Lock-In Amplifier Lab

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I. Introduction

This Labs purpose is to be an introduction to the lock-in amplifier. A lock-in amplifier is a phase sensitive detector that can extract a signal from a noisy environment. The lock-in amplifier we will be using is produced by Stanford Research Systems, model SRS-810. We will be using the lock-in amplifier to measure the resistance and inductance per unit length of a short piece of copper wire with a resistor attached.

II. Layout and equipment

Our Function Generator, HP-8904A Multi-function Synthesizer, must be connected across the length of the wire and the resistor. While our lock-in amplifier measures the voltage drop across either the wire itself, or the wire and resistor. We will also be using an oscilloscope, Tektronix 2213A, to further serve our observations. The oscilloscopes measurements will take place across the wire and resistor much like the lock-in amplifier.

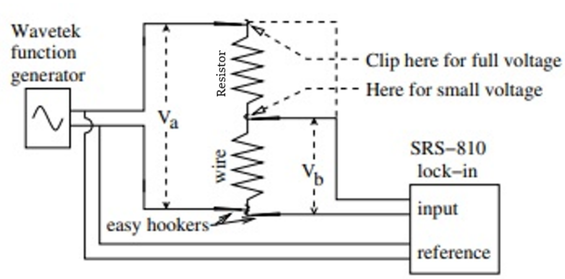

Fig. 1: Our Experimental arrangement for measuring the voltage drop across a wire and resistor. The Resistor we will be using is measured to be .9737 kΩ

III. Procedure

A. Experimental Setup

To begin, we must first measure the length of the copper wire and the resistor, along with the resistance of the resistor attached to the wire.

Next, we must generate an oscillating current in the wire using the function generator. At this point the output easy hooks should connect across the full length of the wire and resistor. Now, we must disconnect the BNC cables from the reference input of the lock-in to leave the path to the oscilloscope engaged.

Now, we must generate a 1 kHz sine wave with an amplitude of 140 μV which we should be able to see on the oscilloscope. This sine wave should have a frequency of 1 kHz, however, the amplitude would be much larger than 1 V. This is due to an error in the function generator. To fix this, we must note what the amplitude is and estimate what we should set the generator to so that the actual amplitude on the oscilloscope is 1 V, or 2 V peak-to-peak.

Next we must pay attention to our lock-in amplifier. At this point there should be a red light on labeled “UNLOCK". This is due to the lock-in being unable to measure the voltage as is has no reference. To solve this, we must re-connect the signal from the function generator to the lock-in (as seen in Figure 1). To see the frequency of the sine wave on the lock-in, press “Freq" in the input buttons located on the left side of the machine, this would give a more accurate signal of our frequency, more so than the function generator itself.

Now we must measure the voltage across the wire and resistor. We already know it should be approximately 1 V, but to observe this on the lock-in, we must adjust the time-constant and sensitivity on the lock-in. First, adjust the time-constant to be 100 ms, next adjust the input sensitivity to 1 V. Now we must use a cable with easy-hooks to connect across the wire and resistor to the input amplifier of the lock in, this will give us the voltage. If done correctly, we should observe a voltage different than that of 1 V, that is because this is the RMS of the voltage, this is V_a in Figure 1.

Now move one of the easy-hooks so that the lock-in input is only looking across the wire. This will be a much lower signal so we must increase our sensitivity from 1 V, in steps down, until we get a clear reading of the RMS voltage across the wire. At this low range the signal will not be very stable so we must increase the time-constant of the lock-in from 100 ms to around 1-3 s. The voltage should be around 8 μV, this is V_b in Figure 1.

B. Complex Voltage

To observe complex voltage, we must change the phase of the lock-in by +90 degrees, this can be done by pressing the “Phase" button to the left of the reference input and pressing the +90° button. The resulting voltage should be very small, this indicates that most of the voltage across the wire is in phase. We can use a complex function to imagine the voltage across the wire.

$$V = Ae^{i\omega t} \tag{1}$$

Where Re[A] is the zero phase shifted voltage and Im[A] is the voltage held at +90°. The voltage measured after the phase shift should be roughly 1 μV.

C. Measurements and Equations

Now, return the lock-in back to in-phase measurements and repeat the measurements of V_a and V_b of different (lower) frequencies of the sine wave. The frequencies we will use include 1000, 750, 500, 250, and 100 Hz.

We wish to find the resistance and inductance of the wire. We can find the resistance by using the equation:

$$V_b = \frac{V_a}{R_1 + R_2} R_2 \tag{2}$$

Where R_1 is the resistance of the resistor, and R_2 is the resistance of the wire, we can simplify this equation by making the approximation that $R_2 \ll R_1$ and rearranging:

$$R_2 = R_1 \frac{V_b}{V_a} \tag{3}$$

Which would give us the resistance of the wire. To calculate it correctly, we should plot this value as a function of frequency to see how stable it is.

Now, to find the Inductance of the wire, we must recall that the out-of-phase voltage was increasing function of frequency, and about 1 μV when the frequency was 1 kHz. We can assume this out-of-phase voltage is caused by the inductance of the wire, therefore it would be proportional to frequency. So starting at 100 kHz, as this is the maximum frequency the lock-in amplifier can handle, we will measure the in-phase voltage across the wire and resistor (V_a), and keeping the same frequency, we will measure the out-of-phase voltage across the wire (V_b). We will do this again with 80, 60, 40, 20, and 5 kHz to get an accurate measurement of inductance as a function of frequency.

To find the inductance in the wire, we must recall that the EMF produced by the inductance is:

$$\mathcal{E} = -L\frac{dI}{dt} \tag{4}$$

If we write the current (I) as $V_a(t)/R_1$ where $V_a(t) = V_a e^{i\omega t}$, we will get a voltage:

$$|V_b| = L\omega \frac{|V_a|}{R_1} \tag{5}$$

So we can plot $R_1 V_b / V_a$ as a function of frequency, where the slope would be our inductance.

IV. Data/Analysis

When conducting this experiment we were able to find R_2 by using equations (2) and (3) with our measured data. Plotting our R_2 against frequency:

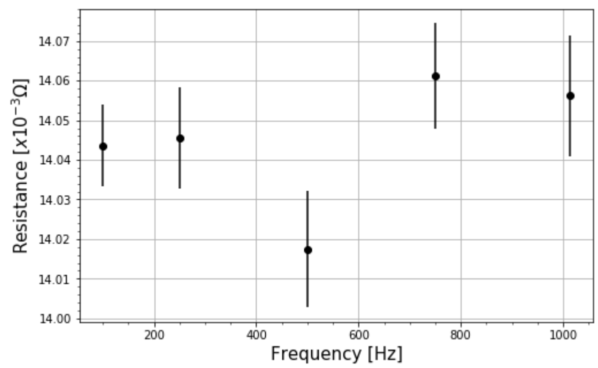

Fig. 2: R_2 vs. Frequency.

As seen in Figure 2, R_2 does not seem to be dependent on frequency, which is as expected. Due to this, we saw it reasonable to take the mean of the points instead of fitting a line to the data. This resulted in us finding the weighted mean resistance of our wire (R_2) equal to $14.045 \times 10^{-3}\Omega$ with an uncertainty of $5.9 \times 10^{-5}\Omega$.

Now, in attempts to find the inductance of our wire, we used our data to plot $R_1 V_b / V_a$ as a function of frequency, we achieved the following result:

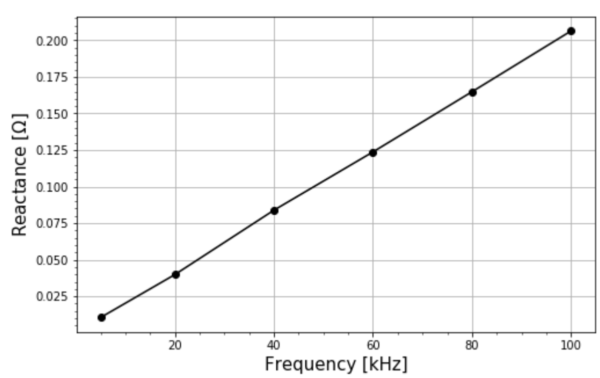

Fig. 3: Reactance vs. Frequency.

However when viewing this, it is clear that the error bars are too small to see on the plot, so we preformed a weighted least-squares fit to this line that gave us a slope of $2\pi L = 2.06561(7) \times 10^{-3}\Omega/\text{kHz}$ or an inductance of 0.27038(7) μH. The factor of 2π in our slope equation is there as the voltage we measured is out of phase. Furthermore, to clearly see the errors, we plotted the residuals of Figure 3 below.

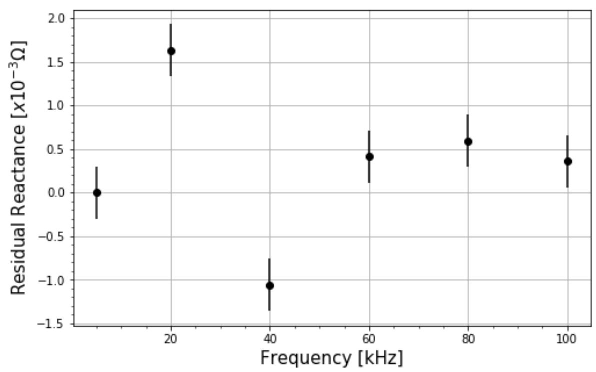

Fig. 4: Residual Reactance vs. Frequency.

As the fluctuations in Figure 4 are occurring beyond that of our measured uncertainties, we can conclude that there is some other systematic effect that is influencing our data. We believe that the self inductance of the wire could be causing these fluctuations.

In attempts to find the conductivity of our copper wire, we can use equation (6):

$$\sigma = \frac{l}{R_2 A} \tag{6}$$

Where σ is the conductivity, l is the length of the wire, R_2 is the resistance of our wire, and A is the total area of the wire.

With our copper wire being a $l = 31.5$ cm, w4 e gauge wirefound the conductivit, 2y of our copper wire to be $\sigma = 1.09 \times 10^8 \frac{1}{\Omega\text{m}}$. We can compare this to the known, acceptable, conductivity of copper, found to be, $\sigma = 2.9 \times 10^8 \frac{1}{\Omega m}$. Our measurement is off by nearly 37.58%.

V. Conclusion

In conclusion, when conducting the first part of the experiment, the Voltage reading on the lock-in held stable when measuring V_a. This was caused as most of the voltage was held in phase. This can be confirmed by observing the voltage that was out of phase, which was held in the micro-Volt range. The out-of-phase voltage is very small in comparison to the in-phase voltage, keeping the in-phase voltage fairly constant, by allowing little to no out-of-phase fluctuations.

Furthermore, when observing our solved inductance, in part two of the experiment, we can conclude that some systematic effect caused our data to fluctuate more than expected. This is more than likely due to the wires own self inductance, producing an electromotive force.

Finally, when addressing error, we cannot help but notice a major offset when observing the conductivity of our wire. The offset could be explained away through issues in measuring the wire, as when observing our resistance and inductance, the values for these measurements seem to match what we expect. When applying the measured length of our wire l into equation (6), we have a resulting conductance that is 37.58% off of what is the known, accepted, amount. However, when running through the same equation, increasing l, our resulting conductance would be closer to the accepted amount. This leads us to believe there was some human error in measuring the copper wire. When taking the measurement, we believe that the bending of the wire lead to us receiving a length l that is shorter than what the wires true length actually was.