Lock-in Amplifier

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1 Introduction

The purpose of this laboratory is to introduce the student to the lock-in amplifier. A lock-in amplifier is a nearly ubiquitous piece of laboratory equipment, and can serve several functions. It is also the most sensitive piece of equipment that is commonly found in modern labs. Certainly there are more sensitive devices, but they are specialized. Lock-in amplifers are common.

References:

- Melissinos and Napolitano, p. 144 (section 3.8).
- "A basic lock-in amplifier experiment for the undergraduate laboratory," Librecht, Black, and Hirata, Am. J. Phys. **71**, 1208, (2003). (Attached.)

The lock-in amplifier we will use is produced by Stanford Reserch Systems, model SRS-810. It is a "digital signal processor" lock-in, meaning that much of the circuitry and processing inside is digital, whereas the classical lock-in is a purely analog device. Do not be confused, though. The input of this device is capable of **nano-Volt** measurements!

WARNING: This is probably the most sensitive device you will use this semester. Treat it carefully!

You will use the lock-in amplifier to **measure the resistance and inductance per unit length** of a short piece of copper wire that I will supply.

The resistance that you will measure is some few milli-Ohms. This is much too small to see with an ordinary hand-held meter. Nonetheless, you will find that you can make the measurement with an uncertainty of only a percent or so. I will not tell you the inductance of the wire, but you will find that it is just as easy to measure.

2 Measurements

Follow these instructions carefully!

You have been given a short piece of 24 gauge copper wire. You are to measure the resistance and inductance per unit length of the wire. For your convenience, I stripped both ends, and soldered an $\approx$1.0 kΩ resistor to one end of the wire.

First, turn on the SRS-810 lock-in amplfier, so it can warm up. It will make a series of beeps and run a diagnostic self-check.

Measure the length of the copper wire. You will find a ruler on the table.

Measure the resistance of the resistor that I soldered onto this wire. (Never trust the colored bands – always check it with a meter.) You can use the hand-held Wavetek Volt-Ohm meter that is on the table, but you will get a better measurement if you ask the people at the Johnson Noise experiment to let you use their incredible 6.5-digit Fluke device. It is by far the best device in the lab for this. Just keep in mind that they are very busy with their experiment

Next you need to generate an oscillating current in the wire you are to measure. Unfortunately, the HP-8904A Multifunction Synthesizer is probably not the optimum device for this, but it is what we have. This device is capable of synthesizing a wide array of waveforms, so using it to generate a sine wave is overkill.

To start, Look at the HP-8904A and make sure its "Output-1-High" is connected, through a series of BNC-tees, to the reference input of the lock-in (labeled REF IN), to channel 1 of the Tektronix 2213A oscilloscope, and finally with a set of "easy-hooks" to the wire and resistor. The output easy hooks should connect across the full length of the wire and the resistor. Now, disconnect the BNC from the reference input of the lock-in, leaving the path to the oscillocope engaged.

Turn on the HP-8904A. It will tell you to press f1 to configure a channel. The f1 button is on the upper left. The display will then say to press the NEXT/LAST key to proceed. Press NEXT. You will see that Channel A is preconfigured for a 1 kHz sine wave with an amplitude of 140 μV. Press the "AMPTD" button. The cursor will now sit in front of the amplitude. Type 1 on the keypad, and then press the button labeled "kHz / V". The label means this button sets kHz when you are changing frequency and Volts when you are setting amplitude.

You should now see a 1 kHz sine wave on the oscilloscope. However, you will notice that the amplitude is much larger than 1 V. This means that either the function generator or the oscilloscope, *or both*, are incorrect. It turns out that the scope is pretty accurate, and that the function generator needs servicing.

To fix this, note what the amplitude is and estimate what you should set the generator to so that the actual amplitude on the scope is approximately one Volt. You may iterate this until the amplitude is what you want. Remember that a 1 V amplitude means 2 V peak-to-peak.

You may also find that the amplitude generated by this device is different if your wire-and-resistor are not connected. This is because the resistor you are using is approximately 1 kΩ but the output impedence of the function generator is 600 Ω.

Now look at the lock-in amplifier. You will see a red light on the far right that says "UNLOCK." This means the lock-in is not able to measure because it has no reference. Re-connect the signal from the HP function generator to the reference input of the lock-in, and observe that the UNLOCK light goes off, indicating that the lock-in is ready to measure.

To the left of the reference input are a series of buttons. Press "Freq". The lock-in will now measure and display the frequency of the reference. This will be very close to what you have programmed the HP-8904A to produce. If they differ, I would trust the lock-in, but your mileage may vary.

You now want to measure the voltage you are producing across the wire and resistor with the lock-in. You know it is approximately one volt because that is what you see on the oscilloscope.

On the upper left of the lock-in are buttons to set the time-constant and sensitivity. First choose a time constant. This must be long compared to the period of your reference waveform, but short enough that it does not try your patience. Remember that the time constant is like the decay time in an exponential. You will need to wait several times this amount of time any time you make a change to your experiment, but the longer your time constant the more precise will be your measurement. To get started, I recommend a time constant of 100 ms.

Now set the input sensitivity. It does not go above 1 V, so set it to that.

Now use a cable with easy-hooks to connect the voltage across the wire and resistor to the input amplifier of the lock-in. The reading will fluctuate, but will settle down in a few times your time constant to a stable value.

You will note that the lock-in is **not** reading 1 Volt. Why? It is very important for you to understand why. Spend a few minutes thinking about it, and then ask either the professor or teaching assistant to make sure you understand correctly.

Write down the measured voltage. This is V_a in the Figure.

Now, move one of the easy-hooks that is connected to the lock-in input so that it is looking only across your wire. The signal will be very low. Slowly increase the sensitivity, from 1 V, in steps down, until you get a clean reading of the RMS voltage across your wire. The signal probably will not be very stable. Now, *increase the time-constant* of the lock-in from 100 ms to something like 1–3 s. You will now always have to wait about 10-15 seconds (3-5 time constants) after any change before making a measurement. But now the voltage read by the lock-in should be stable, and roughly 8 micro-volts.

You have just measured an 8 micro-volt signal with high precision! The set-up looks like this:

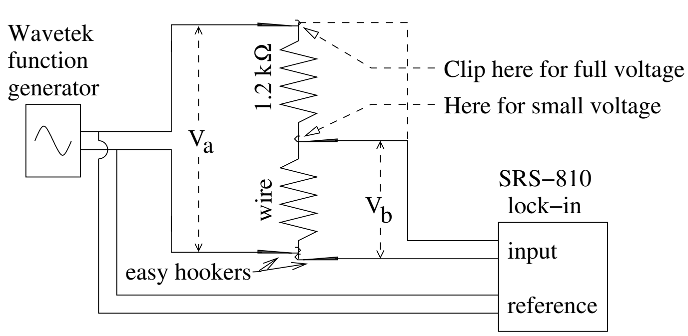


Figure 1: Experimental arrangement for measuring voltage across a short piece of wire.

Complex voltage

Now, change the phase of the lock-in by +90 degrees. To do this, press the "Phase" button to the left of the reference input. It better read 0.00 degrees. If it does not, use the wheel on the far right to set it to zero, and go back and repeat your voltage measurements above. Now, simply press the +90° button.

You are now looking at the voltage that is 90 degress out of phase from the reference. This should be fairly small, indicating that the voltage across your wire is mostly in phase with the driving voltage. You should think of the voltage across the wire as a complex function, $V = Ae^{i\omega t}$, then the voltage measured at zero phase is $\mathrm{Re}[A]$ and the voltage measured at +90° is $\mathrm{Im}[A]$.

Although the out-of-phase voltage is small, it is not zero. It should be approximately 1 μV. Because you want a very clean measurement of the in-phase voltage, you should worry a little bit that harmonic distortion of the reference might shunt some of this signal into the in-phase measurement.

First, verify that this out-of-phase voltage is due to inductance of the wire by doubling the frequency. (Just press "FREQ" then 2 then kHz on the HP.) This should convince you that the out-of-phase voltage is approximately proportional to the frequency.

In order to get a clean measurement of the in-phase voltage, it would be safest to go to lower frequencies. Return the lock-in to in-phase measurements by pressing the −90° button (or by pressing the +90° button three times). You should now repeat these measurements for different (lower) frequencies of the sine wave. You already have 1 kHz, so I recommend doing it at 750 Hz, 500 Hz, 250 Hz, and 100 Hz. As you go to lower frequency, you will have to keep the time-constant many times the period, and you will have to wait for many time-constants for each measurement.

Be careful when you change the hooks from the large voltage to the small – it is good to avoid putting a large voltage on the lock-in when the sensitivity is set for a low voltage.

You expect the data to be something like this:

Freqency (Hz)	V_a (Volts)	V_b (Volts)
1000.0(1)	0.6187(1)	8.74(2)
750.0(1)	0.6187(1)	8.706(10)
499.9(1)	0.6186(1)	8.692(10)
249.9(1)	0.6184(1)	8.735(10)
99.9(1)	0.6177(1)	8.702(10)

Now we recall that the out-of-phase voltage was an increasing function of frequency, and was about 1 microvolt when the frequency was 1 kHz. If we assume this out-of-phase voltage is caused by the inductance of the wire, then it will be proportional to frequency. This is good, because it means higher frequencies will produce higher voltages, and also require less measurement time.

The maximum frequency that the lock-in amplifier can handle is 100 kHz, so let's start there. Make sure the input amplifier of the lock-in is set to 1 V, and then set the HP-8904A to 100 kHz. Look carefully at the lock-in to make sure it has not lost lock. The UNLOCK light must not be on. If it is, then lower the frequency to 99 kHz. I would set the time constant to something like 300 ms, so you get clean readings but don't have to wait more than a few seconds for each one.

Now you want to measure the in-phase voltage across the wire+resistor, and the out-of-phase voltage across the wire. Do you understand why?

When you measure the in-phase voltage across the wire+resistor, you may find that it is lower than in the measurements above. This is unexpected, and is because the HP-8904A is not keeping its amplitude constant as you increased the frequency. That is okay because you will measure it as you change the frequency. You will probably find it is something like 0.5 V.

Now set the phase of the lock-in to +90° and carefully measure the voltage across the wire. At 100 kHz you will find something like 100 μV. To be careful, check the in-phase voltage. It should still be close to what you measured above unless a lot of this signal is leaking in.

Repeat this at 80, 60, 40, 20, and 5 kHz. You expect the data to be something like this:

Freqency (kHz)	V_a at 0° (Volts)	V_b at 90° (Volts)
4.99(1)	0.6187(1)	4.87(5)
19.99(1)	0.6165(1)	19.40(5)
39.99(1)	0.6132(1)	38.56(5)
59.99(1)	0.6063(1)	57.23(5)
79.99(1)	0.5913(1)	74.54(5)
99.99(1)	0.5634(1)	89.08(5)

Now, notice that the wire is stripped near its midpoint. Using this shorter length, repeat enough of the previous the measurement (both kinds) to convince your reader that your results (both resitance and inductance) are proportional to the length of the wire.

3 Analysis

We want to use our data to calculate the resitance of the wire and its inductance. From the circuit in Fig. 1 we see that:

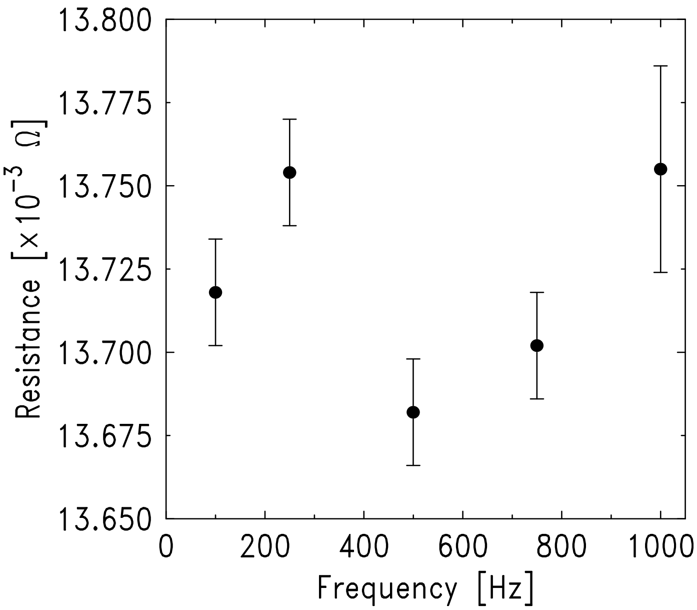
$$V_b = \frac{V_a}{R_1 + R_2} R_2$$

where R_1 is the resistance of the 1.0 kΩ resistor and R_2 is the wire. Making the approximation that $R_2 \ll R_1$ we see

$$R_2 = R_1 \frac{V_b}{V_a}$$

This gives you the resistance of the wire. Your answer should be smaller than anything you could have measured with an ordinary instrument!

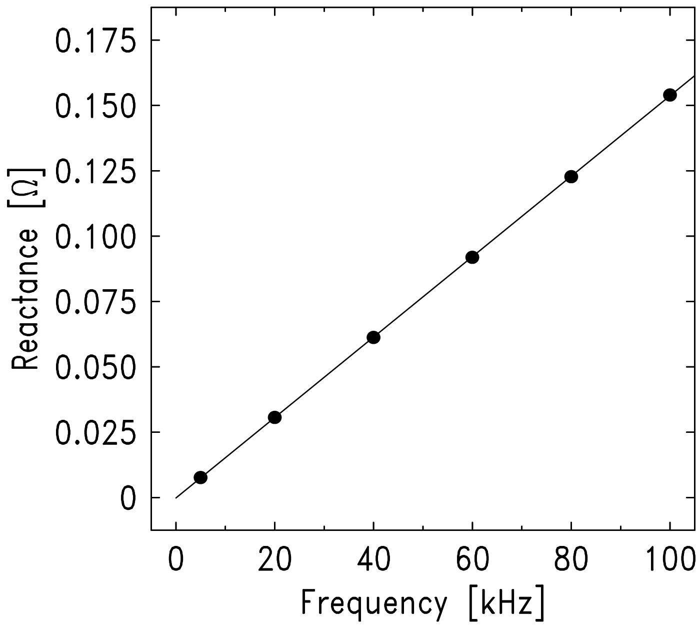
In order to calculate this, we should plot this value as a function of frequency to see how stable it is. You will need to use your uncertainties in R_1, V_a, and V_b to calculate your uncertainty in R_2. For the example data above, your plot should look something like this:



It seems that there is no systematic trend with frequency in this measurement, which is what we expect. Therefore there is no reason to fit a line, but rather we should just take a weighted mean of the points to get a value for the resistance and its uncertainty.

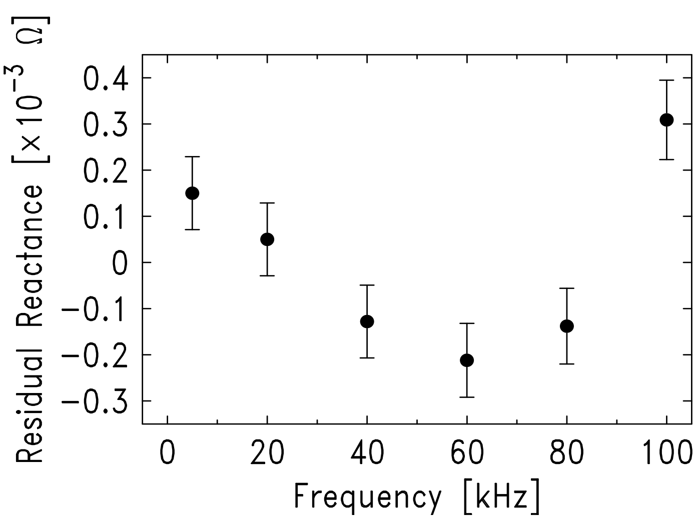
Do you think the error bars are reasonable?

Next, let us think about the inductance of the wire. We know that the EMF produced by such an inductance is given by $\mathcal{E} = -L\,dI/dt$. Writing the current as $V_a(t)/R_1$ with $V_a(t) = V_a e^{i\omega t}$ gives us $|V_b| = L\omega|V_a|/R_1$. So, we want to plot $R_1 V_b/V_a$ as a function of frequency, and the slope should be the inductance. Again, for the example data above, your plot should look something like this:



The error bars are too small to see on this plot. Performing a weighted least-squares fit to this line gives us that the slope is $2\pi L = 1.6989(20) \times 10^{-3}\ \Omega/\mathrm{kHz}$, or an inductance $L = 0.24486(16)\ \mu\mathrm{H}$. **Do you understand the factor of 2 pi?**

Because these uncertainties are so small, and because we could not see the error bars on the plot, we should plot residuals. That is, we plot the difference between our data and our fit for each of the frequencies studied:



This is an extremely important plot becuase it shows that some fairly systematic effect is causing our data to fluctuate by more than our estimated uncertainty. What could be causing this? When we measured V_a etc., the reading on the lock-in was quite stable. What is causing this trend?

What can we conclude about our value for the inductance above?

How can you verify that you are measuring an inductance and not some other frequency-dependent effect, such as capacitance?

Look up the conductivity of copper. Estimate the diameter of the wire. What number does this give for the resistance per unit length, and how does that compare to your measured value?

Make **sure** you understand that these are questions to ask for this sample data. Your data will be different, and the question you need to ask may also be!