Simplicial surfaces in GAP

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RWTH Aachen University

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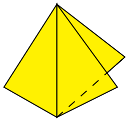

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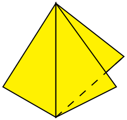

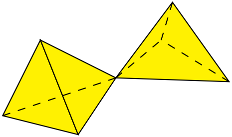

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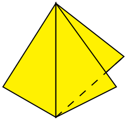

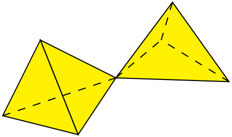

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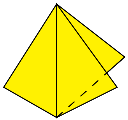

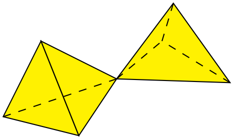

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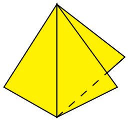

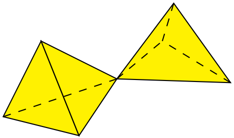

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$\rightsquigarrow$ incidence geometry

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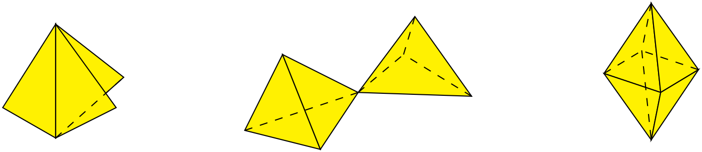

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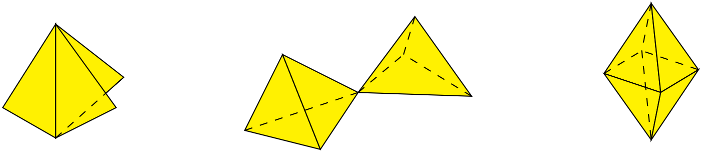

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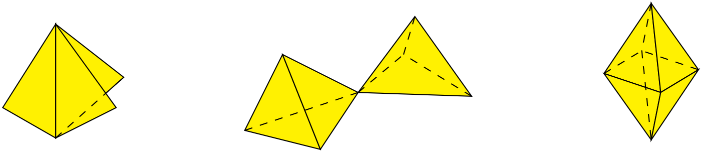

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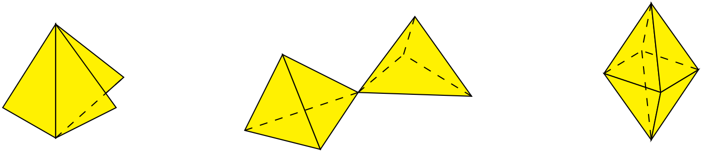

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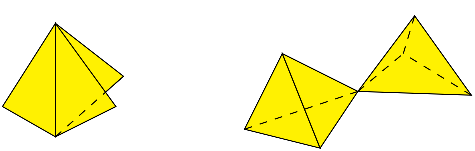

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- difference to `FinInG`–package by De Beule, Neunhöffer et al.
  - only two dimensions but it can work with colourings and foldings

1. General simplicial surfaces

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2. Edge colouring and group properties

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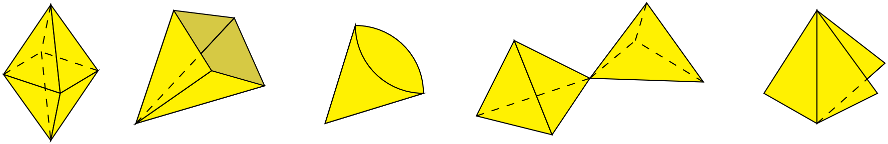

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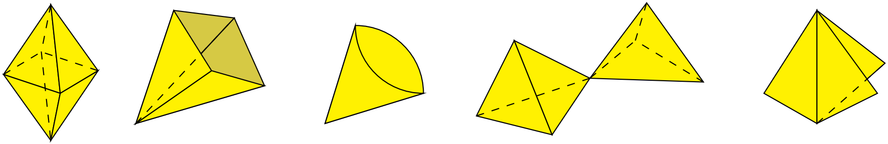

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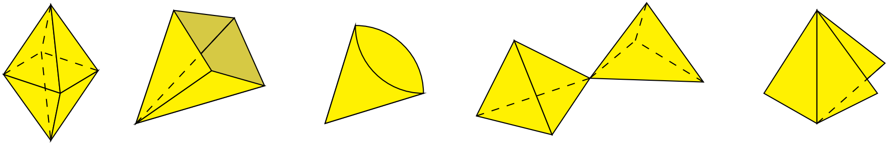

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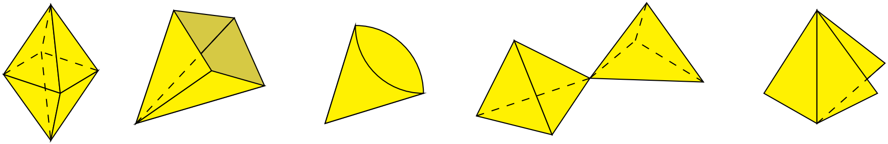

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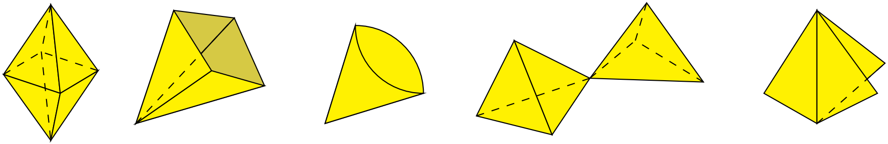

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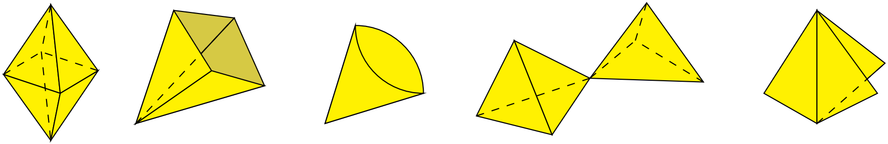

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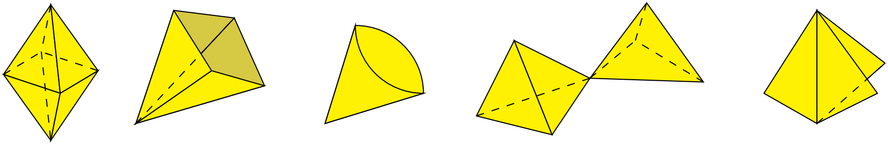

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Incidence structures can be interpreted as coloured graphs:

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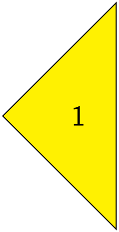

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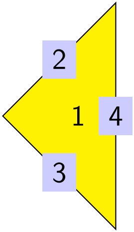

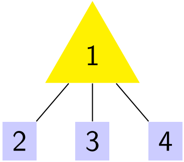

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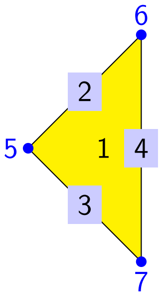

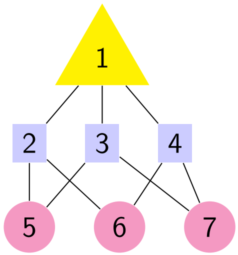

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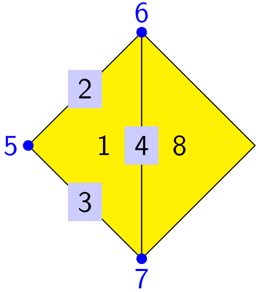

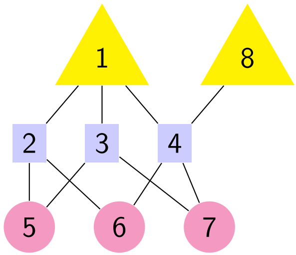

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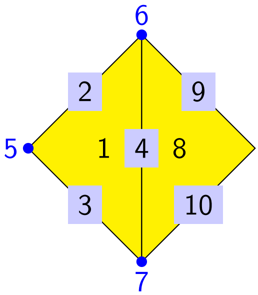

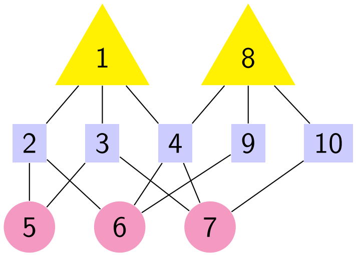

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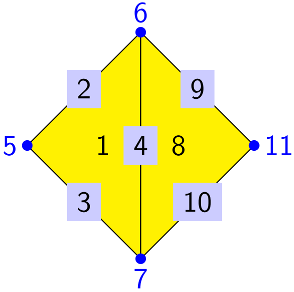

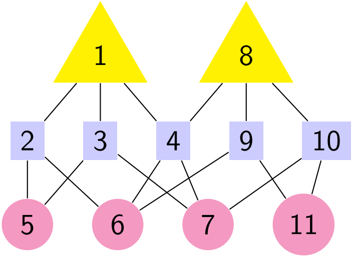

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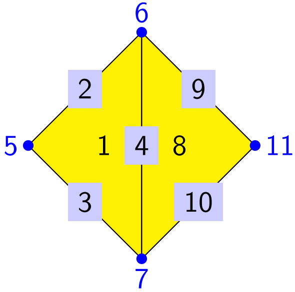

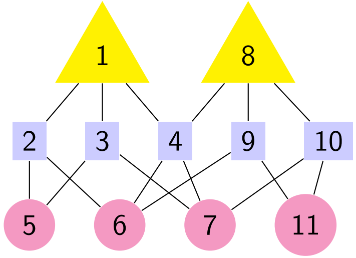

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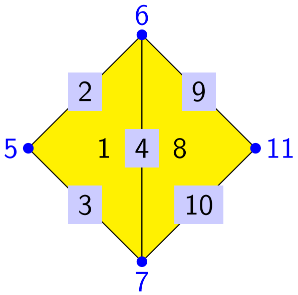

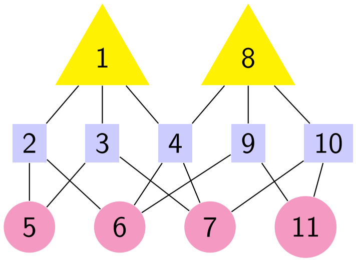

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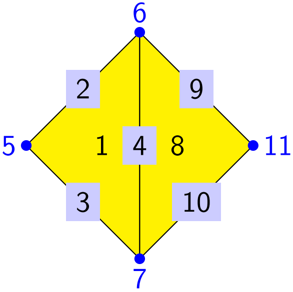

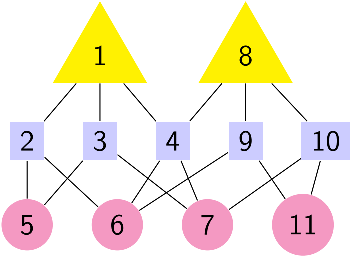

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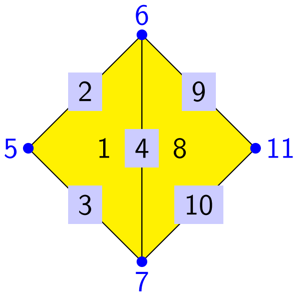

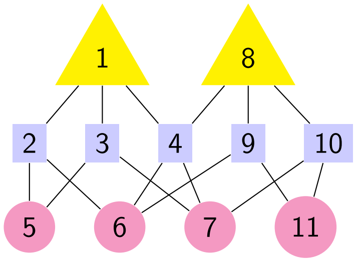

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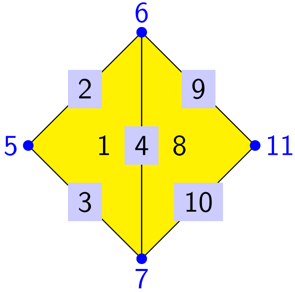

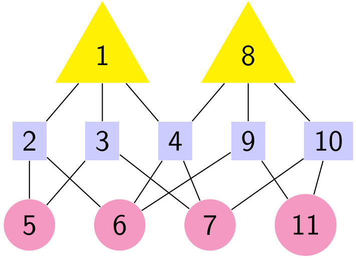

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- also returns automorphism group

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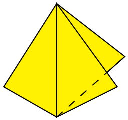

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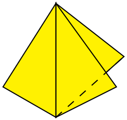

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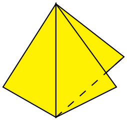

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$\rightsquigarrow$ **ramified simplicial surfaces**

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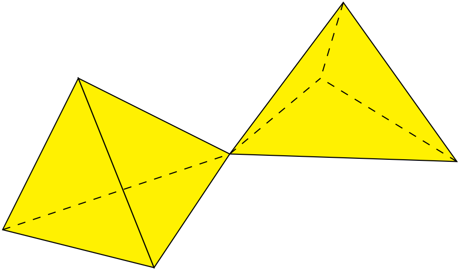

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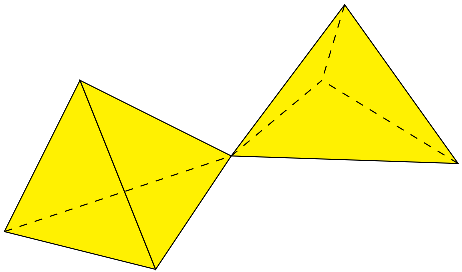

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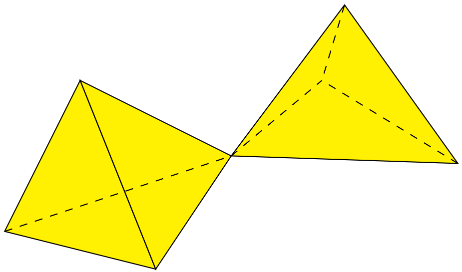

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A **simplicial surface** does not have these ramifications.

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Classification

Plesken/Strzelczyk classified all closed simplicial surfaces up to 20 triangles.

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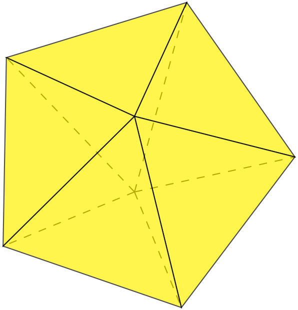

Progress report of triangulated complexes

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Embedding questions

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Given: A triangular complex

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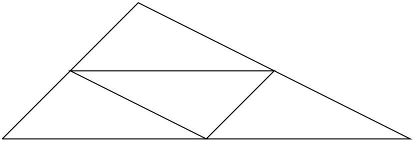

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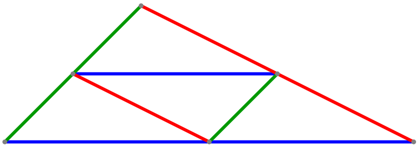
$\rightsquigarrow$ Edge–colouring encodes different lengths



Colouring as permutation

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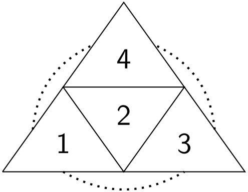
Consider tetrahedron



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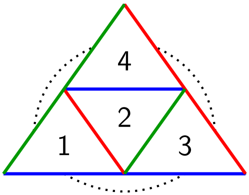

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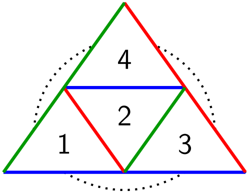

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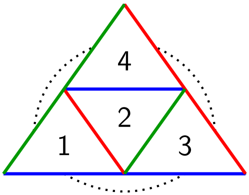

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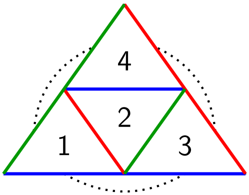

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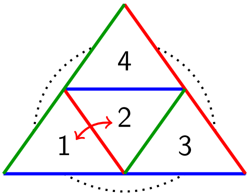

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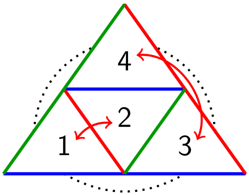

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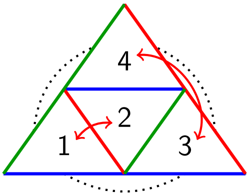

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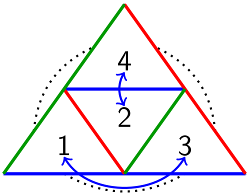

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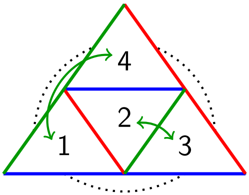

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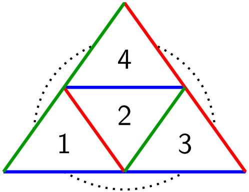

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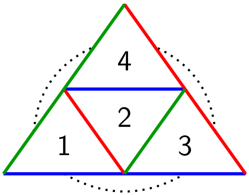

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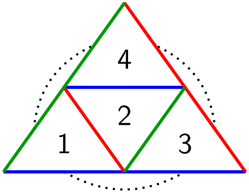

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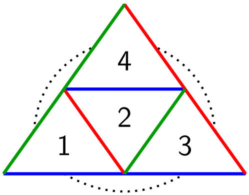

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How do faces fit together?

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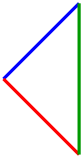
Consider a face of the surface



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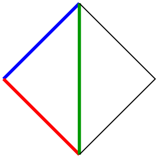

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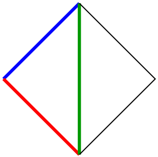

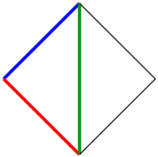

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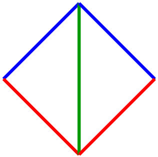

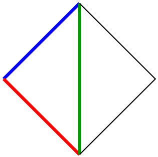

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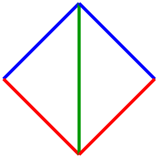

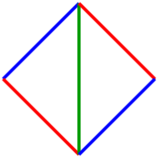

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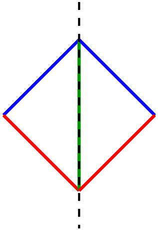
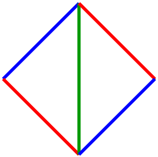


mirror (m)



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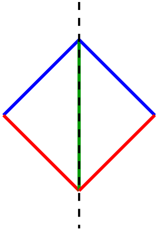

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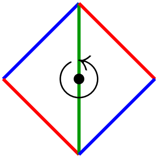

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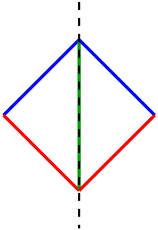

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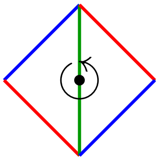

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Lemma

Permutations and mr–assignment uniquely determine the surface.

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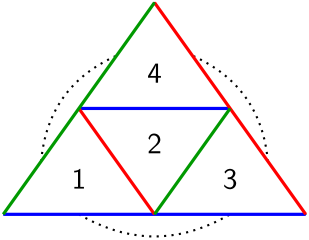

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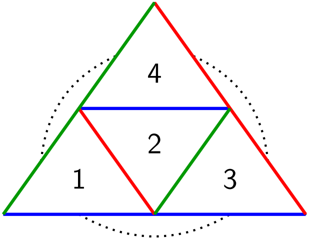

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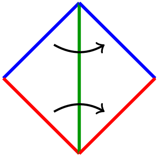

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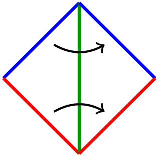

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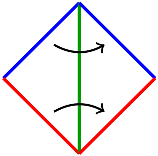

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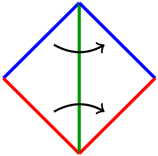

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Construction from permutations

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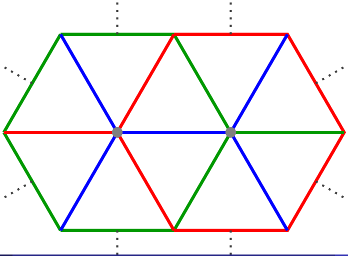

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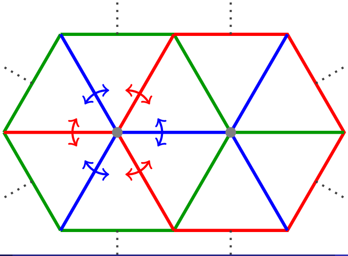

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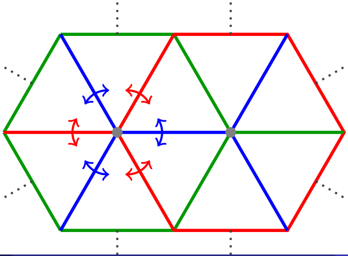

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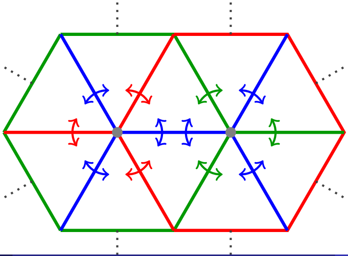

Construction example

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$\sigma_a = (1,2)(3,4)(5,6)(7,8)$

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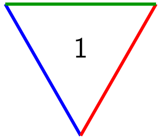

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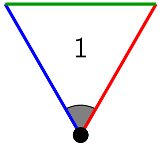

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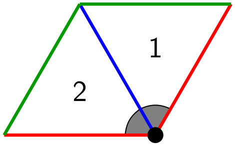

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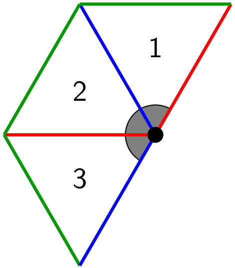

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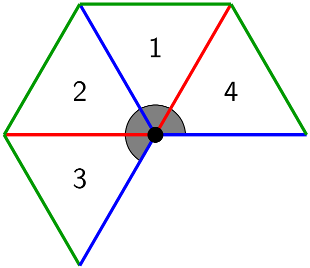

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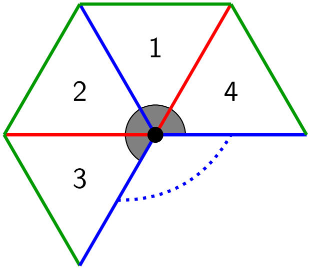

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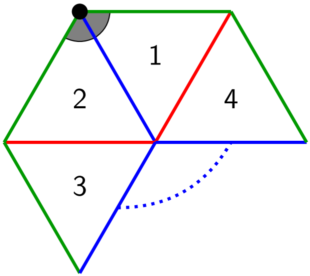

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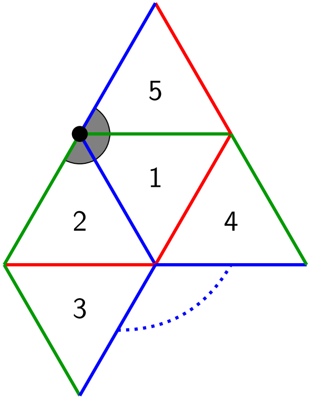

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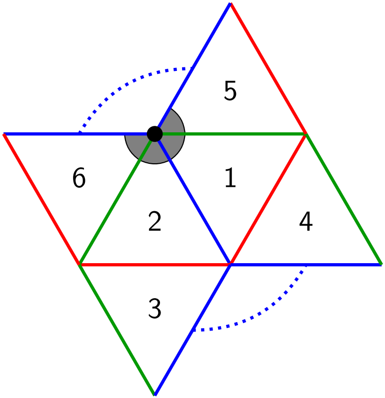

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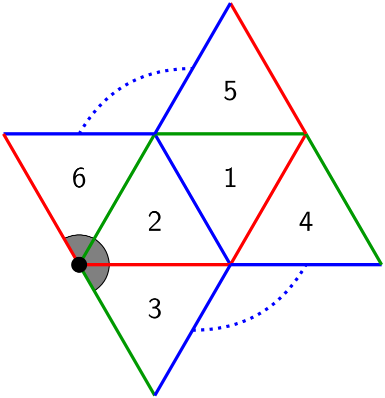

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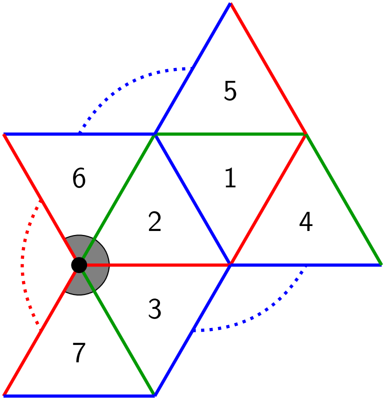

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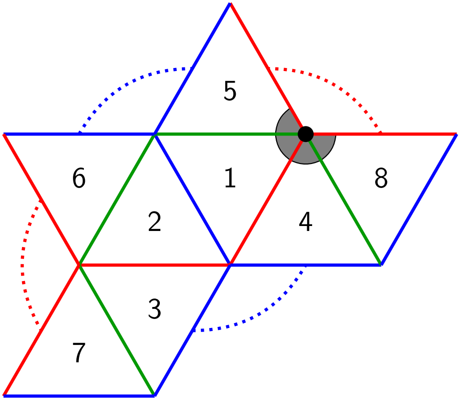

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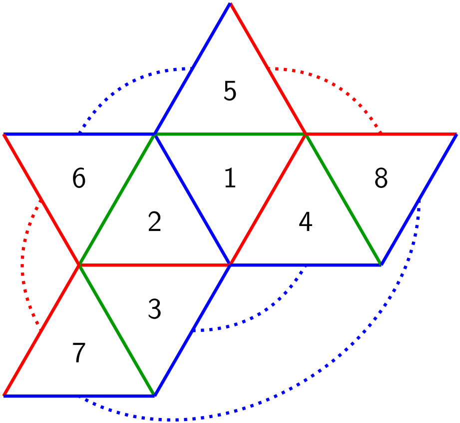

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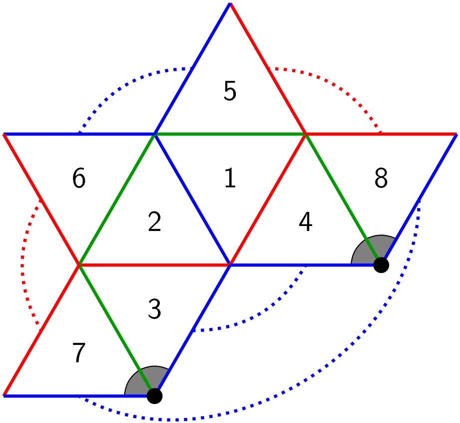

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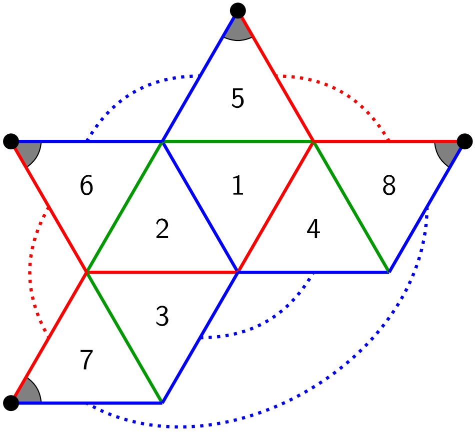

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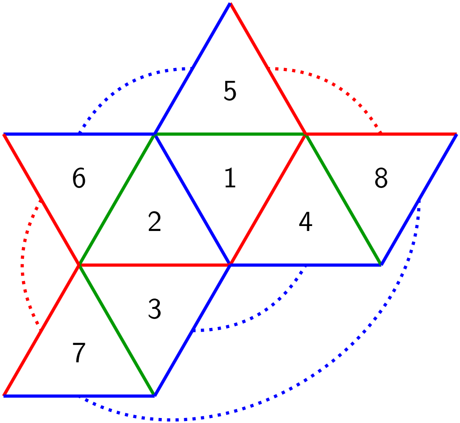

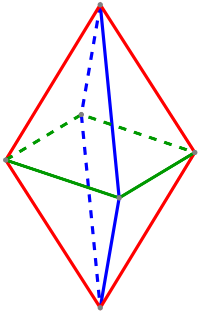

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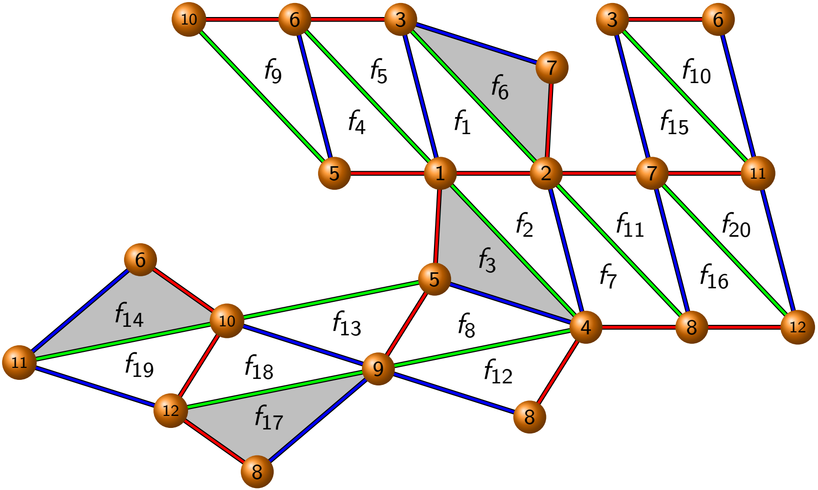

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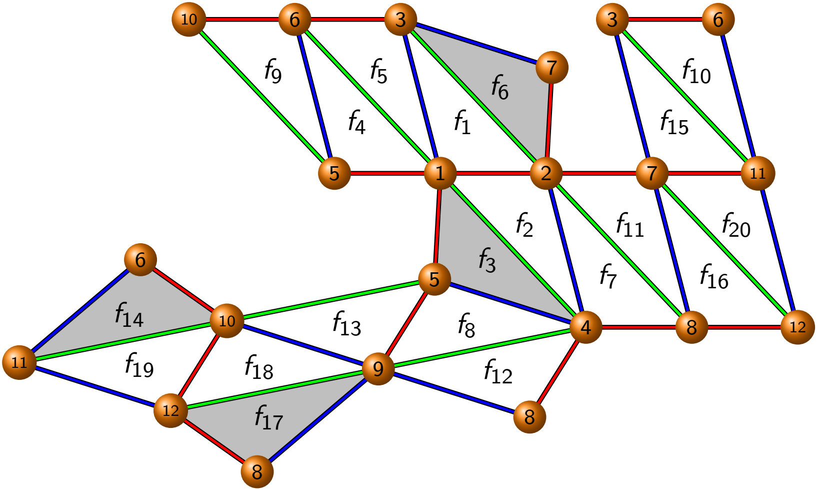

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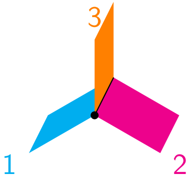

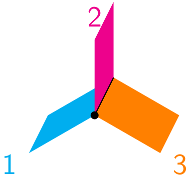

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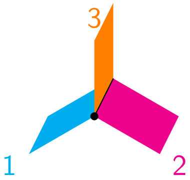

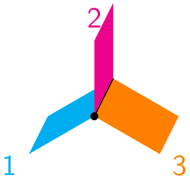

$\rightsquigarrow$ **folding complex**

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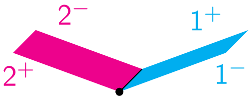

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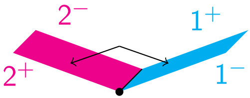

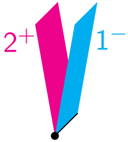

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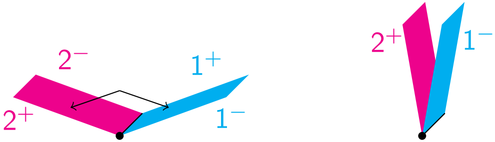

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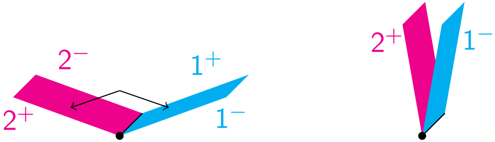

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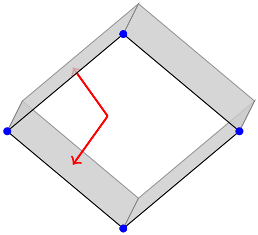

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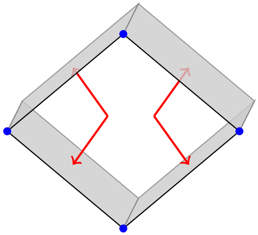

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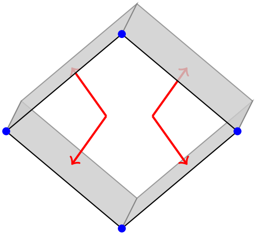

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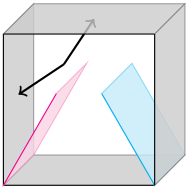

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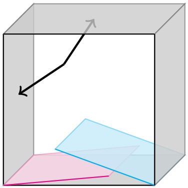

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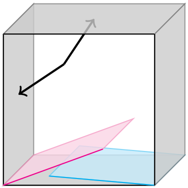

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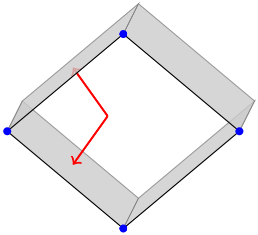

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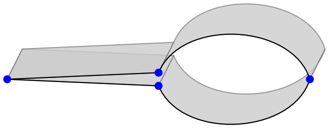

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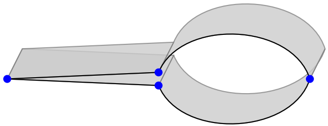

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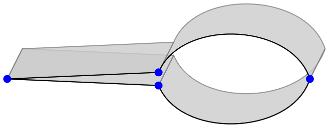

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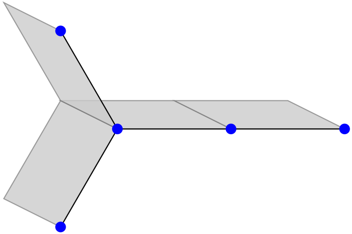
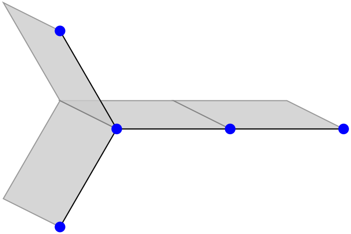
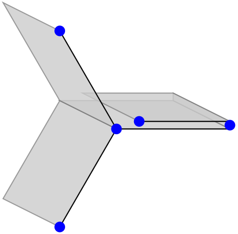


Structure of multiple foldings

With folding plans we can perform the same folding in different folding complexes





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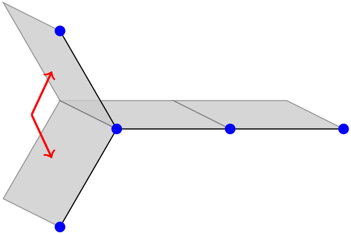

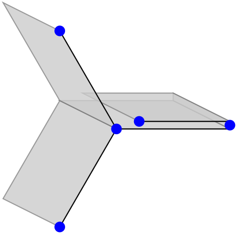

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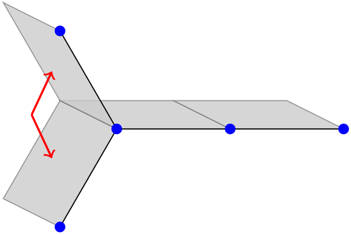
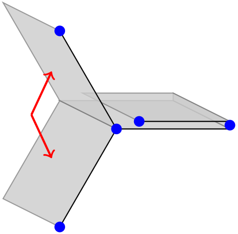

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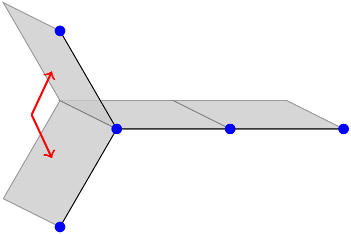

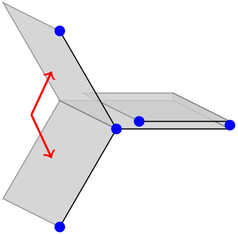

$\rightsquigarrow$ more structure on the set of possible foldings

Folding graph

Folding graph

- Vertices are folding complexes (modelling folding states)

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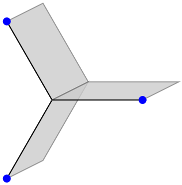

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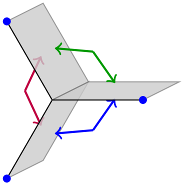

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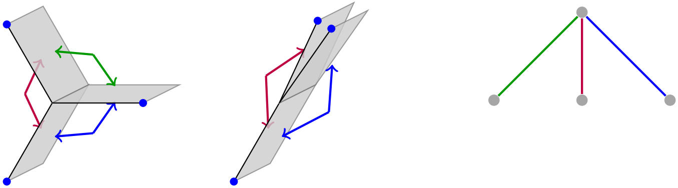

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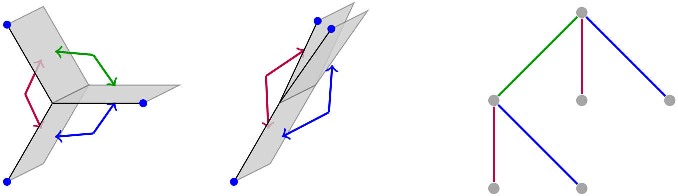

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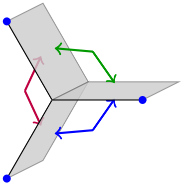

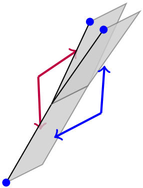

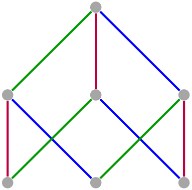

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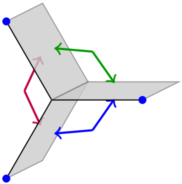

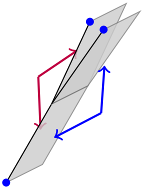

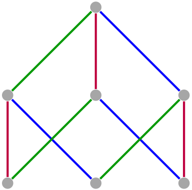

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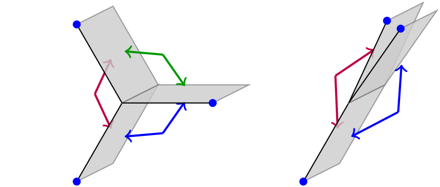

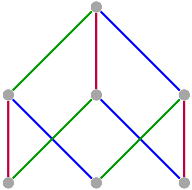

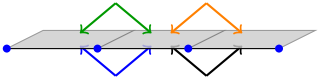

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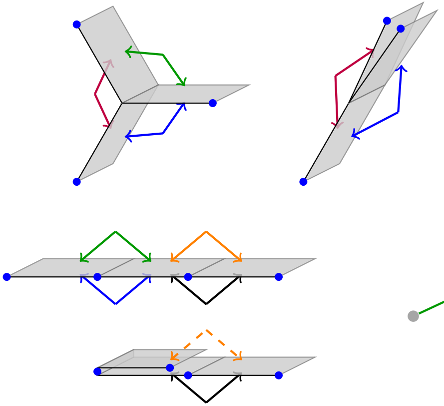

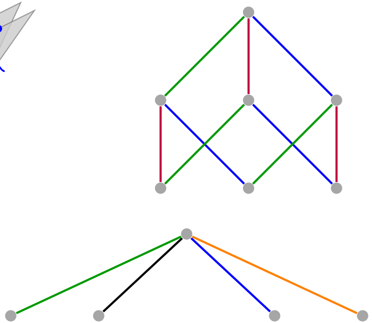

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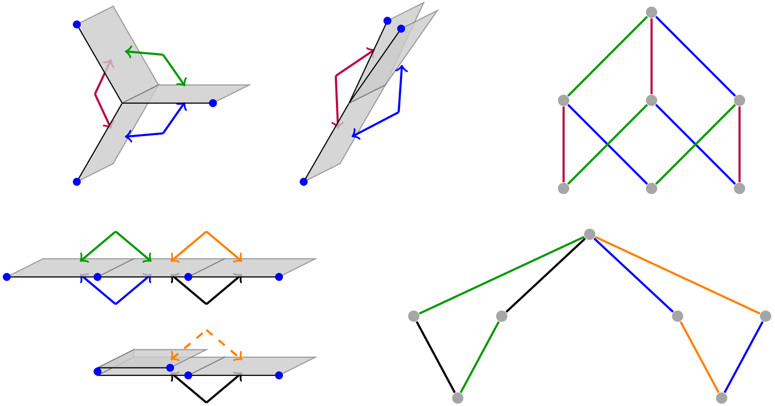

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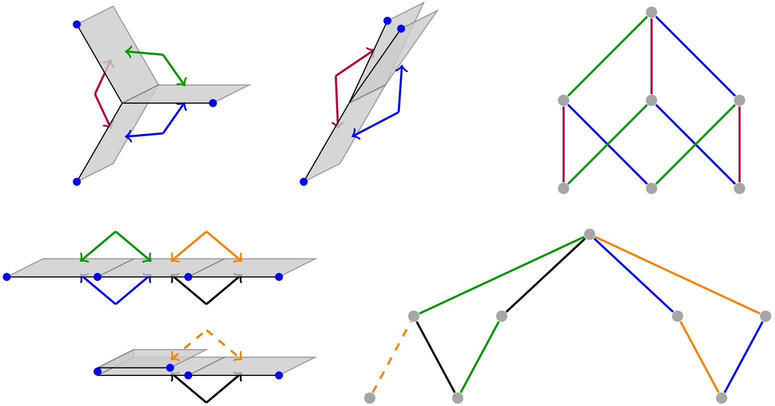

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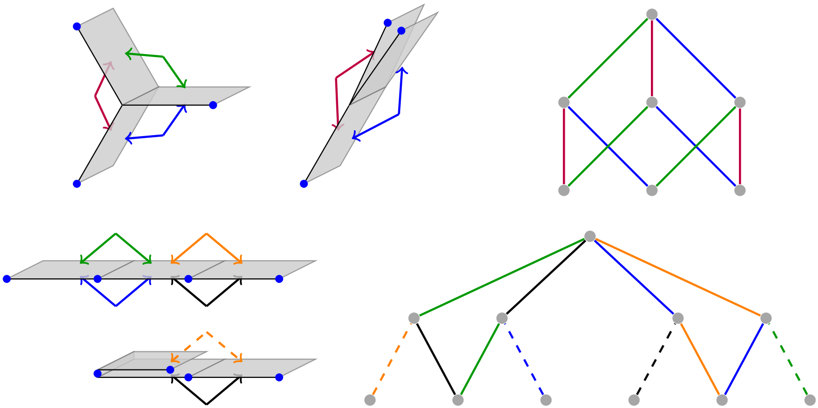

Larger graph

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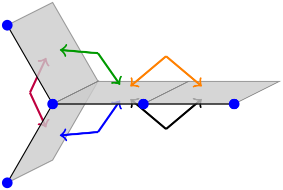

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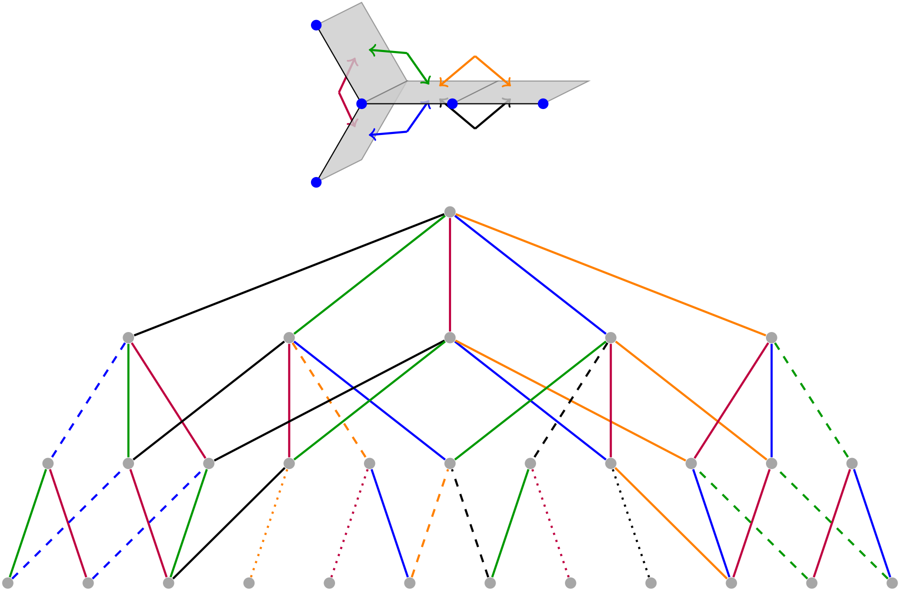

Drawback of folding plans

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Some foldings that “should” be the same, aren’t:

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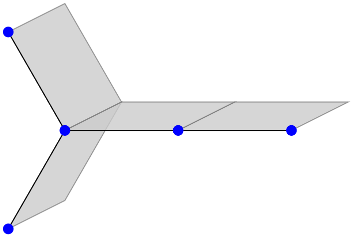

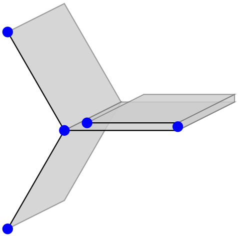

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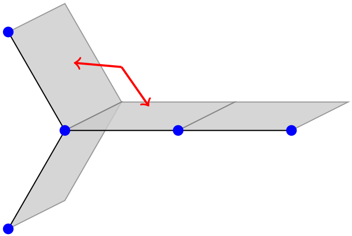

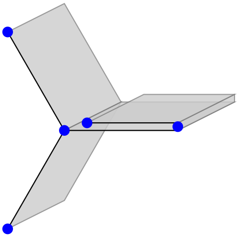

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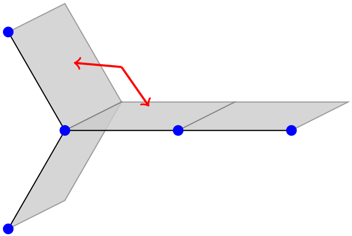

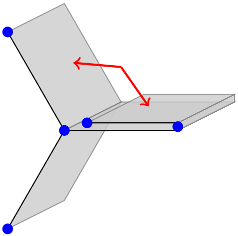

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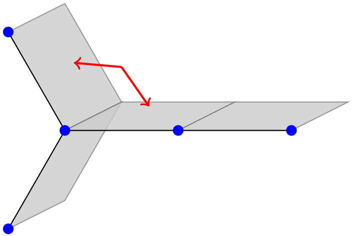

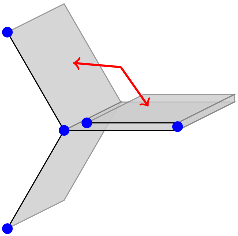

$\Rightarrow$ If you know the folding structure of a small complex,

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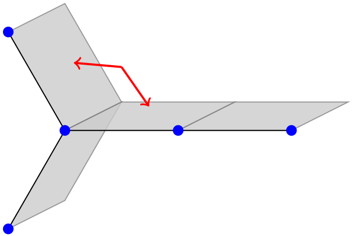

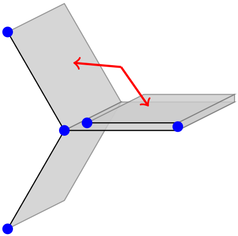

$\Rightarrow$ If you know the folding structure of a small complex, you can't easily find the folding structure of an extended complex

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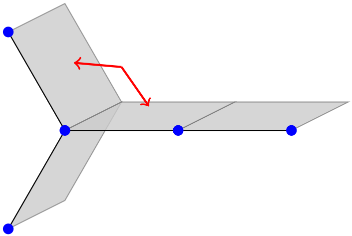

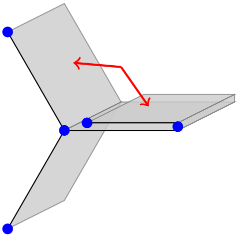

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$\rightsquigarrow$ Folding plans are not optimal to model folding

Progress report of abstract folding

Progress report of abstract folding

In development:

Progress report of abstract folding

In development:

- folding complex

Progress report of abstract folding

In development:

- folding complex
- folding plans

Progress report of abstract folding

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Missing:

Progress report of abstract folding

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Missing:

- better folding description

Progress report of abstract folding

In development:

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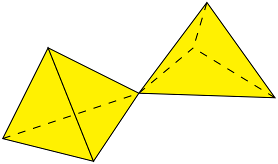
Missing:

- better folding description
- properties of folding graphs

Summary `SimplicialSurfaces`

Summary SimplicialSurfaces

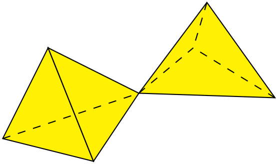
Triangulated complexes



Summary `SimplicialSurfaces`

Triangulated complexes

- mostly complete



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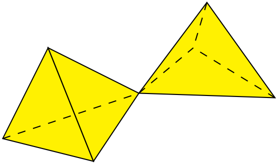
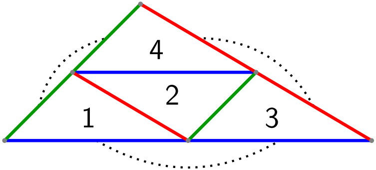


Edge colouring



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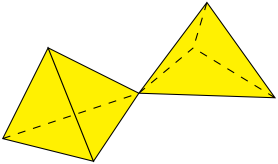
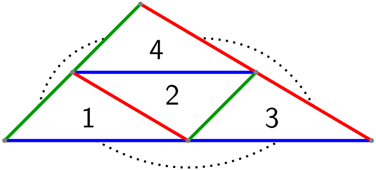


Edge colouring

- current theory implemented



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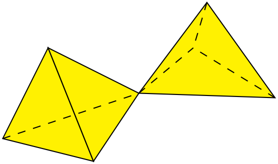

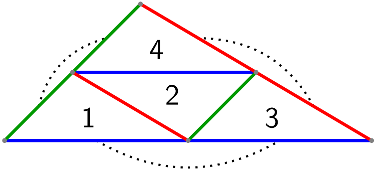

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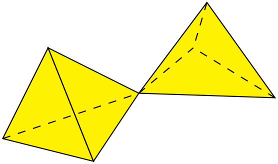

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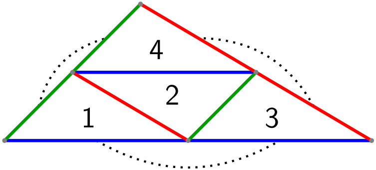
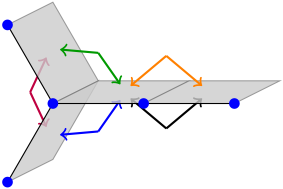


Abstract folding



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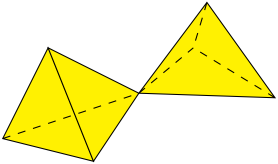

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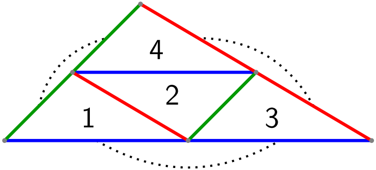
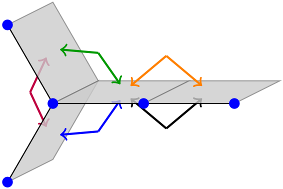


Abstract folding

- framework exists



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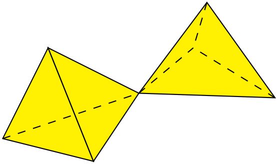

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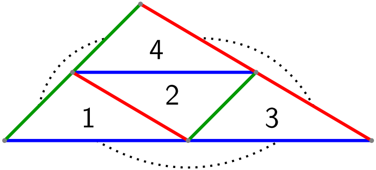

Abstract folding

- framework exists
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