Frank Cline
Math 307
HW 2

3.1

4. The following relations are defined on N.

   (a) Write the relation R_1 defined by $(m, n) \in R_1$ if $m + n = 5$ as a set of ordered pairs.
   $R_1 = \{(0, 5), (1, 4), (2, 3), (3, 2), (4, 1), (5, 0)\}$

   (b) Do the same for R_2 defined by $\max\{m, n\} = 2$.
   $R_2 = \{(0, 2), (1, 2), (2, 2), (2, 1), (2, 0)\}$

   (c) The relations R_3 defined by $\min\{m, n\} = 2$ consists of infinitely many ordered pairs. List five of them.
   {(2,3),(2,4),(2,5),(2,6),(2,7)}

6. Consider the relation R on $\mathbb{Z}$ defined by $(m, n) \in R$ if and only if $m^3 - n^2 \equiv 0 \bmod(5)$. Which of the properties $(R), (AR), (S), (AS)$, and (T) are satisfied by R?

   - Not (R) because if $(m, n) = (3, 3)$ then $3^3 - 3^2 = 27 - 9 = 18$ and $5 \nmid 18$, so 3 is not related to itself. Thus it's not reflexive.
   - Not (AR) because if $m, n = 5$ then $5^3 - 5^2 = 125 - 25 = 100$ and $5 \mid 100$, so 5 is related to itself. Thus it's not antireflexive.
   - Not (S) because if $m = 1, n = 4$ then $1^3 - 4^2 = 1 - 16 = -15$ and $5 \mid -15$, so 1 is related to 4. if $m = 4, n = 1$ then $4^3 - 1^2 = 64 - 1 = 63$ and $5 \nmid 63$, so 4 is not related to 1. Since 1 is related to 4 and 4 is not related to 1, it's not symmetric.
   - Not (AS) because if $m = 5, n = 0$ then $5^3 - 0^2 = 125$ and $5 \mid 125$, so 5 is related to 0. if $m = 0, n = 5$ then $0^3 - 5^2 = -25$ and $5 \mid -25$, so 0 is related to 5. Since 0 is related to 5 and 5 is related to 0, but $5 \neq 0$ it's not antisymmetric.
   - It is (T).