Specific Charge of the Electron

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1 Introduction

The purpose of this laboratory is to measure the specific charge of the electron, that is, the ratio of its charge to mass e/m. This is done by producing a beam of electrons with known energy (by measuring their acceleration potential) and directing the beam perpendicularly to a static uniform magnetic field so that the electrons move in a circular orbit.

Reference:

- Leybold Physics Leaflets P6.1.3.1: Determination of the Specific Charge of the Electron. Attached.

The theory is simple. Let E be the energy of electrons moving in a circle of radius r perpendicularly to a magnetic field B. We set the centripetal acceleration equal to the Lorentz force divided by the mass

$$\frac{evB}{m} = \frac{v^2}{r}$$

so

$$\frac{e^2B^2r^2}{2m} = \frac{1}{2}mv^2$$

Setting the kinetic energy equal to eV, where V is the electric potential through which the electrons were accelerated, gives

$$\frac{e}{m} = \frac{2V}{B^2r^2}$$

Our magnetic field is produced by a pair of Helmholtz coils driven by a constant-current source, and the beam of electrons moves inside an expensive and delicate fine beam tube that has a small partial pressure of hydrogen. It is quite difficult to measure V, B, and r accurately simultaneously with our apparatus. In particular, it is difficult to measure the radius of the electron's path once it becomes comparable to the size of the tube because of refraction effects produced by the approximately spherical shape of the tube. For this reason, we use the current in the Holtzholtz coils as a parameter, varying the acceleration voltage while keeping the radius of the electrons' path constant and small enough that refraction by the glass is negligible.

Thus, we must perform two sets of measurements. In one set we assume that the magnetic field increases linearly with current

$$B = k_1 I$$

and perform a series of measurements of B as a function of I in order to obtain k_1. If we plot our measured values of B as a function of I, we get the slope k_1 and its uncertainty with a least-squares fit.

In the other, we perform a series of measurements varying V and I while keeping r fixed. Because the fractional uncertainty in I is probably larger than the fractional uncertainty in V, it is best to plot I^2 as a function of V. Solving the above gives

$$I^2 = \frac{2}{k^2 r^2} \frac{m}{e} V \equiv k_2 V$$

If we plot our measured values of I^2 as a function of V, the slope is just k_2.

Putting this together gives

$$\frac{e}{m} = \frac{2}{k_1^2 k_2 r^2} \tag{1}$$

and

$$\Delta \frac{e}{m} = \frac{e}{m} \left(\left(2\frac{\Delta k_1}{k_1} \right)^2 + \left(\frac{\Delta k_2}{k_2} \right)^2 + \left(2\frac{\Delta r}{r} \right)^2 \right)^{\frac{1}{2}} \tag{2}$$

2 Measurements

Follow these instructions carefully!

First read the Leybold document. Set up the experiment as shown in Figures 2 and 3 of that manual. We do not have the beam tube clamped in place with brackets as described in that document. This is unnecessary and inconvenient, but be careful with the apparatus.

Make sure that the tube is centered in its cradle. Make sure that the mirror assembly is aligned horizontally and centered vertically with the Helmholtz coil that is furthest from you. Attache the measurment device (plastic with sliders) to the Helmholtz coil nearest you. The function of these two devices is to allow you to set the beam diameter without being fooled by parallax.

Ensure that all the knobs on the Leybold power supply are fully counter clockwise, and turn its power switch on. This immediately starts curent to the filament in the electron source. Turn up the acceleration voltage to approximately 300 Volts. This is the rightmost knob, which has been hand-labeled U. In a few minutes you should be able to see a strong beam of electrons emitted from the electron gun and striking the top of the glass tube. (You will need the lights out, or very low, for this.)

Now turn on the current to the Helmholtz coils. In this experiment you will vary the current between one and two amps. It is not safe to leave the current higher than 2 amps for very long. If you need to go over 2 amps, please try to work quickly and keep the current this high for no more than a few minutes. For now, Set the current to approximately 1.9 amps.

You should now observe circular motion of the electron beam. If the beam is curving the wrong way, then the circuit has been assembled backwards and you need to go back to the beginning, referring again to Figures 2 and 3 in the manual.

Now look at the path of the electron beam from above. If it is helical instead of circular, rotate the beam tube slightly about its long axis. Try to avoid touching it except by the glass at the end opposite its plug, and don't slide it back and forth.

The other knob on the Leybold power supply controls a small voltage to the Weynelt cylinder in the beam source, and should allow you to focus the beam to a narrow and well-defined beam. You want the best edge definition on the side of the circular path opposite the electron source.

As described in the manual move the left slide of the measuring device so that its inner edge, mirror image, and escape aperture of the electron beam come to lay on one line of sight. Set the right slide so that its inside edge is separated by 8 cm from the other one. It is not critical to make this exactly 8.0 cm, but it should be close to that. It is critical to measure the distance as accurately as possible and to understand how accurate it is.

Adjust the current driving the Helmholtz coils until the circular orbit of the electron beam aligns with the slider on the right side of the measurement device and its mirror image; ensuring that the electrons are moving in a circle of radius 4 cm.

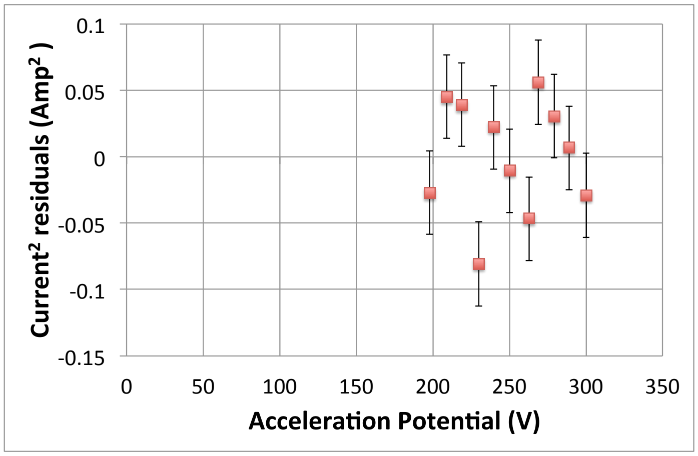
Record the acceleration potential and the current, and estimate your uncertainty in each. Decrease the acceleration potential in approximately 10 V steps, and repeat the above measurements. It should be easy to get down to 200 V. You expect the data to be something like this:

Potential (V)	Current (Amp)
198(1)	1.58(1)
209(1)	1.65(1)
219(1)	1.69(1)
230(1)	1.70(1)
240(1)	1.77(1)
250(1)	1.80(1)
263(1)	1.84(1)
269(1)	1.89(1)
279(1)	1.92(1)
289(1)	1.95(1)
300(1)	1.98(1)

Perform a least-squares fit to potential V as a function of current squared, being careful with error propagation. That is, make sure you correctly calculate the uncertainty in I^2

$$\Delta(I^2) = 2I\Delta I$$

For the data above, I get a slope $k_2 = 0.01398(34)$ Amps^2/V with an intercept of $-0.245(83)$ Volts. This intercept is significant but difficult to interpret. It corresponds to an x-axis intercept of 17 Volts.



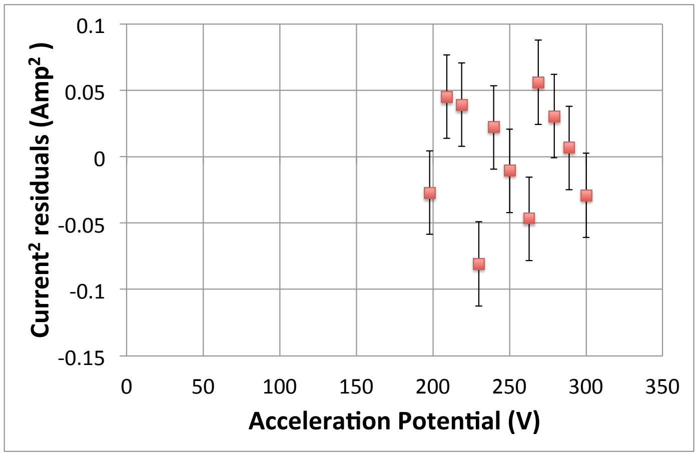
Now reduce the settings on the Leybold power supply to zero and turn it off. Reduce the current to the Helmholtz coils to zero.

Remove the measurement slide from the front Helmholtz coil. Very carefully disconnect the socket from the fine beam tube, and slide the tube out of the device through the now open front Helmholtz coil. Place the tube in the padded box in the cabinet at the back of the room.

You now want to measure the magnetic field in the center of the Helmholtz coils as a function of current. I **strongly** recommend using the magnetometer in a smart phone for this, since they are accurate and precise. If you insist, you can use the Bell gaussmeter that is on the table. The point is to measure the magnetic field at several points in the range of current that you used in the above experiment. You expect the data to be something like this:

Current (Amp)	Magnetic Field (μT)
1.27(1)	930(4)
1.51(1)	1097(4)
1.60(1)	1163(4)
1.70(1)	1230(4)
1.81(1)	1303(4)
1.90(1)	1370(4)
2.02(1)	1463(4)

Perform a least-squares fit to magnetic field B as a function of current I. For the data above, I get a slope $k_1 = 7.048(65) \times 10^2$ μT/A. The intercept is just 33 μT, which is quite comparable to the component of the Earth's magnetic field in the plane of cyclotron motion. The fit is good, and the residuals look like this:



3 Analysis

We are now in a position to extract a value for e/m and its uncertainty. If we take a value of $r = .040(1)$ m then Eqs 1 and 2 give:

$$\frac{e}{m} = 1.80(11) \times 10^{11}\text{As/kg}$$

The accepted value is 1.75882×10^{11} As/kg. Our measurement is 2.2% off, but is well within our experimental uncertainty of the accepted value.

Which measurements provide the majority of our uncertainty? What could be done to increase our precision, and what sort of systematic effects should we think about?

Both of the fits were two-parameter fits, namely a slope and an intercept. Why not use a one-parameter fit? How might the data tell you if that is a good idea or not?

As always, these are generic questions related to the sample data presented here. Your data will be different, and so may be the questions you should ask and things you consider.