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ASTR 401 Homework 1

1) 

$$U = \int_0^M -\frac{GM}{r}\,dm \qquad M = \frac{4}{3}\pi r^3 \rho$$

$$dm = 4\pi r^2 \rho$$

a) $$U = -\int_0^R \frac{G\left(\frac{4}{3}\pi r^3 \rho_0\right)\left(4\pi r^2 \rho_0 dr\right)}{r}$$

$$U = -\frac{16}{3}\pi^2 \rho_0^2 G \int_0^R r^4 dr \Rightarrow \boxed{-\frac{16}{15}\pi^2 \rho_0^2 G R^5}$$

b) $$\rho(r) = \rho_0 c^2\left(1 - \frac{r}{R}\right)$$

$$U = -\int_0^R \frac{G\left(\frac{4}{3}\pi r^3 \rho\right)\left(4\pi r^2 \rho dr\right)}{r} \Rightarrow -\frac{16}{3}\pi c^2 G \int_0^R \rho^2 r^4 dr$$

$$\Rightarrow -\rho_0^2 c^2 \frac{16}{3}\pi^2 G \int_0^R r^4\left(1 - \frac{2r}{R} + \frac{r^2}{R^2}\right) dr \rightarrow \int_0^R r^4 - \frac{2r^5}{R} + \frac{r^6}{R^2}\,dr$$

$$\boxed{U = -\frac{16}{3}\pi^2 \rho_0^2 c^2 G R^5\left(\frac{1}{5} - \frac{1}{3} + \frac{1}{7}\right)}$$

c) $$\frac{dP}{dr} = -\frac{GM\rho_0}{r^2}, \quad \text{for (a):}$$

$$dP = -\frac{4G}{3}\pi \rho_0^2 \frac{r^3}{r^2} dr$$

$$P = -\frac{4}{3}\pi G \rho_0^2\left(\frac{r^2}{2}\right) \rightarrow \boxed{P = -\frac{2}{3}\pi G \rho_0^2 r^2}$$

for (b)

$$dP = -\frac{4}{3} G\pi \rho_0^2 c^2 \frac{r^3}{r^2}\left(1 - \frac{2r}{R} + \frac{r^2}{R^2}\right) dr$$

$$\boxed{P = -\frac{4}{3} G\pi \rho_0^2 c^2\left(\frac{r^2}{2} - \frac{2r^3}{3R} + \frac{r^4}{4R^2}\right)}$$

2| Recall from 1a) $U = -\frac{16}{15}\pi^2\rho^2 G R^5$

a) $\rho = \frac{3M}{4\pi r^3} \quad \rho^2 = \frac{9M^2}{16\pi^2 r^6}$

$U = -\frac{16}{15}\pi^2 G R^5 \frac{9M^2}{16\pi^2 R^6} \Rightarrow U = -\frac{3}{5}\frac{M^2 G}{R}$

$\vec{E} = \frac{1}{2}U \rightarrow \vec{E} = -\frac{3}{10}\frac{GM^2}{R}$

$G = 6.67\times10^{-11}\ \frac{N m^3}{kg^2}$

$M_\odot = 1.989\times10^{30} kg$

$R_\odot = 6.955\times10^{8} m$

$\vec{E} = -\frac{3}{10}\frac{GM_\odot^2}{R_\odot} = -1.139\times10^{41} J$

Total Energy radiated: 1.39×10^{41} J

b) Time taken to reach the main sequence:

$L = \frac{E}{t} \rightarrow t = \frac{E_{radiated}}{L}$

$L_\odot = 3.839\times10^{26} W$

$t = \frac{1.39\times10^{41}}{3.839\times10^{26}} = 3.62\times10^{14} s \left|\frac{1 yr}{3.154\times10^{7} s}\right|$

$t = 1.147\times10^{7} yrs$

3]

a) $m - M_B = 5\log(d) - 5$ $\quad M_B = 4.72$

$m = 6$

$L = 4 \times 10^{26}\ W$

$d = 10^{\left(\frac{m - M_B}{5} + 1\right)}$

$\boxed{d = 18.03\ pc}$

b) photons/sec. in eye = $\frac{\text{Area of Eye}}{\text{SA of sphere where } r = d}$

$n = \frac{\frac{\pi}{4} D_{eye}^2}{(4\pi d^2)}$ $\quad D_{eye}$ = diameter of eye $= .0242\ m$

$1\ pc = 3.086 \times 10^{16}\ m$

$\boxed{n = 6.574 \times 10^{-23}}$

4] $\frac{1}{\mu_I} = \sum_i \frac{X_i}{A_i}$ $\quad X_H = .71$ $\quad m_H = 1$

$X_{He} = .27$ $\quad m_{He} = 4$

$X_{other} = .02$ $\quad m_{other} \approx 20$

$\frac{1}{\mu_I} = \frac{.71}{1} + \frac{.27}{4} + \frac{.02}{20}$

$\frac{1}{\mu_I} = .7785 \rightarrow \boxed{\mu_I = 1.2845}$

Now for All Hydrogen ($X_H = 1$):

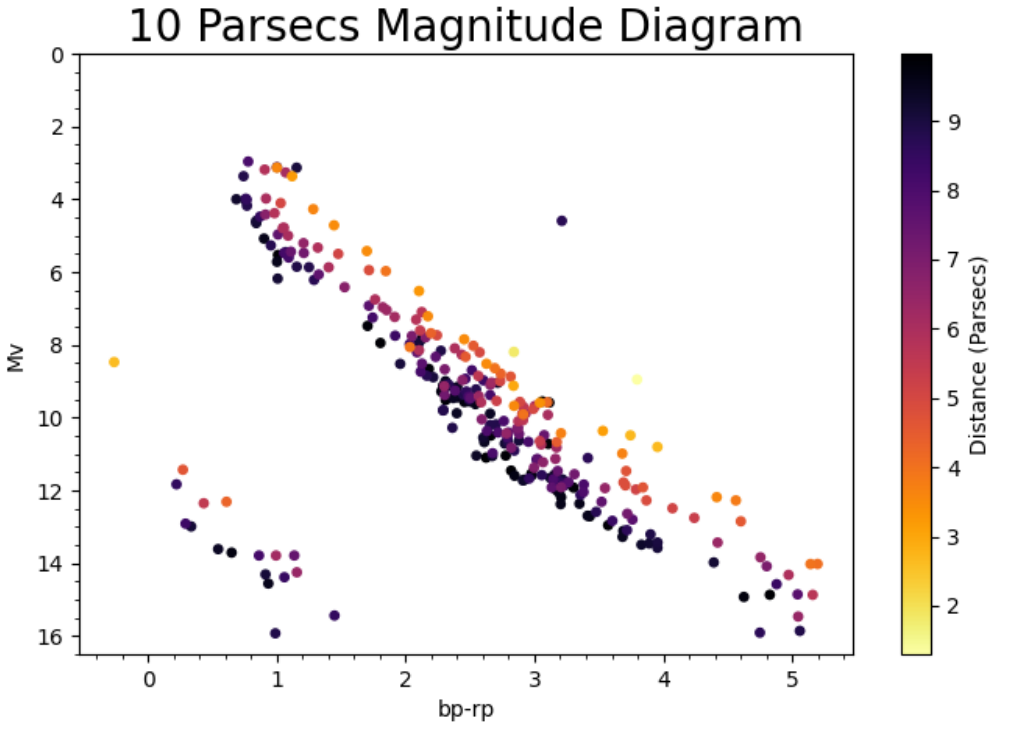
$\frac{1}{\mu_I} = \frac{1}{1} \Rightarrow \boxed{\mu_I = 1}$

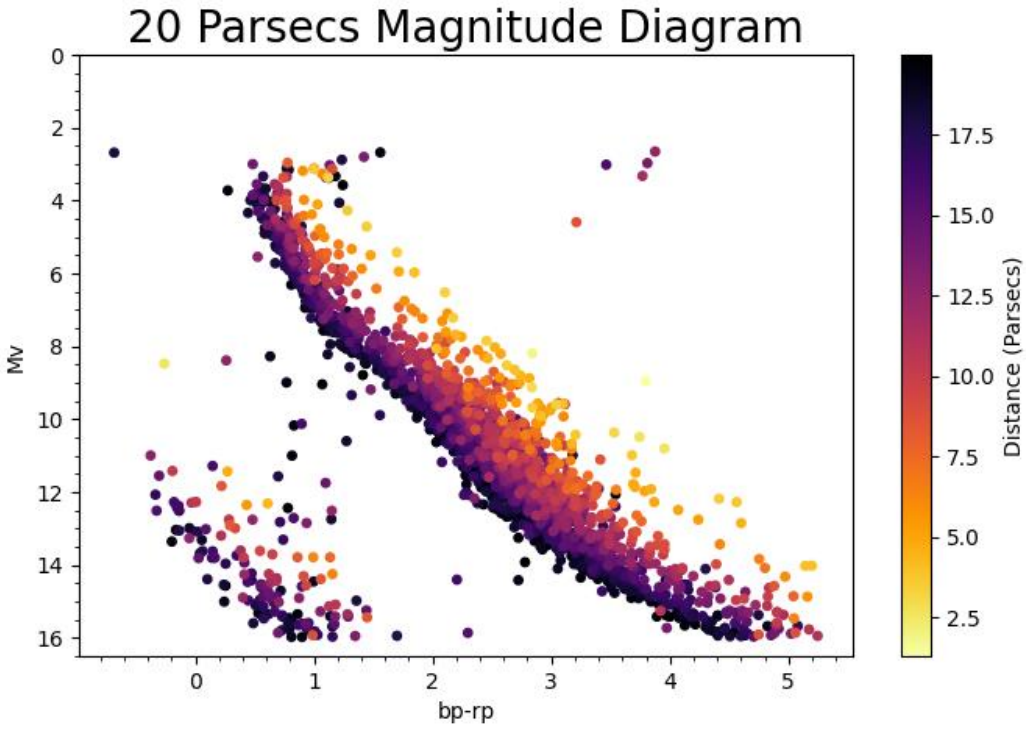
Now for All Helium ($X_{He} = 1$)

$\frac{1}{\mu_I} = \frac{1}{4} \Rightarrow \boxed{\mu_I = 4}$

Now for All Heavy Elements ($X_{other} = 1$)

$\frac{1}{\mu_I} = \frac{1}{20}$ $\quad \boxed{\mu_I = 20}$


10 Parsecs Magnitude Diagram
Mv
bp-rp
Distance (Parsecs)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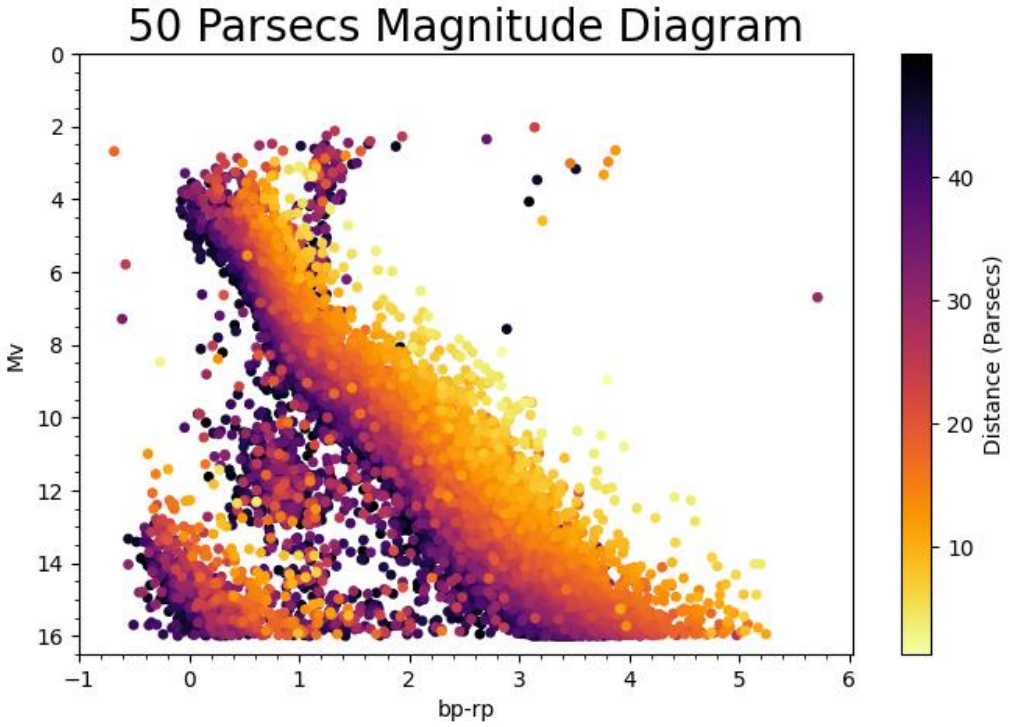

When making the Color Magnitude Diagrams, I filtered out any stats with a magnitude above 16, this was because Gaia's scientific performance sheet stated that any magnitudes above this measurement were subjected to errors due to how they collected the data.

Also there is a large difference when looking at the 20 Parsec CMD vs. the 50 Parsec CMD. When scaling far out to 50 Parsecs, there seems to be an overlap of different stellar groups, as the uplift from the main sequence begins at around Mv 6, and bp-rp 1.2, much different than the Mv 4 and bp-rp of .6 as shown in the 20 Parsec diagram. Furthermore there seems to be a larger scattering of white dwarfs in the 50 Parsec diagram, with a unique characteristic of the closer stars on the main sequence having on average a higher Mv or a higher bp-rp than the further stars (having a gradient scale from the bottom left to the top right of the graph). This could be a result to the inclusion of another group of stars farther out that 20 pc, in a separate group than that of our Sun.