Johnson Noise Lab

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I. Introduction

This lab will study Johnson Noise and measure the Boltzmann constant k. We can relate k to microscopic degrees of freedom to temperature using the ideal gas law, which relates the pressure P, volume V, number of particles N, and temperature T:

$$pV = NkT \tag{1}$$

As electrons in a metal can be thought of as a “gas", their random motion can bring in Boltzmann’s constant. Meaning there is some electrical noise associated with a resistor that cannot be removed except by cooling. This was first observed by Johnson and analyzed by Nyquist in 1928. Below is the Johnson-Nyquist formula.

$$dP = 4kT\,df \tag{2}$$

Where dP is the amount of power in a frequency interval of width dF, k is the Boltzmann’s constant, and T is the temperature. If we consider a resistor, the power is voltage squared divided by resistance, thus the root-mean-square voltage across the resistor is:

$$d\langle V^2 \rangle = 4kTR\,df \tag{3}$$

This means that if we have an amplifier with frequency dependent on a gain $g(f)$, then the voltage we measure will be:

$$\langle V^2 \rangle = 4kTR \int g^2(f)\ df, \quad \text{where,} \quad B \equiv \int g^2(f) \tag{4}$$

We will begin by measuring the integral in equation (4) by measuring $\langle V^2 \rangle$ for a series of different resistors and plotting $\langle V^2 \rangle$ vs. R, the slope of which would be $4kT \times$ integral.

II. Layout and equipment

Below is a simple diagram of our apparatus.

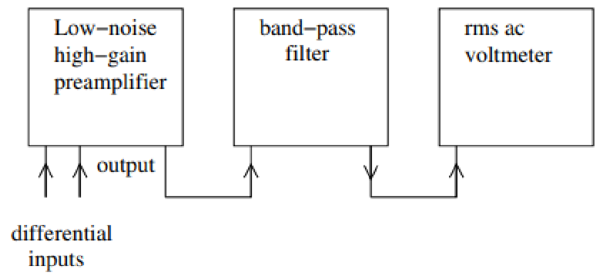

Fig. 1: A simple block diagram of our apparatus, Credit to Dr. George R. Welch.

We will be using the Signal Recovery model 5113 preamplifier, a Fluke 8846A digital multimeter, a Krohn-Hite model 3362 4-pole filter, a Wavetek Model 182A function generator, and two Bud-boxes setups, Box A is setup to receive an output across two resistors, while Box B is setup to measure one resistor.

III. Procedure

A. *Setup*

To begin the experiment, we must connect the output of the preamp to the “CH1 +" filter input, then connect the CH1 output to the “CH 2+" filter input, then connect the CH2 output to the Fluke 8846A multimeter.

Now, to set up Channel 1, we must press the “Mode" button until the “Cutoff Frequency" display reads “H.P." (High pass), we must also press the “Type" button until the display reads “bu" (Butterworth). Finally, ensure that the frequency is set up so that 10 kHz is the cutoff. We will perform a similar procedure to set Channel 2 for a low pass (L.P.) filter with a cutoff at 20 kHz.

Before we can start taking measurements, we must examine Bud-Box A closely. Below is an image of the inner workings of the Box.

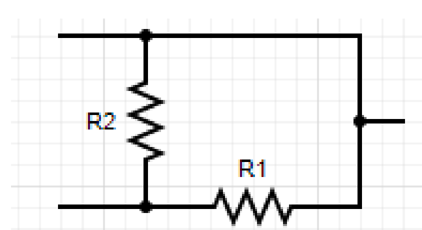

Fig. 2: The circuitry of Bud-Box A.

Where the individual wire on the right is the input from the function generator, and the pair of wires on the left is the output towards the preamplifier. We must measure the two resistors within the box using the fluke multimeter and BNC cables to make sure that the box is designed to divide the input by approximately a factor of 1000. Once it passes this test, connect the Bud-box to the preamplifier.

B. *Calibration*

Now, we will look for the “Calibrate Filter" icon on the computer desktop and start the program. Once opened, we should see a display similar to figure 3.

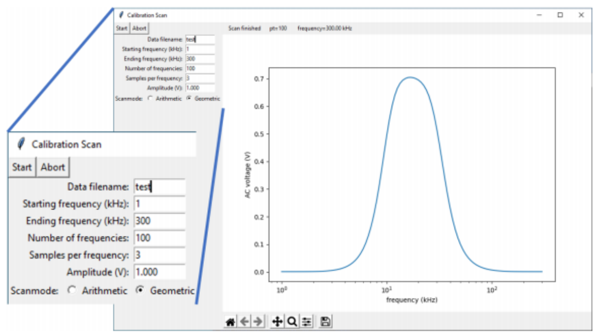

Fig. 3: A python program for calibrating the filter, Credit to Dr. George R. Welch.

This program interfaces with eh Stanford Research Systems DS-345 function generator and the Fluke DMM. It programs the function generator to generate a sine wave with a known amplitude and frequency and then reads the RMS voltage out from the Fluke DMM. It is designed to sweep the frequency form a low starting frequency to a high final frequency. Figure 2 also shows a result of a test sweep of 100 different frequencies from a starting frequency of 100 Hz to a final frequency of 300,000 Hz. At each frequency, the Fluke DMM was read three times, and the results were averaged. The SRS function generator was programmed with a 1 Volt Amplitude sine wave.

This program displays the voltage being measured by the Fluke DMM. In order to convert this to gain $g(f)$, we will need to recall that the gain is the voltage output from the filter divided by the voltage input to the preamp. This meas we will need to run the frequency scan twice. Once with the output of the function generator connected to the voltage divider input to the preamp, and once again with he output of the function generator connected directly to the DMM. The gain is then the ratio of these two.

Now, starting out measurements for $g(f)$, we will be measuring a series of different band-pass settings. We will start by entering “1" for the “Starting frequency" and “200" for the “Ending frequency", these values are measured in kHz, this must be done as the filter has a lower bound set to 10 kHz, with an upper bound of 20 kHz, we will be adjusting the low pass settings a factor of 10 greater than what is programmed on the filter. We will then set the “Number of frequencies" to 100, with the “Samples per frequency" to 3. Ensuring to give our file a name, and double checking that our filter has a high pass set to 10 kHz and a low pass set to 20 kHz, we will begin the scan by pressing “Start".

Ensuring to save the output .csv file, we will perform additional scants but while changing the low-pass filter settings to 25 kHz, 30 kHz, 35 kHz, 40 kHz, 45 kHz, and 50 kHz. With the settings on the “Calibrate Filter" “Ending Frequency" changing as well to 250, 300, 350, 400, 450, and 500 respectively.

Once done performing the original sweep of scans, we must measure the voltage output of the band-pass filter for the seven different low-pass settings. We will do this by connecting the output of the function generator directly to the input of the DMM while repeating the scans, changing the “Ending frequency" each time to the same values used int he previous scans.

C. *Fixed B and vary R*

Once we have determined the integral B for several different band-pass settings, we will pick a particular band-pass and then measure the Johnson noise on a series of resistors. This will allow us to plot $< V^2 >$ vs. R. Where the slope will then be $4kTB$. We will be using the low-pass filter setting of 30 kHz for this section.

We will begin by measuring the resistance of the resistors we will use. We will then connect Bud-box B to the preamplifer. We will then gently clamp one of the resistors with the alligator clips and measure the output of the filter with the Fluke multimeter. We can use the “Analyze" feature of the multimeter to get a precise value of our AC voltage, ensuring to record our standard deviation (our error).

D. *Fixed R and vary B*

Now, using the 10 kΩ resistor in Bud-box B. We will set up the Krohn-Hite filter to the same settings we took when we were calibrating the device (setting the pass band from 10 kHz to 20 kHz). We will then make sure that the output of the filter is connected to the Fluke DMM and measure the AC voltage just like we did in the previous section. We will then repeat this step for each of our low pass settings of 25 kHz, 30 kHz, 35 kHz, 40 kHz, 45 kHz, and 50 kHz.

Finally, as our equation (4) is dependant on temperature, we will make sure to measure the temperature of our lab once we complete our experiment.

IV. Data/Analysis

When measuring Bud-Box A, we found that R_1 = 1.00389(5) MΩ and that R_2 = 0.996632(1) KΩ. These values do indeed result in dividing the input by roughly 1000.

When performing the Calibration step, we surveyed the gain voltage for different low pass settings with our band-pass filter. Below are our results:

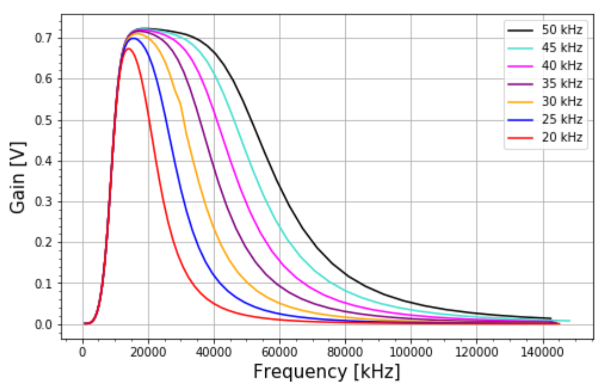

Fig. 4: Our Gain Calibration for our different low pass settings. It is cropped until 1500 kHz for better viewing.

Furthermore, we needed to ensure to include our voltage our function generator used for each of our low pass settings. Below are our results:

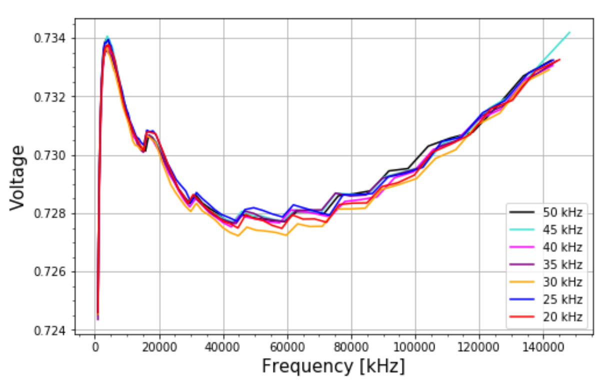

Fig. 5: Our Voltage for our different low pass settings. It is cropped until 1500 kHz for better viewing.

As can be seen by figure 3, as we increase our low pass frequency limit, the gain increases, and then flattens off at around 0.72 Volts, before tapering off towards a gain of zero. Furthermore, we can be confident that these measurements are accurate by observing figure 4. The voltage used for our calibration is roughly the same when used across each of the different low pass frequency settings.

Now, measuring the integral in equation (4) by dividing taking the integral of our gain calibration, squaring it, and then dividing by the integral of our voltage used for each of the low pass frequencies.

$$B = \frac{\left(\int gain\right)^2}{\int volt} \tag{5}$$

We found our results shown in the table below.

Frequency (kHz)	Integral B (V/kHz)
20	1003.05598
25	1532.1623
30	2094.37133
35	2660.29376
40	3244.15938
45	3847.0626
50	4473.28879

TABLE I: Solving for the Integral for each of our low pass frequencies.

As can be seen, the value of our integral does increase as we increase our low pass frequencies, this is obvious when looking at the area beneath each of the curves in figure 4.

A. *Fixed B and vary R*

When measuring the AC voltage for each of our resistors, we received the following results.

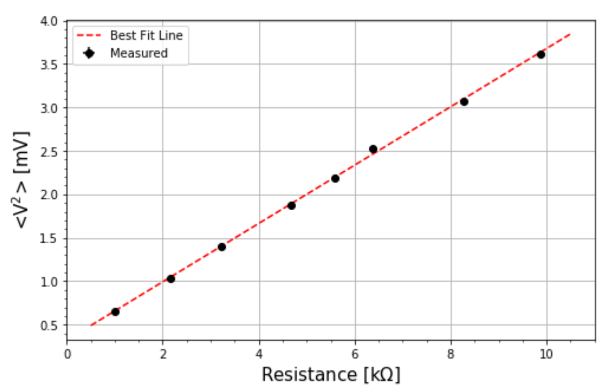

Fig. 6: $<V^2>$ vs. R, the slope of which is equal to $4kBT$.

Now, using our fixed Integral value for 30 kHz. We were able to find the Boltzmann constant by setting our slope equal to $4kBT$, where we measured the temperature of the lab to be at 70° F (294.2611 Kelvin).

$$k_{B1} = 1.3617938539123514 \times 10^{-23} \text{ J/K}$$

With the uncertainty of:

$$\Delta k_{B1} = 1.3998026970031517 \times 10^{-25} \text{ J/K}$$

B. *Fixed R and vary B*

Now keeping R fixed at 9.9866 kΩ and measuring the AC voltage while changing the frequency for the low pass filter, we received the following results.

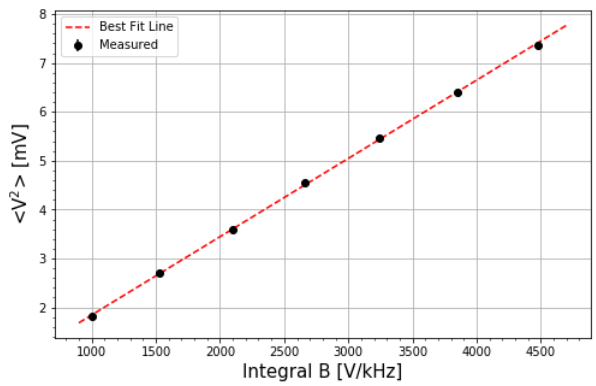

Fig. 7: <V^2> vs. B, the slope of which is equal to $4kRT$.

Now, using our fixed R, and solving for the Boltzmann constant by letting our slope equal $4kRT$ (using the same temperature of 294.2611 Kelvin).

$$k_{B2} = 1.3796530153484708 \times 10^{-23} \text{ J/K}$$

With the uncertainty of:

$$\Delta k_{B2} = 8.781545954721356 \times 10^{-27} \text{ J/K}$$

V. Conclusion

Comparing our two different Boltzmann results to that of the accepted value $k_B = 1.380649 \times 10^{-23}$ J/K. We found that our first measurement (k_{B1}) was only 1.366% below that of the accepted value. We also found that our second measurement (k_{B2}) was only 0.072% below that of the accepted value. Both of the quantities we measured were very close to the accepted value and are well within our margins of error.

How close we were able to measure these values in comparison to the accepted Boltzmann constant leads us to believe there is very few ways to improve the experiment other than conducting it underground, in a temperature controlled environment, free of any outside noise.