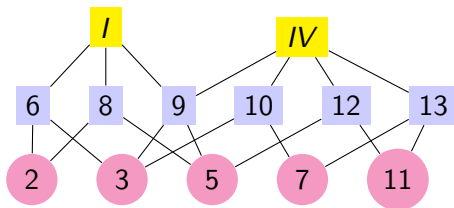
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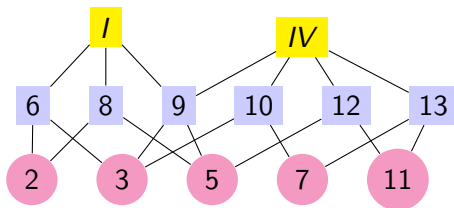
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Solved by NautyTracesInterface (by Gutsche, Niemeyer, Schweitzer)

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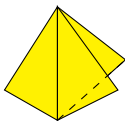
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⇔ **ramified polygonal surfaces**

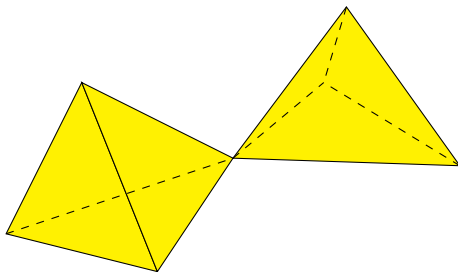
Why ramified?

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Typical example of ramified polygonal surface:

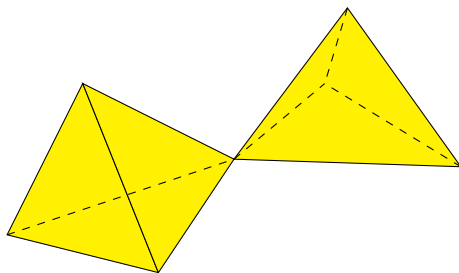
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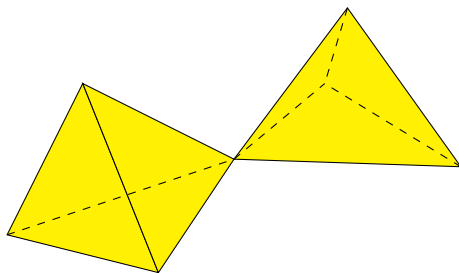
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Typical example of ramified polygonal surface:



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A **polygonal surface** does not have these ramifications.

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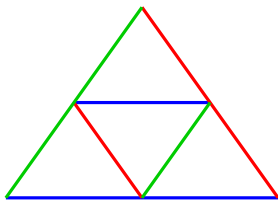
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⇒ Edge-colouring encodes different lengths



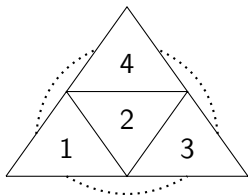
Colouring as permutation

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Consider tetrahedron

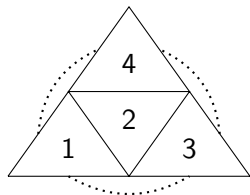
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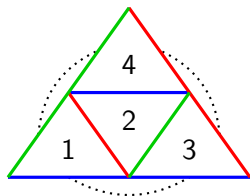
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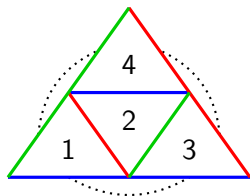
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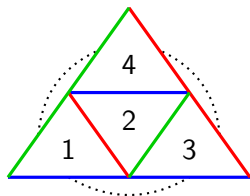
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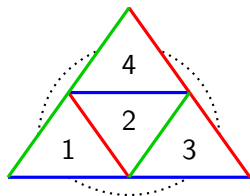
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simplicial surface \Rightarrow at most two faces at each edge

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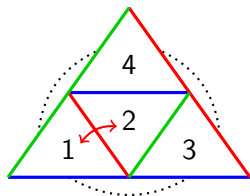


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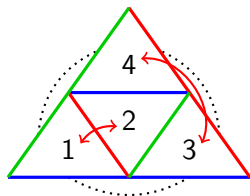
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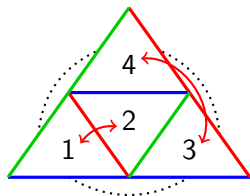
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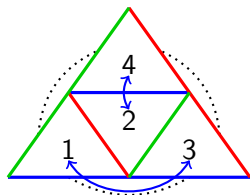


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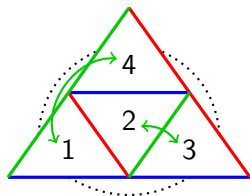


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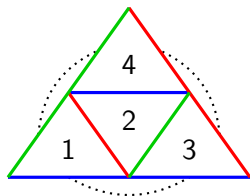


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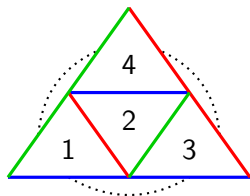


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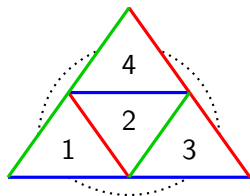


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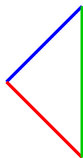
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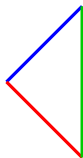
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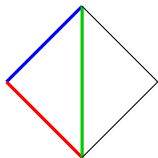
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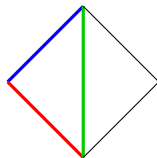
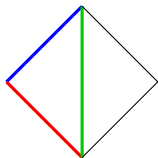
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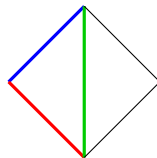
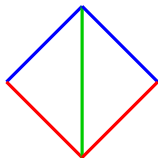
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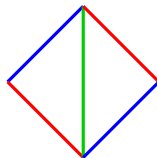
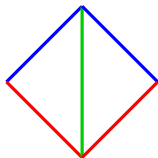
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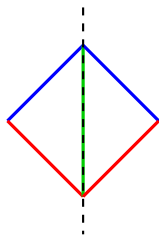
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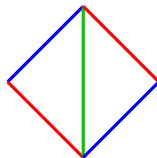


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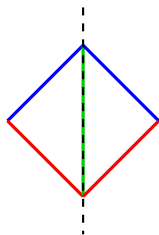


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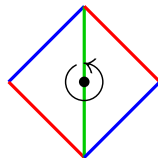


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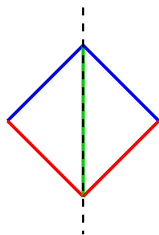
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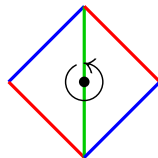
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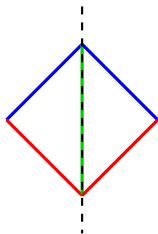


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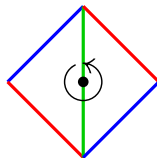
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Permutations and mr-assignment uniquely determine the surface.

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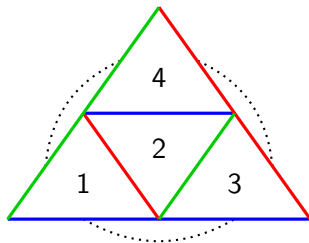
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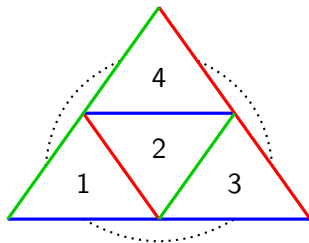
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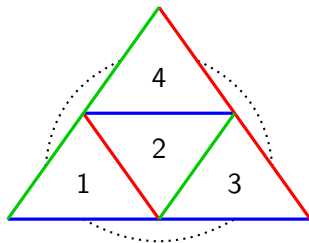


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The easiest structure is an mmm-structure.

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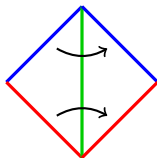
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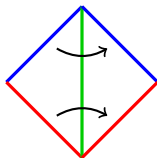
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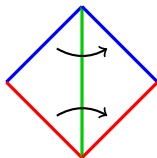
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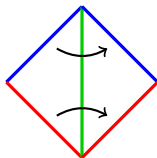
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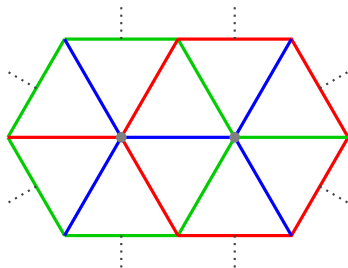
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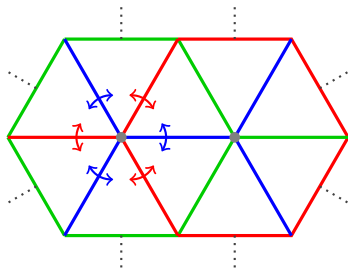
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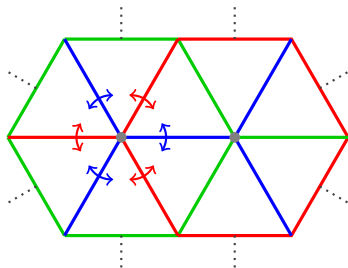
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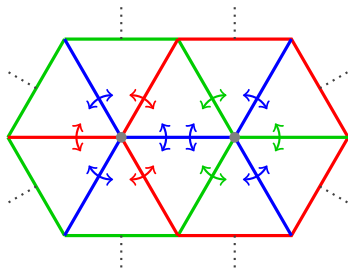
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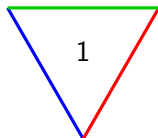
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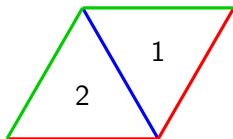


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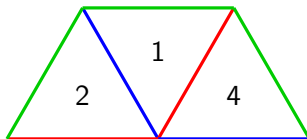


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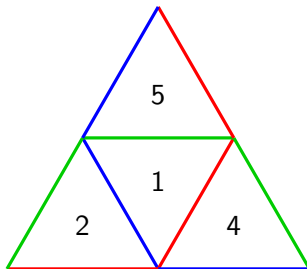


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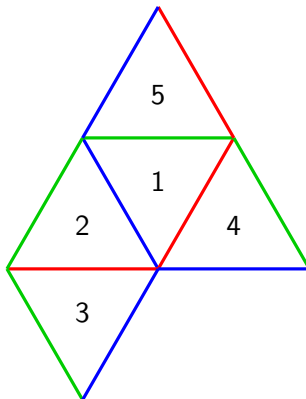


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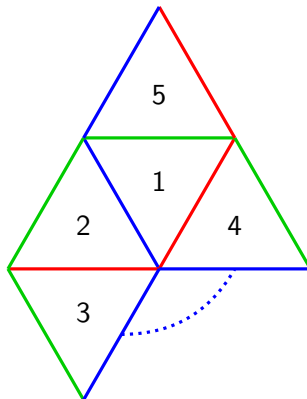


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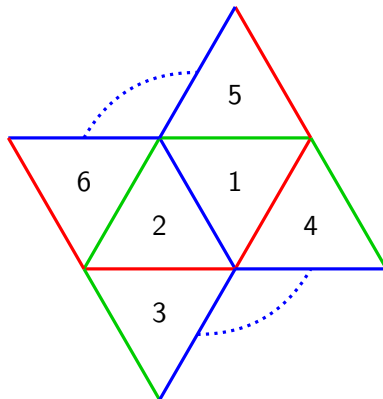


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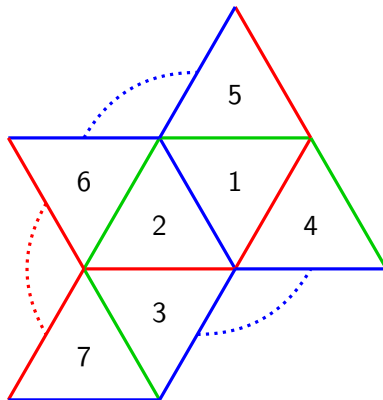


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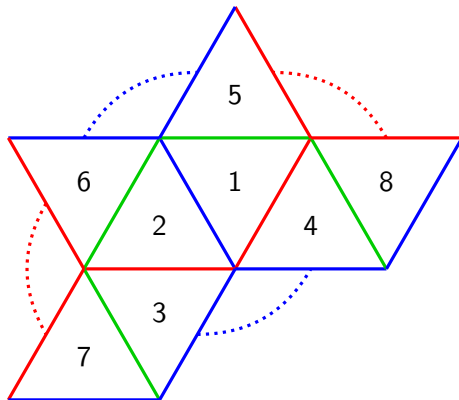


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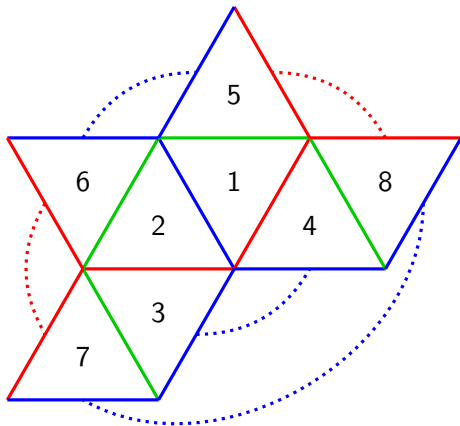


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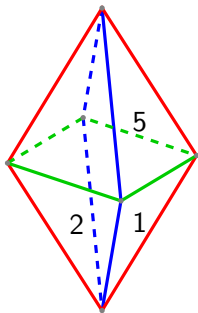
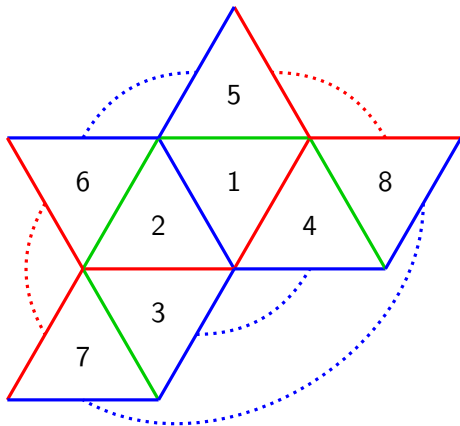


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- 2 Edge colouring and group properties
- 3 Abstract folding

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Important properties of folding

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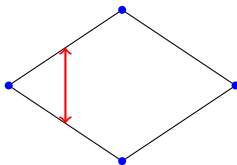
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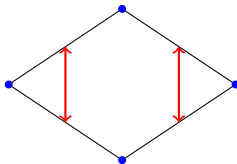
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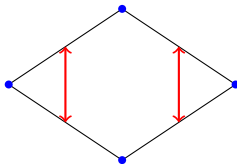
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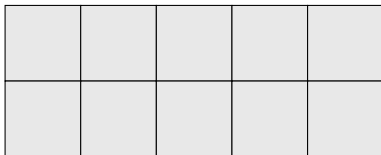
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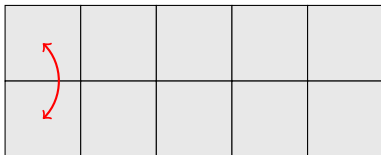
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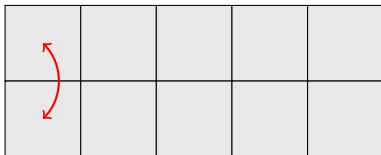
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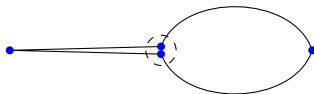
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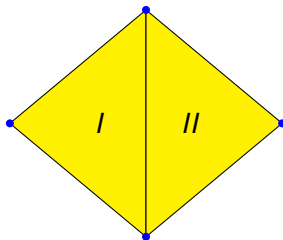
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 - ▶ Modify to include face order relations

Unordered Folding (Covering)

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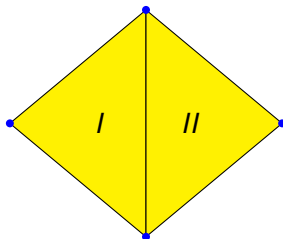
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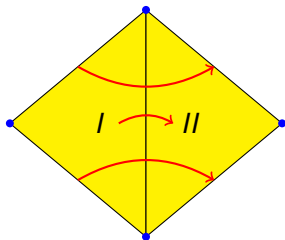
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Naive folding definition: surjective map that respects incidence

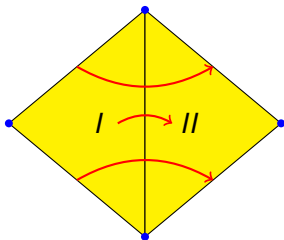
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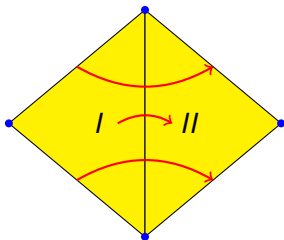


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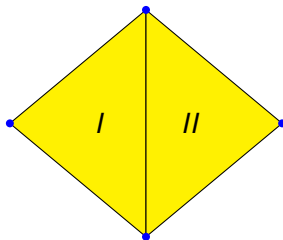
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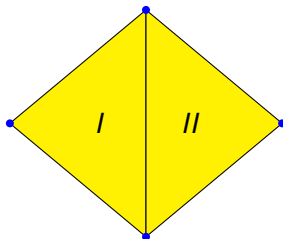
⇒ Folding state should not forget original structure

Unordered Folding (Covering)



Represent folding by equivalence relation

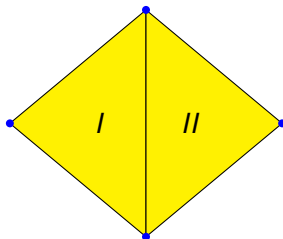
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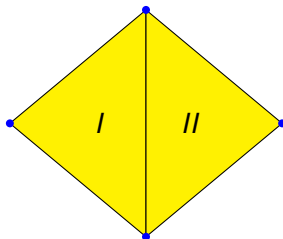
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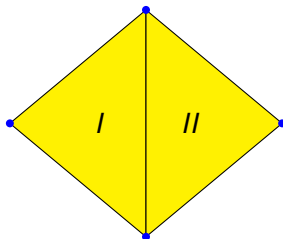
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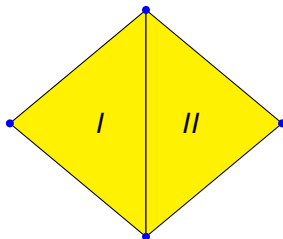
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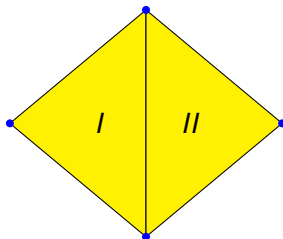
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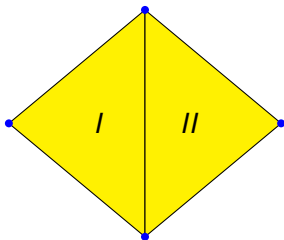
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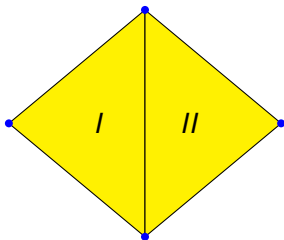
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⇒ Unordered folding is coarsening of equivalence relation

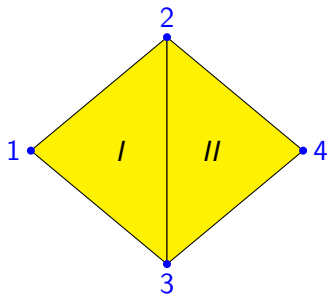
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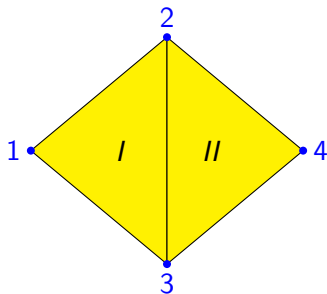
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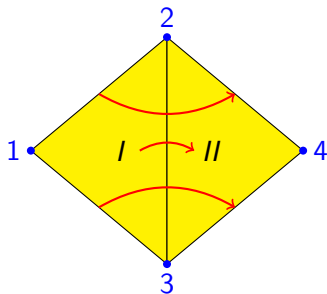
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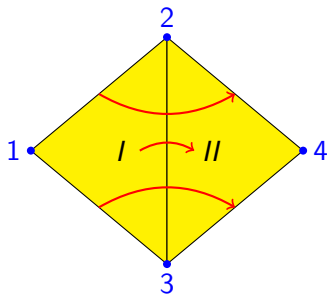
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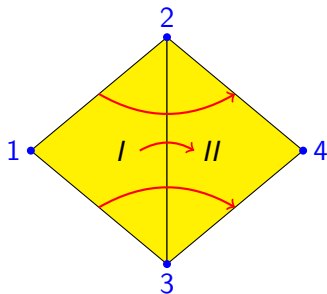
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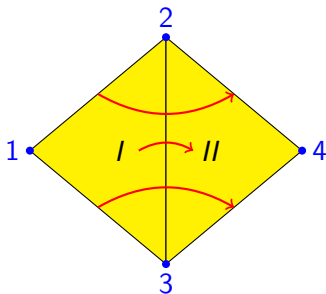
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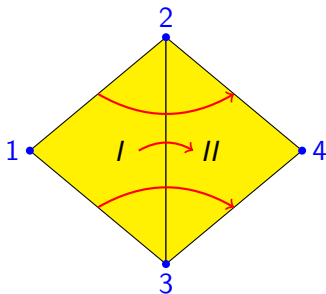
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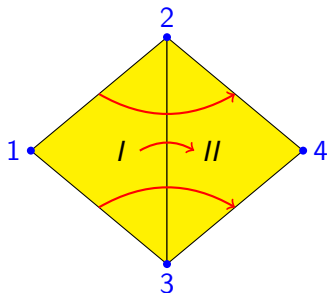


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Two vertices in an edge can't be identified

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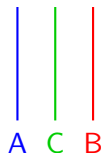
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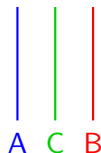
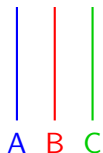
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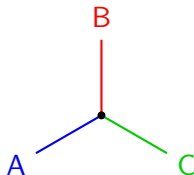
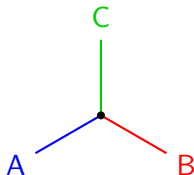
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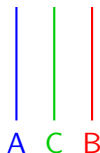
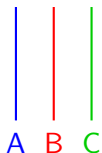


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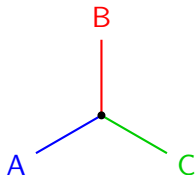
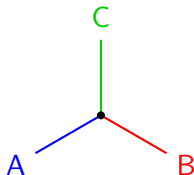


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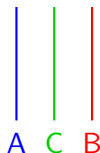
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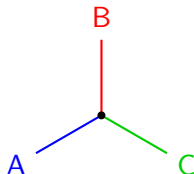
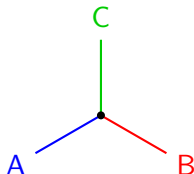
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To identify faces with each other, we have to combine those orderings.

- linear orderings get concatenated
- cyclical orderings are opened at one point and combined
- !! compatibility is not easily transferred,

Folding complex

Definition

A **folding complex** is a polygonal complex together with

- ① *An equivalence relation on vertices, edges and faces (“is folded together”)*
- ② *A linear ordering on each face equivalence class*
- ③ *A cyclical ordering of the faces around each edge equivalence class such that the orderings are compatible (in an appropriate sense).*

To identify faces with each other, we have to combine those orderings.

- linear orderings get concatenated
- cyclical orderings are opened at one point and combined
- !! compatibility is not easily transferred, but can be calculated

Changed definition of folding

olding with ordering:

Changed definition of folding

Before (no ordering): olding with ordering:

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- 1 Choose two faces that are not folded together

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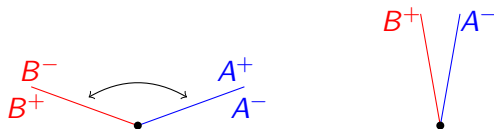


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↪ Each face has two sides

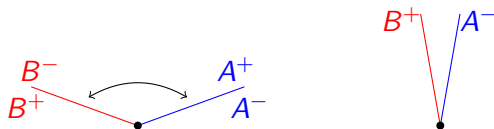


Changed definition of folding

Folding with ordering:

- 1 Choose two faces that are not folded together
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- 3 Choose the sides of the faces that will meet and modify the orderings

→ Each face has two sides

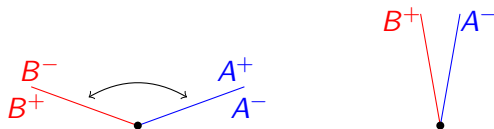


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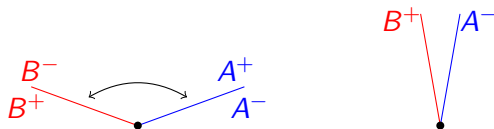


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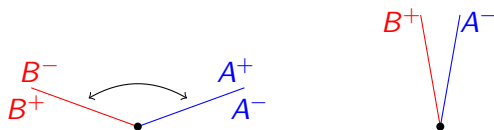
⇒ Define folding by two face sides (**folding plan**)

Changed definition of folding

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⇒ Define folding by two face sides (**folding plan**)

↪ Allows reversible (un)folding

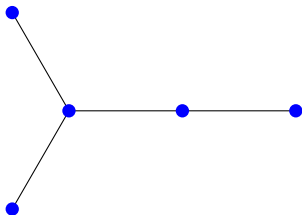
Structure of multiple foldings

Structure of multiple foldings

With folding plans we can perform the same folding in different folding complexes

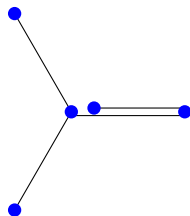
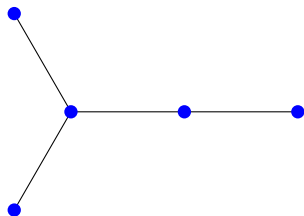
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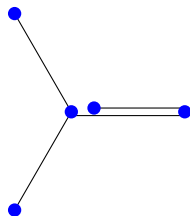
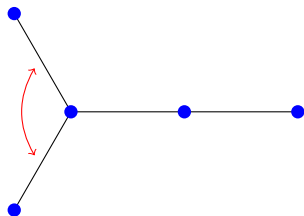
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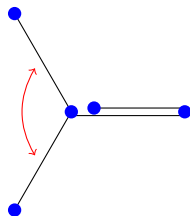
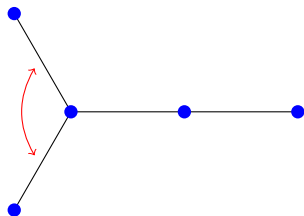
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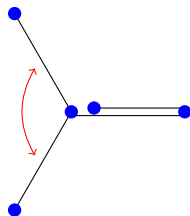
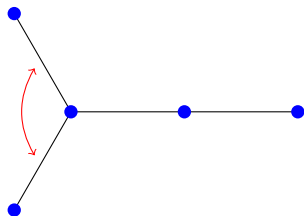
Structure of multiple foldings

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Structure of multiple foldings

With folding plans we can perform the same folding in different folding complexes



\rightsquigarrow more structure on the set of possible foldings

Folding graph

Folding graph

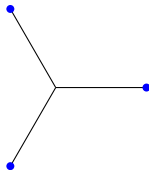
- Vertices are folding complexes (modelling folding states)

Folding graph

- Vertices are folding complexes (modelling folding states)
- Edges are folding plans connecting two folding complexes

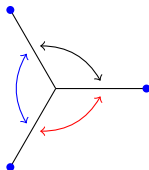
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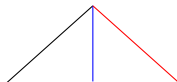
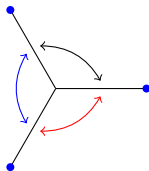
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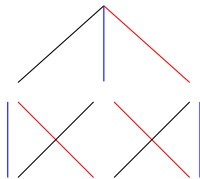
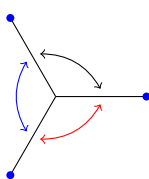
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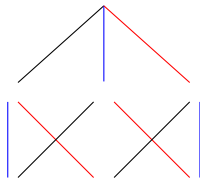
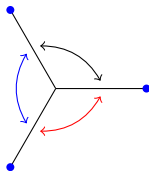
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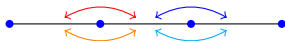
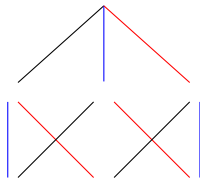
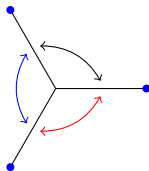
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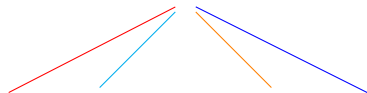
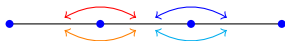
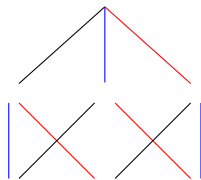
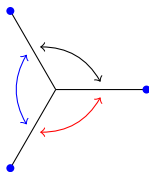
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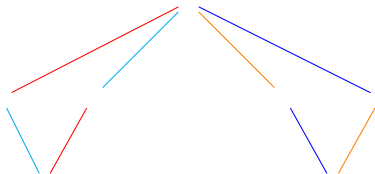
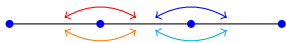
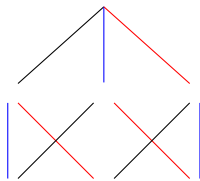
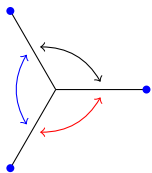
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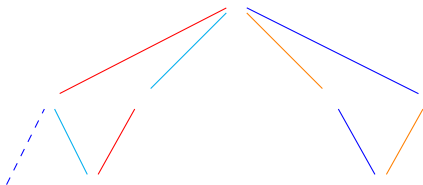
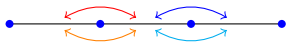
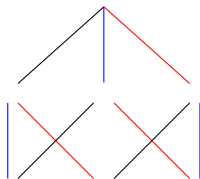
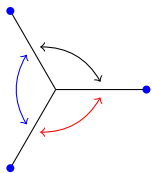
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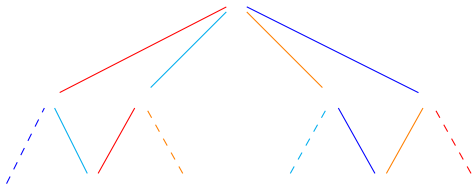
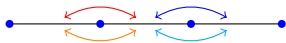
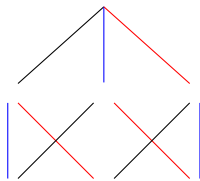
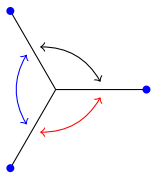
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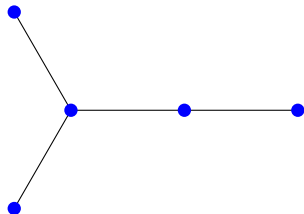
Drawback of folding plans

Drawback of folding plans

Some foldings that “should” be the same, aren’t:

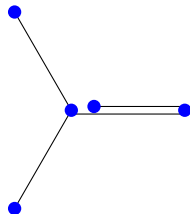
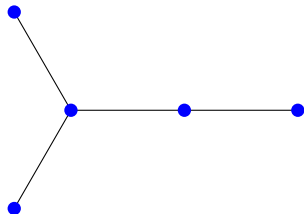
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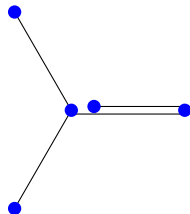
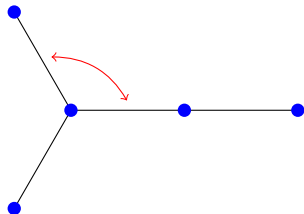
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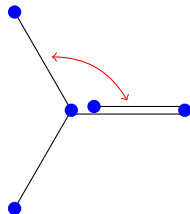
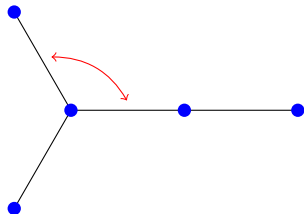
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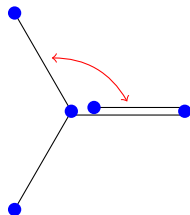
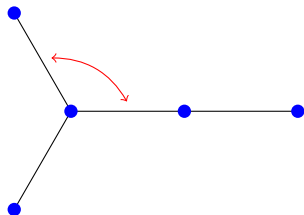
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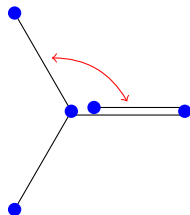
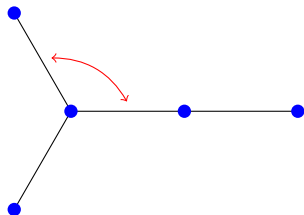
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⇒ If you know the folding structure of a small complex,

Drawback of folding plans

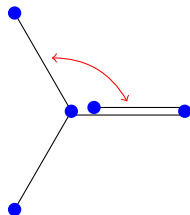
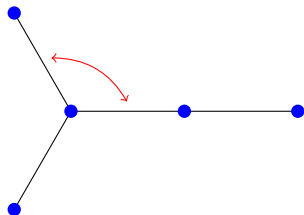
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Drawback of folding plans

Some foldings that “should” be the same, aren’t:



- ⇒ If you know the folding structure of a small complex, you can’t easily find the folding structure of an extended complex
- ⇒ Folding plans are not optimal to model folding.

Questions?