

Hydrogen Spectrum/Rydberg Constant

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NOTE: These instructions are good, but incomplete. Feel free to get started, but check with the instructor before running the experiment.

1 Introduction

The purpose of this laboratory is to introduce the student to spectroscopic measurements. We will use a commercial spectrometer to measure several lines in the spectrum of atomic hydrogen, and from these data obtain a value for the Rydberg constant. The Rydberg constant is a measure of the binding energy of the hydrogen atom, and is the most accurately known physical quantity.

References:

- Melissinos and Napolitano, p. 20 forward (sections 1.4 and 1.5).

In elementary quantum mechanics, we solve the Schrödinger equation for hydrogen, ignoring electron and nuclear spin and relativistic effects, and obtain the formula

$$E_n = -\frac{\mu e^4}{8\epsilon_0^2 h^2 n^2} \quad (1)$$

where $n = 1, 2, 3, \dots$ is called the “principal quantum number,” h is Planck’s constant, e is the electron charge, ϵ_0 is the permittivity of free space, and μ is the reduced mass of the electron:

$$\mu = m \frac{1}{1 + m/M} \quad (2)$$

where m is the mass of the electron and M is the mass of the nucleus. It is common to re-write the energy formula as

$$E_n = -hcR_M \frac{1}{n^2} \quad \text{where} \quad R_M = R_\infty \frac{1}{1 + m/M} \quad (3)$$

and R_∞ is called the *Rydberg constant* or sometimes the “infinite mass” Rydberg constant:

$$R_\infty = \frac{me^4}{8\epsilon_0^2 h^3 c} \quad (4)$$

You should show that R_∞ has units of inverse length. The currently accepted best value is:

$$R_\infty = 109737.31568508(65) \text{ cm}^{-1} \quad (5)$$

(<http://physics.nist.gov/cgi-bin/cuu/Value?ryd>).

It is easy to see how the Rydberg constant characterizes the spectrum of hydrogen. Suppose a hydrogen atom undergoes a radiative transition from principal quantum number n_1 to n_2 . Then the energy of the emitted photon will be

$$\Delta E_{n_1 n_2} = hcR_M \left(\frac{1}{n_2^2} - \frac{1}{n_1^2} \right) \quad (6)$$

and so

$$\frac{1}{\lambda_{n_1 n_2}} = R_M \left(\frac{1}{n_2^2} - \frac{1}{n_1^2} \right) \quad (7)$$

2 Hydrogen Spectral Series

If we consider transitions that end in the ground state ($n_2 = 1$ in Eq. (7)), then we find a series of wavelengths,

$$\lambda_{2 \rightarrow 1} = 121.6 \text{ nm}, \quad \lambda_{3 \rightarrow 1} = 102.5 \text{ nm}, \quad \lambda_{4 \rightarrow 1} = 97.2 \text{ nm}, \quad \dots \quad (8)$$

This is called the “Lyman” series. The first (longest wavelength) transition is called “Lyman- α ,” then “Lyman- β ,” etc.

Looking at transitions from higher levels to $n = 2$ we find the “Balmer series”

$$\lambda_{3 \rightarrow 2} = 656.3 \text{ nm}, \quad \lambda_{4 \rightarrow 2} = 486.1 \text{ nm}, \quad \lambda_{5 \rightarrow 2} = 434.1 \text{ nm}, \quad \dots \quad (9)$$

and again, these are labeled as “Balmer- α ,” “Balmer- β ,” and so on.

The next series (terminating at $n = 3$ is called “Paschen” and even higher series have names, but the names are seldom used. You can see a great picture of this in Fig. 1.13 in your book. You should calculate some of the Paschen series to see if they overlap any of the Balmer series.

The goal of this lab is to record enough spectral lines of hydrogen to do two things: first, to identify which ones they are, and second to use the measured wavelengths to obtain an experimental value for the Rydberg constant.

3 Apparatus

3.1 Lamp

A hydrogen discharge tube is an interesting piece of equipment – It must do two very different things. First, it must *disassociate* hydrogen molecules into atomic hydrogen, and second, it must *excite* the resulting hydrogen atoms to high lying states so we can observe the emission spectrum.

Our hydrogen lamp is old, and we do not currently have a spare. *Be careful* with it. Do not bang it, and make sure we remember to turn it off when we are done with it.

3.2 Spectrometer

Our spectrometer is the centerpiece of this experiment. It is a model USB-4000 produced by Ocean Optics, Inc. (www.oceanoptics.com). This is a modern, sensitive, accurate, and versatile device, and ones like it are very common in research laboratories all over the world. The spectral range of the device is rated to be 350 nm to 1050 nm. This will limit what hydrogen series you can detect. For example, it is clear from the discussion above that none of the Lyman series will be detected, but that the Balmer series is right in the range of the device. What about the Paschen series?

The spectrometer is a sealed, monolithic device, (i.e., a “black box”) but you should understand how it works. Light comes into the box via an optical fiber, and is dispersed by a grating onto a CCD array. The CCD array is controlled by a microprocessor, and the output connected to a computer by a USB connection. The device is powered through the USB connection. The software program we use is provided by Ocean Optics and is called “Spectra Suite.” It is fairly straightforward.

To use the device, first unplug it from the computer. Next check the computer to see if the Spectra-Suite program is running, and if it is, terminate it. Next, connect the spectrometer to the USB port. Wait a few seconds, and then start Spectra-Suite (there is a short-cut on the desktop). If Spectra-Suite does not detect the spectrometer and show it on the side bar, then something is wrong and you should contact the instructor.

Play with the elements of the tool bar, so that you can expand the horizontal and vertical ranges. If you get the program into a state where you can’t get data anymore, just quit the program and re-start it.

3.3 Optical Fiber

One end of the fiber is connected to the spectrometer, and you use the other end to collect light. You can mount the free end with the stand on the table so it points at the hydrogen lamp, but *be careful* with it. Do not clamp down on the fiber coupling or it will break.

Arrange the free end of the fiber so that light from the hydrogen lamp enters the fiber.

4 Measurements and Data collection

Now you should see a spectrum of the light displayed on the computer screen by Spectra-Suite. An example of this is shown in Fig. 1.

Note that in this figure some of the spectral lines are very small, while another is actually clipped. The lamp does not emit enough light to damage the CCD array, but you cannot get an accurate measure of the wavelength if a line is too weak (and therefore noisy) or too strong (and therefore clipped.)

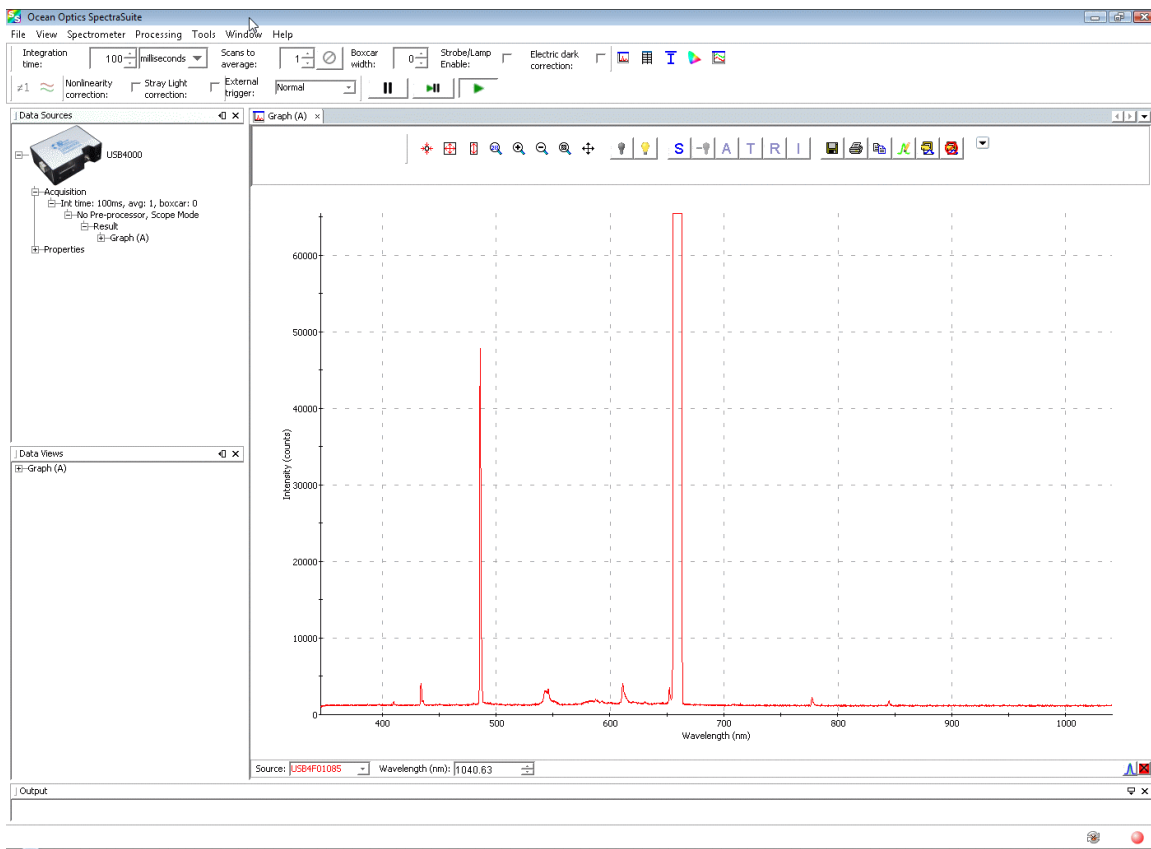


Figure 1: Screen shot of Spectra-suite running.

So, for each spectral line you want to measure, move the fiber so that you get as much light into the spectrometer as possible without clipping the line. When you try to find the very weak lines, you will want the fiber as close to the hydrogen discharge tube as possible. The lab jack that the fiber is taped to is very convenient for this.

You can adjust the integration time and number of averages if you like. If you are looking at a weak line, it is OK if the strong ones are clipped. This is an advantage of CCD detectors. You can control the scale using the various tools on the tool bar. Play with these, or ask the instructor for help. When you have a single spectral line isolated, you can print it, and save this data for your lab notebook. An example might look as shown in Fig. 2.

You can also play with the background subtraction feature of the software. This is especially useful when looking at the very weak lines at low wavelength. I have personally observed the transition from $n = 10$ to $n = 3$ of the Balmer series, and present that data in Fig. 3:

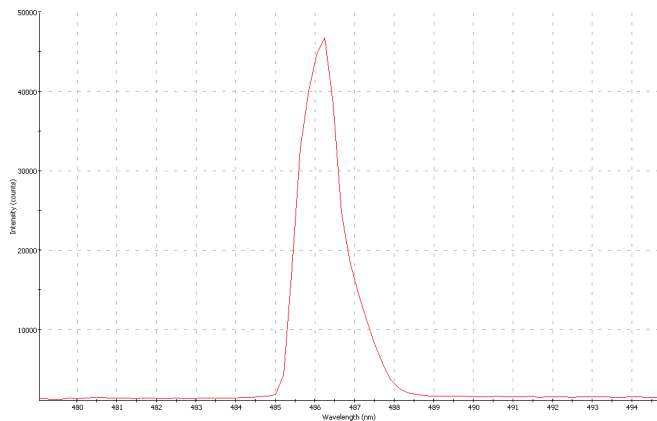


Figure 2: Example of a spectral line that is expanded so that its wavelength can be measured.

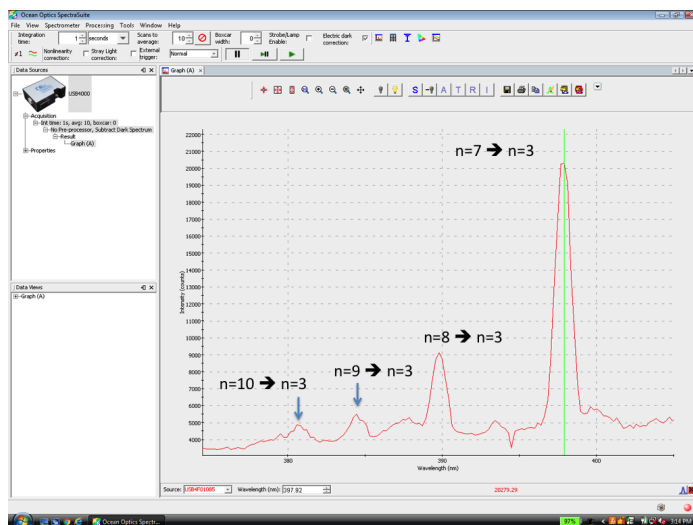


Figure 3: Very weak lines with adequate signal to noise ratio

5 Analysis

First, decide if you need to correct your data. You have measured the wavelength in air, but Eq. (7) is for the wavelength in vacuum. You may use 1.00029 for the index of refraction of the air in the lab. Are your measured wavelengths accurate enough for this correction to matter?

What about the spectrometer? How accurate is it? (Hint, check out the Ocean Optics web site!)

For each value of n_2 (see Eq. (7)) that you think you have, plot the inverse of the wavelength that you measured versus $1/n_2^2 - 1/n_1^2$. According to Eq. (7), this should form a straight line, and the slope should be R_M . If you have several spectral lines, you should get a good least-squares fit to the data. Because there is only one parameter to fit, we should use the least-squares formula for fitting to a line constrained to pass through the origin.

Once you have obtained a value for R_M you need to calculate R_∞ . To do this, you need the mass of the nucleus and the mass of the electron, as described in Eq. (3).

How accurate is your result?