Chapter 1

Integers

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1.1 Properties of Addition and Subtraction of Integers

We have learnt about whole numbers and integers in Class VI. We have also learnt about addition and subtraction of integers.

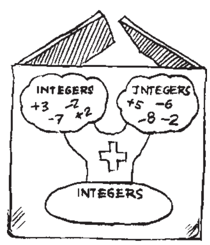
1.1.1 Closure under Addition

We have learnt that sum of two whole numbers is again a whole number. For example, $17 + 24 = 41$ which is again a whole number. We know that, this property is known as the closure property for addition of the whole numbers.

Let us see whether this property is true for integers or not.

Following are some pairs of integers. Observe the following table and complete it.

	Statement	Observation
(i)	$17 + 23 = 40$	Result is an integer
(ii)	$(-10) + 3 =$ _____	__________
(iii)	$(-75) + 18 =$ _____	__________
(iv)	$19 + (-25) = -6$	Result is an integer
(v)	$27 + (-27) =$ _____	__________
(vi)	$(-20) + 0 =$ _____	__________
(vii)	$(-35) + (-10) =$ _____	__________



What do you observe? Is the sum of two integers always an integer?

Did you find a pair of integers whose sum is not an integer?

Since addition of integers gives integers, we say **integers are closed under addition**.

In general, **for any two integers *a* and *b*, *a* + *b* is an integer.**

1.1.2 Closure under Subtraction

What happens when we subtract an integer from another integer? Can we say that their difference is also an integer?

Observe the following table and complete it:

Statement	Observation
(i) $7 - 9 = -2$	Result is an integer
(ii) $17 - (-21) =$ _______	_______
(iii) $(-8) - (-14) = 6$	Result is an integer
(iv) $(-21) - (-10) =$ _______	_______
(v) $32 - (-17) =$ _______	_______
(vi) $(-18) - (-18) =$ _______	_______
(vii) $(-29) - 0 =$ _______	_______

What do you observe? Is there any pair of integers whose difference is not an integer? Can we say integers are closed under subtraction? Yes, we can see that *integers are closed under subtraction*.

Thus, *if a and b are two integers then a – b is also an intger*. Do the whole numbers satisfy this property?

1.1.3 Commutative Property

We know that $3 + 5 = 5 + 3 = 8$, that is, the whole numbers can be added in any order. In other words, addition is commutative for whole numbers.

Can we say the same for integers also?

We have $5 + (-6) = -1$ and $(-6) + 5 = -1$

So, $5 + (-6) = (-6) + 5$

Are the following equal?

(i) $(-8) + (-9)$ and $(-9) + (-8)$

(ii) $(-23) + 32$ and $32 + (-23)$

(iii) $(-45) + 0$ and $0 + (-45)$

Try this with five other pairs of integers. Do you find any pair of integers for which the sums are different when the order is changed? Certainly not. We say that *addition is commutative for integers*.

In general, for any two integers a and b, we can say

$$a + b = b + a$$

- We know that subtraction is not commutative for whole numbers. Is it commutative for integers?

  Consider the integers 5 and (–3).

  Is 5 – (–3) the same as (–3) –5? No, because 5 – (–3) = 5 + 3 = 8, and (–3) – 5 = – 3 – 5 = – 8.

  Take atleast five different pairs of integers and check this.

  We conclude that subtraction is not commutative for integers.

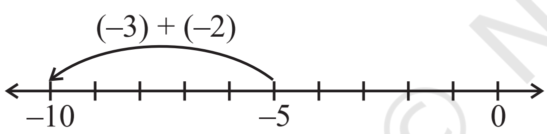
1.1.4 Associative Property

Observe the following examples:

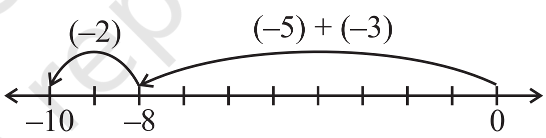
Consider the integers –3, –2 and –5.

Look at (–5) + [(–3) + (–2)] and [(–5) + (–3)] + (–2).

In the first sum (–3) and (–2) are grouped together and in the second (–5) and (–3) are grouped together. We will check whether we get different results.



(–5) + [(–3) + (–2)]



[(–5) + (–3)] + (–2)

In both the cases, we get –10.

i.e., (–5) + [(–3) + (–2)] = [(–5) + (–2)] + (–3)

Similarly consider –3 , 1 and –7.

(–3) + [1 + (–7)] = –3 + ___________ = ___________

[(–3) + 1] + (–7) = –2 + ___________ = ___________

Is (–3) + [1 + (–7)] same as [(–3) + 1] + (–7)?

Take five more such examples. You will not find any example for which the sums are different. *Addition is associative for integers*.

In general for any integers a, b and c, we can say

$$a + (b + c) = (a + b) + c$$

1.1.5 Additive Identity

When we add zero to any whole number, we get the same whole number. Zero is an additive identity for whole numbers. Is it an additive identity again for integers also?

Observe the following and fill in the blanks:

(i) (– 8) + 0 = – 8	(ii) 0 + (– 8) = – 8
(iii) (–23) + 0 = ______	(iv) 0 + (–37) = –37
(v) 0 + (–59) = ______	(vi) 0 + ______ = – 43
(vii) – 61 + ______ = – 61	(viii) ______ + 0 = ______

The above examples show that zero is an additive identity for integers.

You can verify it by adding zero to any other five integers.

In general, for any integer a

$$a + 0 = a = 0 + a$$

TRY THESE

1. Write a pair of integers whose sum gives

(a) a negative integer (b) zero

(c) an integer smaller than both the integers. (d) an integer smaller than only one of the integers.

(e) an integer greater than both the integers.

2. Write a pair of integers whose difference gives

(a) a negative integer. (b) zero.

(c) an integer smaller than both the integers. (d) an integer greater than only one of the integers.

(e) an integer greater than both the integers.

EXAMPLE 1 Write down a pair of integers whose

(a) sum is –3 (b) difference is –5

(c) difference is 2 (d) sum is 0

SOLUTION (a) (–1) + (–2) = –3 or (–5) + 2 = –3

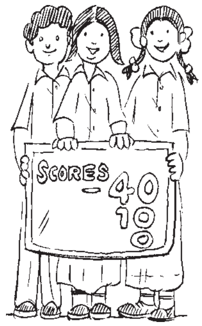
(b) (–9) – (– 4) = –5 or (–2) – 3 = –5

(c) (–7) – (–9) = 2 or 1 – (–1) = 2

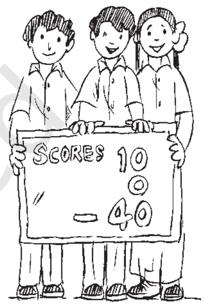
(d) (–10) + 10 = 0 or 5 + (–5) = 0

Can you write more pairs in these examples?

Exercise 1.1



1. Write down a pair of integers whose:
   (a) sum is –7 (b) difference is –10 (c) sum is 0
2. (a) Write a pair of negative integers whose difference gives 8.
   (b) Write a negative integer and a positive integer whose sum is –5.
   (c) Write a negative integer and a positive integer whose difference is –3.
3. In a quiz, team A scored – 40, 10, 0 and team B scored 10, 0, – 40 in three successive rounds. Which team scored more? Can we say that we can add integers in any order?



4. Fill in the blanks to make the following statements true:
   (i) (–5) + (– 8) = (– 8) + (............)
   (ii) –53 + ... = –53
   (iii) 17 + ... = 0
   (iv) [13 + (– 12)] + (............) = 13 + [(–12) + (–7)]
   (v) (– 4) + [15 + (–3)] = [– 4 + 15] + ...

1.2 Multiplication of Integers

We can add and subtract integers. Let us now learn how to multiply integers.

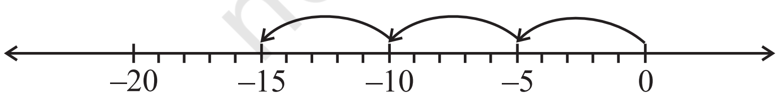
1.2.1 Multiplication of a Positive and a Negative Integer

We know that multiplication of whole numbers is repeated addition. For example,

$$5 + 5 + 5 = 3 \times 5 = 15$$

Can you represent addition of integers in the same way?

We have from the following number line, (–5) + (–5) + (–5) = –15



But we can also write

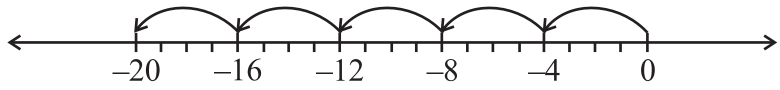
$$(-5) + (-5) + (-5) = 3 \times (-5)$$

Therefore, $3 \times (-5) = -15$

Try These

Find:
$4 \times (-8)$,
$8 \times (-2)$,
$3 \times (-7)$,
$10 \times (-1)$
using number line.

Similarly $(-4) + (-4) + (-4) + (-4) + (-4) = 5 \times (-4) = -20$



And $(-3) + (-3) + (-3) + (-3) =$ __________ = __________

Also, $(-7) + (-7) + (-7) =$ __________ = __________

Let us see how to find the product of a positive integer and a negative integer without using number line.

Let us find $3 \times (-5)$ in a different way. First find 3×5 and then put minus sign (–) before the product obtained. You get –15. That is we find $-(3 \times 5)$ to get –15.

Similarly, $5 \times (-4) = -(5 \times 4) = -20$.

Find in a similar way,

$4 \times (-8) =$ _____ = _____, $3 \times (-7) =$ _____ = _____

$6 \times (-5) =$ _____ = _____, $2 \times (-9) =$ _____ = _____

TRY THESE

Find:

(i) $6 \times (-19)$

(ii) $12 \times (-32)$

(iii) $7 \times (-22)$

Using this method we thus have,

$10 \times (-43) =$ _____ $-(10 \times 43) = -430$

Till now we multiplied integers as (positive integer) × (negative integer).

Let us now multiply them as (negative integer) × (positive integer).

We first find -3×5.

To find this, observe the following pattern:

We have,

$3 \times 5 = 15$

$2 \times 5 = 10 = 15 - 5$

$1 \times 5 = 5 = 10 - 5$

$0 \times 5 = 0 = 5 - 5$

So,

$-1 \times 5 = 0 - 5 = -5$

$-2 \times 5 = -5 - 5 = -10$

$-3 \times 5 = -10 - 5 = -15$

We already have $3 \times (-5) = -15$

So we get $(-3) \times 5 = -15 = 3 \times (-5)$

Using such patterns, we also get $(-5) \times 4 = -20 = 5 \times (-4)$

Using patterns, find $(-4) \times 8$, $(-3) \times 7$, $(-6) \times 5$ and $(-2) \times 9$

Check whether, $(-4) \times 8 = 4 \times (-8)$, $(-3) \times 7 = 3 \times (-7)$, $(-6) \times 5 = 6 \times (-5)$

and $(-2) \times 9 = 2 \times (-9)$

Using this we get, $(-33) \times 5 = 33 \times (-5) = -165$

We thus find that while *multiplying a positive integer and a negative integer, we multiply them as whole numbers and put a minus sign (–) before the product. We thus get a negative integer.*

TRY THESE

1. Find: (a) $15 \times (-16)$ (b) $21 \times (-32)$
   (c) $(-42) \times 12$ (d) -55×15
2. Check if (a) $25 \times (-21) = (-25) \times 21$ (b) $(-23) \times 20 = 23 \times (-20)$
   Write five more such examples.

In general, for any two positive integers a and b we can say

$$a \times (-b) = (-a) \times b = -(a \times b)$$

1.2.2 Multiplication of two Negative Integers

Can you find the product $(-3) \times (-2)$?

Observe the following:

$-3 \times 4 = -12$

$-3 \times 3 = -9 = -12 - (-3)$

$-3 \times 2 = -6 = -9 - (-3)$

$-3 \times 1 = -3 = -6 - (-3)$

$-3 \times 0 = 0 \ = -3 - (-3)$

$-3 \times -1 = 0 - (-3) = 0 + 3 = 3$

$-3 \times -2 = 3 - (-3) = 3 + 3 = 6$

Do you see any pattern? Observe how the products change.

Based on this observation, complete the following:

$-3 \times -3 =$ ______ $-3 \times -4 =$ ______

Now observe these products and fill in the blanks:

$-4 \times 4 = -16$

$-4 \times 3 = -12 \ = -16 + 4$

$-4 \times 2 =$ ______ $= -12 + 4$

$-4 \times 1 =$ ______

$-4 \times 0 =$ ______

$-4 \times (-1) =$ ______

$-4 \times (-2) =$ ______

$-4 \times (-3) =$ ______

TRY THESE

(i) Starting from $(-5) \times 4$, find $(-5) \times (-6)$

(ii) Starting from $(-6) \times 3$, find $(-6) \times (-7)$

From these patterns we observe that,

$(-3) \times (-1) = 3 = 3 \times 1$

$(-3) \times (-2) = 6 = 3 \times 2$

$(-3) \times (-3) = 9 = 3 \times 3$

and $(-4) \times (-1) = 4 = 4 \times 1$

So, $(-4) \times (-2) = 4 \times 2 =$ ______

$(-4) \times (-3) =$ ______ $=$ ______

So observing these products we can say that the *product of two negative integers is a positive integer. We multiply the two negative integers as whole numbers and put the positive sign before the product.*

Thus, we have $(-10) \times (-12) = +120 = 120$

Similarly $(-15) \times (-6) = +90 = 90$

In general, for any two positive integers a and b,

$$(-a) \times (-b) = a \times b$$

TRY THESE Find: $(-31) \times (-100)$, $(-25) \times (-72)$, $(-83) \times (-28)$

Game 1

(i) Take a board marked from –104 to 104 as shown in the figure.

(ii) Take a bag containing two blue and two red dice. Number of dots on the blue dice indicate positive integers and number of dots on the red dice indicate negative integers.

(iii) Every player will place his/her counter at zero.

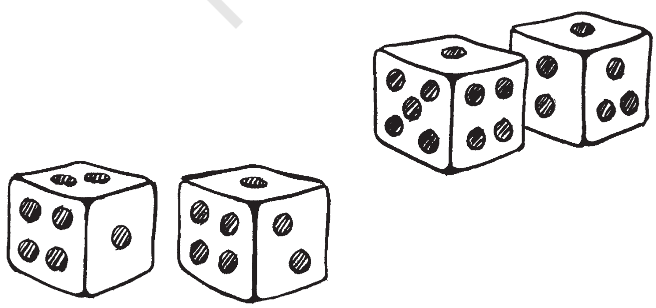
(iv) Each player will take out two dice at a time from the bag and throw them.

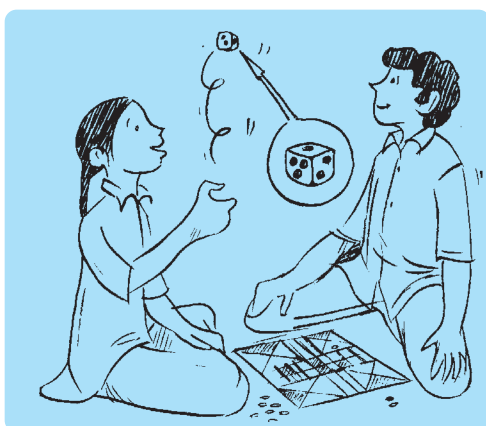
(v) After every throw, the player has to multiply the numbers marked on the dice.

104	103	102	101	100	99	98	97	96	95	94
83	84	85	86	87	88	89	90	91	92	93
82	81	80	79	78	77	76	75	74	73	72
61	62	63	64	65	66	67	68	69	70	71
60	59	58	57	56	55	54	53	52	51	50
39	40	41	42	43	44	45	46	47	48	49
38	37	36	35	34	33	32	31	30	29	28
17	18	19	20	21	22	23	24	25	26	27
16	15	14	13	12	11	10	9	8	7	6
–5	–4	–3	–2	–1	0	1	2	3	4	5
–6	–7	–8	–9	–10	–11	–12	–13	–14	–15	–16
–27	–26	–25	–24	–23	–22	–21	–20	–19	–18	–17
–28	–29	–30	–31	–32	–33	–34	–35	–36	–37	–38
–49	–48	–47	–46	–45	–44	–43	–42	–41	–40	–39
–50	–51	–52	–53	–54	–55	–56	–57	–58	–59	–60
–71	–70	–69	–68	–67	–66	–65	–64	–63	–62	–61
–72	–73	–74	–75	–76	–77	–78	–79	–80	–81	–82
–93	–92	–91	–90	–89	–88	–87	–86	–85	–84	–83
–94	–95	–96	–97	–98	–99	–100	–101	–102	–103	–104

(vi) If the product is a positive integer then the player will move his counter towards 104; if the product is a negative integer then the player will move his counter towards –104.

(vii) The player who reaches either -104 or 104 first is the winner.





1.3 Properties of Multiplication of Integers

1.3.1 Closure under Multiplication

1. Observe the following table and complete it:

Statement	Inference
$(-20) \times (-5) = 100$	Product is an integer
$(-15) \times 17 = -255$	Product is an integer
$(-30) \times 12 =$ _____	
$(-15) \times (-23) =$ _____	
$(-14) \times (-13) =$ _____	
$12 \times (-30) =$ _____	

What do you observe? Can you find a pair of integers whose product is not an integer? No. This gives us an idea that the product of two integers is again an integer. So we can say that *integers are closed under multiplication*.

In general,

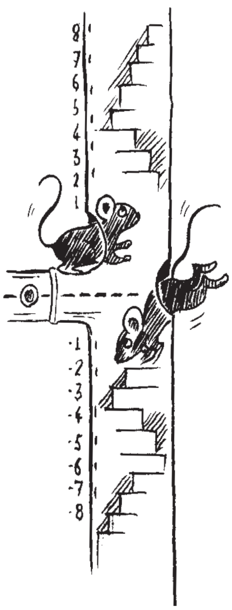
$a \times b$ is an integer, for all integers a and b.

Find the product of five more pairs of integers and verify the above statement.

1.3.2 Commutativity of Multiplication

We know that multiplication is commutative for whole numbers. Can we say, multiplication is also commutative for integers?

Observe the following table and complete it:



Statement 1	Statement 2	Inference
$3 \times (-4) = -12$	$(-4) \times 3 = -12$	$3 \times (-4) = (-4) \times 3$
$(-30) \times 12 =$ _____	$12 \times (-30) =$ _____	
$(-15) \times (-10) = 150$	$(-10) \times (-15) = 150$	
$(-35) \times (-12) =$ _____	$(-12) \times (-35) =$	
$(-17) \times 0 =$ _____		
__________ = _____	$(-1) \times (-15) =$	

What are your observations? The above examples suggest *multiplication is commutative for integers*. Write five more such examples and verify.

In general, for any two integers a and b,

$$a \times b = b \times a$$

1.3.3 Multiplication by Zero

We know that any whole number when multiplied by zero gives zero. Observe the following products of negative integers and zero. These are obtained from the patterns done earlier.

$(-3) \times 0 = 0$

$0 \times (-4) = 0$

$-5 \times 0 =$ _____

$0 \times (-6) =$ _____

This shows that the product of a negative integer and zero is zero.

In general, for any integer a,

$$a \times 0 = 0 \times a = 0$$

1.3.4 Multiplicative Identity

We know that 1 is the multiplicative identity for whole numbers.

Check that 1 is the multiplicative identity for integers as well. Observe the following products of integers with 1.

$(-3) \times 1 = -3$	$1 \times 5 = 5$
$(-4) \times 1 =$ _____	$1 \times 8 =$ _____
$1 \times (-5) =$ _____	$3 \times 1 =$ _____
$1 \times (-6) =$ _____	$7 \times 1 =$ _____

This shows that 1 is the multiplicative identity for integers also.

In general, for any integer a we have,

$$a \times 1 = 1 \times a = a$$

What happens when we multiply any integer with –1? Complete the following:

$(-3) \times (-1) = 3$

$3 \times (-1) = -3$

$(-6) \times (-1) =$ _____

$(-1) \times 13 =$ _____

$(-1) \times (-25) =$ _____

$18 \times (-1) =$ _____

0 is the additive identity whereas 1 is the multiplicative identity for integers. We get additive inverse of an integer a when we multiply (–1) to a, i.e. $a \times (-1) = (-1) \times a = -a$

What do you observe?

Can we say –1 is a multiplicative identity of integers? No.

1.3.5 Associativity for Multiplication

Consider –3, –2 and 5.

Look at $[(-3) \times (-2)] \times 5$ and $(-3) \times [(-2) \times 5]$.

In the first case (–3) and (–2) are grouped together and in the second (–2) and 5 are grouped together.

We see that $[(-3) \times (-2)] \times 5 = 6 \times 5 = 30$

and $(-3) \times [(-2) \times 5] = (-3) \times (-10) = 30$

So, we get the same answer in both the cases.

Thus, $[(-3) \times (-2)] \times 5 = (-3) \times [(-2) \times 5]$

Look at this and complete the products:

$[(7) \times (-6)] \times 4 =$ __________ $\times 4 =$ __________

$7 \times [(-6) \times 4] = 7 \times$ __________ $=$ __________

Is $[7 \times (-6)] \times 4 = 7 \times [(-6) \times 4]$?

Does the grouping of integers affect the product of integers? No.

In general, for any three integers a, b and c

$$(a \times b) \times c = a \times (b \times c)$$

Take any five values for a, b and c each and verify this property.

Thus, like whole numbers, *the product of three integers does not depend upon the grouping of integers and this is called the associative property for multiplication of integers.*

1.3.6 Distributive Property

We know

$16 \times (10 + 2) = (16 \times 10) + (16 \times 2)$ [Distributivity of multiplication over addition]

Let us check if this is true for integers also.

Observe the following:

(a) $(-2) \times (3 + 5) = -2 \times 8 = -16$

and $[(-2) \times 3] + [(-2) \times 5] = (-6) + (-10) = -16$

So, $(-2) \times (3 + 5) = [(-2) \times 3] + [(-2) \times 5]$

(b) $(-4) \times [(-2) + 7] = (-4) \times 5 = -20$

and $[(-4) \times (-2)] + [(-4) \times 7] = 8 + (-28) = -20$

So, $(-4) \times [(-2) + 7] = [(-4) \times (-2)] + [(-4) \times 7]$

(c) $(-8) \times [(-2) + (-1)] = (-8) \times (-3) = 24$

and $[(-8) \times (-2)] + [(-8) \times (-1)] = 16 + 8 = 24$

So, $(-8) \times [(-2) + (-1)] = [(-8) \times (-2)] + [(-8) \times (-1)]$

Can we say that the distributivity of multiplication over addition is true for integers also? Yes.

In general, for any integers a, b and c,

$$a \times (b + c) = a \times b + a \times c$$

Take atleast five different values for each of a, b and c and verify the above Distributive property.

TRY THESE

(i) Is $10 \times [(6 + (-2)] = 10 \times 6 + 10 \times (-2)$?

(ii) Is $(-15) \times [(-7) + (-1)] = (-15) \times (-7) + (-15) \times (-1)$?

Now consider the following:

Can we say $4 \times (3 - 8) = 4 \times 3 - 4 \times 8$?

Let us check:

$$4 \times (3 - 8) = 4 \times (-5) = -20$$

$$4 \times 3 - 4 \times 8 = 12 - 32 = -20$$

So, $4 \times (3 - 8) = 4 \times 3 - 4 \times 8$.

Look at the following:

$$(-5) \times [(-4) - (-6)] = (-5) \times 2 = -10$$

$$[(-5) \times (-4)] - [(-5) \times (-6)] = 20 - 30 = -10$$

So, $(-5) \times [(-4) - (-6)] = [(-5) \times (-4)] - [(-5) \times (-6)]$

Check this for $(-9) \times [10 - (-3)]$ and $[(-9) \times 10] - [(-9) \times (-3)]$

You will find that these are also equal.

In general, for any three integers a, b and c,

$$a \times (b - c) = a \times b - a \times c$$

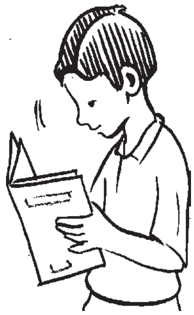
Take atleast five different values for each of a, b and c and verify this property.

TRY THESE

(i) Is $10 \times (6 - (-2)] = 10 \times 6 - 10 \times (-2)$?

(ii) Is $(-15) \times [(-7) - (-1)] = (-15) \times (-7) - (-15) \times (-1)$?

EXERCISE 1.2



1. Find each of the following products:

(a) $3 \times (-1)$ (b) $(-1) \times 225$

(c) $(-21) \times (-30)$ (d) $(-316) \times (-1)$

(e) $(-15) \times 0 \times (-18)$ (f) $(-12) \times (-11) \times (10)$

(g) $9 \times (-3) \times (-6)$ (h) $(-18) \times (-5) \times (-4)$

(i) $(-1) \times (-2) \times (-3) \times 4$ (j) $(-3) \times (-6) \times (-2) \times (-1)$

2. Verify the following:

(a) $18 \times [7 + (-3)] = [18 \times 7] + [18 \times (-3)]$

(b) $(-21) \times [(-4) + (-6)] = [(-21) \times (-4)] + [(-21) \times (-6)]$

3. (i) For any integer a, what is $(-1) \times a$ equal to?

(ii) Determine the integer whose product with (-1) is

(a) -22 (b) 37 (c) 0

4. Starting from $(-1) \times 5$, write various products showing some pattern to show $(-1) \times (-1) = 1$.

1.4 DIVISION OF INTEGERS

We know that division is the inverse operation of multiplication. Let us see an example for whole numbers.

Since $3 \times 5 = 15$

So $15 \div 5 = 3$ and $15 \div 3 = 5$

Similarly, $4 \times 3 = 12$ gives $12 \div 4 = 3$ and $12 \div 3 = 4$

We can say for each multiplication statement of whole numbers there are two division statements.

Can you write multiplication statement and its corresponding divison statements for integers?

- Observe the following and complete it.

Multiplication Statement	Corresponding Division Statements
$2 \times (-6) = (-12)$	$(-12) \div (-6) = 2$, $(-12) \div 2 = (-6)$
$(-4) \times 5 = (-20)$	$(-20) \div 5 = (-4)$, $(-20) \div (-4) = 5$
$(-8) \times (-9) = 72$	$72 \div$ ______ = ______ , $72 \div$ ______ = ______
$(-3) \times (-7) =$ ______	______ $\div (-3) =$ ______ , ______________
$(-8) \times 4 =$ _____	______________ , ______________
$5 \times (-9) =$ ______	______________ , ______________
$(-10) \times (-5) =$	______________ , ______________

From the above we observe that :

$(-12) \div 2 = (-6)$

$(-20) \div 5 = (-4)$

$(-32) \div 4 = (-8)$

$(-45) \div 5 = (-9)$

TRY THESE

Find:

(a) $(-100) \div 5$ (b) $(-81) \div 9$

(c) $(-75) \div 5$ (d) $(-32) \div 2$

We observe that *when we divide a negative integer by a positive integer, we divide them as whole numbers and then put a minus sign (–) before the quotient.*

- We also observe that:

$72 \div (-8) = -9$ and $50 \div (-10) = -5$

$72 \div (-9) = -8$ $50 \div (-5) = -10$

So we can say that *when we divide a positive integer by a negative integer, we first divide them as whole numbers and then put a minus sign (–) before the quotient.*

In general, for any two positive integers *a* and *b*

$$a \div (-b) = (-a) \div b \quad \text{where } b \neq 0$$

Can we say that

$(-48) \div 8 = 48 \div (-8)$?

Let us check. We know that

$(-48) \div 8 = -6$

and $48 \div (-8) = -6$

So $(-48) \div 8 = 48 \div (-8)$

Check this for

(i) $90 \div (-45)$ and $(-90) \div 45$

(ii) $(-136) \div 4$ and $136 \div (-4)$

TRY THESE

Find: (a) $125 \div (-25)$ (b) $80 \div (-5)$ (c) $64 \div (-16)$

- Lastly, we observe that

$(-12) \div (-6) = 2$; $(-20) \div (-4) = 5$; $(-32) \div (-8) = 4$; $(-45) \div (-9) = 5$

So, we can say that when we divide a negative integer by a negative integer, we first divide them as whole numbers and then put a positive sign (+).

In general, for any two positive integers *a* and *b*

$$(-a) \div (-b) = a \div b \quad \text{where } b \neq 0$$

TRY THESE

Find: (a) $(-36) \div (-4)$ (b) $(-201) \div (-3)$ (c) $(-325) \div (-13)$

1.5 PROPERTIES OF DIVISION OF INTEGERS

Observe the following table and complete it:

What do you observe? We observe that integers are not closed under division.

Statement	Inference	Statement	Inference
$(-8) \div (-4) = 2$	Result is an integer	$(-8) \div 3 = \frac{-8}{3}$	__________________
$(-4) \div (-8) = \frac{-4}{-8}$	Result is not an integer	$3 \div (-8) = \frac{3}{-8}$	__________________

Justify it by taking five more examples of your own.

- We know that division is not commutative for whole numbers. Let us check it for integers also.

  You can see from the table that $(-8) \div (-4) \neq (-4) \div (-8)$.

  Is $(-9) \div 3$ the same as $3 \div (-9)$?

  Is $(-30) \div (-6)$ the same as $(-6) \div (-30)$?

  Can we say that division is commutative for integers? No.

  You can verify it by taking five more pairs of integers.

- Like whole numbers, any integer divided by zero is meaningless and zero divided by an integer other than zero is equal to zero i.e., *for any integer a,* $a \div 0$ *is not defined but* $0 \div a = 0$ *for* $a \neq 0$.

- When we divide a whole number by 1 it gives the same whole number. Let us check whether it is true for negative integers also.

  Observe the following :

  $(-8) \div 1 = (-8)$ $\quad$ $(-11) \div 1 = -11$ $\quad$ $(-13) \div 1 = -13$

  $(-25) \div 1 =$ ____ $\quad$ $(-37) \div 1 =$ ____ $\quad$ $(-48) \div 1 =$ ____

  This shows that negative integer divided by 1 gives the same negative integer. So, *any integer divided by 1 gives the same integer.*

  In general, for any integer a,

$$a \div 1 = a$$

- What happens when we divide any integer by (–1)? Complete the following table

  $(-8) \div (-1) = 8$ $\quad$ $11 \div (-1) = -11$ $\quad$ $13 \div (-1) =$ ____

  $(-25) \div (-1) =$ ____ $\quad$ $(-37) \div (-1) =$ ____ $\quad$ $-48 \div (-1) =$ ____

  What do you observe?

  We can say that if any integer is divided by (–1) it does not give the same integer.

- Can we say $[(-16) \div 4] \div (-2)$ is the same as $(-16) \div [4 \div (-2)]$?

  We know that $\quad [(-16) \div 4] \div (-2) = (-4) \div (-2) = 2$

  and $\quad (-16) \div [4 \div (-2)] = (-16) \div (-2) = 8$

  So $\quad [(-16) \div 4] \div (-2) \neq (-16) \div [4 \div (-2)]$

  Can you say that division is associative for integers? No.

  Verify it by taking five more examples of your own.

TRY THESE

Is (i) $1 \div a = 1$?

(ii) $a \div (-1) = -a$? for any integer a.

Take different values of a and check.

EXAMPLE 2 In a test (+5) marks are given for every correct answer and (–2) marks are given for every incorrect answer. (i) Radhika answered all the questions and scored 30 marks though she got 10 correct answers. (ii) Jay also answered all the questions and scored (–12) marks though he got 4 correct answers. How many incorrect answers had they attempted?

SOLUTION

(i) Marks given for one correct answer = 5

So, marks given for 10 correct answers = 5 × 10 = 50

Radhika's score = 30

Marks obtained for incorrect answers = 30 – 50 = – 20

Marks given for one incorrect answer = (–2)

Therefore, number of incorrect answers = (–20) ÷ (–2) = 10

(ii) Marks given for 4 correct answers = 5 × 4 = 20

Jay's score = –12

Marks obtained for incorrect answers = –12 – 20 = – 32

Marks given for one incorrect answer = (–2)

Therefore number of incorrect answers = (–32) ÷ (–2) = 16

EXAMPLE 3 A shopkeeper earns a profit of ₹ 1 by selling one pen and incurs a loss of 40 paise per pencil while selling pencils of her old stock.

(i) In a particular month she incurs a loss of ₹ 5. In this period, she sold 45 pens. How many pencils did she sell in this period?

(ii) In the next month she earns neither profit nor loss. If she sold 70 pens, how many pencils did she sell?

SOLUTION

(i) Profit earned by selling one pen = ₹ 1

Profit earned by selling 45 pens = ₹ 45, which we denote by + ₹ 45

Total loss given = ₹ 5, which we denote by – ₹ 5

Profit earned + Loss incurred = Total loss

Therefore, Loss incurred = Total Loss – Profit earned

= ₹ (– 5 – 45) = ₹ (–50) = –5000 paise

Loss incurred by selling one pencil = 40 paise which we write as – 40 paise

So, number of pencils sold = (–5000) ÷ (– 40) = 125

(ii) In the next month there is neither profit nor loss.

So, Profit earned + Loss incurred = 0

i.e., Profit earned = – Loss incurred.

Now, profit earned by selling 70 pens = ₹ 70

Hence, loss incurred by selling pencils = ₹ 70 which we indicate by – ₹ 70 or – 7,000 paise.

Total number of pencils sold = (–7000) ÷ (– 40) = 175 pencils.

EXERCISE 1.3

1. Evaluate each of the following:

(a) (–30) ÷ 10 (b) 50 ÷ (–5) (c) (–36) ÷ (–9)

(d) (– 49) ÷ (49) (e) 13 ÷ [(–2) + 1] (f) 0 ÷ (–12)

(g) (–31) ÷ [(–30) + (–1)]

(h) [(–36) ÷ 12] ÷ 3 (i) [(– 6) + 5)] ÷ [(–2) + 1]

2. Verify that $a \div (b + c) \neq (a \div b) + (a \div c)$ for each of the following values of a, b and c.

(a) $a = 12, b = -4, c = 2$ (b) $a = (-10), b = 1, c = 1$

3. Fill in the blanks:

(a) 369 ÷ _____ = 369 (b) (–75) ÷ _____ = –1

(c) (–206) ÷ _____ = 1 (d) – 87 ÷ _____ = 87

(e) _____ ÷ 1 = – 87 (f) _____ ÷ 48 = –1

(g) 20 ÷ _____ = –2 (h) _____ ÷ (4) = –3

4. Write five pairs of integers (a, b) such that $a \div b = -3$. One such pair is (6, –2) because 6 ÷ (–2) = (–3).

5. The temperature at 12 noon was 10°C above zero. If it decreases at the rate of 2°C per hour until midnight, at what time would the temperature be 8°C below zero? What would be the temperature at mid-night?

6. In a class test (+ 3) marks are given for every correct answer and (–2) marks are given for every incorrect answer and no marks for not attempting any question. (i) Radhika scored 20 marks. If she has got 12 correct answers, how many questions has she attempted incorrectly? (ii) Mohini scores –5 marks in this test, though she has got 7 correct answers. How many questions has she attempted incorrectly?

7. An elevator descends into a mine shaft at the rate of 6 m/min. If the descent starts from 10 m above the ground level, how long will it take to reach – 350 m.

What have We Discussed?

1. We now study the properties satisfied by addition and subtraction.
   (a) Integers are closed for addition and subtraction both. That is, $a + b$ and $a - b$ are again integers, where a and b are any integers.
   (b) Addition is commutative for integers, i.e., $a + b = b + a$ for all integers a and b.
   (c) Addition is associative for integers, i.e., $(a + b) + c = a + (b + c)$ for all integers a, b and c.
   (d) Integer 0 is the identity under addition. That is, $a + 0 = 0 + a = a$ for every integer a.
2. We studied, how integers could be multiplied, and found that product of a positive and a negative integer is a negative integer, whereas the product of two negative integers is a positive integer. For example, $-2 \times 7 = -14$ and $-3 \times -8 = 24$.
3. Product of even number of negative integers is positive, whereas the product of odd number of negative integers is negative.
4. Integers show some properties under multiplication.
   (a) Integers are closed under multiplication. That is, $a \times b$ is an integer for any two integers a and b.
   (b) Multiplication is commutative for integers. That is, $a \times b = b \times a$ for any integers a and b.
   (c) The integer 1 is the identity under multiplication, i.e., $1 \times a = a \times 1 = a$ for any integer a.
   (d) Multiplication is associative for integers, i.e., $(a \times b) \times c = a \times (b \times c)$ for any three integers a, b and c.
5. Under addition and multiplication, integers show a property called distributive property. That is, $a \times (b + c) = a \times b + a \times c$ for any three integers a, b and c.
6. The properties of commutativity, associativity under addition and multiplication, and the distributive property help us to make our calculations easier.
7. We also learnt how to divide integers. We found that,
   (a) When a positive integer is divided by a negative integer, the quotient obtained is negative and vice-versa.
   (b) Division of a negative integer by another negative integer gives positive as quotient.
8. For any integer a, we have
   (a) $a \div 0$ is not defined (b) $a \div 1 = a$

Fractions and Decimals

Chapter 2

2.1 Multiplication of Fractions

You know how to find the area of a rectangle. It is equal to length × breadth. If the length and breadth of a rectangle are 7 cm and 4 cm respectively, then what will be its area? Its area would be $7 \times 4 = 28$ cm^2.

What will be the area of the rectangle if its length and breadth are $7\frac{1}{2}$ cm and $3\frac{1}{2}$ cm respectively? You will say it will be $7\frac{1}{2} \times 3\frac{1}{2} = \frac{15}{2} \times \frac{7}{2}$ cm^2. The numbers $\frac{15}{2}$ and $\frac{7}{2}$ are fractions. To calculate the area of the given rectangle, we need to know how to multiply fractions. We shall learn that now.

2.1.1 Multiplication of a Fraction by a Whole Number

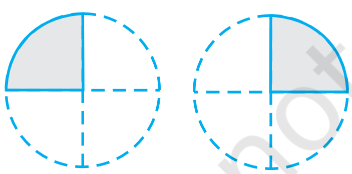


Fig 2.1

Observe the pictures at the left (Fig 2.1). Each shaded part is $\frac{1}{4}$ part of a circle. How much will the two shaded parts represent together? They will represent $\frac{1}{4} + \frac{1}{4} = 2 \times \frac{1}{4}$.

Combining the two shaded parts, we get Fig 2.2 . What part of a circle does the shaded part in Fig 2.2 represent? It represents $\frac{2}{4}$ part of a circle .

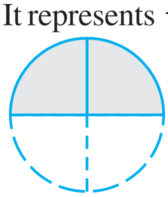


Fig 2.2

The shaded portions in Fig 2.1 taken together are the same as the shaded portion in Fig 2.2, i.e., we get Fig 2.3.

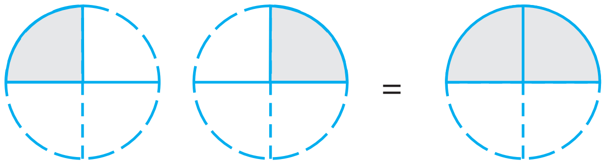


Fig 2.3

or $$2 \times \frac{1}{4} = \frac{2}{4}.$$

Can you now tell what this picture will represent? (Fig 2.4)

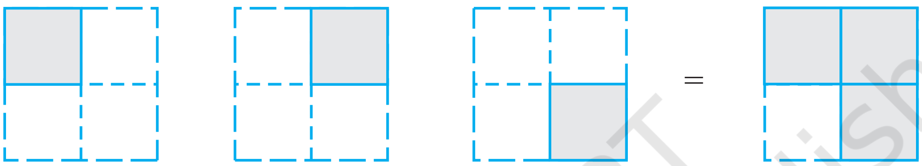


Fig 2.4

And this? (Fig 2.5)

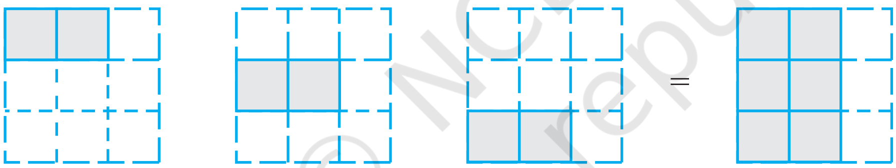


Fig 2.5

Let us now find $3 \times \frac{1}{2}$.

We have $$3 \times \frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$$

We also have $$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{1+1+1}{2} = \frac{3 \times 1}{2} = \frac{3}{2}$$

So $$3 \times \frac{1}{2} = \frac{3 \times 1}{2} = \frac{3}{2}$$

Similarly $$\frac{2}{3} \times 5 = \frac{2 \times 5}{3} = ?$$

Can you tell $3 \times \frac{2}{7} = ?$ $\quad 4 \times \frac{3}{5} = ?$

The fractions that we considered till now, i.e., $\frac{1}{2}, \frac{2}{3}, \frac{2}{7}$ and $\frac{3}{5}$ were proper fractions.

For improper fractions also we have,

$$2 \times \frac{5}{3} = \frac{2 \times 5}{3} = \frac{10}{3}$$

Try, $3 \times \frac{8}{7} = ?$ $4 \times \frac{7}{5} = ?$

Thus, *to multiply a whole number with a proper or an improper fraction, we multiply the whole number with the numerator of the fraction, keeping the denominator same.*

Try These

1. Find: (a) $\frac{2}{7} \times 3$ (b) $\frac{9}{7} \times 6$ (c) $3 \times \frac{1}{8}$ (d) $\frac{13}{11} \times 6$

If the product is an improper fraction express it as a mixed fraction.

2. Represent pictorially : $2 \times \frac{2}{5} = \frac{4}{5}$

Try These

Find: (i) $5 \times 2\frac{3}{7}$

(ii) $1\frac{4}{9} \times 6$

To multiply a mixed fraction to a whole number, first convert the mixed fraction to an improper fraction and then multiply.

Therefore, $3 \times 2\frac{5}{7} = 3 \times \frac{19}{7} = \frac{57}{7} = 8\frac{1}{7}$.

Similarly, $2 \times 4\frac{2}{5} = 2 \times \frac{22}{5} = ?$

Fraction as an operator 'of'

Observe these figures (Fig 2.6)

The two squares are exactly similar.

Each shaded portion represents $\frac{1}{2}$ of 1.

So, both the shaded portions together will represent $\frac{1}{2}$ of 2.

Combine the 2 shaded $\frac{1}{2}$ parts. It represents 1.

So, we say $\frac{1}{2}$ of 2 is 1. We can also get it as $\frac{1}{2} \times 2 = 1$.

Thus, $\frac{1}{2}$ of $2 = \frac{1}{2} \times 2 = 1$

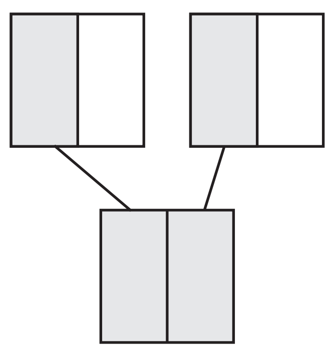


Fig 2.6

Also, look at these similar squares (Fig 2.7).

Each shaded portion represents $\frac{1}{2}$ of 1.

So, the three shaded portions represent $\frac{1}{2}$ of 3.

Combine the 3 shaded parts.

It represents $1\frac{1}{2}$ i.e., $\frac{3}{2}$.

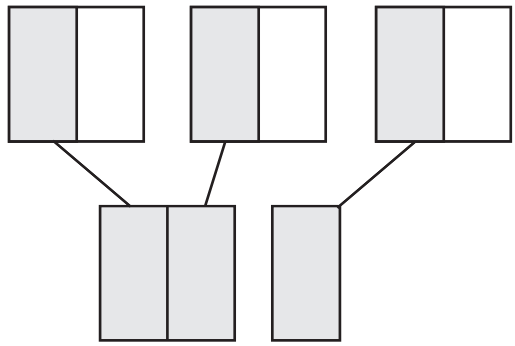


Fig 2.7

So, $\frac{1}{2}$ of 3 is $\frac{3}{2}$. Also, $\frac{1}{2} \times 3 = \frac{3}{2}$.

Thus, $\frac{1}{2}$ of $3 = \frac{1}{2} \times 3 = \frac{3}{2}$.

So we see that **'of' represents multiplication.**

Farida has 20 marbles. Reshma has $\frac{1}{5}$th of the number of marbles what Farida has. How many marbles Reshma has? As, 'of' indicates multiplication, so, Reshma has $\frac{1}{5} \times 20 = 4$ marbles.

Similarly, we have $\frac{1}{2}$ of 16 is $\frac{1}{2} \times 16 = \frac{16}{2} = 8$.

TRY THESE

Can you tell, what is (i) $\frac{1}{2}$ of 10?, (ii) $\frac{1}{4}$ of 16?, (iii) $\frac{2}{5}$ of 25?

EXAMPLE 1 In a class of 40 students $\frac{1}{5}$ of the total number of studetns like to study English, $\frac{2}{5}$ of the total number like to study Mathematics and the remaining students like to study Science.

(i) How many students like to study English?

(ii) How many students like to study Mathematics?

(iii) What fraction of the total number of students like to study Science?

SOLUTION Total number of students in the class = 40.

(i) Of these $\frac{1}{5}$ of the total number of students like to study English.

Thus, the number of students who like to study English $= \frac{1}{5}$ of $40 = \frac{1}{5} \times 40 = 8$.

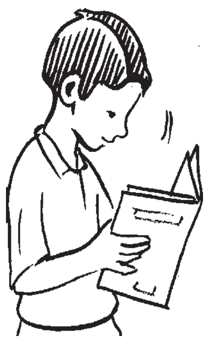
(ii) Try yourself.

(iii) The number of students who like English and Mathematics $= 8 + 16 = 24$. Thus, the number of students who like Science $= 40 - 24 = 16$.

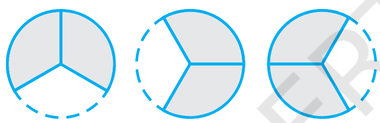
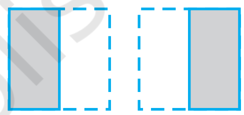
Thus, the required fraction is $\frac{16}{40}$.

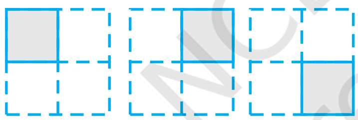
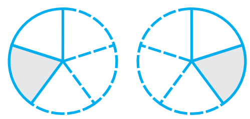
Exercise 2.1

1. Which of the drawings (a) to (d) show :



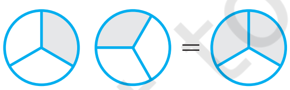
(i) $2 \times \frac{1}{5}$ (ii) $2 \times \frac{1}{2}$ (iii) $3 \times \frac{2}{3}$ (iv) $3 \times \frac{1}{4}$

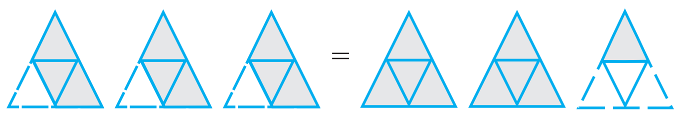
(a)

(b)


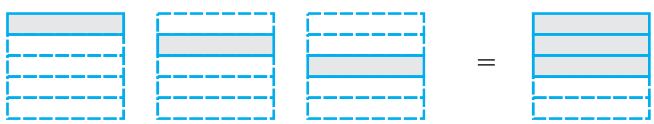
(c)

(d)


2. Some pictures (a) to (c) are given below. Tell which of them show:

(i) $3 \times \frac{1}{5} = \frac{3}{5}$ (ii) $2 \times \frac{1}{3} = \frac{2}{3}$ (iii) $3 \times \frac{3}{4} = 2\frac{1}{4}$


(a)


(b)


(c)

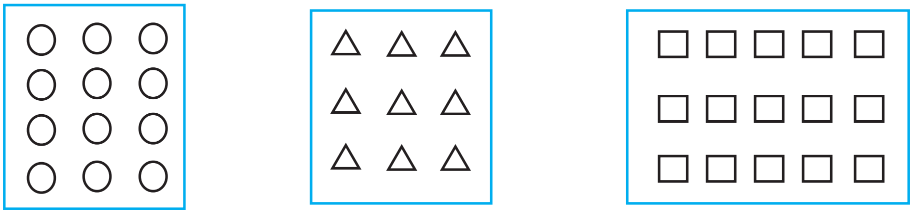
3. Multiply and reduce to lowest form and convert into a mixed fraction:

(i) $7 \times \frac{3}{5}$ (ii) $4 \times \frac{1}{3}$ (iii) $2 \times \frac{6}{7}$ (iv) $5 \times \frac{2}{9}$ (v) $\frac{2}{3} \times 4$

(vi) $\frac{5}{2} \times 6$ (vii) $11 \times \frac{4}{7}$ (viii) $20 \times \frac{4}{5}$ (ix) $13 \times \frac{1}{3}$ (x) $15 \times \frac{3}{5}$

4. Shade: (i) $\frac{1}{2}$ of the circles in box (a) (ii) $\frac{2}{3}$ of the triangles in box (b)

(iii) $\frac{3}{5}$ of the squares in box (c).



(a) (b) (c)

5. Find:

(a) $\frac{1}{2}$ of (i) 24 (ii) 46 (b) $\frac{2}{3}$ of (i) 18 (ii) 27

(c) $\frac{3}{4}$ of (i) 16 (ii) 36 (d) $\frac{4}{5}$ of (i) 20 (ii) 35

6. Multiply and express as a mixed fraction :

(a) $3\times5\frac{1}{5}$ (b) $5\times6\frac{3}{4}$ (c) $7\times2\frac{1}{4}$

(d) $4\times6\frac{1}{3}$ (e) $3\frac{1}{4}\times6$ (f) $3\frac{2}{5}\times8$

7. Find: (a) $\frac{1}{2}$ of (i) $2\frac{3}{4}$ (ii) $4\frac{2}{9}$ (b) $\frac{5}{8}$ of (i) $3\frac{5}{6}$ (ii) $9\frac{2}{3}$

8. Vidya and Pratap went for a picnic. Their mother gave them a water bottle that contained 5 litres of water. Vidya consumed $\frac{2}{5}$ of the water. Pratap consumed the remaining water.

(i) How much water did Vidya drink?

(ii) What fraction of the total quantity of water did Pratap drink?

2.1.2 Multiplication of a Fraction by a Fraction

Farida had a 9 cm long strip of ribbon. She cut this strip into four equal parts. How did she do it? She folded the strip twice. What fraction of the total length will each part represent? Each part will be $\frac{9}{4}$ of the strip. She took one part and divided it in two equal parts by folding the part once. What will one of the pieces represent? It will represent $\frac{1}{2}$ of $\frac{9}{4}$ or $\frac{1}{2} \times \frac{9}{4}$.

Let us now see how to find the product of two fractions like $\frac{1}{2} \times \frac{9}{4}$.

To do this we first learn to find the products like $\frac{1}{2} \times \frac{1}{3}$.

Fig 2.8

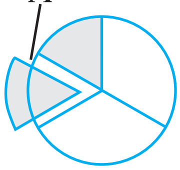


Fig 2.9

(a) How do we find $\frac{1}{3}$ of a whole? We divide the whole in three equal parts. Each of the three parts represents $\frac{1}{3}$ of the whole. Take one part of these three parts, and shade it as shown in Fig 2.8.

(b) How will you find $\frac{1}{2}$ of this shaded part? Divide this one-third ($\frac{1}{3}$) shaded part into two equal parts. Each of these two parts represents $\frac{1}{2}$ of $\frac{1}{3}$ i.e., $\frac{1}{2} \times \frac{1}{3}$ (Fig 2.9).

Take out 1 part of these two and name it 'A'. 'A' represents $\frac{1}{2} \times \frac{1}{3}$.

(c) What fraction is 'A' of the whole? For this, divide each of the remaining $\frac{1}{3}$ parts also in two equal parts. How many such equal parts do you have now?

There are six such equal parts. 'A' is one of these parts.

So, 'A' is $\frac{1}{6}$ of the whole. Thus, $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.

How did we decide that 'A' was $\frac{1}{6}$ of the whole? The whole was divided in $6 = 2 \times 3$ parts and $1 = 1 \times 1$ part was taken out of it.

Thus, $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6} = \frac{1\times1}{2\times3}$

or $\frac{1}{2} \times \frac{1}{3} = \frac{1\times1}{2\times3}$

The value of $\frac{1}{3}\times\frac{1}{2}$ can be found in a similar way. Divide the whole into two equal parts and then divide one of these parts in three equal parts. Take one of these parts. This will represent $\frac{1}{3}\times\frac{1}{2}$ i.e., $\frac{1}{6}$.

Therefore $\frac{1}{3}\times\frac{1}{2} = \frac{1}{6} = \frac{1\times1}{3\times2}$ as discussed earlier.

Hence $\frac{1}{2}\times\frac{1}{3} = \frac{1}{3}\times\frac{1}{2} = \frac{1}{6}$

Find $\frac{1}{3}\times\frac{1}{4}$ and $\frac{1}{4}\times\frac{1}{3}$; $\frac{1}{2}\times\frac{1}{5}$ and $\frac{1}{5}\times\frac{1}{2}$ and check whether you get

$$\frac{1}{3}\times\frac{1}{4} = \frac{1}{4}\times\frac{1}{3};\ \frac{1}{2}\times\frac{1}{5} = \frac{1}{5}\times\frac{1}{2}$$

Try These

Fill in these boxes:

(i) $\frac{1}{2}\times\frac{1}{7} = \frac{1\times1}{2\times7} = \square$

(ii) $\frac{1}{5}\times\frac{1}{7} = \square = \square$

(iii) $\frac{1}{7}\times\frac{1}{2} = \square = \square$

(iv) $\frac{1}{7}\times\frac{1}{5} = \square = \square$

Example 2 Sushant reads $\frac{1}{3}$ part of a book in 1 hour. How much part of the book will he read in $2\frac{1}{5}$ hours?

Solution The part of the book read by Sushant in 1 hour $= \frac{1}{3}$.

So, the part of the book read by him in $2\frac{1}{5}$ hours $= 2\frac{1}{5}\times\frac{1}{3}$

$$= \frac{11}{5}\times\frac{1}{3} = \frac{11\times1}{5\times3} = \frac{11}{15}$$

Let us now find $\frac{1}{2}\times\frac{5}{3}$. We know that $\frac{5}{3} = \frac{1}{3}\times5$.

$$\text{So, } \frac{1}{2}\times\frac{5}{3} = \frac{1}{2}\times\frac{1}{3}\times5 = \frac{1}{6}\quad 5 = \frac{5}{6}$$

Also, $\frac{5}{6} = \frac{1 \times 5}{2 \times 3}$. Thus, $\frac{1}{2} \times \frac{5}{3} = \frac{1 \times 5}{2 \times 3} = \frac{5}{6}$.

This is also shown by the figures drawn below. Each of these five equal shapes (Fig 2.10) are parts of five similar circles. Take one such shape. To obtain this shape we first divide a circle in three equal parts. Further divide each of these three parts in two equal parts. One part out of it is the shape we considered. What will it represent?

It will represent $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$. The total of such parts would be $5 \times \frac{1}{6} = \frac{5}{6}$.

Fig 2.10

Try These

Find: $\frac{1}{3} \times \frac{4}{5}$; $\frac{2}{3} \times \frac{1}{5}$

Similarly $\frac{3}{5} \times \frac{1}{7} = \frac{3 \times 1}{5 \times 7} = \frac{3}{35}$.

We can thus find $\frac{2}{3} \times \frac{7}{5}$ as $\frac{2}{3} \times \frac{7}{5} = \frac{2 \times 7}{3 \times 5} = \frac{14}{15}$.

So, we find that we multiply two fractions as $\frac{\text{Product of Numerators}}{\text{Product of Denominators}}$.

Value of the Products

Try These

Find: $\frac{8}{3} \times \frac{4}{7}$; $\frac{3}{4} \times \frac{2}{3}$.

You have seen that the product of two whole numbers is bigger than each of the two whole numbers. For example, $3 \times 4 = 12$ and $12 > 4$, $12 > 3$. What happens to the value of the product when we multiply two fractions?

Let us first consider the product of two proper fractions.

We have,

$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$	$\frac{8}{15} < \frac{2}{3}, \frac{8}{15} < \frac{4}{5}$	Product is less than each of the fractions
$\frac{1}{5} \times \frac{2}{7}$ = ---------	--------,--------	------------------------------------
$\frac{3}{5} \times \frac{\square}{8}$ =	--------,--------	------------------------------------
$\frac{2}{\square} \times \frac{4}{9} = \frac{8}{45}$	--------,--------	------------------------------------

You will find that *when two proper fractions are multiplied, the product is less than each of the fractions*. Or, *we say the value of the product of two proper fractions is smaller than each of the two fractions.*

Check this by constructing five more examples.

Let us now multiply two improper fractions.

$\frac{7}{3}\times\frac{5}{2}=\frac{35}{6}$	$\frac{35}{6}>\frac{7}{3},\frac{35}{6}>\frac{5}{2}$	Product is greater than each of the fractions
$\frac{6}{5}\times\frac{\square}{3}=\frac{24}{15}$	--------,--------	--
$\frac{9}{2}\times\frac{7}{\square}=\frac{63}{8}$	--------,--------	--
$\frac{3}{\square}\times\frac{8}{7}=\frac{24}{14}$	--------,--------	--

We find that *the product of two improper fractions is greater than each of the two fractions.*

Or, *the value of the product of two improper fractions is more than each of the two fractions*.

Construct five more examples for yourself and verify the above statement.

Let us now multiply a proper and an improper fraction, say $\frac{2}{3}$ and $\frac{7}{5}$.

We have $\frac{2}{3}\times\frac{7}{5}=\frac{14}{15}$. Here, $\frac{14}{15}<\frac{7}{5}$ and $\frac{14}{15}>\frac{2}{3}$

The product obtained is less than the improper fraction and greater than the proper fraction involved in the multiplication.

Check it for $\frac{6}{5}\times\frac{2}{8}$, $\frac{8}{3}\times\frac{4}{5}$.

EXERCISE 2.2

1. Find:

(i) $\frac{1}{4}$ of (a) $\frac{1}{4}$ (b) $\frac{3}{5}$ (c) $\frac{4}{3}$

(ii) $\frac{1}{7}$ of (a) $\frac{2}{9}$ (b) $\frac{6}{5}$ (c) $\frac{3}{10}$

2. Multiply and reduce to lowest form (if possible) :

(i) $\frac{2}{3} \times 2\frac{2}{3}$ (ii) $\frac{2}{7} \times \frac{7}{9}$ (iii) $\frac{3}{8} \times \frac{6}{4}$ (iv) $\frac{9}{5} \times \frac{3}{5}$

(v) $\frac{1}{3} \times \frac{15}{8}$ (vi) $\frac{11}{2} \times \frac{3}{10}$ (vii) $\frac{4}{5} \times \frac{12}{7}$

3. Multiply the following fractions:

(i) $\frac{2}{5} \times 5\frac{1}{4}$ (ii) $6\frac{2}{5} \times \frac{7}{9}$ (iii) $\frac{3}{2} \times 5\frac{1}{3}$ (iv) $\frac{5}{6} \times 2\frac{3}{7}$

(v) $3\frac{2}{5} \times \frac{4}{7}$ (vi) $2\frac{3}{5} \times 3$ (vii) $3\frac{4}{7} \times \frac{3}{5}$

4. Which is greater:

(i) $\frac{2}{7}$ of $\frac{3}{4}$ or $\frac{3}{5}$ of $\frac{5}{8}$ (ii) $\frac{1}{2}$ of $\frac{6}{7}$ or $\frac{2}{3}$ of $\frac{3}{7}$

5. Saili plants 4 saplings, in a row, in her garden. The distance between two adjacent saplings is $\frac{3}{4}$ m. Find the distance between the first and the last sapling.

6. Lipika reads a book for $1\frac{3}{4}$ hours everyday. She reads the entire book in 6 days. How many hours in all were required by her to read the book?

7. A car runs 16 km using 1 litre of petrol. How much distance will it cover using $2\frac{3}{4}$ litres of petrol.

8. (a) (i) Provide the number in the box $\square$, such that $\frac{2}{3} \times \square = \frac{10}{30}$.

(ii) The simplest form of the number obtained in $\square$ is _____.

(b) (i) Provide the number in the box $\square$, such that $\frac{3}{5} \times \square = \frac{24}{75}$.

(ii) The simplest form of the number obtained in $\square$ is _____.

2.2 Division of Fractions

John has a paper strip of length 6 cm. He cuts this strip in smaller strips of length 2 cm each. You know that he would get 6 ÷ 2 =3 strips.

John cuts another strip of length 6 cm into smaller strips of length $\frac{3}{2}$ cm each. How many strips will he get now? He will get $6 \div \frac{3}{2}$ strips.

A paper strip of length $\frac{15}{2}$ cm can be cut into smaller strips of length $\frac{3}{2}$ cm each to give $\frac{15}{2} \div \frac{3}{2}$ pieces.

So, we are required to divide a whole number by a fraction or a fraction by another fraction. Let us see how to do that.

2.2.1 Division of Whole Number by a Fraction

Let us find $1 \div \frac{1}{2}$.

We divide a whole into a number of equal parts such that each part is half of the whole. The number of such half ($\frac{1}{2}$) parts would be $1 \div \frac{1}{2}$. Observe the figure (Fig 2.11). How many half parts do you see?

There are two half parts.

So, $1 \div \frac{1}{2} = 2$. Also, $1 \times \frac{2}{1} = 1 \times 2 = 2$. Thus, $1 \div \frac{1}{2} = 1 \times \frac{2}{1}$

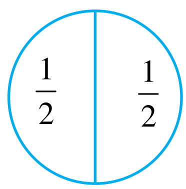


Fig 2.11

Similarly, $3 \div \frac{1}{4}$ = number of $\frac{1}{4}$ parts obtained when each of the 3 whole, are divided into $\frac{1}{4}$ equal parts = 12 (From Fig 2.12)

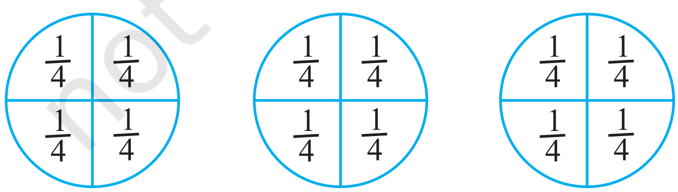


Fig 2.12

Observe also that, $3 \times \frac{4}{1} = 3 \times 4 = 12$. Thus, $3 \div \frac{1}{4} = 3 \times \frac{4}{1} = 12$.

Find in a similar way, $3 \div \frac{1}{2}$ and $3 \times \frac{2}{1}$.

Reciprocal of a fraction

The number $\frac{2}{1}$ can be obtained by interchanging the numerator and denominator of $\frac{1}{2}$ or by inverting $\frac{1}{2}$. Similarly, $\frac{3}{1}$ is obtained by inverting $\frac{1}{3}$.

Let us first see about the inverting of such numbers.

Observe these products and fill in the blanks :

$7\times\frac{1}{7} = 1$	$\frac{5}{4}\times\frac{4}{5} =$ ---------
$\frac{1}{9}\times 9 =$ ------	$\frac{2}{7}\times$ ------- $= 1$
$\frac{2}{3}\times\frac{3}{2} = \frac{2\times 3}{3\times 2} = \frac{6}{6} = 1$	------ $\times\frac{5}{9} = 1$

Multiply five more such pairs.

The non-zero numbers whose product with each other is 1, are called the reciprocals of each other. So reciprocal of $\frac{5}{9}$ is $\frac{9}{5}$ and the reciprocal of $\frac{9}{5}$ is $\frac{5}{9}$. What is the receiprocal of $\frac{1}{9}$? $\frac{2}{7}$?

You will see that the reciprocal of $\frac{2}{3}$ is obtained by inverting it. You get $\frac{3}{2}$.

Think, Discuss and Write

(i) Will the reciprocal of a proper fraction be again a proper fraction?

(ii) Will the reciprocal of an improper fraction be again an improper fraction?

Therefore, we can say that

$1 \div \frac{1}{2} = 1\times\frac{2}{1} = 1\times$ reciprocal of $\frac{1}{2}$.

$3 \div \frac{1}{4} = 3\times\frac{4}{1} = 3\times$ reciprocal of $\frac{1}{4}$.

$3 \div \frac{1}{2} =$ ------ $=$ ----------------------.

So, $2 \div \frac{3}{4} = 2\times$ reciprocal of $\frac{3}{4} = 2\times\frac{4}{3}$.

$5 \div \frac{2}{9} = 5\times$ ------------------- $= 5\times$ -------------

Thus, *to divide a whole number by any fraction, multiply that whole number by the reciprocal of that fraction.*

TRY THESE

Find : (i) $7 \div \frac{2}{5}$ (ii) $6 \div \frac{4}{7}$ (iii) $2 \div \frac{8}{9}$

- *While dividing a whole number by a mixed fraction, first convert the mixed fraction into improper fraction and then solve it.*

Thus, $4 \div 2\frac{2}{5} = 4 \div \frac{12}{5} = ?$ Also, $5 \div 3\frac{1}{3} = 3 \div \frac{10}{3} = ?$

TRY THESE

Find: (i) $6 \div 5\frac{1}{3}$

(ii) $7 \div 2\frac{4}{7}$

2.2.2 Division of a Fraction by a Whole Number

- What will be $\frac{3}{4} \div 3$?

Based on our earlier observations we have: $\frac{3}{4} \div 3 = \frac{3}{4} \div \frac{3}{1} = \frac{3}{4} \times \frac{1}{3} = \frac{3}{12} = \frac{1}{4}$

So, $\frac{2}{3} \div 7 = \frac{2}{3} \times \frac{1}{7} = ?$ What is $\frac{5}{7} \div 6$, $\frac{2}{7} \div 8$?

- While dividing mixed fractions by whole numbers, convert the mixed fractions into improper fractions. That is,

$2\frac{2}{3} \div 5 = \frac{8}{3} \div 5 =$ ------ ; $4\frac{2}{5} \div 3 =$ ------ $=$ ------; $2\frac{3}{5} \div 2 =$ ------ $=$ ------

2.2.3 Division of a Fraction by Another Fraction

We can now find $\frac{1}{3} \div \frac{6}{5}$.

$\frac{1}{3} \div \frac{6}{5} = \frac{1}{3} \times$ reciprocal of $\frac{6}{5} = \frac{1}{3} \times \frac{5}{6} = \frac{5}{18}$.

Similarly, $\frac{8}{5} \div \frac{2}{3} = \frac{8}{5} \times$ reciprocal of $\frac{2}{3} = ?$ and, $\frac{1}{2} \div \frac{3}{4} = ?$

TRY THESE

Find: (i) $\frac{3}{5} \div \frac{1}{2}$ (ii) $\frac{1}{2} \div \frac{3}{5}$ (iii) $2\frac{1}{2} \div \frac{3}{5}$ (iv) $5\frac{1}{6} \div \frac{9}{2}$

Exercise 2.3

1. Find:

   (i) $12 \div \frac{3}{4}$ (ii) $14 \div \frac{5}{6}$ (iii) $8 \div \frac{7}{3}$ (iv) $4 \div \frac{8}{3}$

   (v) $3 \div 2\frac{1}{3}$ (vi) $5 \div 3\frac{4}{7}$

2. Find the reciprocal of each of the following fractions. Classify the reciprocals as proper fractions, improper fractions and whole numbers.

   (i) $\frac{3}{7}$ (ii) $\frac{5}{8}$ (iii) $\frac{9}{7}$ (iv) $\frac{6}{5}$

   (v) $\frac{12}{7}$ (vi) $\frac{1}{8}$ (vii) $\frac{1}{11}$

3. Find:

   (i) $\frac{7}{3} \div 2$ (ii) $\frac{4}{9} \div 5$ (iii) $\frac{6}{13} \div 7$ (iv) $4\frac{1}{3} \div 3$

   (v) $3\frac{1}{2} \div 4$ (vi) $4\frac{3}{7} \div 7$

4. Find:

   (i) $\frac{2}{5} \div \frac{1}{2}$ (ii) $\frac{4}{9} \div \frac{2}{3}$ (iii) $\frac{3}{7} \div \frac{8}{7}$ (iv) $2\frac{1}{3} \div \frac{3}{5}$ (v) $3\frac{1}{2} \div \frac{8}{3}$

   (vi) $\frac{2}{5} \div 1\frac{1}{2}$ (vii) $3\frac{1}{5} \div 1\frac{2}{3}$ (viii) $2\frac{1}{5} \div 1\frac{1}{5}$

2.3 Multiplication of Decimal Numbers

Reshma purchased 1.5kg vegetable at the rate of ₹ 8.50 per kg. How much money should she pay? Certainly it would be ₹ (8.50 × 1.50). Both 8.5 and 1.5 are decimal numbers. So, we have come across a situation where we need to know how to multiply two decimals. Let us now learn the multiplication of two decimal numbers.

First we find 0.1 × 0.1.

Now, $0.1 = \frac{1}{10}$. So, $0.1 \times 0.1 = \frac{1}{10} \times \frac{1}{10} =$

$\frac{1 \times 1}{10 \times 10} = \frac{1}{100} = 0.01.$

Let us see it's pictorial representation (Fig 2.13)

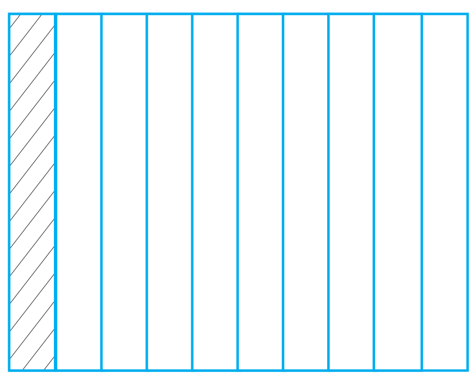


Fig 2.13

The fraction $\frac{1}{10}$ represents 1 part out of 10 equal parts.

The shaded part in the picture represents $\frac{1}{10}$.

We know that,

$\frac{1}{10} \times \frac{1}{10}$ means $\frac{1}{10}$ of $\frac{1}{10}$. So, divide this $\frac{1}{10}^{\text{th}}$ part into 10 equal parts and take one part out of it.

Thus, we have, (Fig 2.14).

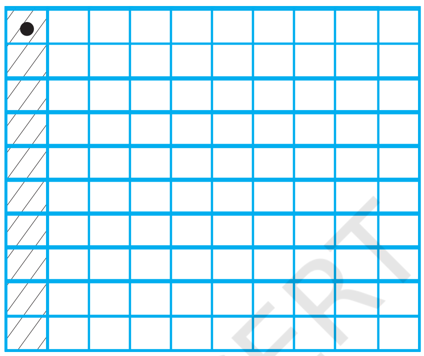


Fig 2.14

The dotted square is one part out of 10 of the $\frac{1}{10}^{\text{th}}$ part. That is, it represents $\frac{1}{10} \times \frac{1}{10}$ or 0.1×0.1.

Can the dotted square be represented in some other way?

How many small squares do you find in Fig 2.14?

There are 100 small squares. So the dotted square represents one out of 100 or 0.01.

Hence, $0.1 \times 0.1 = 0.01$.

Note that 0.1 occurs two times in the product. In 0.1 there is one digit to the right of the decimal point. In 0.01 there are two digits (i.e., 1 + 1) to the right of the decimal point.

Let us now find 0.2×0.3.

We have, $0.2 \times 0.3 = \frac{2}{10} \times \frac{3}{10}$

As we did for $\frac{1}{10} \times \frac{1}{10}$, let us divide the square into 10 equal parts and take three parts out of it, to get $\frac{3}{10}$. Again divide each

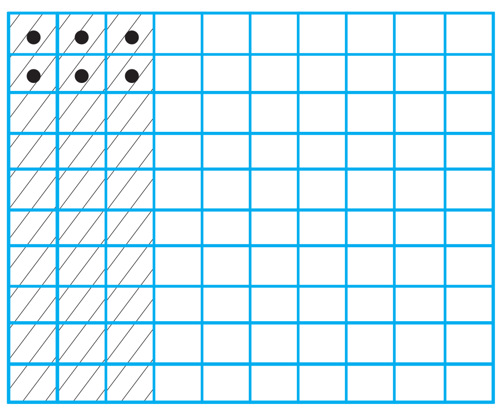


Fig 2.15

of these three equal parts into 10 equal parts and take two from each. We get $\frac{2}{10} \times \frac{3}{10}$.

The dotted squares represent $\frac{2}{10} \times \frac{3}{10}$ or 0.2×0.3. (Fig 2.15)

Since there are 6 dotted squares out of 100, so they also reprsent 0.06.

Thus, $0.2 \times 0.3 = 0.06$.

Observe that $2 \times 3 = 6$ and the number of digits to the right of the decimal point in 0.06 is 2 (= 1 + 1).

Check whether this applies to 0.1×0.1 also.

Find 0.2×0.4 by applying these observations.

While finding 0.1×0.1 and 0.2×0.3, you might have noticed that first we multiplied them as whole numbers ignoring the decimal point. In 0.1×0.1, we found 01×01 or 1×1. Similarly in 0.2×0.3 we found 02×03 or 2×3.

Then, we counted the number of digits starting from the rightmost digit and moved towards left. We then put the decimal point there. The number of digits to be counted is obtained by adding the number of digits to the right of the decimal point in the decimal numbers that are being multiplied.

Let us now find 1.2×2.5.

Multiply 12 and 25. We get 300. Both, in 1.2 and 2.5, there is 1 digit to the right of the decimal point. So, count $1 + 1 = 2$ digits from the rightmost digit (i.e., 0) in 300 and move towards left. We get 3.00 or 3.

Find in a similar way 1.5×1.6, 2.4×4.2.

While multiplying 2.5 and 1.25, you will first multiply 25 and 125. For placing the decimal in the product obtained, you will count $1 + 2 = 3$ (Why?) digits starting from the rightmost digit. Thus, $2.5 \times 1.25 = 3.225$

Find 2.7×1.35.

TRY THESE

1. Find: (i) 2.7×4 (ii) 1.8×1.2 (iii) 2.3×4.35
2. Arrange the products obtained in (1) in descending order.

EXAMPLE 3 The side of an equilateral triangle is 3.5 cm. Find its perimeter.

SOLUTION All the sides of an equilateral triangle are equal.

So, length of each side = 3.5 cm

Thus, perimeter = 3 × 3.5 cm = 10.5 cm

Example 4 The length of a rectangle is 7.1 cm and its breadth is 2.5 cm. What is the area of the rectangle?

Solution Length of the rectangle = 7.1 cm

Breadth of the rectangle = 2.5 cm

Therefore, area of the rectangle = 7.1 × 2.5 cm^2 = 17.75 cm^2

2.3.1 Multiplication of Decimal Numbers by 10, 100 and 1000

Reshma observed that $2.3 = \frac{23}{10}$ whereas $2.35 = \frac{235}{100}$. Thus, she found that depending on the position of the decimal point the decimal number can be converted to a fraction with denominator 10 or 100. She wondered what would happen if a decimal number is multiplied by 10 or 100 or 1000.

Let us see if we can find a pattern of multiplying numbers by 10 or 100 or 1000.

Have a look at the table given below and fill in the blanks:

$1.76 \times 10 = \frac{176}{100} \times 10 = 17.6$	2.35 ×10 =___	12.356 × 10 =___
$1.76 \times 100 = \frac{176}{100} \times 100 = 176$ or 176.0	2.35 ×100 = ___	12.356 × 100 =___
$1.76 \times 1000 = \frac{176}{100} \times 1000 = 1760$ or 1760.0	2.35 ×1000 = ___	12.356 × 1000 = ___
$0.5 \times 10 = \frac{5}{10} \times 10 = 5$; 0.5 × 100 = ___ ; 0.5 × 1000 = ___		

Observe the shift of the decimal point of the products in the table. Here the numbers are multiplied by 10,100 and 1000. In 1.76 × 10 = 17.6, the digits are same i.e., 1, 7 and 6. Do you observe this in other products also? Observe 1.76 and 17.6. To which side has the decimal point shifted, right or left? The decimal point has shifted to the right by one place. Note that 10 has one zero over 1.

In 1.76×100 = 176.0, observe 1.76 and 176.0. To which side and by how many digits has the decimal point shifted? The decimal point has shifted to the right by two places.

Note that 100 has two zeros over one.

Do you observe similar shifting of decimal point in other products also?

So we say, *when a decimal number is multiplied by 10, 100 or 1000, the digits in the product are same as in the decimal number but the decimal point in the product is shifted to the right by as, many of places as there are zeros over one.*

TRY THESE

Find: (i) 0.3×10
(ii) 1.2×100
(iii) 56.3×1000

Based on these observations we can now say

$$0.07 \times 10 = 0.7,\ 0.07 \times 100 = 7 \text{ and } 0.07 \times 1000 = 70.$$

Can you now tell $2.97 \times 10 = ?$ $2.97 \times 100 = ?$ $2.97 \times 1000 = ?$

Can you now help Reshma to find the total amount i.e., ₹ 8.50×150, that she has to pay?

EXERCISE 2.4

1. Find:
   (i) 0.2×6 (ii) 8×4.6 (iii) 2.71×5 (iv) 20.1×4
   (v) 0.05×7 (vi) 211.02×4 (vii) 2×0.86
2. Find the area of rectangle whose length is 5.7cm and breadth is 3 cm.
3. Find:
   (i) 1.3×10 (ii) 36.8×10 (iii) 153.7×10 (iv) 168.07×10
   (v) 31.1×100 (vi) 156.1×100 (vii) 3.62×100 (viii) 43.07×100
   (ix) 0.5×10 (x) 0.08×10 (xi) 0.9×100 (xii) 0.03×1000
4. A two-wheeler covers a distance of 55.3 km in one litre of petrol. How much distance will it cover in 10 litres of petrol?
5. Find:
   (i) 2.5×0.3 (ii) 0.1×51.7 (iii) 0.2×316.8 (iv) 1.3×3.1
   (v) 0.5×0.05 (vi) 11.2×0.15 (vii) 1.07×0.02
   (viii) 10.05×1.05 (ix) 101.01×0.01 (x) 100.01×1.1

2.4 DIVISION OF DECIMAL NUMBERS

Savita was preparing a design to decorate her classroom. She needed a few coloured strips of paper of length 1.9 cm each. She had a strip of coloured paper of length 9.5 cm. How many pieces of the required length will she get out of this strip? She thought it would be $\frac{9.5}{1.9}$ cm. Is she correct?

Both 9.5 and 1.9 are decimal numbers. So we need to know the division of decimal numbers too!

2.4.1 Division by 10, 100 and 1000

Let us find the division of a decimal number by 10, 100 and 1000.
Consider $31.5 \div 10$.

$$31.5 \div 10 = \frac{315}{10} \times \frac{1}{10} = \frac{315}{100} = 3.15$$

$$\text{Similarly, } 31.5 \div 100 = \frac{315}{10} \times \frac{1}{100} = \frac{315}{1000} = 0.315$$

Let us see if we can find a pattern for dividing numbers by 10, 100 or 1000. This may help us in dividing numbers by 10, 100 or 1000 in a shorter way.

$31.5 \div 10 = 3.15$	$231.5 \div 10 =$___	$1.5 \div 10 =$___	$29.36 \div 10 =$___
$31.5 \div 100 = 0.315$	$231.5 \div 10 =$___	$1.5 \div 100 =$___	$29.36 \div 100 =$___
$31.5 \div 1000 = 0.0315$	$231.5 \div 1000 =$___	$1.5 \div 1000 =$___	$29.36 \div 1000 =$___

Take $31.5 \div 10 = 3.15$. In 31.5 and 3.15, the digits are same i.e., 3, 1, and 5 but the decimal point has shifted in the quotient. To which side and by how many digits? The decimal point has shifted to the left by one place. Note that 10 has one zero over 1.

Try These

Find: (i) $235.4 \div 10$

(ii) $235.4 \div 100$

(iii) $235.4 \div 1000$

Consider now $31.5 \div 100 = 0.315$. In 31.5 and 0.315 the digits are same, but what about the decimal point in the quotient? It has shifted to the left by two places. Note that 100 has two zeros over1.

So we can say that, *while dividing a number by 10, 100 or 1000, the digits of the number and the quotient are same but the decimal point in the quotient shifts to the left by as many places as there are zeros over 1.* Using this observation let us now quickly find: $2.38 \div 10 = 0.238$, $2.38 \div 100 = 0.0238$, $2.38 \div 1000 = 0.00238$

2.4.2 Division of a Decimal Number by a Whole Number

Let us find $\frac{6.4}{2}$. Remember we also write it as $6.4 \div 2$.

So, $6.4 \div 2 = \frac{64}{10} \div 2 = \frac{64}{10} \times \frac{1}{2}$ as learnt in fractions.

$$= \frac{64 \times 1}{10 \times 2} = \frac{1 \times 64}{10 \times 2} = \frac{1}{10} \times \frac{64}{2} = \frac{1}{10} \times 32 = \frac{32}{10} = 3.2$$

Try These

(i) $35.7 \div 3 = ?$;

(ii) $25.5 \div 3 = ?$

Or, let us first divide 64 by 2. We get 32. There is one digit to the right of the decimal point in 6.4. Place the decimal in 32 such that there would be one digit to its right. We get 3.2 again.

To find $19.5 \div 5$, first find $195 \div 5$. We get 39. There is one digit to the right of the decimal point in 19.5. Place the decimal point in 39 such that there would be one digit to its right. You will get 3.9.

Try These

(i) $43.15 \div 5 = ?$;

(ii) $82.44 \div 6 = ?$

TRY THESE

Find: (i) $15.5 \div 5$

(ii) $126.35 \div 7$

Now, $12.96 \div 4 = \frac{1296}{100} \div 4 = \frac{1296}{100} \times \frac{1}{4} = \frac{1}{100} \times \frac{1296}{4} = \frac{1}{100} \times 324 = 3.24$

Or, divide 1296 by 4. You get 324. There are two digits to the right of the decimal in 12.96. Making similar placement of the decimal in 324, you will get 3.24.

Note that here and in the next section, we have considered only those divisions in which, ignoring the decimal, the number would be completely divisible by another number to give remainder zero. Like, in $19.5 \div 5$, the number 195 when divided by 5, leaves remainder zero.

However, there are situations in which the number may not be completely divisible by another number, i.e., we may not get remainder zero. For example, $195 \div 7$. We deal with such situations in later classes.

EXAMPLE 5 Find the average of 4.2, 3.8 and 7.6.

SOLUTION The average of 4.2, 3.8 and 7.6 is $\frac{4.2+3.8+7.6}{3} = \qquad = 5.2$.

2.4.3 Division of a Decimal Number by another Decimal Number

Let us find $\frac{25.5}{0.5}$ i.e., $25.5 \div 0.5$.

We have $25.5 \div 0.5 = \frac{255}{10} \div \frac{5}{10} = \frac{255}{10} \times \frac{10}{5} = 51$. Thus, $25.5 \div 0.5 = 51$

What do you observe? For $\frac{25.5}{0.5}$, we find that there is one digit to the right of the decimal in 0.5. This could be converted to whole number by dividing by 10. Accordingly 25.5 was also converted to a fraction by dividing by 10.

Or, we say the decimal point was shifted by one place to the right in 0.5 to make it 5. So, there was a shift of one decimal point to the right in 25.5 also to make it 255.

Thus, $22.5 \div 1.5 = \frac{22.5}{1.5} = \frac{225}{15} = 15$

Find $\frac{20.3}{0.7}$ and $\frac{15.2}{0.8}$ in a similar way.

TRY THESE

Find: (i) $\frac{7.75}{0.25}$ (ii) $\frac{42.8}{0.02}$ (iii) $\frac{5.6}{1.4}$

Let us now find $20.55 \div 1.5$.

We can write it is as $205.5 \div 15$, as discussed above. We get 13.7. Find $\frac{3.96}{0.4}$, $\frac{2.31}{0.3}$.

Consider now, $\frac{33.725}{0.25}$. We can write it as $\frac{3372.5}{25}$ (How?) and we get the quotient as 134.9. How will you find $\frac{27}{0.03}$? We know that 27 can be written as 27.00.

So, $\frac{27}{0.03} = \frac{27.00}{0.03} = \frac{2700}{3} = 900$

Example 6 Each side of a regular polygon is 2.5 cm in length. The perimeter of the polygon is 12.5cm. How many sides does the polygon have?

Solution The perimeter of a regular polygon is the sum of the lengths of all its equal sides = 12.5 cm.

Length of each side = 2.5 cm. Thus, the number of sides = $\frac{12.5}{2.5} = \frac{125}{25} = 5$

The polygon has 5 sides.

Example 7 A car covers a distance of 89.1 km in 2.2 hours. What is the average distance covered by it in 1 hour?

Solution Distance covered by the car = 89.1 km.

Time required to cover this distance = 2.2 hours.

So distance covered by it in 1 hour $= \frac{89.1}{2.2} = \frac{891}{22} = 40.5$ km.

Exercise 2.5

1. Find:

(i) 0.4 ÷ 2 (ii) 0.35 ÷ 5 (iii) 2.48 ÷ 4 (iv) 65.4 ÷ 6

(v) 651.2 ÷ 4 (vi) 14.49 ÷ 7 (vii) 3.96 ÷ 4 (viii) 0.80 ÷ 5

2. Find:

(i) 4.8 ÷ 10 (ii) 52.5 ÷ 10 (iii) 0.7 ÷ 10 (iv) 33.1 ÷ 10

(v) 272.23 ÷ 10 (vi) 0.56 ÷ 10 (vii) 3.97 ÷10

3. Find:

(i) 2.7 ÷ 100 (ii) 0.3 ÷ 100 (iii) 0.78 ÷ 100

(iv) 432.6 ÷ 100 (v) 23.6 ÷100 (vi) 98.53 ÷ 100

4. Find:

(i) 7.9 ÷ 1000 (ii) 26.3 ÷ 1000 (iii) 38.53 ÷ 1000

(iv) 128.9 ÷ 1000 (v) 0.5 ÷ 1000

5. Find:

(i) $7 \div 3.5$ (ii) $36 \div 0.2$ (iii) $3.25 \div 0.5$ (iv) $30.94 \div 0.7$

(v) $0.5 \div 0.25$ (vi) $7.75 \div 0.25$ (vii) $76.5 \div 0.15$ (viii) $37.8 \div 1.4$

(ix) $2.73 \div 1.3$

6. A vehicle covers a distance of 43.2 km in 2.4 litres of petrol. How much distance will it cover in one litre of petrol?

What have We Discussed?

1. We have learnt how to multiply fractions. Two fractions are multiplied by multiplying their numerators and denominators seperately and writing the product as $\frac{\text{product of numerators}}{\text{product of denominators}}$. For example, $\frac{2}{3} \times \frac{5}{7} = \frac{2 \times 5}{3 \times 7} = \frac{10}{21}$.

2. A fraction acts as an operator 'of'. For example, $\frac{1}{2}$ of 2 is $\frac{1}{2} \times 2 = 1$.

3. (a) The product of two proper fractions is less than each of the fractions that are multiplied.

(b) The product of a proper and an improper fraction is less than the improper fraction and greater than the proper fraction.

(c) The product of two imporper fractions is greater than the two fractions.

4. A reciprocal of a fraction is obtained by inverting it upside down.

5. We have seen how to divide two fractions.

(a) While dividing a whole number by a fraction, we multiply the whole number with the reciprocal of that fraction.

For example, $2 \div \frac{3}{5} = 2 \times \frac{5}{3} = \frac{10}{3}$

(b) While dividing a fraction by a whole number we multiply the fraction by the reciprocal of the whole number.

For example, $\frac{2}{3} \div 7 = \frac{2}{3} \times \frac{1}{7} = \frac{2}{21}$

(c) While dividing one fraction by another fraction, we multuiply the first fraction by the reciprocal of the other. So, $\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \times \frac{7}{5} = \frac{14}{15}$.

6. We also learnt how to multiply two decimal numbers. While multiplying two decimal numbers, first multiply them as whole numbers. Count the number of digits to the right of the decimal point in both the decimal numbers. Add the number of digits counted. Put the decimal point in the product by counting the digits from its rightmost place. The count should be the sum obtained earlier.

For example, $0.5 \times 0.7 = 0.35$

7. To multiply a decimal number by 10, 100 or 1000, we move the decimal point in the number to the right by as many places as there are zeros over 1.

Thus $0.53 \times 10 = 5.3$, $0.53 \times 100 = 53$, $0.53 \times 1000 = 530$

8. We have seen how to divide decimal numbers.

(a) To divide a decimal number by a whole number, we first divide them as whole numbers. Then place the decimal point in the quotient as in the decimal number.

For example, $8.4 \div 4 = 2.1$

Note that here we consider only those divisions in which the remainder is zero.

(b) To divide a decimal number by 10, 100 or 1000, shift the digits in the decimal number to the left by as many places as there are zeros over 1, to get the quotient.

So, $23.9 \div 10 = 2.39$,$23.9 \div 100 = 0.239$, $23.9 \div 1000 = 0.0239$

(c) While dividing two decimal numbers, first shift the decimal point to the right by equal number of places in both, to convert the divisor to a whole number. Then divide. Thus, $2.4 \div 0.2 = 24 \div 2 = 12$.