ASTR 401 Final

Felix Martinez

1 Mass-Radius Relationship for Exoplanets

1.1 Obtaining the Data

To find the Mass-Radius relationship for exoplanets, we used the exoplanet database to find as many planets as we could to fill our sample. Our minimum requirements for a planet to be included in our sampler were: Planet Radius [Earth Radius], **AND** Planet Radius [Jupiter Radius] to both have a value greater than zero. The reasoning behind this is we did not want to acquire a lower bound Earth (or Jupiter) radius planet that had the errors in its measurement that would put its opposing Jupiter (or Earth) radius equal to, or below zero. We also included Planet Mass [Earth Mass] and Planet Mass [Jupiter Mass] to both be set above zero for the same reasoning as our previous two requirements. Furthermore, we included the Number of Planets per Star System tool in our sample from the database (for Section 2). This left us with a 1946 planets downloaded from the exoplanet database.

We also included our own Solar System Planets (Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, and Neptune) in our data. We acquired their measurements by finding their mass (in kilograms) and radius (in kilometers), and converting their values into Earth mass and Earth radius. We will be using Earth Mass and radii throughout the rest of the project for consistency.

After combining the Solar System Planets into our data set, we now have **a total of 1955 planets in our sample**.

1.2 Planet Density Curves

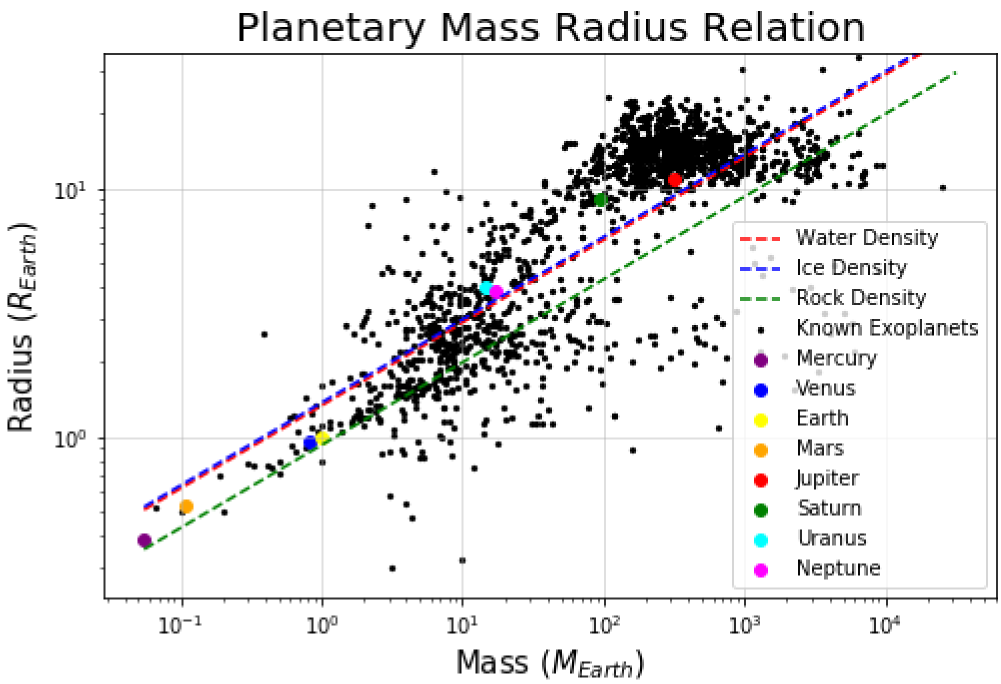
Before plotting our data, we first are going to find functions that match the density for planets composed entirely of rock, water, and ice. We will be looking at the Mass-Density relationship function:

$$\rho = \frac{3M}{4\pi R^3} \tag{1}$$

Where ρ is the density, M is mass, and R is our radius. After rearranging our function for radius to be a function of mass, we have:

$$R = \left(\frac{3M}{4\pi\rho}\right)^{\frac{1}{3}} \tag{2}$$

We will be using the densities of Water ($1.00\frac{g}{cm^3}$), Ice ($0.92\frac{g}{cm^3}$), and Rock ($\approx 3.00\frac{g}{cm^3}$) as our ρ constants in our 3 curves.



1.3 Plotting and Analysis

Now, below is our sample along with our established planet density curves on the same figure:

When analyzing this graph, the first thing that jumps out is that there seems to be two ”clumps” of planets, one very dense clump that has a large mass and radius, and another wider but less dense clump that has an intermediate mass and radius. The first, larger clump of planets seem to align with the types of Jupiter and Saturn, these planets are very low in physical density, much less dense than that of ice and water. This relationship could give way to these planets likely being gas giants, much like Jupiter and Saturn among them. The second, wider but less dense clump, has a larger range of physical densities, with a majority of its volume within the range of the rocky planets like Earth and Venus, and more icy-gaseous planets like Uranus and Neptune.

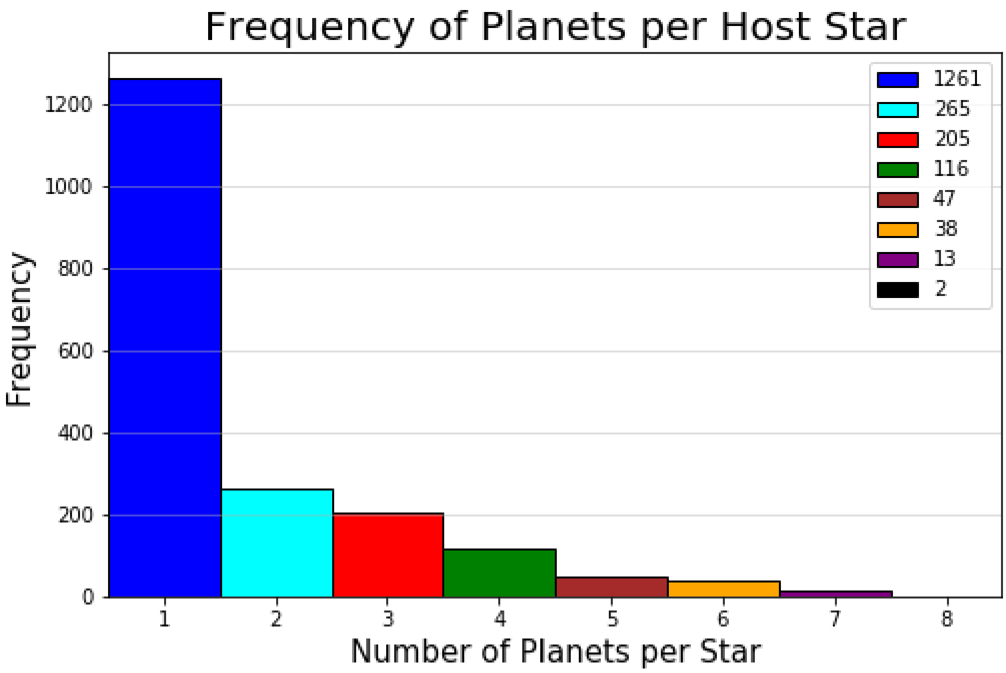
A very interesting note is that the high mass and radius clump seems to be very dense, which could be a result of the detection methods used, where more massive larger planets are easier to find around their host stars than smaller, less massive planets. This relationship seems to carry as we lower into the second, wider but less massive clump. These planets are harder to find than their larger, more massive counter parts but exponentially easier to find than the Earth to Mercury sized planets that seem to disappear as we follow the trend down. It is also worthy to note that our inner solar system planets fall towards the bottom of this trend, which could be explained as Earth to Mercury sized planets may naturally be located closer to their host star, further increasing the difficulty in detection rate.

A final interesting note comes from looking at the density curves and noting where our solar system planets and the other exoplanets land among them. There seems to be a correlation that as a planet decreases in mass, a higher makeup of their mass is made up of a more dense material, such as rock.

2 Exoplanet Multiplicity Function

2.1 Obtaining the Multiplicity Function

As explained in **Section 1.1**, when we extracted planets from the exoplanet database we included the Number of Planets per Star System in our sample. We will be using this data to find m, where m is defined as the number of planets around a star. We will use m to obtain the Exoplanet Multiplicity Function, to do this, we first will create a histogram of our sample as shown below:

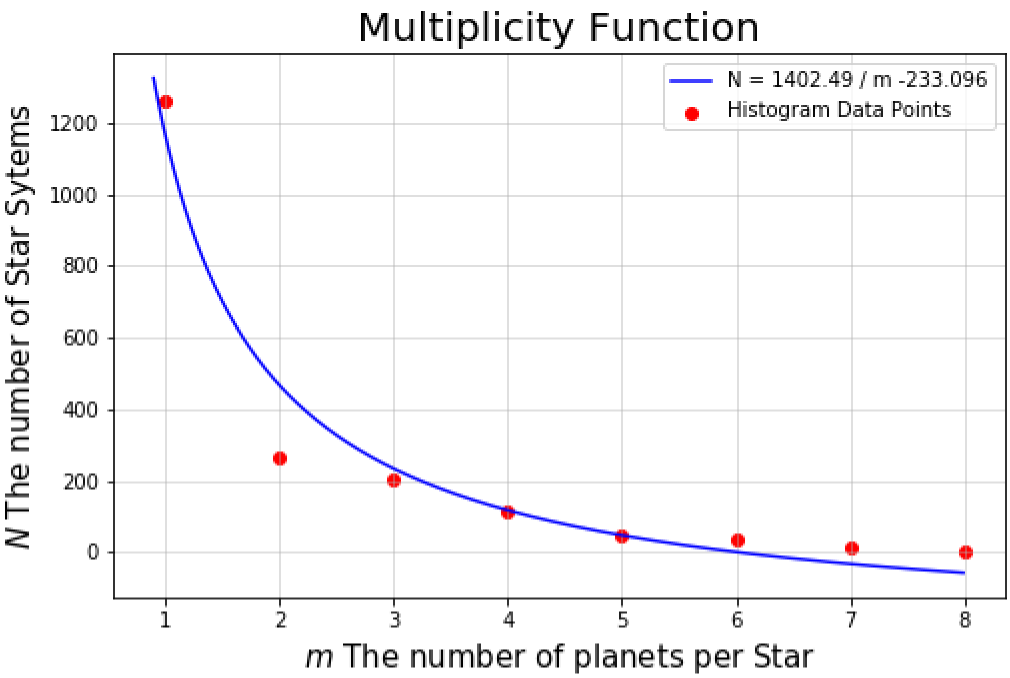


It is seen with this chart that by far the most common number of planets per star is 1 with over 1261 star systems in our sample having only one planet, it then exponentially decreases with a maximum of 8 planets per star system appearing only twice. **The mean of this distribution is 1.7668, with a variance of 1.6703.**

Now we will find the Exoplanet Multiplicity function for the whole sample using the data from our histogram. Just looking at the histogram, it is obvious the data is not linear, so in attempts to find an appropriate model for the function, we will use $\frac{1}{x}$ as a parent function to best fit the curve as it appears to fit the data decently. After putting our m values in the $\frac{1}{m}$ equation, we performed a linear regression on the transformed data to find the slope and y-intercept coefficients. This is the equation we achieved:

$$N = \frac{1402.49}{m} - 233.096 \tag{3}$$

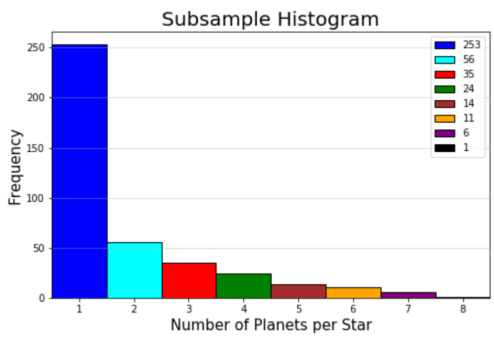
Where N is the number of star systems, and m is the number of planets per star. We will prove this equation is accurate by plotting the equation alongside the data from our histogram.

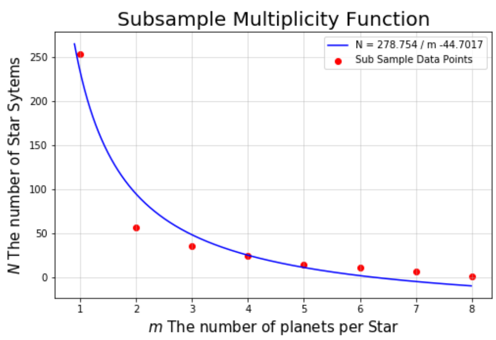


2.2 Working with a Sub Sample

We will now replicate what we have done for the full sample but on a smaller scale and find the Multiplicity function for the sub sample. First, we decided to pick a random sample of 400 star systems, in attempts to keep the Multiplicity Function of the sub sample similar to that of the full sample. We then proceeded to create a histogram for the sub sample and followed the same steps in creating a model using $\frac{1}{x}$ as a parent function for our m, to then run a linear regression on the sub sample to find the coefficients. Below is our calculated sub sample Multiplicity Function and our sub sample histogram and Multiplicity Function plotted:

$$N = \frac{278.754}{m} - 44.7017 \tag{4}$$





As is obvious, the sub sample of 400 star systems closely follows that of the full sample. This is probably due to the sub sample size being large enough to hold the same characteristics of the full sample.

2.3 Issues with the Multiplicity Function

Unfortunately, the multiplicity function has many biases in it with the largest one being the ease of observation. As it is easier to find massive planets around a star (covered in **Section 1.3**) smaller planets may go unnoticed. For example, in our solar system, the inner solar system planets are smaller and less massive than a majority of the planets from our sample. This relationship suggests that smaller, less massive planets that may be harder to detect. Furthermore, the proximity to the planets host star could also prove challenging in planetary detection, while it may help for large and massive planets to be close to their host star, it may hinder detection rates for smaller planets held at the same distance. Continuing, due to the types of detection methods available, as a planet is located further from their host star, it becomes even harder to identify a planet. Due to these issues in detecting smaller planets, and planets located far from their host star, many of these low numbered planetary star systems may have a higher m value than previously thought. With these confounding problems, we are unable to determine the true Multiplicity Function with just observational data due to the many levels of biases held when taking the data itself.