Methods and findings for Planck's constant $\hbar$ and the electron charge e and mass m_e

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Abstract

In this paper, we discuss how we found values for Planck's constant $\hbar$, the electron charge e, and the electron mass m_e. We did this by conducting four separate experiments in order to correctly measure and identify four separate fundamental constants, the specific charge of the electron e/m, the Rydberg's constant R_∞, the speed of light c, and the photoelectric effect h/e. We then used these constants to mathematically solve for $\hbar$, the e, and m_e. Our results are $\hbar = 1.06756 \times 10^{-34}$ J s with an uncertainty of $\Delta\hbar = 1.710819 \times 10^{-36}$ J s for Planck's constant, $e = 1.57081 \times 10^{-19}$ C with an uncertainty of $\Delta e = 2.51602 \times 10^{-21}$ C for the electron charge, and $m_e = 9.23769 \times 10^{-31}$ kg with an uncertainty of $\Delta m_e = 1.47963423 \times 10^{-33}$ kg for the electron mass. Including our uncertainty, all of our measurements fall closely to that of the accepted values for each desired constant. As we found h/e, e/m, R_∞, and c separate from one another, and used these results to solve for $\hbar$, e, and m_e, we have reason to believe that our findings are void of systemic errors due to our experiments having little to no confounding factors.

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I. INTRODUCTION

In this paper we found Planck's constant $\hbar$, the electron charge e, and the electron mass m_e, by conducting four separate experiments to independently measure the fundamental constants h/e, e/m, R_∞, and c.

II. EXPERIMENTS

Below are the four experiments used to measure h/e, e/m, R_∞, and c. For each measurment, we first introduce the experiment we did, then explain our methodology, and finally share our results.

A. Measurement of e/m

1. Introduction

Starting with e/m, we will be taking two measurements to calculate our constant. We will be using the equation below, which is just the kinetic energy of electrons moving in a circle of radius, r, perpendicular to a magnetic field, B [4][6].

$$\frac{e}{m} = \frac{2V}{B^2r^2} \tag{1}$$

Our first measurment will consist of us finding B by assuming that the magnetic field increases linearly with current, I, and take multiple measurements of B as a function of I in order to obtain a constant k_1.

$$B = k_1 I \tag{2}$$

Our second measurment will consist on finding a value for I, which we can do by taking a series of measurements varying the voltage, V, and current while keeping r fixed. This can be found with the equation below.

$$I^2 = \frac{2m}{k^2r^2e}V \equiv k_2 V \tag{3}$$

Equations (2) and (3) can be compressed into equation (1) where we can get the following result.

$$\frac{e}{m} = \frac{2}{k_1^2 k_2 r^2} \tag{4}$$

2. Methodology

To find our constants k_1 and k_2, we will be using a fine beam tube surrounded by Helmholtz coils and configured so that we could use an adjustable measuring device to measure the radius of the electron beam. Figure 1 is a simple cartoon that shows all of the details in our setup.

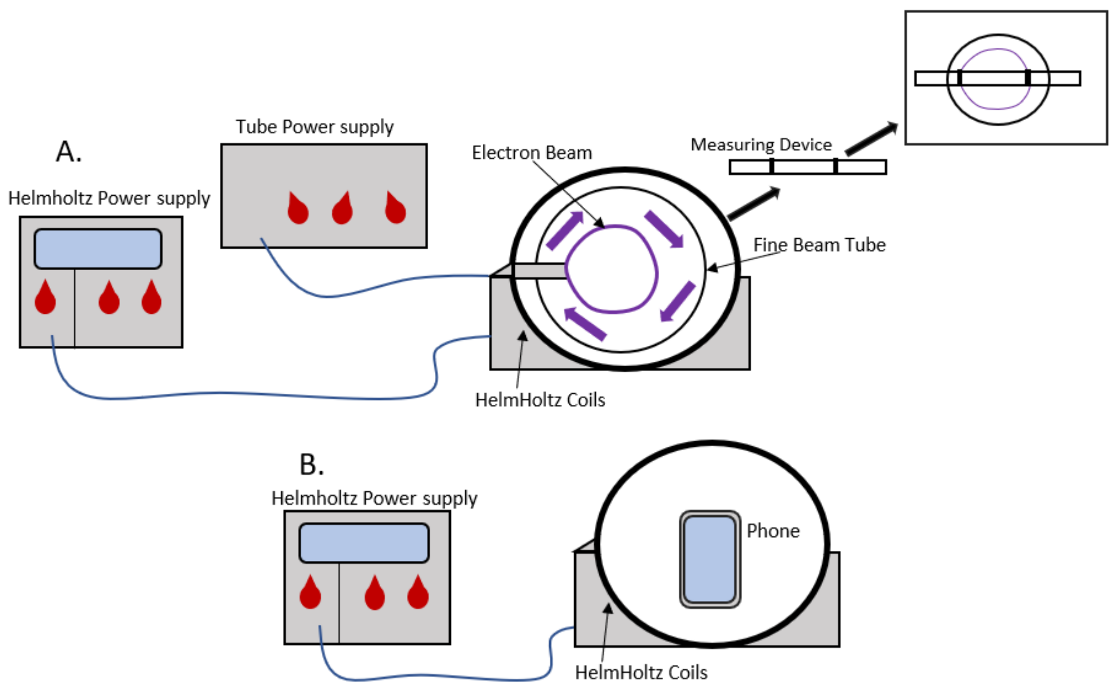

FIG. 1. Our setup for measuring e/m with parts A. and B.

We conducted this experiment in a dark room so that we could see the electron beam more accurately within the fine beam tube. The Electron Beam in Figure 1 was powered by a Leybold power supply, set to an acceleration voltage of 300 Volts. The Helmholtz coils were also given a current of 1.9 Amps by a DC power supply.

This setup allowed for our electron beam to start a circular motion curving away from us. We then focused the beam by adjusting the voltage emitting from the Leybold power supply. We did this until we found a voltage that resulted in a strong edge defining the circular motion of our beam.

We then attached the measuring device shown in Figure 1 to give off a gap of 8.0 cm, and aligned it to be parallel to that of our electron beam. Now, adjusting the current driving the Helmholtz coils until the circular orbit of the electron beam falls to the same gap of our measuring device, we were left with a circle that has a radius of $\sim$ 4.0 cm (3.937 cm to be exact).

Writing down our potential (V) and current (Amp), we then decreased the potential in steps of approximately 10 V, and adjusted the current at each of these positions to keep the electron beam at $\sim$4.0 cm. We did this procedure a total of 11 times, ensuring to markdown our error for each measurment.

For part B of our e/m experiment, we shut down our Leybold power supply and removed the fine beam tube. We then used a magnetometer app on our smartphone called "Gauges" [8] to measure the magnetic field in the center of the Helmholtz coils. We did 7 measurements, ensuring to cover the same range to that our current covered in part A.

3. *Results*

Now, performing a least squares fit on our data in part A to find k_2:

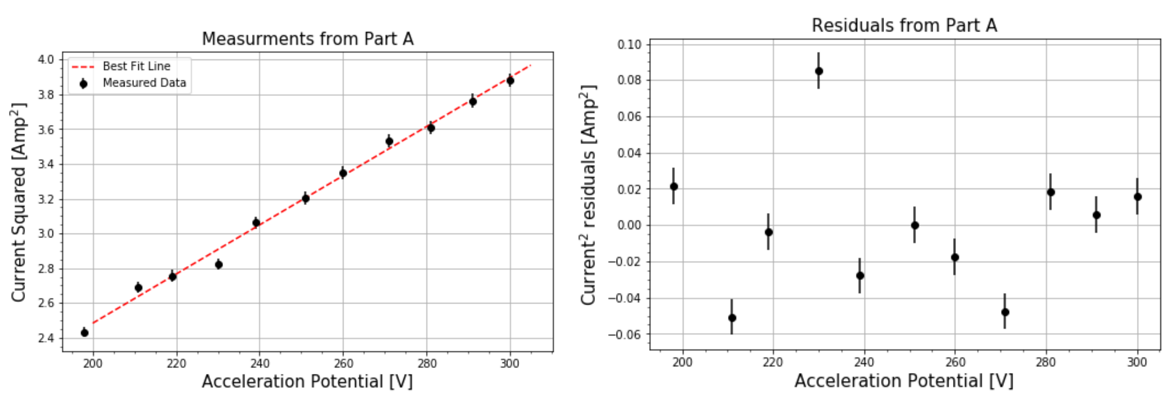

FIG. 2. Our Results from Experiment A, we found our slope k_2 to be: $k_2 = 0.01413(28)$ Amps^2/V.

Similarly, we were able to find k_1 by performing a least squares fit on our data from part B, the slope of which will be our desired value.

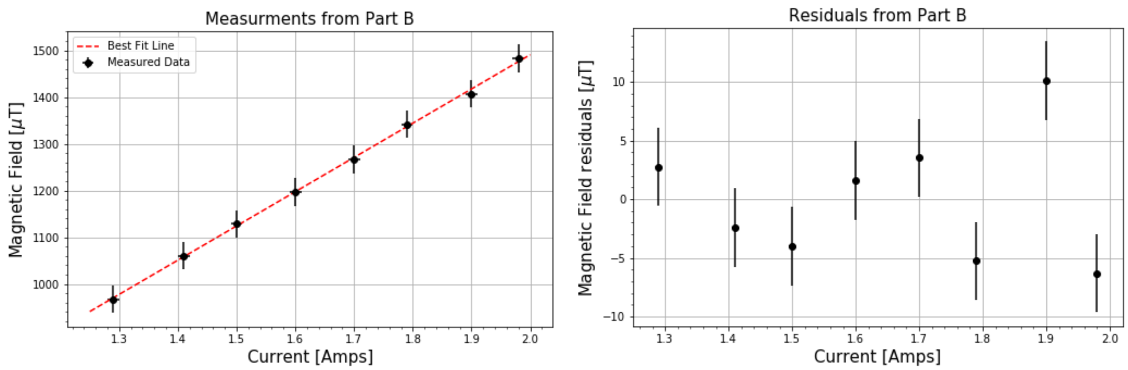

FIG. 3. Our Results from Experiment B, we found our slope k_1 to be: $k_1 = 732.821(61)$ μT/Amps.

Now using equation (4), we were able to find our constant e/m to be:

$$\frac{e}{m} = 1.700436 \times 10^{11} \text{ C/kg}$$

With an uncertainty of:

$$\Delta\frac{e}{m} = 0.0990137 \times 10^{11} \text{ C/kg}$$

B. Measurement of R_∞

1. Introduction

We can find the Rydberg constant by examining Schrödinger's equation for hydrogen [7]:

$$E_n = -hcR_M\frac{1}{n} \quad \text{where} \quad R_M = R_\infty\frac{1}{1+m/M} \tag{5}$$

Where R_∞ in equation (5) is the Rydberg Constant, m is the mass of the electron, M is the mass of the Nucleus, E_n is the energy, and n is the principle quantum number. We can rearrange this equation to find the energy emitted by a photon changing from the principle quantum number n_1 to n_2 as shown below.

$$\Delta E_{n_1n_2} = hcR_M\Big(\frac{1}{n_2^2} - \frac{1}{n_1^2}\Big) \tag{6}$$

Where the frequency is then:

$$\frac{1}{\lambda_{n_1 n_2}} = R_M\Big(\frac{1}{n_2^2} - \frac{1}{n_1^2}\Big) \tag{7}$$

2. Methodology

To conduct the measurement for the Rydberg constant (R_∞), we will be measuring the wavelength of the Balmer series, where our principle quantum number n_2 is 2. We will be using a much simpler apparatus shown below in Figure 4. Our apparatus includes a hydrogen discharge tube, an optical fiber (or our spectroscope), a spectrometer, and a computer using "Ocean View" software [3].

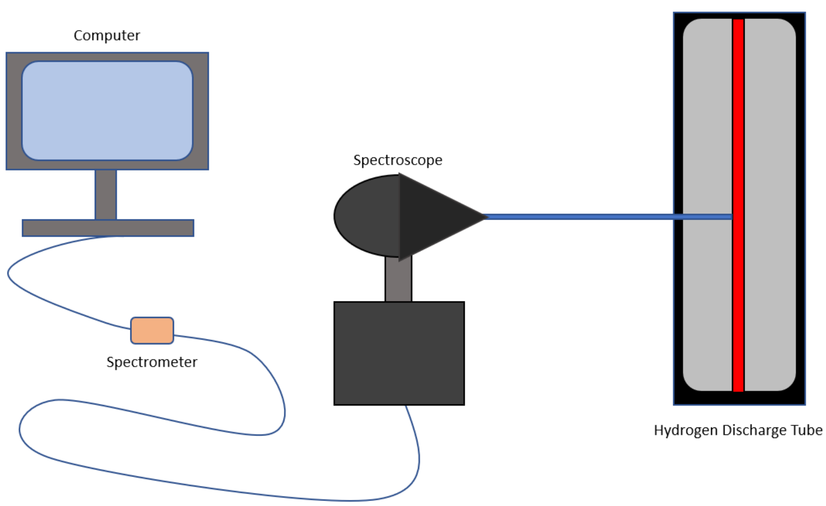

FIG. 4. Our setup for measuring the Rydberg Constant.

The hydrogen discharge tube disassociates hydrogen molecules into atomic hydrogen and excites the resulting hydrogen atoms to high lying states so we can observe the emission spectrum. This spectrum is then collected by our optical fiber, and then translated into a CCD array from our spectrometer (Ocean Optics model USB-4000). We then can see our spectrum on the computer using the "Ocean View" software.

After booting up our equipment and our "Ocean view" software, we were able to observe and record the Balmer series transition lines in the spectral lines of hydrogen. We did this by adjusting the integration time, number of averages, and the distance our optical fiber is held from the hydrogen discharge tube to more accurately detect the wavelength of our spectral lines.

3. Results

We were then able to find the slope, R_M, by plotting the wavelengths of the Balmer series lines we detected vs. $\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$ and performing a least square fit.

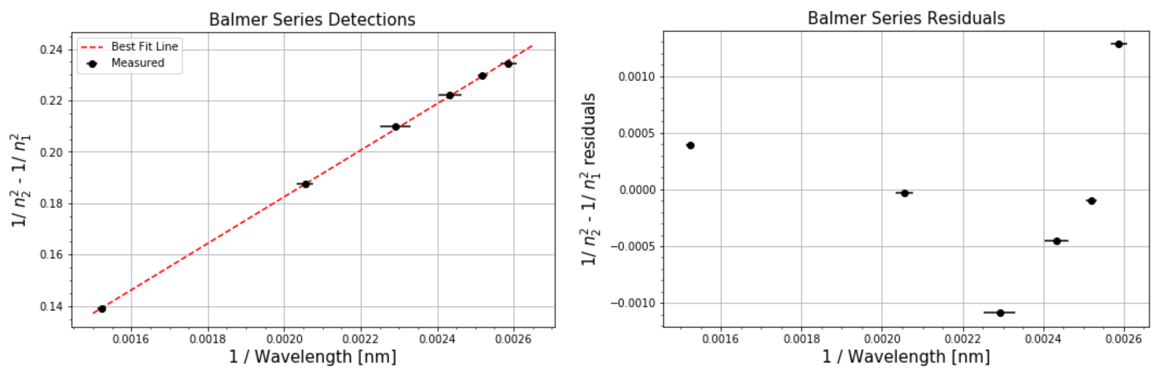

FIG. 5. Our detection's and residuals from measuring the Balmer Series transitions, we found our slope R_M to be: $R_M = 90.628\ \Delta n/\mathrm{nm}$.

We then were able to use equation (5) along with the mass of the proton and electron to calculate R_∞.

$$R_\infty = 100752.73966630323\ \mathrm{cm}^{-1}$$

With an error of:

$$\Delta R_\infty = 1120.91149\ \mathrm{cm}^{-1}$$

C. Measurement of h/e

1. Introduction

We can find h/e by relating the kinetic energy of a light photon to its stopping potential using the equation below [5].

$$KE_{max} = Ve \tag{8}$$

Where we can then use Einstein's equation,

$$hf = Ve + W_o \tag{9}$$

Solving for our voltage V, we can rearrange equation (9) to be:

$$V = \frac{h}{e}f - \frac{W_o}{e} \tag{10}$$

This means, if we plot V vs our frequency f of our lights radiation, the slope of our best-fit line would be equivalent to h/e.

2. Methodology

To to this, we used the PASCO AP-9368. The apparatus for which can be seen in Figure 6, includes a Mercury lamp with a lens grating attached, a photo diode held held within the h/e apparatus, a voltmeter, and yellow and green colored filters.

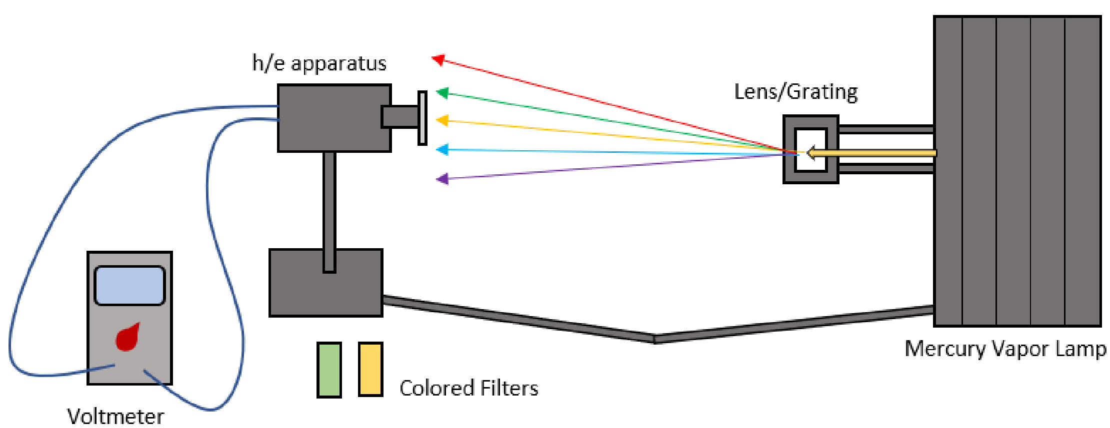

FIG. 6. Our setup for measuring h/e.

We then found the stopping potential with our voltmeter for each color refracted from our Mercury lamp. To accurately measure our data, we ensured to use the yellow and green filters when measuring the stopping potential of our yellow and green emission lines respectively.

3. *Results*

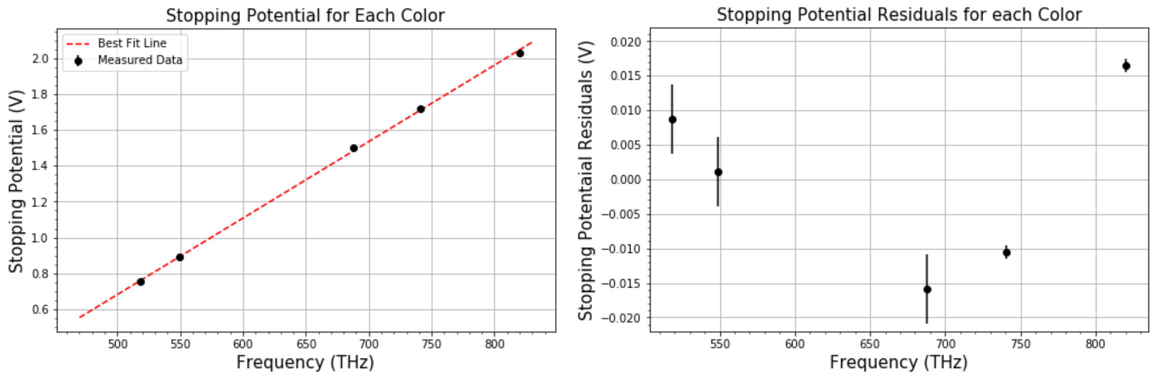

FIG. 7. Our Detection and residuals from each color we observed.

We then found h/e by measuring the slope of our Best-fit line and converting our units from V/THz to J/A, giving us a value of:

$$\frac{h}{e} = 4.270204 \times 10^{-15} \text{ J/A}$$

With the error:

$$\Delta\frac{h}{e} = 6.035508 \times 10^{-17} \text{ J/A}$$

D. Measurement of c

1. *Introduction*

Our final experiment concludes in measuring the speed of light. To do this, we will be measuring the time delay between when a laser is shut off, to when the laser gets reflected off a movable mirror and then back onto a detector. If we adjust the movable mirror and plot its position as a function to the time delay from our laser, we could then find the speed of light by finding the slope of our data's best-fit line and setting that equal to the value $2/c$. We used $2/c$ as it takes the laser to travel twice the distance to the detector (to the mirror and back) than it does to the mirror.

2. Methodology

To do do this we used a solid state laser which is a Power Technology model LDCU5/5894, an oscilloscope, a photo detector, two 50 Ω terminators, and a setup of mirrors seen below in Figure 8.

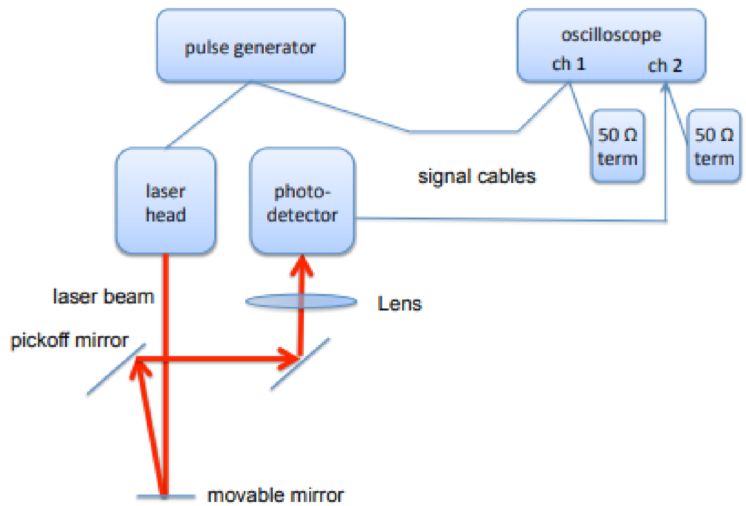

FIG. 8. The setup for speed of light measurement. Thanks to Dr. George R. Welch [1].

The output of the laser is held at $\lambda = 639$ nm and pulsed and controlled with the Global Specialties model 4001 electronic pulse generator. The pulse generator is made to make a rectangular waveform with a frequency in the 1-100 kHz range with a very high duty cycle. This allowed us to measure the transition from on to off. The resulting settings allowed us to get the following result in Figure 9 from our Tektronix TDS 2024B Oscilloscope.

We decided to determine a set position to be the section where the blue curve (the input from our detector) makes a stark drop down and becomes almost inline with our yellow curve (the input form the pulse generator). We then used the cursor function on our oscilloscope to measure the time delay by finding the time difference between where the yellow and blue curves first intersect, and then where they pass our set position.

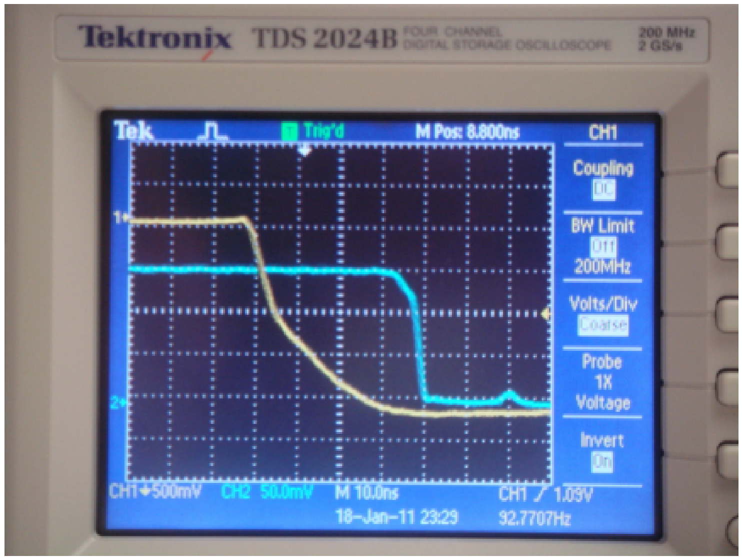

FIG. 9. Our Oscilloscope display with the movable mirror measured 1.5 meters from the laser. Thanks to Dr. George R. Welch [1].

We did this procedure a total of 9 times while adjusting the mirror position between 0.5 and 2.0 meters. Each time finding the time delay between our intersection, and our changing set position.

3. Results

Now, plotting our data and performing a least squares fit on it to find the slope of its best-fit line:

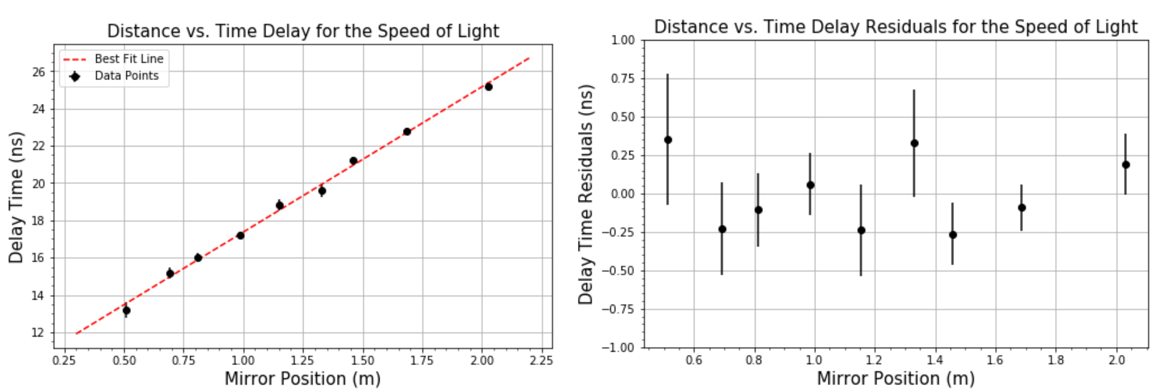

FIG. 10. Our Detection and residuals measuring the speed of light, we found our slope to be equal to $2/c = 7.798851$ m/ns.

Now, letting our slope equal to $2/c$, we found c to be:

$$c = 294913946.276 \text{ m/s}$$

With the uncertainty of:

$$\Delta c = 7050192.801 \text{ m/s}$$

III. ANALYSIS AND RESULTS

Our results for the four experiments we conducted are compressed together onto the table listed below:

TABLE I. Results from our four experiments.

Constant	Value	Uncertainty
e/m	1.700436×10^{11} C/kg	0.0990137×10^{11} C/kg
R_∞	100752.73966630323 cm^{-1}	1120.91149 cm^{-1}
h/e	4.270209×10^{-15} J/A	6.035508×10^{-17} J/A
c	294913946.276 m/s	7050192.801 m/s

We can find our constants $\hbar$, e and m_e by letting each of those values be equivalent to the constants we found. An example can be seen below:

$$Q = a\Big(\frac{h}{e}\Big)^{\alpha}\Big(\frac{e}{m}\Big)^{\beta}(c)^{\delta}(R_\infty)^{\lambda} \tag{11}$$

where,

$$R_\infty = \frac{me^4}{8\epsilon_0^2 h^3 c} \tag{12}$$

Using equation (12) with equation (11), we can find our constants by first solving for h by letting $Q = h$.

$$h = a\Big(\frac{h}{e}\Big)^{\alpha}\Big(\frac{e}{m}\Big)^{\beta}(c)^{\delta}\Big(\frac{me^4}{8\epsilon_0^2 h^3 c}\Big)^{\lambda} \tag{13}$$

This gives us:

$$1 = \alpha - 3\lambda, \quad 0 = -\alpha + \beta + 4\lambda, \quad 0 = -\beta + \lambda \quad 0 = -\lambda + \delta, \text{ and } \quad a = (8\epsilon_0^2)^\lambda$$

When solving for h, e, m, c, and a constant respectively. Where $\lambda = \beta = \delta = 1/2$, $\alpha = 5/2$, and $a = \sqrt{8}\epsilon_0$. We can use these values, and the values of our constants found in Table I, with equation (13) to get an estimate for h.

$$h = 6.70769 \times 10^{-34} \text{ J s}, \quad \text{with the uncertainty,} \quad \Delta h = 1.07439 \times 10^{-36} \text{ J s}$$

We can now use this value for h to find Planck's constant $\hbar$ by dividing h by 2π as $\hbar = h/2\pi$. Our value for $\hbar$ is:

$$\hbar = 1.06756 \times 10^{-34} \text{ J s}, \quad \text{with the uncertainty,} \quad \Delta\hbar = 1.710819 \times 10^{-36} \text{ J s}$$

We could also use our measurment for h to find e by dividing h by h/e, resulting in just the electron charge. Our value for e is:

$$e = 1.57081 \times 10^{-19} \text{ C}, \quad \text{with the uncertainty,} \quad \Delta e = 2.51602 \times 10^{-21} \text{ C}$$

Now, using our value for e, we can find the mass of the electron, m_e, by using the same method we just employed. Dividing our value for e by our value for e/m, we will be left with the electron mass of:

$$m_e = 9.23769 \times 10^{-31} \text{ kg}, \quad \text{with the uncertainty,} \quad \Delta m_e = 1.47963423 \times 10^{-33} \text{ kg}$$

Combining our findings together onto another table:

TABLE II. The values and Uncertainties of our desired constants.

Constant	Value	Uncertainty
$\hbar$	1.06756×10^{-34} J s	1.710819×10^{-36} J s
e	1.57081×10^{-19} C	2.51602×10^{-21} C
m_e	9.23769×10^{-31} kg	$1.47963423 \times 10^{-33}$ kg

IV. CONCLUSION

We conducted four separate experiments in order to correctly measure and identify the specific charge of the electron e/m, Rydberg's constant R_∞, the speed of light c, and the photoelectric effect h/e. All four of these experiments were conducted separately with different experimental setups and equipment. This means that our methodology ensured the minimization of confounding effects when obtaining values for Planck's constant $\hbar$, the electron charge e, and the electron mass m_e. Additionally, when we include our uncertainty and the values we measured in Table II for $\hbar$, e, and m_e, we can see that our measurements fall closely to that of the accepted values for each constant. Overall, we are fairly confident that, due to the reasons listed above, our results suffer from little to no systemic error.

ACKNOWLEDGMENTS

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[2] George R. Welch, 2018. "Photoelectric effect — h/e".

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