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ASTR 401 Homework 1

1) 

$$U = \int_0^M -\frac{GM}{r}\,dm \qquad M = \frac{4}{3}\pi r^3 \rho$$

$$dm = 4\pi r^2 \rho$$

a) $$U = -\int_0^R \frac{G\left(\frac{4}{3}\pi r^3 \rho_0\right)\left(4\pi r^2 \rho_0 dr\right)}{r}$$

$$U = -\frac{16}{3}\pi^2 \rho_0^2 G \int_0^R r^4 dr \Rightarrow \boxed{-\frac{16}{15}\pi^2 \rho_0^2 G R^5}$$

b) $$\rho(r) = \rho_0 c^2\left(1 - \frac{r}{R}\right)$$

$$U = -\int_0^R \frac{G\left(\frac{4}{3}\pi r^3 \rho\right)\left(4\pi r^2 \rho dr\right)}{r} \Rightarrow -\frac{16}{3}\pi c^2 G \int_0^R \rho^2 r^4 dr$$

$$\Rightarrow -\rho_0^2 c^2 \frac{16}{3}\pi^2 G \int_0^R r^4\left(1 - \frac{2r}{R} + \frac{r^2}{R^2}\right) dr \rightarrow \int_0^R r^4 - \frac{2r^5}{R} + \frac{r^6}{R^2}\,dr$$

$$\boxed{U = -\frac{16}{3}\pi^2 \rho_0^2 c^2 G R^5\left(\frac{1}{5} - \frac{1}{3} + \frac{1}{7}\right)}$$

c) $$\frac{dP}{dr} = -\frac{GM\rho_0}{r^2}, \quad \text{for (a):}$$

$$dP = -\frac{4G}{3}\pi \rho_0^2 \frac{r^3}{r^2} dr$$

$$P = -\frac{4}{3}\pi G \rho_0^2\left(\frac{r^2}{2}\right) \rightarrow \boxed{P = -\frac{2}{3}\pi G \rho_0^2 r^2}$$

for (b)

$$dP = -\frac{4}{3} G\pi \rho_0^2 c^2 \frac{r^3}{r^2}\left(1 - \frac{2r}{R} + \frac{r^2}{R^2}\right) dr$$

$$\boxed{P = -\frac{4}{3} G\pi \rho_0^2 c^2\left(\frac{r^2}{2} - \frac{2r^3}{3R} + \frac{r^4}{4R^2}\right)}$$

2| Recall from 1a) $U = -\frac{16}{15}\pi^2\rho^2 G R^5$

a) $\rho = \frac{3M}{4\pi r^3} \quad \rho^2 = \frac{9M^2}{16\pi^2 r^6}$

$U = -\frac{16}{15}\pi^2 G R^5 \frac{9M^2}{16\pi^2 R^6} \Rightarrow U = -\frac{3}{5}\frac{M^2 G}{R}$

$\vec{E} = \frac{1}{2}U \rightarrow \vec{E} = -\frac{3}{10}\frac{GM^2}{R}$

$G = 6.67\times10^{-11}\ \frac{N m^3}{kg^2}$

$M_\odot = 1.989\times10^{30} kg$

$R_\odot = 6.955\times10^{8} m$

$\vec{E} = -\frac{3}{10}\frac{GM_\odot^2}{R_\odot} = -1.139\times10^{41} J$

Total Energy radiated: 1.39×10^{41} J

b) Time taken to reach the main sequence:

$L = \frac{E}{t} \rightarrow t = \frac{E_{radiated}}{L}$

$L_\odot = 3.839\times10^{26} W$

$t = \frac{1.39\times10^{41}}{3.839\times10^{26}} = 3.62\times10^{14} s \left|\frac{1 yr}{3.154\times10^{7} s}\right|$

$t = 1.147\times10^{7} yrs$

3]

a) $m - M_B = 5\log(d) - 5$ $\quad M_B = 4.72$

$m = 6$

$L = 4 \times 10^{26}\ W$

$d = 10^{\left(\frac{m - M_B}{5} + 1\right)}$

$\boxed{d = 18.03\ pc}$

b) photons/sec. in eye = $\frac{\text{Area of Eye}}{\text{SA of sphere where } r = d}$

$n = \frac{\frac{\pi}{4} D_{eye}^2}{(4\pi d^2)}$ $\quad D_{eye}$ = diameter of eye $= .0242\ m$

$1\ pc = 3.086 \times 10^{16}\ m$

$\boxed{n = 6.574 \times 10^{-23}}$

4] $\frac{1}{\mu_I} = \sum_i \frac{X_i}{A_i}$ $\quad X_H = .71$ $\quad m_H = 1$

$X_{He} = .27$ $\quad m_{He} = 4$

$X_{other} = .02$ $\quad m_{other} \approx 20$

$\frac{1}{\mu_I} = \frac{.71}{1} + \frac{.27}{4} + \frac{.02}{20}$

$\frac{1}{\mu_I} = .7785 \rightarrow \boxed{\mu_I = 1.2845}$

Now for All Hydrogen ($X_H = 1$):

$\frac{1}{\mu_I} = \frac{1}{1} \Rightarrow \boxed{\mu_I = 1}$

Now for All Helium ($X_{He} = 1$)

$\frac{1}{\mu_I} = \frac{1}{4} \Rightarrow \boxed{\mu_I = 4}$

Now for All Heavy Elements ($X_{other} = 1$)

$\frac{1}{\mu_I} = \frac{1}{20}$ $\quad \boxed{\mu_I = 20}$