Hydrogen Spectrum and the Rydberg Constant Lab

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I. Introduction

This lab will be an introduction to conducting spectroscopic measurements. We will use a spectrometer to measure several lines in the atomic spectrum of hydrogen. We will use our data to obtain a value for the Rydberg constant (R_∞). The Rydberg constant is a measure used to express the limiting value of the highest wave number of any photon that can be emitted from an atom, or the wave number of the lowest-energy photon capable of ionizing an atom from its ground state. It is also the most accurately known physical quantity.

The theory behind the Rydberg constant begins in quantum mechanics, where we solve the Schrödinger equation for hydrogen, ignoring electron and nuclear spin and relativistic effects to obtain the formula:

$$E_n = -\frac{\mu e^4}{8\epsilon_0^2 h^2 n^2} \tag{1}$$

Where n = 1, 2, 3,... is called the "principal quantum number", h is Planck's constant, e is the electron charge, ϵ_0 is the permitivity of free space, and μ is the reduced mass of the electron given by the following equation:

$$\mu = m\frac{1}{1+m/M} \tag{2}$$

Where m is the mass of the electron and M is the mass of the nucleus. The energy formula is commonly re-written as:

$$E_n = -hcR_M\frac{1}{n} \quad \text{where} \quad R_M = R_\infty\frac{1}{1+m/M} \tag{3}$$

Where R_∞ is the *Rydberg Constant*, or otherwise known as the "infinite mass" Rydberg constant. It is found with the equation:

$$R_\infty = \frac{me^4}{8\epsilon_0^2 h^3 c} \tag{4}$$

Currently, the accepted best measurement of R_∞ is:

$$R_\infty = 109737.31568508(65)\ \text{cm}^{-1} \tag{5}$$

To see how the Rydberg constant characterizes the spectrum of hydrogen, we must can observe the radiative transition of a hydrogen atom from the principal quantum number n_1 to n_2. The energy of the emitted photon will be given by:

$$\Delta E_{n_1 n_2} = hcR_M\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right) \tag{6}$$

Where the frequency is then:

$$\frac{1}{\lambda_{n_1 n_2}} = R_M\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right) \tag{7}$$

Series of wavelengths are separated by their ending transition state (n_2). For example, the "Lyman series" is defined to have its transition end in the ground state, ($n_2 = 1$) and the "Balmer series" is defined to have its transitions end in the $n_2 = 2$ state. Each series can have its individual wavelengths defined by alpha, beta, and so on based on their first longest wavelength, in descending fashion, (i.e. Balmer-α is the first longest wavelength, following by Balmer-β, its second longest wavelength).

Our goal is to record enough spectral lines of hydrogen to successfully identify which ones they are and use the measured wavelengths to obtain an experimental value for the Rydberg constant.

II. Layout and equipment

A simple apparatus for our experiment is shown below:

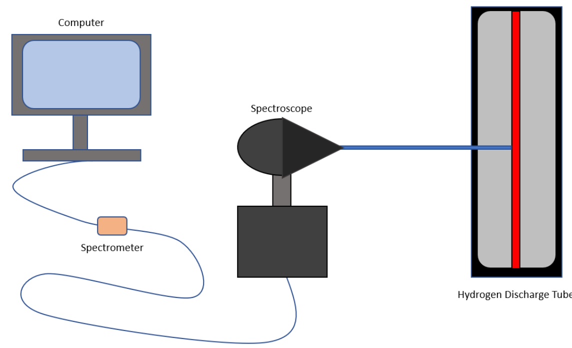

Fig. 1: Our Experimental Setup for our Lab.

Where the object on the far right is our hydrogen discharge tube, it will disassociate hydrogen molecules into atomic hydrogen and excite the resulting hydrogen atoms to high lying states so we can observe the emission spectrum.

We will observe the emission spectrum using our spectroscope, otherwise known as an optical fiber. One end of our optical fiber will connect to our spectrometer, while the other end will be used to collect light. We will mount the data collection end with a stand on the table, pointed at our hydrogen lamp.

Once we have our data, it will then be translated using our spectrometer, Ocean Optics model USB-4000. The spectral range for our spectrometer is rated to be 350 nm to 1050 nm, this will limit what hydrogen series we will be able to detect. The spectrometer will then translate our data by grating onto a CCD array, this array will then be sent to our computer via a USB connection, powered by a microprocessor.

We will then be able to see our displayed data by using our software of "Ocean View" to more clearly analyse our data.

III. Procedure

Once we have our experiment set up as shown in figure 1 with all of our equipment on, we should begin to see a spectrum of light displayed on the computer screen by Ocean-View. An example of this raw data can be seen in figure 2 below.

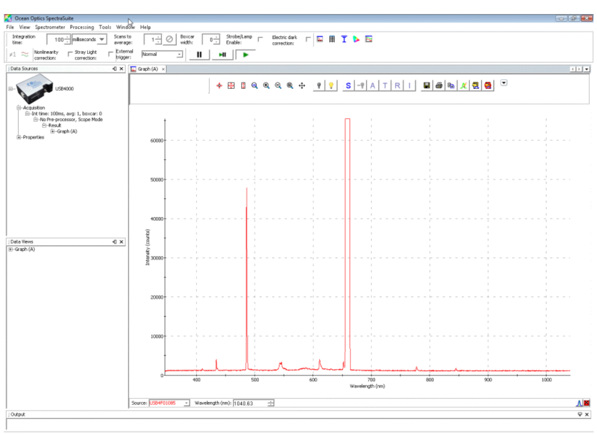

Fig. 2: A Screen shot of Ocean-View running.

We must notice that some of the spectral lines are very small, we can adjust our optical fiber along with the scale to achieve a more focused in result of these minor lines. When observing very weak lines, we will want the fiber to be as close to the hydrogen discharge tube as possible.

We can also adjust the integration time and number of averages to further examine our data, a method we could enable is the clipping of our strong spectral line to better observe our weaker lines. We could also adjust the background subtraction of our data to further our weak line observations.

With the methods mentioned above, we should be able to successfully identify the spectral lines of hydrogen, and use this to calculate our Rydberg constant. Below is a widely excepted table of the Balmer series transition lines that we can use to identify our measurements.

Name	Transitions of n	Wavelength (nm, air)
Ba-α	$3 \longrightarrow 2$	656.279
Ba-β	$4 \longrightarrow 2$	486.135
Ba-γ	$5 \longrightarrow 2$	434.0472
Ba-δ	$6 \longrightarrow 2$	410.1734
Ba-ε	$7 \longrightarrow 2$	397.0075
Ba-ζ	$8 \longrightarrow 2$	388.9064
Ba-η	$9 \longrightarrow 2$	383.5397

IV. Data/Analysis

When observing our data, we were able to cross reference their measurements to the table above. Using this, we were able to observe transitions from $n = 9$ to $n = 3$ of the Balmer series. We have listed each of our measurements below for the Balmer series starting with the $n = 3$ transition.

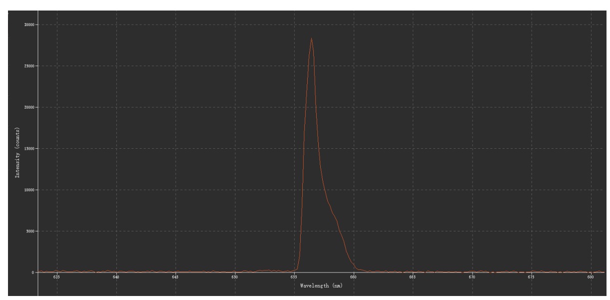

Fig. 3: Balmer transition line of $n = 3$ (Ba-α), measured at 656.43 nm.

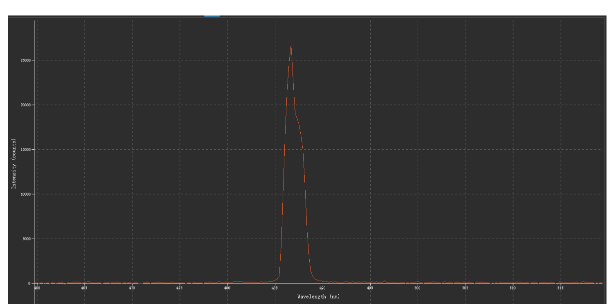

Fig. 4: Balmer transition line of $n = 4$ (Ba-β), measured at 486.47 nm.

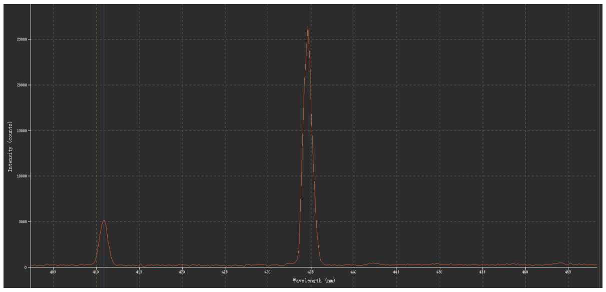

Fig. 5: Balmer transition line of $n = 5$ (Ba-γ), measured at 436.35 nm.

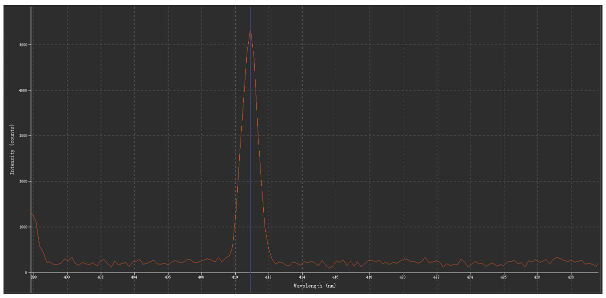

Fig. 6: Balmer transition line of $n = 6$ (Ba-δ), measured at 410.916 nm.

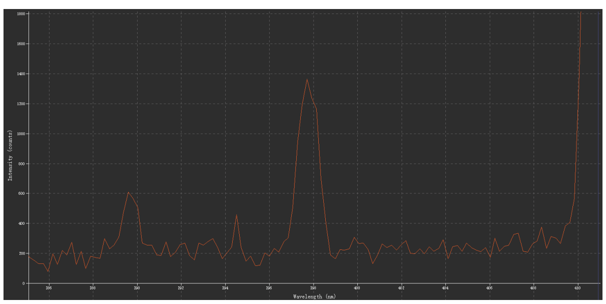

Fig. 7: Balmer transition line centered on $n = 7$ (Ba-ε), measured at 397. nm.

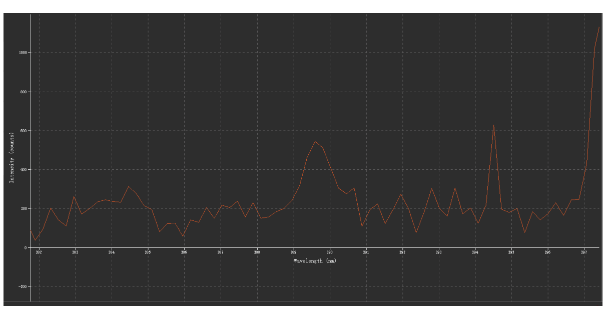

Fig. 8: Balmer transition line centered on $n = 8$ (Ba-ζ), measured at 389.571 nm.

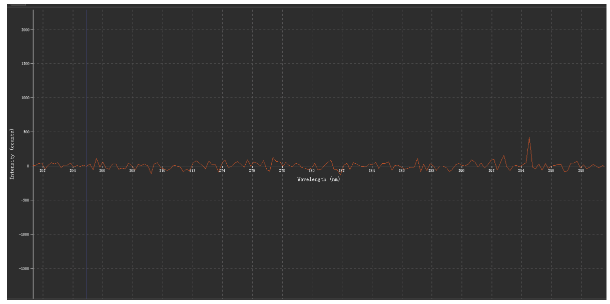

Fig. 9: Lack of a Balmer transition line $n = 9$ (Ba-η).

We have compiled all of our measurements of the Balmer series in the table below.

Name	Transitions of n	Wavelength (nm, air)
Ba-α	$3 \longrightarrow 2$	656.43
Ba-β	$4 \longrightarrow 2$	486.47
Ba-γ	$5 \longrightarrow 2$	436.35
Ba-δ	$6 \longrightarrow 2$	410.916
Ba-ε	$7 \longrightarrow 2$	397.
Ba-ζ	$8 \longrightarrow 2$	386.571
Ba-η	$9 \longrightarrow 2$	NA

In attempts to find the Rydberg Constant, we used equation (7) to find R_M by plotting the inverse frequency of our measured Balmer series transitions against $\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$. The resulting graph would produce a slope that is equivalent to our R_M in inverse nm. Our graph is shown below.

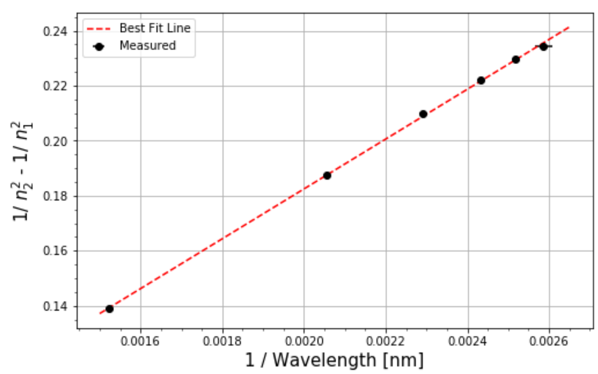

Fig. 10: Our Inverse Wavelength vs. $\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$.

We then used equation (3) along with the slope we found to calculate our Rydberg Constant (R_∞). We found our Rydberg Constant to be:

$$R_\infty = 100752.739666303(23)\ \mathrm{cm}^{-1} \tag{8}$$

Which is only 8.187% off of the known value.

Other interesting observations we were able to take that did not fall within the Balmer series can be seen below.

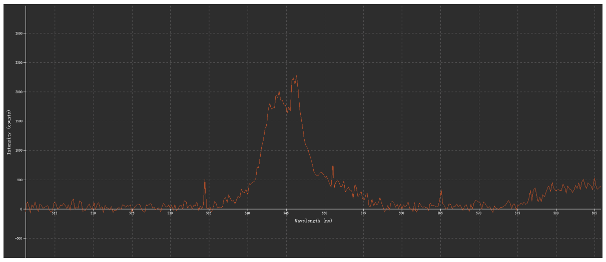

Fig. 11: A peak at 545 nm.

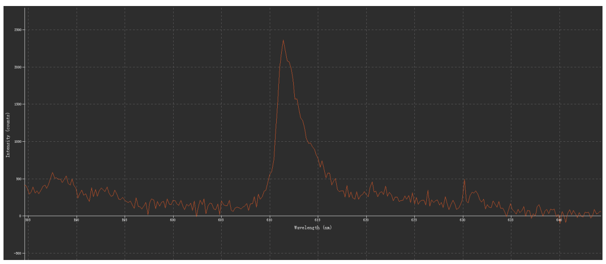

Fig. 12: A peak at 613 nm.

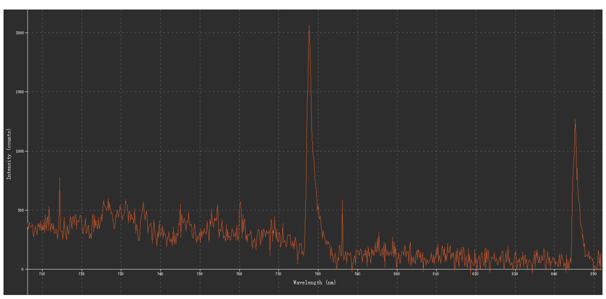

Fig. 13: A peak at 778 nm.

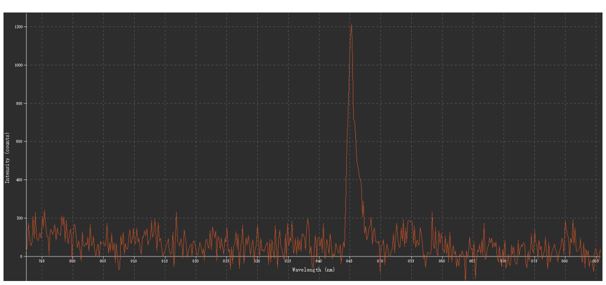

Fig. 14: A peak at 846 nm.

As the Paschen series break is found at 820.4 nm in air, we believe the peak displayed in figure 14 could belong to the Paschen Series family ($n_2 = 3$). Unfortunately, we were unable to determine what transition of n it belonged to.

For the peaks that were displayed in figures 11 - 13, we believe these to be outside interference from another light source somewhere else in the lab. We came to this deduction as Figure 13 is outside the known transition wavelength ranges of both the Balmer and Paschen series. Furthermore, while figures 11 and 12 fit within the transition wavelength range for the Balmer series, they do not fall under any known transition wavelength for this series.

V. Conclusion

This lab found us able to identify the Balmer transition lines in the Hydrogen Spectrum, and allowed us to use their relationship of wavelength and transition of n to find the Rydberg Constant, R_∞.

We found that we were unable to find the Balmer transition line of Ba-η ($n = 9$), we believe this to result from the sensitivity range of our Ocean Optics spectrometer along with the other outside interference we experienced within the lab.

The Rydberg constant we found was only 8.187 % off of the known, accepted value. We believe we could have received a more accurate result if we were able to measure the Ba-η line while also taking multiple measurements of the Balmer transition series. We would then take the average of each transition of n to be our final “Measured" value, which we would then plot against $\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$, much like we did in Figure 10.