Simplicial surfaces in GAP

Markus Baumeister

30.08.2017

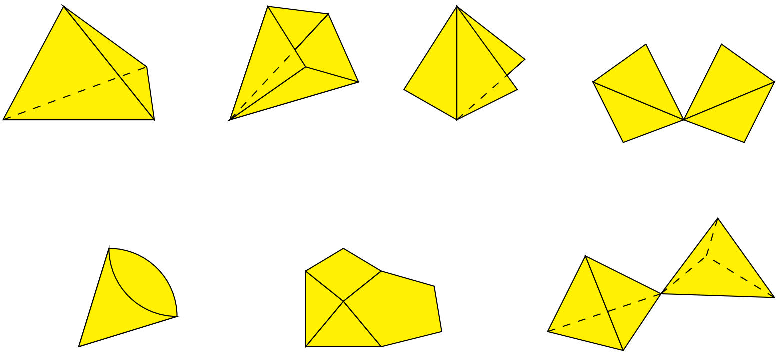
Motivation

Motivation

Goal: simplicial surfaces (and generalisations) in GAP

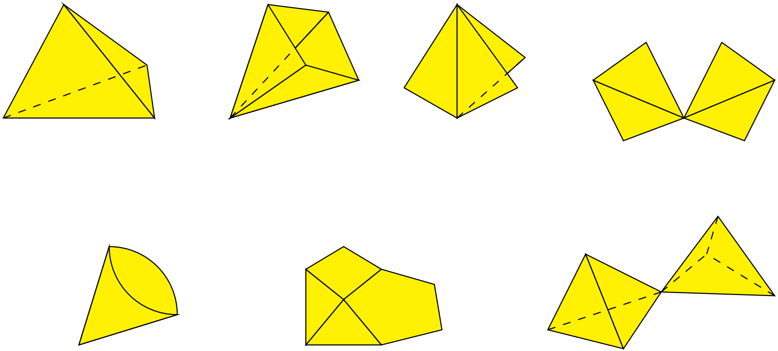
Motivation

Goal: simplicial surfaces (and generalisations) in GAP



Motivation

Goal: simplicial surfaces (and generalisations) in GAP



$\rightsquigarrow$ examples of **polygonal complexes**

No embedding

No embedding

We do **not** work with embeddings (mostly)

No embedding

We do **not** work with embeddings (mostly)

- is very hard to compute

No embedding

We do **not** work with embeddings (mostly)

- is very hard to compute (compare the pentagon flippy)

No embedding

We do **not** work with embeddings (mostly)

- is very hard to compute (compare the pentagon flippy)
- if often unknown for an abstractly constructed surface

No embedding

We do **not** work with embeddings (mostly)

- is very hard to compute (compare the pentagon flippy)
- if often unknown for an abstractly constructed surface
- is mostly independent from *intrinsic structure*

No embedding

We do **not** work with embeddings (mostly)

- is very hard to compute (compare the pentagon flippy)
- if often unknown for an abstractly constructed surface
- is mostly independent from *intrinsic structure*

$\Rightarrow$ lengths and angles are not important

No embedding

We do **not** work with embeddings (mostly)

- is very hard to compute (compare the pentagon flippy)
- if often unknown for an abstractly constructed surface
- is mostly independent from *intrinsic structure*

$\Rightarrow$ lengths and angles are not important

$\rightsquigarrow$ incidence structure is intrinsic

Incidence structure of a polygonal complex

Incidence structure of a polygonal complex

A **polygonal complex** consists of

Incidence structure of a polygonal complex

A **polygonal complex** consists of

- set of vertices $\mathcal{V}$

Incidence structure of a polygonal complex

A **polygonal complex** consists of

- set of vertices $\mathcal{V}$

$\bullet 2 \quad \bullet 3 \quad \bullet 5 \quad \bullet 7 \quad \bullet 11$

Incidence structure of a polygonal complex

A **polygonal complex** consists of

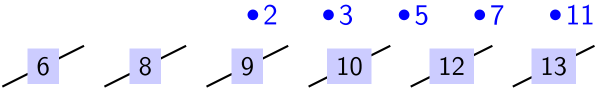
- set of vertices $\mathcal{V}$
- set of edges $\mathcal{E}$

$\bullet 2 \quad \bullet 3 \quad \bullet 5 \quad \bullet 7 \quad \bullet 11$

Incidence structure of a polygonal complex

A **polygonal complex** consists of

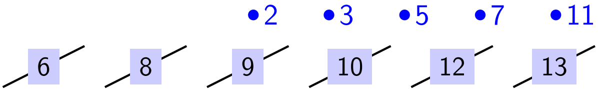
- set of vertices $\mathcal{V}$
- set of edges $\mathcal{E}$



Incidence structure of a polygonal complex

A **polygonal complex** consists of

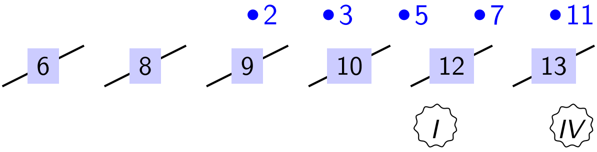
- set of vertices $\mathcal{V}$
- set of edges $\mathcal{E}$
- set of faces $\mathcal{F}$



Incidence structure of a polygonal complex

A **polygonal complex** consists of

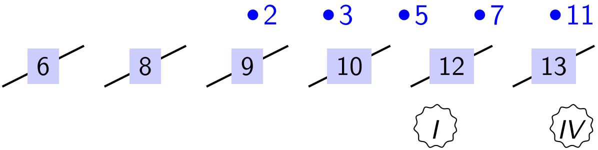
- set of vertices $\mathcal{V}$
- set of edges $\mathcal{E}$
- set of faces $\mathcal{F}$



Incidence structure of a polygonal complex

A **polygonal complex** consists of

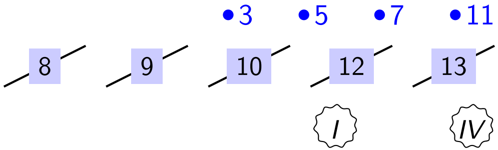
- set of vertices $\mathcal{V}$
- set of edges $\mathcal{E}$
- set of faces $\mathcal{F}$
- transitive relation $\subseteq \big(\mathcal{V} \times \mathcal{E}\big) \uplus \big(\mathcal{V} \times \mathcal{F}\big) \uplus \big(\mathcal{E} \times \mathcal{F}\big)$

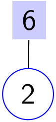


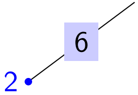
Incidence structure of a polygonal complex

A **polygonal complex** consists of

- set of vertices $\mathcal{V}$
- set of edges $\mathcal{E}$
- set of faces $\mathcal{F}$
- transitive relation $\subseteq \left(\mathcal{V} \times \mathcal{E}\right) \uplus \left(\mathcal{V} \times \mathcal{F}\right) \uplus \left(\mathcal{E} \times \mathcal{F}\right)$



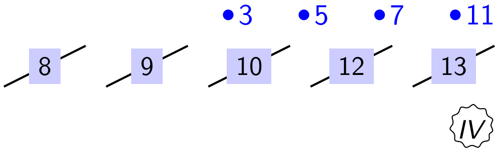


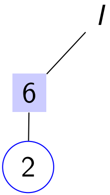


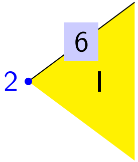
Incidence structure of a polygonal complex

A **polygonal complex** consists of

- set of vertices $\mathcal{V}$
- set of edges $\mathcal{E}$
- set of faces $\mathcal{F}$
- transitive relation $\subseteq \left(\mathcal{V} \times \mathcal{E}\right) \uplus \left(\mathcal{V} \times \mathcal{F}\right) \uplus \left(\mathcal{E} \times \mathcal{F}\right)$



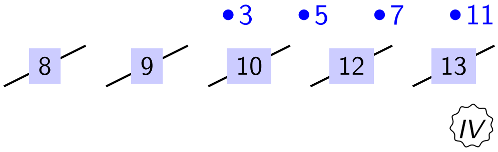


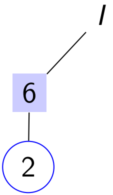


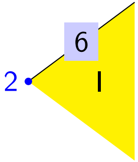
Incidence structure of a polygonal complex

A **polygonal complex** consists of

- set of vertices $\mathcal{V}$
- set of edges $\mathcal{E}$
- set of faces $\mathcal{F}$
- transitive relation $\subseteq \left(\mathcal{V} \times \mathcal{E}\right) \uplus \left(\mathcal{V} \times \mathcal{F}\right) \uplus \left(\mathcal{E} \times \mathcal{F}\right)$





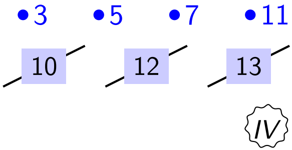


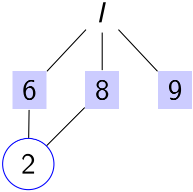
1. Every face is a polygon

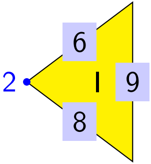
Incidence structure of a polygonal complex

A **polygonal complex** consists of

- set of vertices $\mathcal{V}$
- set of edges $\mathcal{E}$
- set of faces $\mathcal{F}$
- transitive relation $\subseteq \left(\mathcal{V} \times \mathcal{E}\right) \uplus \left(\mathcal{V} \times \mathcal{F}\right) \uplus \left(\mathcal{E} \times \mathcal{F}\right)$





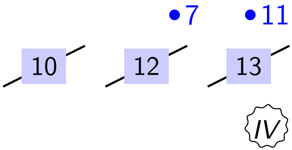


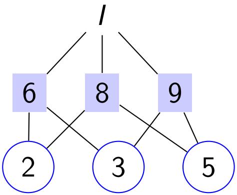
1. Every face is a polygon

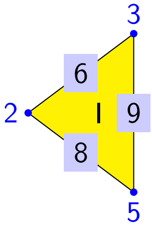
Incidence structure of a polygonal complex

A **polygonal complex** consists of

- set of vertices $\mathcal{V}$
- set of edges $\mathcal{E}$
- set of faces $\mathcal{F}$
- transitive relation $\subseteq \left(\mathcal{V} \times \mathcal{E}\right) \uplus \left(\mathcal{V} \times \mathcal{F}\right) \uplus \left(\mathcal{E} \times \mathcal{F}\right)$





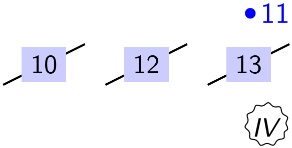


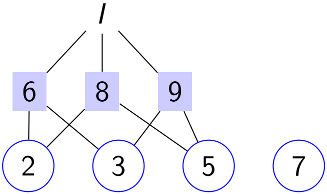
1. Every face is a polygon

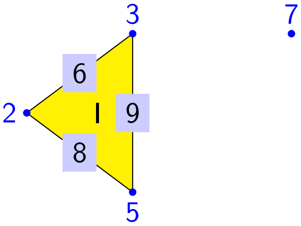
Incidence structure of a polygonal complex

A **polygonal complex** consists of

- set of vertices $\mathcal{V}$
- set of edges $\mathcal{E}$
- set of faces $\mathcal{F}$
- transitive relation $\subseteq \left(\mathcal{V} \times \mathcal{E}\right) \uplus \left(\mathcal{V} \times \mathcal{F}\right) \uplus \left(\mathcal{E} \times \mathcal{F}\right)$



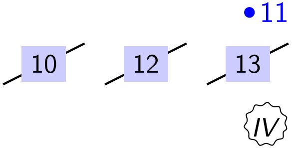




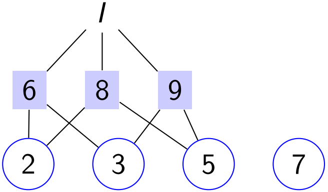
1. Every face is a polygon

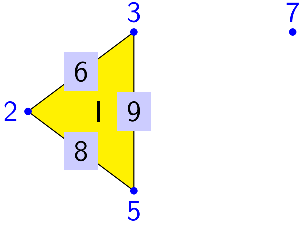
Incidence structure of a polygonal complex

A **polygonal complex** consists of



- set of vertices $\mathcal{V}$
- set of edges $\mathcal{E}$
- set of faces $\mathcal{F}$
- transitive relation $\subseteq \left(\mathcal{V} \times \mathcal{E}\right) \uplus \left(\mathcal{V} \times \mathcal{F}\right) \uplus \left(\mathcal{E} \times \mathcal{F}\right)$



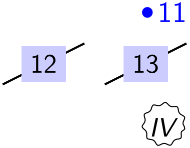


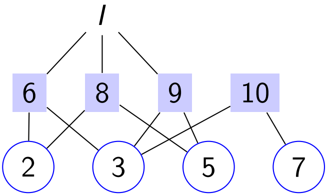
1. Every face is a polygon
2. Every vertex lies in an edge

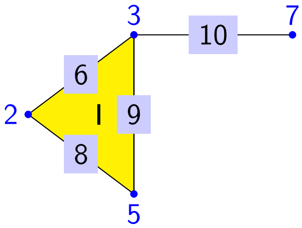
Incidence structure of a polygonal complex

A **polygonal complex** consists of

- set of vertices $\mathcal{V}$
- set of edges $\mathcal{E}$
- set of faces $\mathcal{F}$
- transitive relation $\subseteq (\mathcal{V} \times \mathcal{E}) \uplus (\mathcal{V} \times \mathcal{F}) \uplus (\mathcal{E} \times \mathcal{F})$



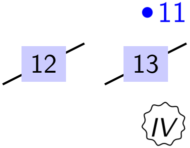




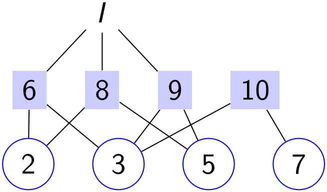
1. Every face is a polygon
2. Every vertex lies in an edge

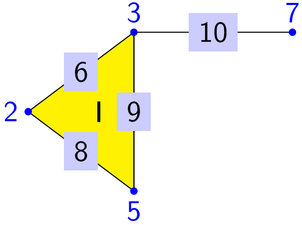
Incidence structure of a polygonal complex

A **polygonal complex** consists of



- set of vertices $\mathcal{V}$
- set of edges $\mathcal{E}$
- set of faces $\mathcal{F}$
- transitive relation $\subseteq \left(\mathcal{V} \times \mathcal{E}\right) \uplus \left(\mathcal{V} \times \mathcal{F}\right) \uplus \left(\mathcal{E} \times \mathcal{F}\right)$



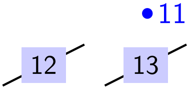


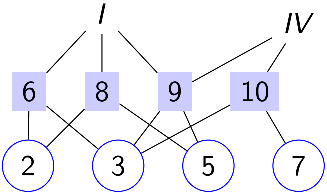
1. Every face is a polygon
2. Every vertex lies in an edge and every edge lies in a face

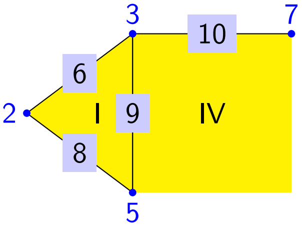
Incidence structure of a polygonal complex

A **polygonal complex** consists of

- set of vertices $\mathcal{V}$
- set of edges $\mathcal{E}$
- set of faces $\mathcal{F}$
- transitive relation $\subseteq \left(\mathcal{V}\times\mathcal{E}\right) \uplus \left(\mathcal{V}\times\mathcal{F}\right) \uplus \left(\mathcal{E}\times\mathcal{F}\right)$





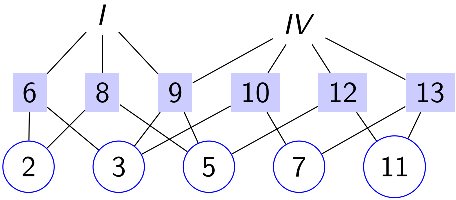


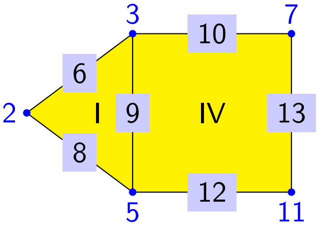
1. Every face is a polygon
2. Every vertex lies in an edge and every edge lies in a face

Incidence structure of a polygonal complex

A **polygonal complex** consists of

- set of vertices $\mathcal{V}$
- set of edges $\mathcal{E}$
- set of faces $\mathcal{F}$
- transitive relation $\subseteq \left(\mathcal{V} \times \mathcal{E}\right) \uplus \left(\mathcal{V} \times \mathcal{F}\right) \uplus \left(\mathcal{E} \times \mathcal{F}\right)$





1. Every face is a polygon
2. Every vertex lies in an edge and every edge lies in a face

Isomorphism testing

Isomorphism testing

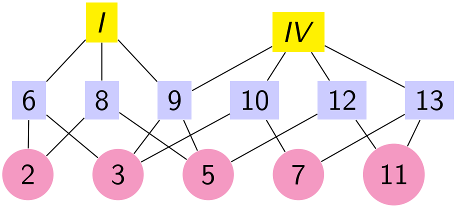
Incidence geometry allows "easy" isomorphism testing.

Isomorphism testing

Incidence geometry allows “easy” isomorphism testing. Incidence structure can be interpreted as a coloured graph:

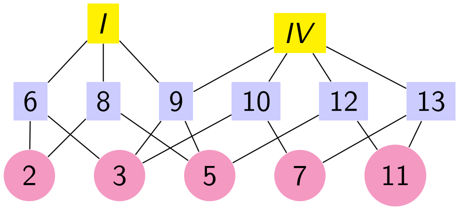
Isomorphism testing

Incidence geometry allows "easy" isomorphism testing. Incidence structure can be interpreted as a coloured graph:



Isomorphism testing

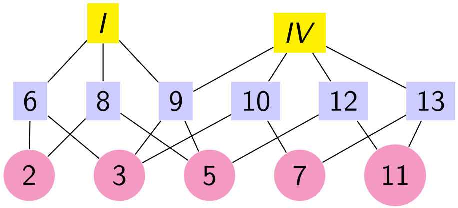
Incidence geometry allows “easy” isomorphism testing. Incidence structure can be interpreted as a coloured graph:



$\rightsquigarrow$ reduce to graph isomorphism problem

Isomorphism testing

Incidence geometry allows “easy” isomorphism testing. Incidence structure can be interpreted as a coloured graph:



$\rightsquigarrow$ reduce to graph isomorphism problem
Solved by NautyTracesInterface (by Gutsche, Niemeyer, Schweitzer)

General properties

General properties

Some properties can be computed for all polygonal complexes:

General properties

Some properties can be computed for all polygonal complexes:

- Connectivity

General properties

Some properties can be computed for all polygonal complexes:

- Connectivity
- Euler–Characteristic

General properties

Some properties can be computed for all polygonal complexes:

- Connectivity
- Euler–Characteristic

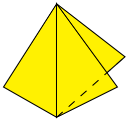
Orientability is **not** one of them.

General properties

Some properties can be computed for all polygonal complexes:

- Connectivity
- Euler–Characteristic

Orientability is **not** one of them. Counterexample:

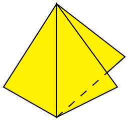


General properties

Some properties can be computed for all polygonal complexes:

- Connectivity
- Euler–Characteristic

Orientability is **not** one of them. Counterexample:



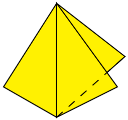
$\Rightarrow$ every edge lies in at most two faces (for well–definedness)

General properties

Some properties can be computed for all polygonal complexes:

- Connectivity
- Euler–Characteristic

Orientability is **not** one of them. Counterexample:



$\Rightarrow$ every edge lies in at most two faces (for well–definedness)
$\rightsquigarrow$ **ramified polygonal surfaces**

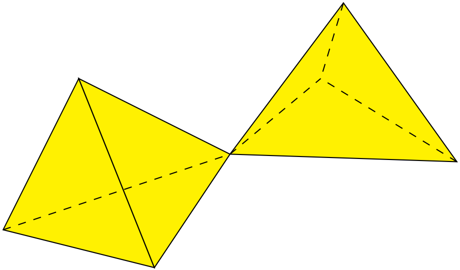
Why ramified?

Why ramified?

Typical example of ramified polygonal surface:

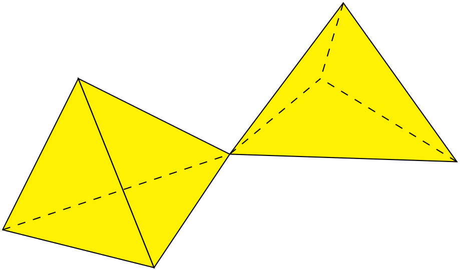
Why ramified?

Typical example of ramified polygonal surface:



Why ramified?

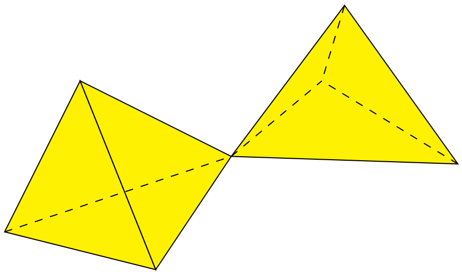
Typical example of ramified polygonal surface:



$\Rightarrow$ It is not a surface – there is a *ramification* at the central vertex

Why ramified?

Typical example of ramified polygonal surface:



$\Rightarrow$ It is not a surface – there is a *ramification* at the central vertex
A **polygonal surface** does not have these ramifications.

Embedding question

Embedding question

Given: A polygonal complex

Embedding question

Given: A polygonal complex

- Can it be embedded?

Embedding question

Given: A polygonal complex

- Can it be embedded?
- In how many ways?

Embedding question

Given: A polygonal complex

- Can it be embedded?
- In how many ways?

Simplifications:

Embedding question

Given: A polygonal complex

- Can it be embedded?
- In how many ways?

Simplifications:

1. Only polygonal surfaces (surface that is build from polygons)

Embedding question

Given: A polygonal complex

- Can it be embedded?
- In how many ways?

Simplifications:

1. Only polygonal surfaces (surface that is build from polygons)
2. All polygons are triangles (**simplicial surfaces**)

Embedding question

Given: A polygonal complex

- Can it be embedded?
- In how many ways?

Simplifications:

1. Only polygonal surfaces (surface that is build from polygons)
2. All polygons are triangles (**simplicial surfaces**)
3. All triangles are isometric

Embedding question

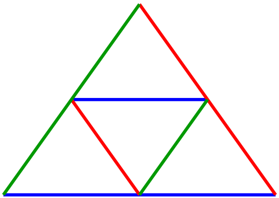
Given: A polygonal complex

- Can it be embedded?
- In how many ways?

Simplifications:

1. Only polygonal surfaces (surface that is build from polygons)
2. All polygons are triangles (**simplicial surfaces**)
3. All triangles are isometric

$\rightsquigarrow$ Edge–colouring encodes different lengths



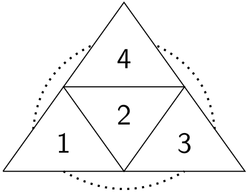
Colouring as permutation

Colouring as permutation

Consider tetrahedron

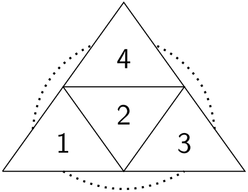
Colouring as permutation

Consider tetrahedron



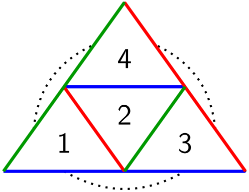
Colouring as permutation

Consider tetrahedron with edge colouring



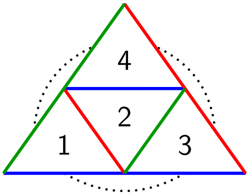
Colouring as permutation

Consider tetrahedron with edge colouring



Colouring as permutation

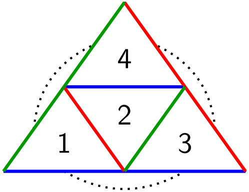
Consider tetrahedron with edge colouring



simplicial surface $\Rightarrow$

Colouring as permutation

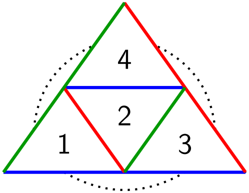
Consider tetrahedron with edge colouring



simplicial surface $\Rightarrow$ at most two faces at each edge

Colouring as permutation

Consider tetrahedron with edge colouring

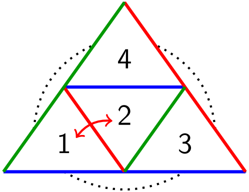


simplicial surface $\Rightarrow$ at most two faces at each edge

$\rightsquigarrow$ every edge defines transposition of incident faces

Colouring as permutation

Consider tetrahedron with edge colouring



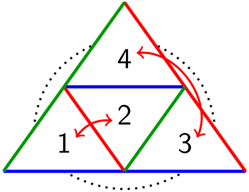
simplicial surface $\Rightarrow$ at most two faces at each edge

$\rightsquigarrow$ every edge defines transposition of incident faces

- (1,2)

Colouring as permutation

Consider tetrahedron with edge colouring



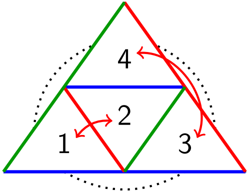
simplicial surface $\Rightarrow$ at most two faces at each edge

$\rightsquigarrow$ every edge defines transposition of incident faces

- (1,2)(3,4)

Colouring as permutation

Consider tetrahedron with edge colouring



simplicial surface $\Rightarrow$ at most two faces at each edge

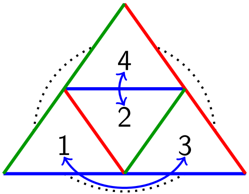
$\rightsquigarrow$ every edge defines transposition of incident faces

$\rightsquigarrow$ every colour class defines permutation of the faces

- (1,2)(3,4)

Colouring as permutation

Consider tetrahedron with edge colouring



simplicial surface ⇒ at most two faces at each edge

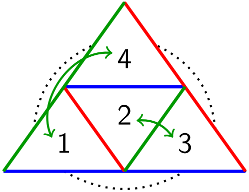
⇝ every edge defines transposition of incident faces

⇝ every colour class defines permutation of the faces

- (1,2)(3,4) , (1,3)(2,4)

Colouring as permutation

Consider tetrahedron with edge colouring

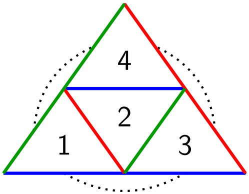


simplicial surface $\Rightarrow$ at most two faces at each edge

- $\rightsquigarrow$ every edge defines transposition of incident faces
- $\rightsquigarrow$ every colour class defines permutation of the faces
- (1,2)(3,4) , (1,3)(2,4) , (1,4)(2,3)

Colouring as permutation

Consider tetrahedron with edge colouring



simplicial surface $\Rightarrow$ at most two faces at each edge

$\rightsquigarrow$ every edge defines transposition of incident faces

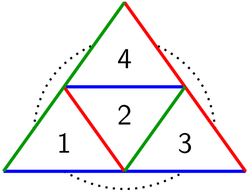
$\rightsquigarrow$ every colour class defines permutation of the faces

- (1,2)(3,4) , (1,3)(2,4) , (1,4)(2,3)

$\rightsquigarrow$ group theoretic considerations

Colouring as permutation

Consider tetrahedron with edge colouring



simplicial surface $\Rightarrow$ at most two faces at each edge

$\rightsquigarrow$ every edge defines transposition of incident faces

$\rightsquigarrow$ every colour class defines permutation of the faces

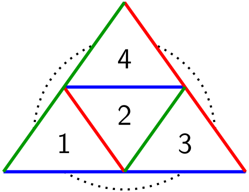
- (1,2)(3,4) , (1,3)(2,4) , (1,4)(2,3)

$\rightsquigarrow$ group theoretic considerations

- The connected components of the surface correspond to

Colouring as permutation

Consider tetrahedron with edge colouring



simplicial surface $\Rightarrow$ at most two faces at each edge

- $\rightsquigarrow$ every edge defines transposition of incident faces
- $\rightsquigarrow$ every colour class defines permutation of the faces
  - (1,2)(3,4) , (1,3)(2,4) , (1,4)(2,3)
- $\rightsquigarrow$ group theoretic considerations
  - The connected components of the surface correspond to the orbits of $\langle \sigma_a, \sigma_b, \sigma_c \rangle$ on the faces

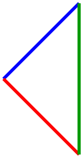
How do faces fit together?

How do faces fit together?

Consider a face of the surface

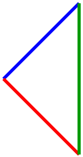
How do faces fit together?

Consider a face of the surface



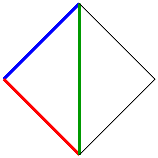
How do faces fit together?

Consider a face of the surface and a neighbouring face



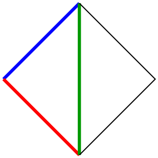
How do faces fit together?

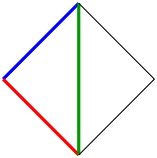
Consider a face of the surface and a neighbouring face



How do faces fit together?

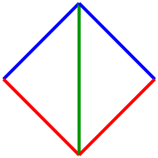
Consider a face of the surface and a neighbouring face
The neighbour can be coloured in two ways:

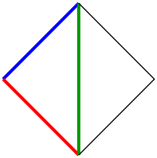




How do faces fit together?

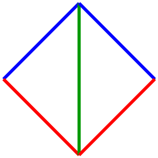
Consider a face of the surface and a neighbouring face
The neighbour can be coloured in two ways:

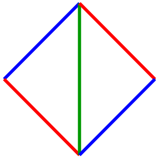




How do faces fit together?

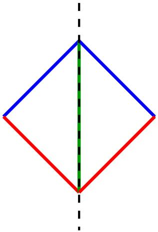
Consider a face of the surface and a neighbouring face
The neighbour can be coloured in two ways:



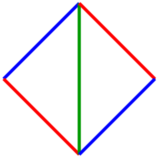


How do faces fit together?

Consider a face of the surface and a neighbouring face
The neighbour can be coloured in two ways:

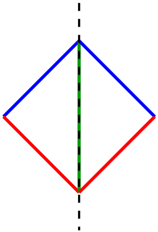


mirror (m)

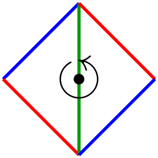


How do faces fit together?

Consider a face of the surface and a neighbouring face
The neighbour can be coloured in two ways:



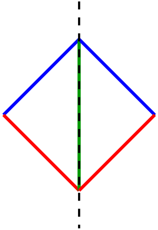
mirror (m)



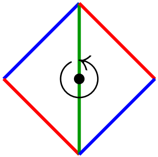
rotation (r)

How do faces fit together?

Consider a face of the surface and a neighbouring face
The neighbour can be coloured in two ways:



mirror (m)

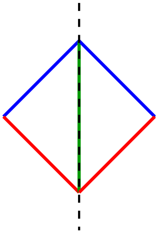


rotation (r)

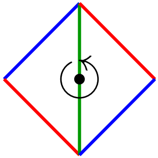
This gives an **mr–assignment** for the edges.

How do faces fit together?

Consider a face of the surface and a neighbouring face
The neighbour can be coloured in two ways:



mirror (m)



rotation (r)

This gives an **mr–assignment** for the edges.
Permutations and mr–assignment uniquely determine the surface.

Constructing surfaces from groups

Constructing surfaces from groups

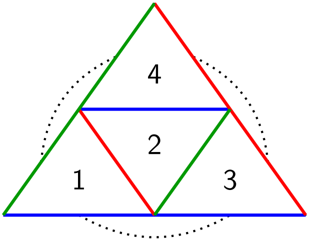
A general mr–assignment leads to complicated surfaces.

Constructing surfaces from groups

A general mr–assignment leads to complicated surfaces.
Simplification: edges of same colour have the same type

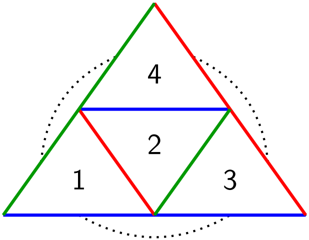
Constructing surfaces from groups

A general mr–assignment leads to complicated surfaces.
Simplification: edges of same colour have the same type
Example



Constructing surfaces from groups

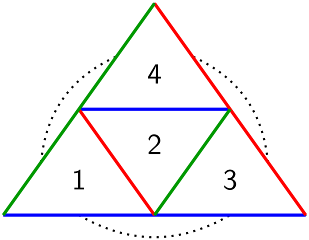
A general mr–assignment leads to complicated surfaces.
Simplification: edges of same colour have the same type
Example



has an rrr–structure

Constructing surfaces from groups

A general mr–assignment leads to complicated surfaces.
Simplification: edges of same colour have the same type
Example



has an rrr–structure
The easiest structure is an mmm–structure.

Covering

Covering

We want to characterize surfaces where all edges are mirrors.

Covering

We want to characterize surfaces where all edges are mirrors.

Lemma

A simplicial surface has an mmm–structure iff it covers a single triangle

Covering

We want to characterize surfaces where all edges are mirrors.

Lemma

A simplicial surface has an mmm–structure iff it covers a single triangle, i. e. there is an incidence–preserving map to the simplicial surface consisting of exactly one face.

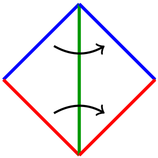
Covering

We want to characterize surfaces where all edges are mirrors.

Lemma

A simplicial surface has an mmm–structure iff it covers a single triangle, i. e. there is an incidence–preserving map to the simplicial surface consisting of exactly one face.

Consider



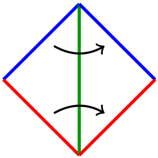
Covering

We want to characterize surfaces where all edges are mirrors.

Lemma

A simplicial surface has an mmm–structure iff it covers a single triangle, i. e. there is an incidence–preserving map to the simplicial surface consisting of exactly one face.

Consider



$\Rightarrow$ Colour of an edge is defined by vertices

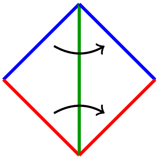
Covering

We want to characterize surfaces where all edges are mirrors.

Lemma

A simplicial surface has an mmm–structure iff it covers a single triangle, i. e. there is an incidence–preserving map to the simplicial surface consisting of exactly one face.

Consider



$\Rightarrow$ Colour of an edge is defined by vertices

- Covering pulls back a colouring of the triangle.

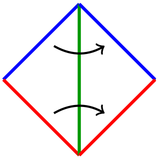
Covering

We want to characterize surfaces where all edges are mirrors.

Lemma

A simplicial surface has an mmm–structure iff it covers a single triangle, i. e. there is an incidence–preserving map to the simplicial surface consisting of exactly one face.

Consider



$\Rightarrow$ Colour of an edge is defined by vertices

- Covering pulls back a colouring of the triangle.
- Colouring defines a map to the triangle.

Construction from permutations

Construction from permutations

Start with three involutions σ_a, σ_b, σ_c

Construction from permutations

Start with three involutions σ_a, σ_b, σ_c (like generators of a finite group)

Construction from permutations

Start with three involutions σ_a, σ_b, σ_c (like generators of a finite group)

Lemma

There exists a coloured surface with the given involutions

Construction from permutations

Start with three involutions σ_a, σ_b, σ_c (like generators of a finite group)

Lemma

There exists a coloured surface with the given involutions where all edges are mirror edges.

Construction from permutations

Start with three involutions σ_a, σ_b, σ_c (like generators of a finite group)

Lemma

There exists a coloured surface with the given involutions where all edges are mirror edges.

- The faces are the points moved by the involutions

Construction from permutations

Start with three involutions σ_a, σ_b, σ_c (like generators of a finite group)

Lemma

There exists a coloured surface with the given involutions where all edges are mirror edges.

- The faces are the points moved by the involutions
- The edges are the cycles of the involutions

Construction from permutations

Start with three involutions σ_a, σ_b, σ_c (like generators of a finite group)

Lemma

There exists a coloured surface with the given involutions where all edges are mirror edges.

- The faces are the points moved by the involutions
- The edges are the cycles of the involutions
- The vertices are

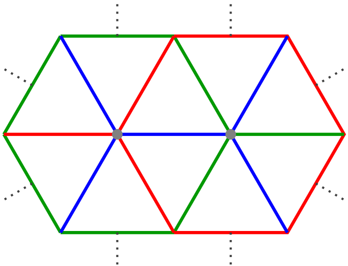
Construction from permutations

Start with three involutions σ_a, σ_b, σ_c (like generators of a finite group)

Lemma

There exists a coloured surface with the given involutions where all edges are mirror edges.

- The faces are the points moved by the involutions
- The edges are the cycles of the involutions
- The vertices are



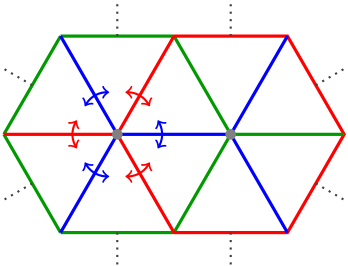
Construction from permutations

Start with three involutions σ_a, σ_b, σ_c (like generators of a finite group)

Lemma

There exists a coloured surface with the given involutions where all edges are mirror edges.

- The faces are the points moved by the involutions
- The edges are the cycles of the involutions
- The vertices are



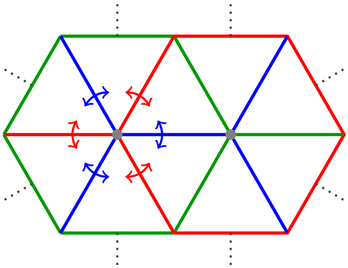
Construction from permutations

Start with three involutions σ_a, σ_b, σ_c (like generators of a finite group)

Lemma

There exists a coloured surface with the given involutions where all edges are mirror edges.

- The faces are the points moved by the involutions
- The edges are the cycles of the involutions
- The vertices are the orbits of $\langle \sigma_a, \sigma_b \rangle$ on the faces



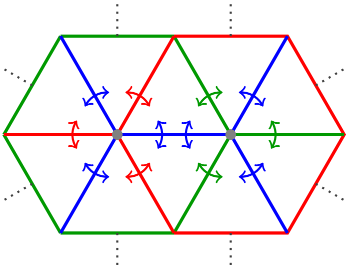
Construction from permutations

Start with three involutions σ_a, σ_b, σ_c (like generators of a finite group)

Lemma

There exists a coloured surface with the given involutions where all edges are mirror edges.

- The faces are the points moved by the involutions
- The edges are the cycles of the involutions
- The vertices are the orbits of $\langle \sigma_a, \sigma_b \rangle$ on the faces (for all pairs)



Construction example

Construction example

$\sigma_a = (1,2)(3,4)(5,6)(7,8)$

Construction example

$\sigma_a = (1,2)(3,4)(5,6)(7,8)$

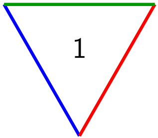
$\sigma_b = (1,4)(2,3)(5,8)(6,7)$

Construction example

$\sigma_a = (1,2)(3,4)(5,6)(7,8)$
$\sigma_b = (1,4)(2,3)(5,8)(6,7)$
$\sigma_c = (1,5)(2,6)(3,7)(4,8)$

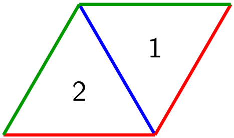
Construction example

$\sigma_a = (1,2)(3,4)(5,6)(7,8)$
$\sigma_b = (1,4)(2,3)(5,8)(6,7)$
$\sigma_c = (1,5)(2,6)(3,7)(4,8)$



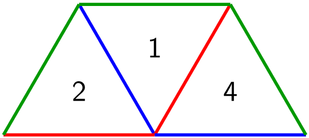
Construction example

$\sigma_a = (1,2)(3,4)(5,6)(7,8)$
$\sigma_b = (1,4)(2,3)(5,8)(6,7)$
$\sigma_c = (1,5)(2,6)(3,7)(4,8)$



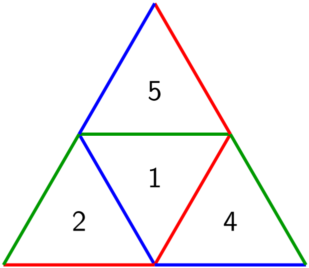
Construction example

$\sigma_a = (1,2)(3,4)(5,6)(7,8)$
$\sigma_b = (1,4)(2,3)(5,8)(6,7)$
$\sigma_c = (1,5)(2,6)(3,7)(4,8)$



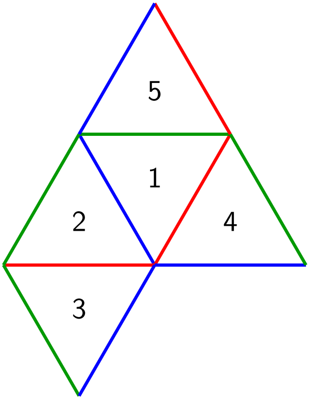
Construction example

$\sigma_a = (1,2)(3,4)(5,6)(7,8)$
$\sigma_b = (1,4)(2,3)(5,8)(6,7)$
$\sigma_c = (1,5)(2,6)(3,7)(4,8)$



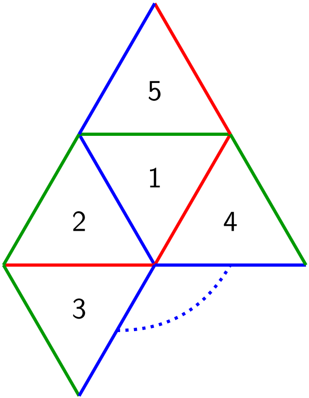
Construction example

$\sigma_a = (1,2)(3,4)(5,6)(7,8)$
$\sigma_b = (1,4)(2,3)(5,8)(6,7)$
$\sigma_c = (1,5)(2,6)(3,7)(4,8)$



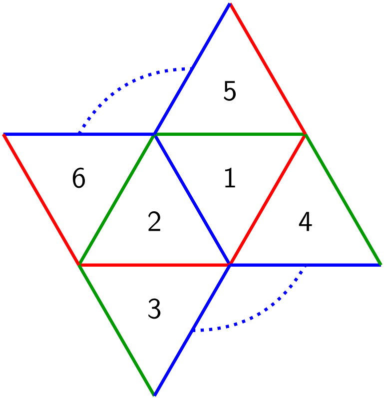
Construction example

$\sigma_a = (1,2)(3,4)(5,6)(7,8)$
$\sigma_b = (1,4)(2,3)(5,8)(6,7)$
$\sigma_c = (1,5)(2,6)(3,7)(4,8)$



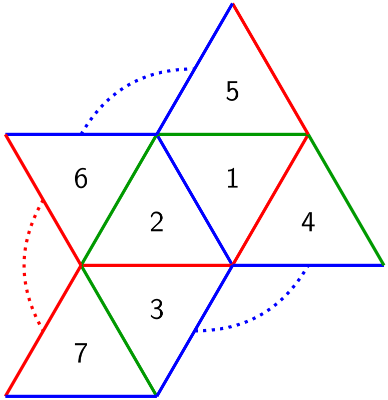
Construction example

$\sigma_a = (1,2)(3,4)(5,6)(7,8)$
$\sigma_b = (1,4)(2,3)(5,8)(6,7)$
$\sigma_c = (1,5)(2,6)(3,7)(4,8)$



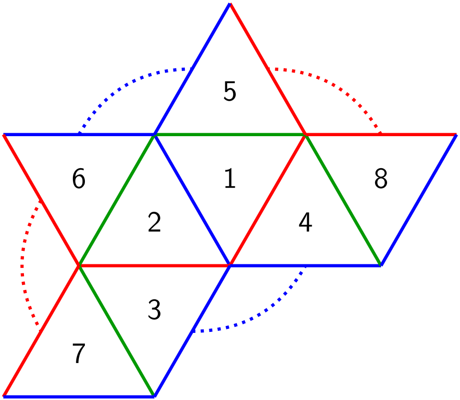
Construction example

$\sigma_a = (1,2)(3,4)(5,6)(7,8)$
$\sigma_b = (1,4)(2,3)(5,8)(6,7)$
$\sigma_c = (1,5)(2,6)(3,7)(4,8)$



Construction example

$\sigma_a = (1,2)(3,4)(5,6)(7,8)$
$\sigma_b = (1,4)(2,3)(5,8)(6,7)$
$\sigma_c = (1,5)(2,6)(3,7)(4,8)$

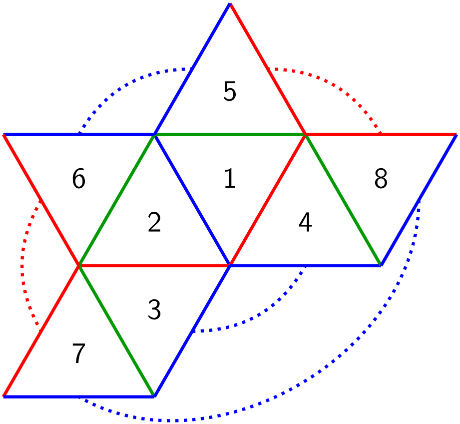


Construction example

$\sigma_a = (1,2)(3,4)(5,6)(7,8)$

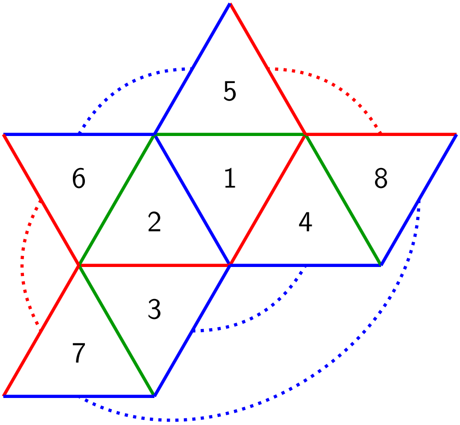
$\sigma_b = (1,4)(2,3)(5,8)(6,7)$

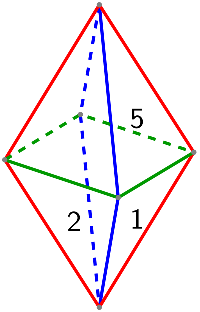
$\sigma_c = (1,5)(2,6)(3,7)(4,8)$



Construction example

$\sigma_a = (1,2)(3,4)(5,6)(7,8)$
$\sigma_b = (1,4)(2,3)(5,8)(6,7)$
$\sigma_c = (1,5)(2,6)(3,7)(4,8)$





What kind of folding?

What kind of folding?

There are many different kinds of folding (e. g. Origami)

What kind of folding?

There are many different kinds of folding (e. g. Origami)

Here:

What kind of folding?

There are many different kinds of folding (e. g. Origami)

Here:

- Folding of surface in $\mathbb{R}^3$

What kind of folding?

There are many different kinds of folding (e. g. Origami)
Here:

- Folding of surface in $\mathbb{R}^3$
- Possible folding edges are fixed

What kind of folding?

There are many different kinds of folding (e. g. Origami)

Here:

- Folding of surface in $\mathbb{R}^3$
- Possible folding edges are fixed
- Folding should be rigid (no curvature)

What kind of folding?

There are many different kinds of folding (e. g. Origami)

Here:

- Folding of surface in $\mathbb{R}^3$
- Possible folding edges are fixed
- Folding should be rigid (no curvature)

Goal: Classify possible folding patterns (given a net)

What kind of folding?

There are many different kinds of folding (e. g. Origami)

Here:

- Folding of surface in $\mathbb{R}^3$
- Possible folding edges are fixed
- Folding should be rigid (no curvature)

Goal: Classify possible folding patterns (given a net)

Why are embeddings hard?

Why are embeddings hard?

Ideally, we would like to have embeddings.

Why are embeddings hard?

Ideally, we would like to have embeddings.

But we want to define folding independently from an embedding, since:

Why are embeddings hard?

Ideally, we would like to have embeddings.

But we want to define folding independently from an embedding, since:

- They are very hard to compute (even for small examples)

Why are embeddings hard?

Ideally, we would like to have embeddings.

But we want to define folding independently from an embedding, since:

- They are very hard to compute (even for small examples)
- We can only show foldability for specific small examples

Why are embeddings hard?

Ideally, we would like to have embeddings.

But we want to define folding independently from an embedding, since:

- They are very hard to compute (even for small examples)
- We can only show foldability for specific small examples
  - Usually using regularity (like crystallographic symmetry)

Why are embeddings hard?

Ideally, we would like to have embeddings.

But we want to define folding independently from an embedding, since:

- They are very hard to compute (even for small examples)
- We can only show foldability for specific small examples
  - Usually using regularity (like crystallographic symmetry)
  - No general method

Why are embeddings hard?

Ideally, we would like to have embeddings.

But we want to define folding independently from an embedding, since:

- They are very hard to compute (even for small examples)
- We can only show foldability for specific small examples
  - Usually using regularity (like crystallographic symmetry)
  - No general method
- It is very hard to define iterated folding in an embedding

Why are embeddings hard?

Ideally, we would like to have embeddings.

But we want to define folding independently from an embedding, since:

- They are very hard to compute (even for small examples)
- We can only show foldability for specific small examples
  - Usually using regularity (like crystallographic symmetry)
  - No general method
- It is very hard to define iterated folding in an embedding

Folding without embedding

Folding without embedding

Central idea:

Folding without embedding

Central idea:

- Don't model folding process (needs embedding)

Folding without embedding

Central idea:

- Don't model folding process (needs embedding)
- Describe starting and final folding state

Folding without embedding

Central idea:

- Don't model folding process (needs embedding)
- Describe starting and final folding state
  - Only consider changes in the topology

Folding without embedding

Central idea:

- Don't model folding process (needs embedding)
- Describe starting and final folding state
  - Only consider changes in the topology (like identification of faces)

Folding without embedding

Central idea:

- Don't model folding process (needs embedding)
- Describe starting and final folding state
  - Only consider changes in the topology (like identification of faces)
  - allows abstraction from embedding

Folding without embedding

Central idea:

- Don't model folding process (needs embedding)
- Describe starting and final folding state
  - Only consider changes in the topology (like identification of faces)
  - allows abstraction from embedding

$\rightsquigarrow$ Incidence geometry (polygonal complex/surface)

Folding without embedding

Central idea:

- Don't model folding process (needs embedding)
- Describe starting and final folding state
  - Only consider changes in the topology (like identification of faces)
  - allows abstraction from embedding

$\rightsquigarrow$ Incidence geometry (polygonal complex/surface)

- Captures some folding restrictions

Folding without embedding

Central idea:

- Don't model folding process (needs embedding)
- Describe starting and final folding state
  - Only consider changes in the topology (like identification of faces)
  - allows abstraction from embedding

$\rightsquigarrow$ Incidence geometry (polygonal complex/surface)

- Captures some folding restrictions (rigidity of tetrahedron)

Folding without embedding

Central idea:

- Don't model folding process (needs embedding)
- Describe starting and final folding state
  - Only consider changes in the topology (like identification of faces)
  - allows abstraction from embedding

$\rightsquigarrow$ Incidence geometry (polygonal complex/surface)

- Captures some folding restrictions (rigidity of tetrahedron)
- Still needs a lot of refinement

Important properties of folding

Important properties of folding

- The class of surfaces is not closed under folding

Important properties of folding

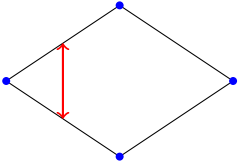
- The class of surfaces is not closed under folding
- Folding can be undone by *unfolding*

Important properties of folding

- The class of surfaces is not closed under folding
- Folding can be undone by *unfolding*
- Identification of two faces might force identification of two other faces

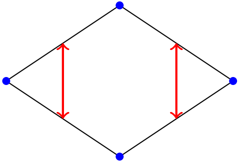
Important properties of folding

- The class of surfaces is not closed under folding
- Folding can be undone by *unfolding*
- Identification of two faces might force identification of two other faces



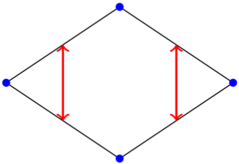
Important properties of folding

- The class of surfaces is not closed under folding
- Folding can be undone by *unfolding*
- Identification of two faces might force identification of two other faces



Important properties of folding

- The class of surfaces is not closed under folding
- Folding can be undone by *unfolding*
- Identification of two faces might force identification of two other faces
  - Can apply to arbitrary many faces



Important properties of folding

- The class of surfaces is not closed under folding
- Folding can be undone by *unfolding*
- Identification of two faces might force identification of two other faces
  - Can apply to arbitrary many faces

Important properties of folding

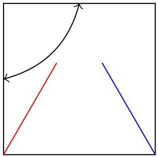
- The class of surfaces is not closed under folding
- Folding can be undone by *unfolding*
- Identification of two faces might force identification of two other faces
  - Can apply to arbitrary many faces

Important properties of folding

- The class of surfaces is not closed under folding
- Folding can be undone by *unfolding*
- Identification of two faces might force identification of two other faces
  - Can apply to arbitrary many faces
  - The forced identification is not unique

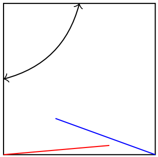
Important properties of folding

- The class of surfaces is not closed under folding
- Folding can be undone by *unfolding*
- Identification of two faces might force identification of two other faces
  - Can apply to arbitrary many faces
  - The forced identification is not unique



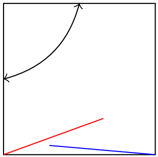
Important properties of folding

- The class of surfaces is not closed under folding
- Folding can be undone by *unfolding*
- Identification of two faces might force identification of two other faces
    - Can apply to arbitrary many faces
    - The forced identification is not unique



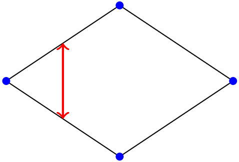
Important properties of folding

- The class of surfaces is not closed under folding
- Folding can be undone by *unfolding*
- Identification of two faces might force identification of two other faces
  - Can apply to arbitrary many faces
  - The forced identification is not unique



Important properties of folding

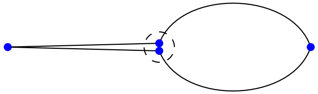
- The class of surfaces is not closed under folding
- Folding can be undone by *unfolding*
- Identification of two faces might force identification of two other faces
  - Can apply to arbitrary many faces
  - The forced identification is not unique
  - $\Rightarrow$ Identify only two faces at a time



Important properties of folding

- The class of surfaces is not closed under folding
- Folding can be undone by *unfolding*
- Identification of two faces might force identification of two other faces
  - Can apply to arbitrary many faces
  - The forced identification is not unique

  $\Rightarrow$ Identify only two faces at a time



How to define abstract folding?

How to define abstract folding?

We need to define two structures:

How to define abstract folding?

We need to define two structures:

1. A folding state

How to define abstract folding?

We need to define two structures:

1. A folding state

2. The folding steps

How to define abstract folding?

We need to define two structures:

1. A folding state
    - Based on polygonal complexes
2. The folding steps

How to define abstract folding?

We need to define two structures:

1. A folding state
    - Based on polygonal complexes
    - Describe "is folded together" by an equivalence relation
2. The folding steps

How to define abstract folding?

We need to define two structures:

1. A folding state
   - Based on polygonal complexes
   - Describe "is folded together" by an equivalence relation
   - Describe order of faces in folding state
2. The folding steps

How to define abstract folding?

We need to define two structures:

1. A folding state
    - Based on polygonal complexes
    - Describe "is folded together" by an equivalence relation
    - Describe order of faces in folding state
2. The folding steps
    - Only two faces at a time

How to define abstract folding?

We need to define two structures:

1. A folding state
   - Based on polygonal complexes
   - Describe "is folded together" by an equivalence relation
   - Describe order of faces in folding state
2. The folding steps
   - Only two faces at a time
   - Explain "unordered folding" (e. g. covering)

How to define abstract folding?

We need to define two structures:

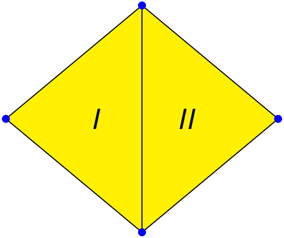
1. A folding state
    - Based on polygonal complexes
    - Describe "is folded together" by an equivalence relation
    - Describe order of faces in folding state
2. The folding steps
    - Only two faces at a time
    - Explain "unordered folding" (e. g. covering)
    - Modify to include face order relations

Unordered Folding (Covering)

Unordered Folding (Covering)

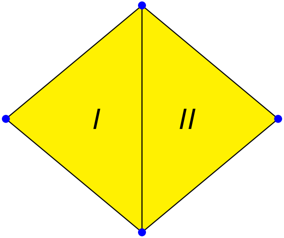
Why do we need more than a polygonal complex?

Unordered Folding (Covering)



Why do we need more than a polygonal complex?

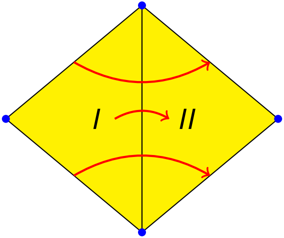
Unordered Folding (Covering)



Why do we need more than a polygonal complex?

Naive folding definition: surjective map that respects incidence

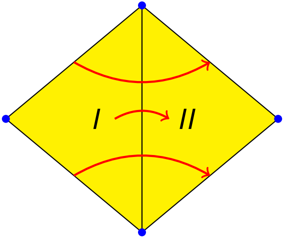
Unordered Folding (Covering)



Why do we need more than a polygonal complex?

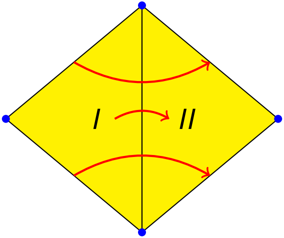
Naive folding definition: surjective map that respects incidence

Unordered Folding (Covering)



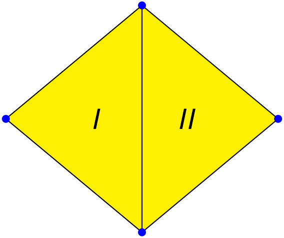
Why do we need more than a polygonal complex?
Naive folding definition: surjective map that respects incidence
Problem: Can't be unfolded

Unordered Folding (Covering)



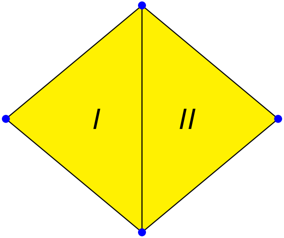
Why do we need more than a polygonal complex?
Naive folding definition: surjective map that respects incidence
Problem: Can't be unfolded
$\Rightarrow$ Folding state should not forget original structure

Unordered Folding (Covering)



Represent folding by equivalence relation

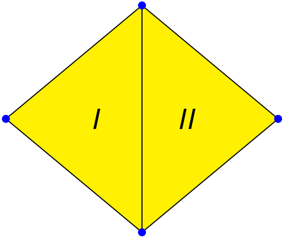
Unordered Folding (Covering)



Represent folding by equivalence relation

- Separate relation on vertices, edges and faces

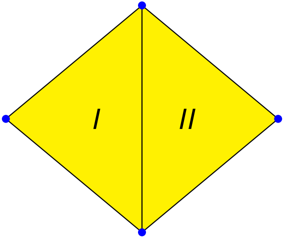
Unordered Folding (Covering)



Represent folding by equivalence relation

- Separate relation on vertices, edges and faces
- Two elements are equivalent if they are folded together

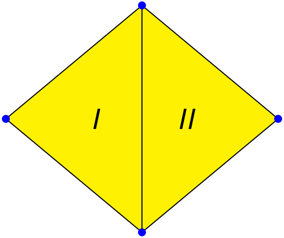
Unordered Folding (Covering)



Represent folding by equivalence relation

- Separate relation on vertices, edges and faces
- Two elements are equivalent if they are folded together
- If two edges are equivalent,

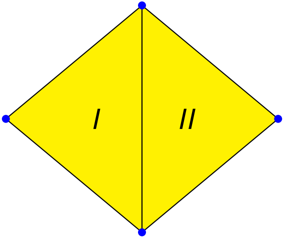
Unordered Folding (Covering)



Represent folding by equivalence relation

- Separate relation on vertices, edges and faces
- Two elements are equivalent if they are folded together
- If two edges are equivalent, then their vertices have to be as well

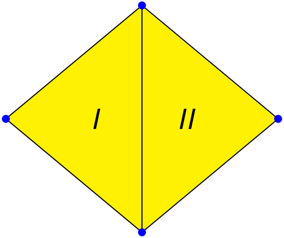
Unordered Folding (Covering)



Represent folding by equivalence relation

- Separate relation on vertices, edges and faces
- Two elements are equivalent if they are folded together
- If two edges are equivalent, then their vertices have to be as well (likewise for faces)

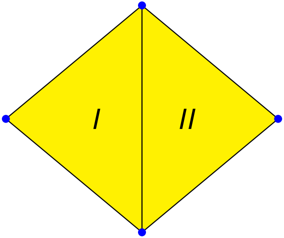
Unordered Folding (Covering)



Represent folding by equivalence relation

- Separate relation on vertices, edges and faces
- Two elements are equivalent if they are folded together
- If two edges are equivalent, then their vertices have to be as well (likewise for faces)
- The vertices of an edge are not equivalent

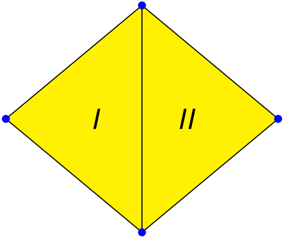
Unordered Folding (Covering)



Represent folding by equivalence relation

- Separate relation on vertices, edges and faces
- Two elements are equivalent if they are folded together
- If two edges are equivalent, then their vertices have to be as well (likewise for faces)
- The vertices of an edge are not equivalent (likewise for faces)

Unordered Folding (Covering)



Represent folding by equivalence relation

- Separate relation on vertices, edges and faces
- Two elements are equivalent if they are folded together
- If two edges are equivalent, then their vertices have to be as well (likewise for faces)
- The vertices of an edge are not equivalent (likewise for faces)

$\Rightarrow$ Unordered folding is coarsening of equivalence relation

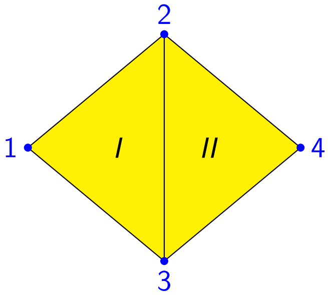
How does folding work?

How does folding work?

1. Choose two faces that are not folded together

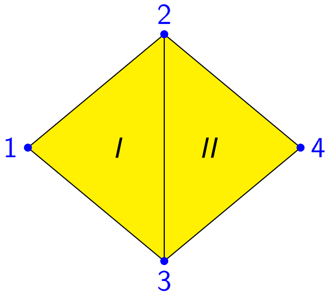
How does folding work?

1. Choose two faces that are not folded together



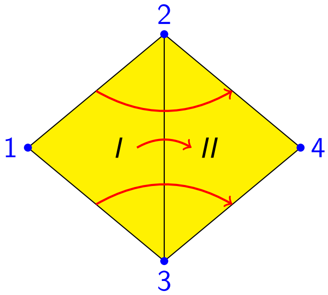
How does folding work?

1. Choose two faces that are not folded together
2. Choose how to identify them



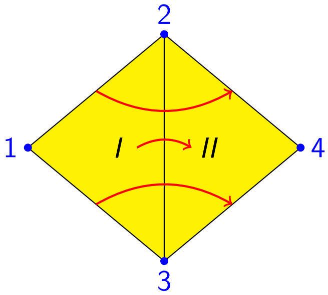
How does folding work?

1. Choose two faces that are not folded together
2. Choose how to identify them



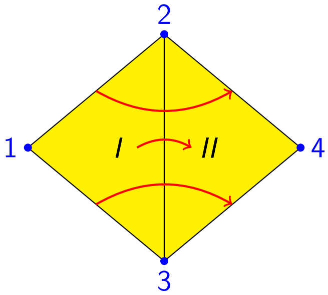
How does folding work?

1. Choose two faces that are not folded together
2. Choose how to identify them (like $I \sim II$ and $1 \sim 4$)



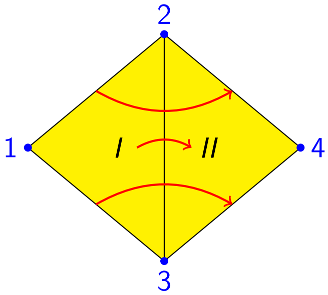
How does folding work?

1. Choose two faces that are not folded together
2. Choose how to identify them (like $I \sim II$ and $1 \sim 4$)
3. Add those pairs to the equivalence relation



How does folding work?

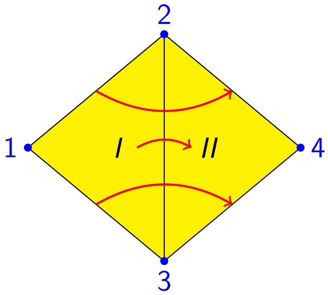
1. Choose two faces that are not folded together
2. Choose how to identify them (like $I \sim II$ and $1 \sim 4$)
3. Add those pairs to the equivalence relation



Only restriction:

How does folding work?

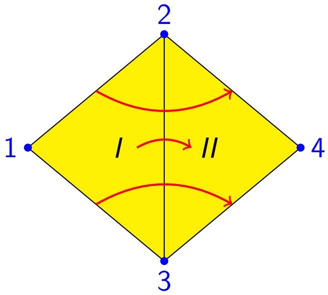
1. Choose two faces that are not folded together
2. Choose how to identify them (like $I \sim II$ and $1 \sim 4$)
3. Add those pairs to the equivalence relation



Only restriction:
Two vertices in an edge can't be identified

How does folding work?

1. Choose two faces that are not folded together
2. Choose how to identify them (like $I \sim II$ and $1 \sim 4$)
3. Add those pairs to the equivalence relation



Only restriction:
Two vertices in an edge can't be identified (slightly generalized)

Limitation of unordered folding

Limitation of unordered folding

We can't work with ordering of faces:

Limitation of unordered folding

We can't work with ordering of faces:

Limitation of unordered folding

We can't work with ordering of faces:

Adding a linear order on each face equivalence class

Limitation of unordered folding

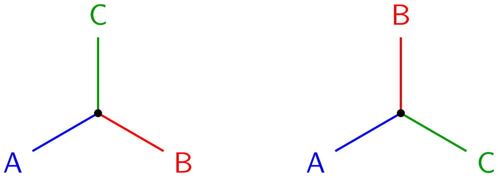
We can't work with ordering of faces:

Adding a linear order on each face equivalence class is not enough:

Limitation of unordered folding

We can't work with ordering of faces:

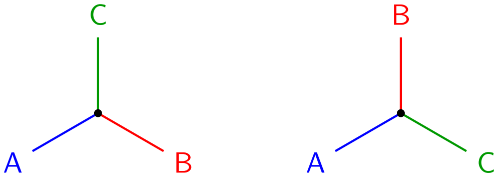
Adding a linear order on each face equivalence class is not enough:



Limitation of unordered folding

We can't work with ordering of faces:

Adding a linear order on each face equivalence class is not enough:

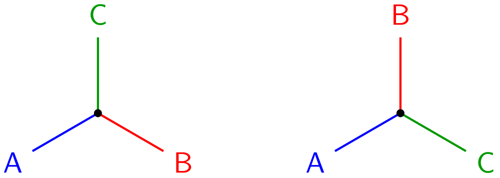


$\rightsquigarrow$ define order of faces around edges

Limitation of unordered folding

We can't work with ordering of faces:

Adding a linear order on each face equivalence class is not enough:



$\rightsquigarrow$ define order of faces around edges (we will skip the details)

Folding complex

Folding complex

Definition

A **folding complex**

Folding complex

Definition

A **folding complex** *is a polygonal complex together with*

Folding complex

Definition

A **folding complex** *is a polygonal complex together with*

1. *An equivalence relation on vertices, edges and faces*

Folding complex

Definition

A **folding complex** *is a polygonal complex together with*

1. *An equivalence relation on vertices, edges and faces ("is folded together")*

Folding complex

Definition

A **folding complex** *is a polygonal complex together with*

1. *An equivalence relation on vertices, edges and faces ("is folded together")*
2. *A linear ordering on each face equivalence class*

Folding complex

Definition

A **folding complex** *is a polygonal complex together with*

1. *An equivalence relation on vertices, edges and faces ("is folded together")*
2. *A linear ordering on each face equivalence class*
3. *A cyclical ordering of the faces around each edge equivalence class*

Folding complex

Definition

A **folding complex** *is a polygonal complex together with*

1. *An equivalence relation on vertices, edges and faces ("is folded together")*
2. *A linear ordering on each face equivalence class*
3. *A cyclical ordering of the faces around each edge equivalence class*

such that the orderings are compatible

Folding complex

Definition

A **folding complex** *is a polygonal complex together with*

1. *An equivalence relation on vertices, edges and faces ("is folded together")*
2. *A linear ordering on each face equivalence class*
3. *A cyclical ordering of the faces around each edge equivalence class*

such that the orderings are compatible (in an appropriate sense).

Folding complex

Definition

A **folding complex** *is a polygonal complex together with*

1. *An equivalence relation on vertices, edges and faces ("is folded together")*
2. *A linear ordering on each face equivalence class*
3. *A cyclical ordering of the faces around each edge equivalence class*

such that the orderings are compatible (in an appropriate sense).

To identify faces with each other, we have to combine those orderings.

Folding complex

Definition

A **folding complex** *is a polygonal complex together with*

1. *An equivalence relation on vertices, edges and faces ("is folded together")*
2. *A linear ordering on each face equivalence class*
3. *A cyclical ordering of the faces around each edge equivalence class*

such that the orderings are compatible (in an appropriate sense).

To identify faces with each other, we have to combine those orderings.

- linear orderings get concatenated

Folding complex

Definition

A **folding complex** *is a polygonal complex together with*

1. *An equivalence relation on vertices, edges and faces ("is folded together")*
2. *A linear ordering on each face equivalence class*
3. *A cyclical ordering of the faces around each edge equivalence class*

such that the orderings are compatible (in an appropriate sense).

To identify faces with each other, we have to combine those orderings.

- linear orderings get concatenated
- cyclical orderings are opened at one point

Folding complex

Definition

A **folding complex** *is a polygonal complex together with*

1. *An equivalence relation on vertices, edges and faces ("is folded together")*
2. *A linear ordering on each face equivalence class*
3. *A cyclical ordering of the faces around each edge equivalence class*

such that the orderings are compatible (in an appropriate sense).

To identify faces with each other, we have to combine those orderings.

- linear orderings get concatenated
- cyclical orderings are opened at one point and combined

Folding complex

Definition

A **folding complex** *is a polygonal complex together with*

1. *An equivalence relation on vertices, edges and faces ("is folded together")*
2. *A linear ordering on each face equivalence class*
3. *A cyclical ordering of the faces around each edge equivalence class*

such that the orderings are compatible (in an appropriate sense).

To identify faces with each other, we have to combine those orderings.

- linear orderings get concatenated
- cyclical orderings are opened at one point and combined

!! compatibility is not easily transfered,

Folding complex

Definition

A **folding complex** *is a polygonal complex together with*

1. *An equivalence relation on vertices, edges and faces ("is folded together")*
2. *A linear ordering on each face equivalence class*
3. *A cyclical ordering of the faces around each edge equivalence class*

such that the orderings are compatible (in an appropriate sense).

To identify faces with each other, we have to combine those orderings.

- linear orderings get concatenated
- cyclical orderings are opened at one point and combined

!! compatibility is not easily transfered, but can be calculated

Changed definition of folding

Changed definition of folding

Before (no ordering):

Changed definition of folding

Before (no ordering):

1. Choose two faces that are not folded together

Changed definition of folding

Before (no ordering):

1. Choose two faces that are not folded together
2. Choose how to identify them and extend the equivalence relation

Changed definition of folding

Folding with ordering:

1. Choose two faces that are not folded together
2. Choose how to identify them and extend the equivalence relation

Changed definition of folding

Folding with ordering:

1. Choose two faces that are not folded together
2. Choose how to identify them and extend the equivalence relation

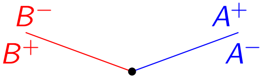
$\rightsquigarrow$ Each face has two sides

Changed definition of folding

Folding with ordering:

1. Choose two faces that are not folded together
2. Choose how to identify them and extend the equivalence relation

$\rightsquigarrow$ Each face has two sides

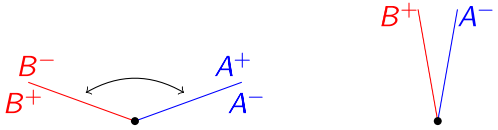


Changed definition of folding

Folding with ordering:

1. Choose two faces that are not folded together
2. Choose how to identify them and extend the equivalence relation

$\rightsquigarrow$ Each face has two sides

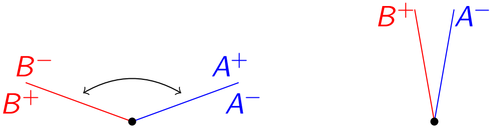


Changed definition of folding

Folding with ordering:

1. Choose two faces that are not folded together
2. Choose how to identify them and extend the equivalence relation
3. Choose the sides of the faces that will meet

$\rightsquigarrow$ Each face has two sides

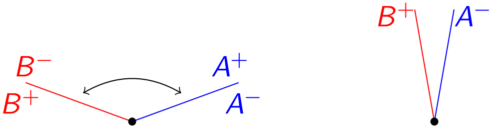


Changed definition of folding

Folding with ordering:

1. Choose two faces that are not folded together
2. Choose how to identify them and extend the equivalence relation
3. Choose the sides of the faces that will meet and modify the orderings

$\rightsquigarrow$ Each face has two sides

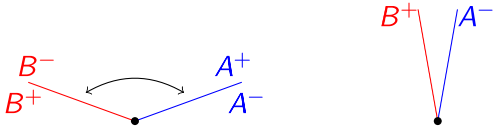


Changed definition of folding

Folding with ordering:

1. Choose two faces that are not folded together
2. Choose how to identify them and extend the equivalence relation
3. Choose the sides of the faces that will meet and modify the orderings

$\rightsquigarrow$ Each face has two sides



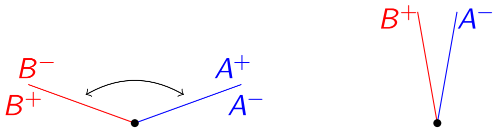
$\Rightarrow$ Define folding by two face sides (**folding plan**)

Changed definition of folding

Folding with ordering:

1. Choose two faces that are not folded together
2. Choose how to identify them and extend the equivalence relation
3. Choose the sides of the faces that will meet and modify the orderings

$\rightsquigarrow$ Each face has two sides



$\Rightarrow$ Define folding by two face sides (**folding plan**)
$\rightsquigarrow$ Allows reversible (un)folding

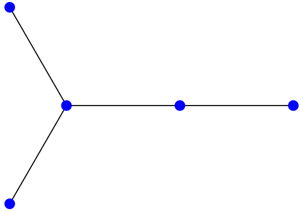
Structure of multiple foldings

Structure of multiple foldings

With folding plans we can perform the same folding in different folding complexes

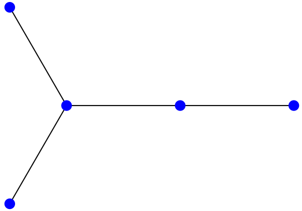
Structure of multiple foldings

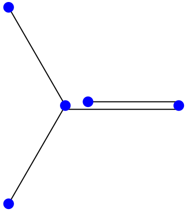
With folding plans we can perform the same folding in different folding complexes



Structure of multiple foldings

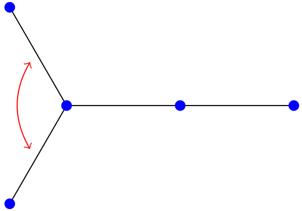
With folding plans we can perform the same folding in different folding complexes

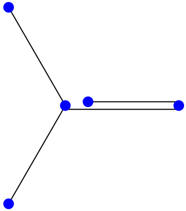




Structure of multiple foldings

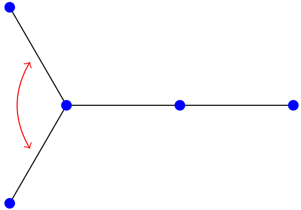
With folding plans we can perform the same folding in different folding complexes

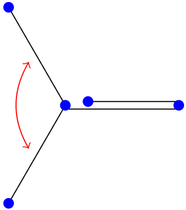




Structure of multiple foldings

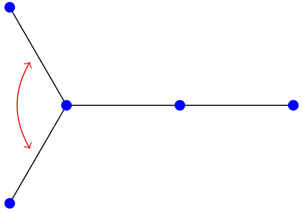
With folding plans we can perform the same folding in different folding complexes

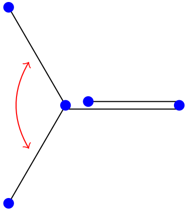




Structure of multiple foldings

With folding plans we can perform the same folding in different folding complexes





$\rightsquigarrow$ more structure on the set of possible foldings

Folding graph

Folding graph

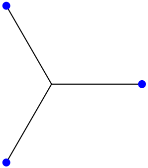
- Vertices are folding complexes (modelling folding states)

Folding graph

- Vertices are folding complexes (modelling folding states)
- Edges are folding plans connecting two folding complexes

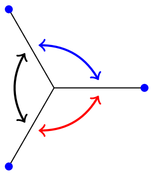
Folding graph

- Vertices are folding complexes (modelling folding states)
- Edges are folding plans connecting two folding complexes



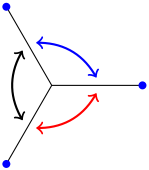
Folding graph

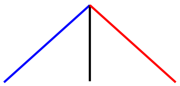
- Vertices are folding complexes (modelling folding states)
- Edges are folding plans connecting two folding complexes



Folding graph

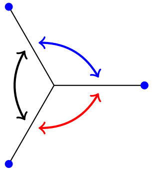
- Vertices are folding complexes (modelling folding states)
- Edges are folding plans connecting two folding complexes

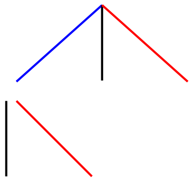




Folding graph

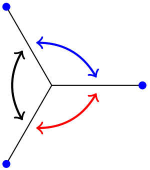
- Vertices are folding complexes (modelling folding states)
- Edges are folding plans connecting two folding complexes

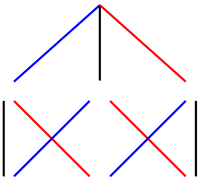




Folding graph

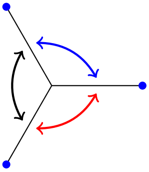
- Vertices are folding complexes (modelling folding states)
- Edges are folding plans connecting two folding complexes

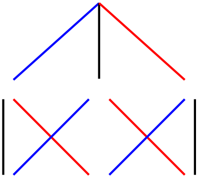


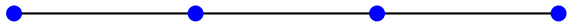


Folding graph

- Vertices are folding complexes (modelling folding states)
- Edges are folding plans connecting two folding complexes

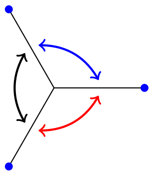


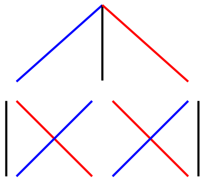


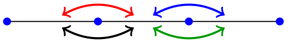


Folding graph

- Vertices are folding complexes (modelling folding states)
- Edges are folding plans connecting two folding complexes

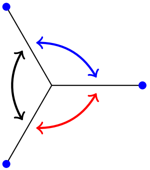


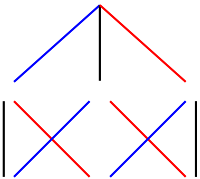


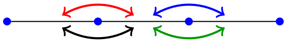


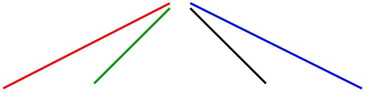
Folding graph

- Vertices are folding complexes (modelling folding states)
- Edges are folding plans connecting two folding complexes



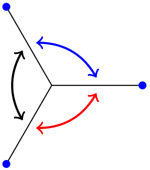


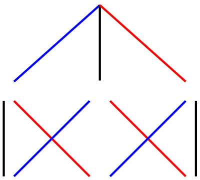


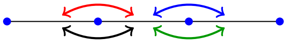


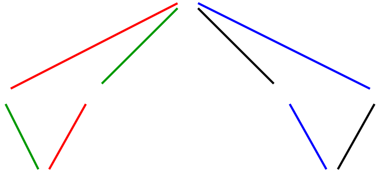
Folding graph

- Vertices are folding complexes (modelling folding states)
- Edges are folding plans connecting two folding complexes



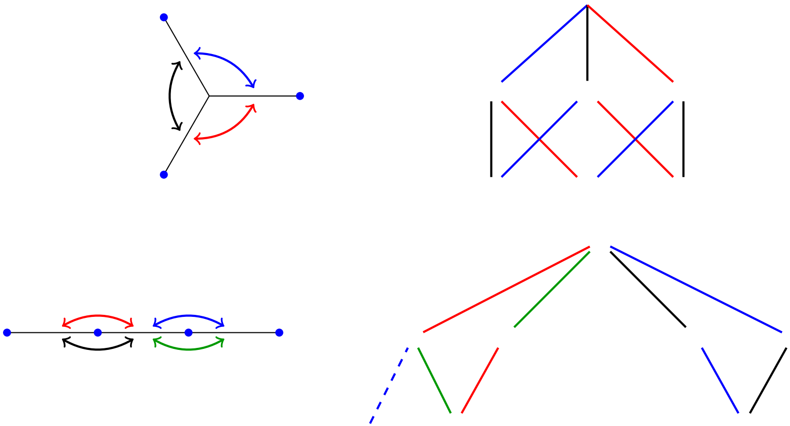






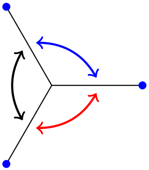
Folding graph

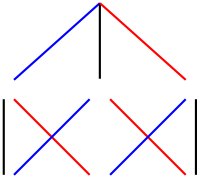
- Vertices are folding complexes (modelling folding states)
- Edges are folding plans connecting two folding complexes

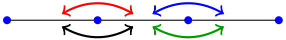


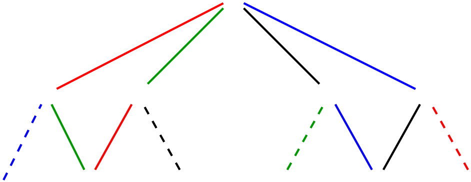
Folding graph

- Vertices are folding complexes (modelling folding states)
- Edges are folding plans connecting two folding complexes









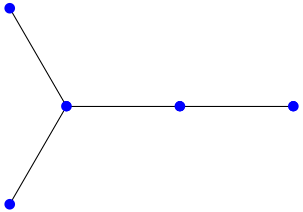
Drawback of folding plans

Drawback of folding plans

Some foldings that "should" be the same, aren't:

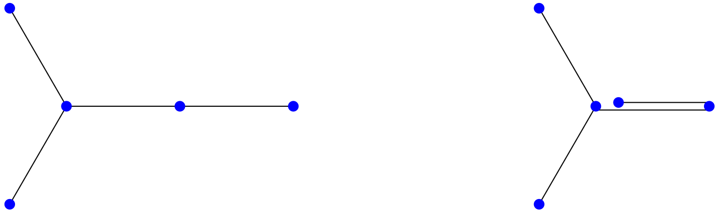
Drawback of folding plans

Some foldings that "should" be the same, aren't:



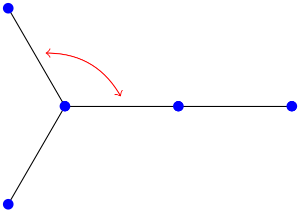
Drawback of folding plans

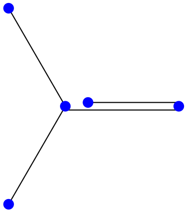
Some foldings that "should" be the same, aren't:



Drawback of folding plans

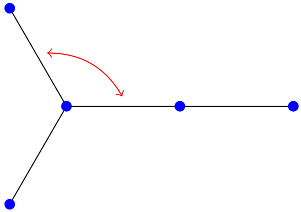
Some foldings that “should” be the same, aren’t:

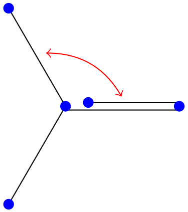




Drawback of folding plans

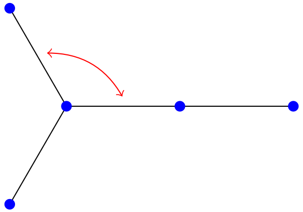
Some foldings that "should" be the same, aren't:

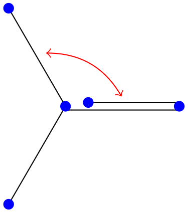




Drawback of folding plans

Some foldings that “should” be the same, aren't:

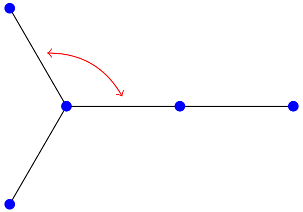


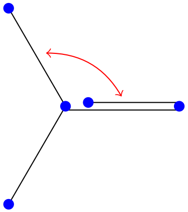


$\Rightarrow$ If you know the folding structure of a small complex,

Drawback of folding plans

Some foldings that "should" be the same, aren't:

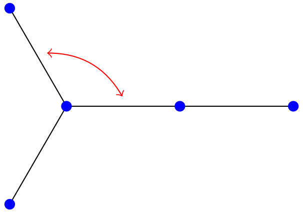


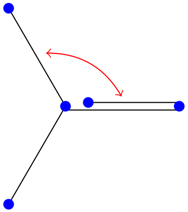


$\Rightarrow$ If you know the folding structure of a small complex, you can't easily find the folding structure of an extended complex

Drawback of folding plans

Some foldings that “should” be the same, aren’t:





$\Rightarrow$ If you know the folding structure of a small complex, you can’t easily find the folding structure of an extended complex

$\rightsquigarrow$ Folding plans are not optimal to model folding.

Questions?