Math F307   Spring 2017
Homework Exercises

Instructions for writing up homework.

- Although you are encouraged to work with your classmates on the homework assignments, you must write up the solutions individually.

- Read the section!

- Use pen or non-smeary pencil. Do not have little scritchies on the side.

- Staple your homework.

- Write legibly, and leave lots of whitespace. Make sure your writing is dark enough to be readable. If this is problematic for you, consider typing your homework.

- Answer the question in complete sentences, where appropriate.

- **Write the** boxed:problem statements **as well as the answers.** This is not optional. Points will be deducted if you do not do this.

- Have headings indicating the section.

- **In the upper right hand corner of your top page,** write your name (first and last) on your assignment, along with the course number (Math 307) and which assignment it is.

- **You are expected to ask questions in class about the homework problems!**

HOMEWORK SET 8

DUE Friday, March 31 at the beginning of class.

Graph coloring and planar graphs. To turn in:

(1) Find the chromatic number $\chi(G)$ of each of the graphs shown in Figure 1. To do so: (1) provide a coloring (with actual colors) for each of the graphs using $\chi(G)$ colors, and (2) explain why you cannot use fewer colors.

(2) Give examples of a graph (that are not examples from class or the worksheet) for which

   (a) $\chi(G) = \Delta(G)$

   (b) $\chi(G) < \Delta(G)$.

(3) Suppose a graph G has chromatic number 1. What can you say about G?

(4) (a) Use the greedy algorithm to color the graph G in Figure 2. How many colors did you use?

   (b) Determine the chromatic number of G. Justify that the number you find really is the number of colors needed.

(5) The math department is trying to schedule focus group interviews with students for certain classes outside of the ordinary class time. The courses have the following (entirely made up) student overlaps:

	Math 265	Math 314	Math 307	Math 490	Math 405	Math 422
Math 265		1	5	0	0	2
Math 314	1		0	0	0	0
Math 307	5	0		1	0	2
Math 490	0	0	1		4	2
Math 405	0	0	0	4		4
Math 422	2	0	2	2	4	

(a) Construct a graph representing the student overlaps (that is, assign the vertices to be the classes, and connect the vertices with an edge if there are students in both classes, labelled with the number of students in both classes).

(b) How many meeting times are needed? Explain, briefly.

(c) Suppose only three meeting times are available, at 9AM, 10AM and 11AM, and furthermore, suppose that only one class is allowed to meet at 9AM. Is it possible to schedule the focus groups with this restriction? If so, give a possible schedule. If not, explain why not.

(6) There are seven tour bus companies in the Los Angeles Area. During a particular day, each visits at most three locations from among Hollywood, Beverly Hills, Disneyland, and Universal Studios. The same location cannot be visited by more than one company on the same day. The first tour company visits only Hollywood, the second only Hollywood and Disneyland, the third only Universal Studios, the fourth only Disneyland and Universal Studios, the fifth Hollywood and Beverly Hills, the sixth Beverly Hills and Universal Studios, and the seventh Disneyland and Beverly Hills. Can these tours be scheduled only on Monday, Wednesday and Friday? Support/explain your answer.

(7) Prove that the Grötzsch graph G_5, shown in Figure 3a, has $\chi(G) = 4$ by doing the following:

(a) Find a 4-coloring of G.

(b) Suppose that G has a 3-coloring, say using red, green, blue. Without loss of generality, we may suppose the center vertex is colored red. Explain why this forces a contradiction.

(c) The Grötzsch graph is a member of an infinite family of triangle-free graphs, . The graph G_6 is shown in Figures 3b. What is $\chi(G_6)$?
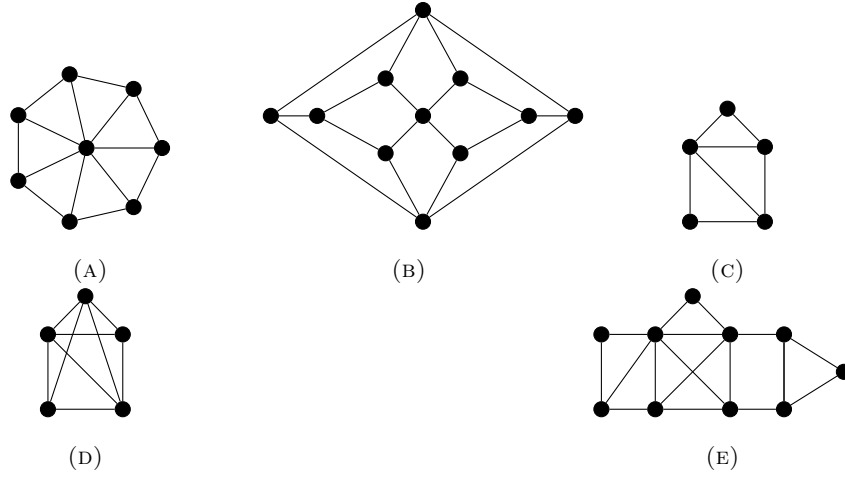
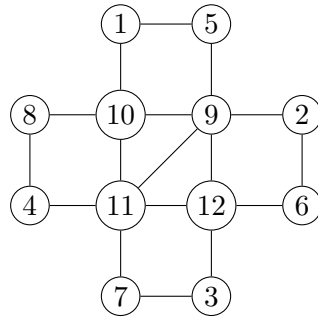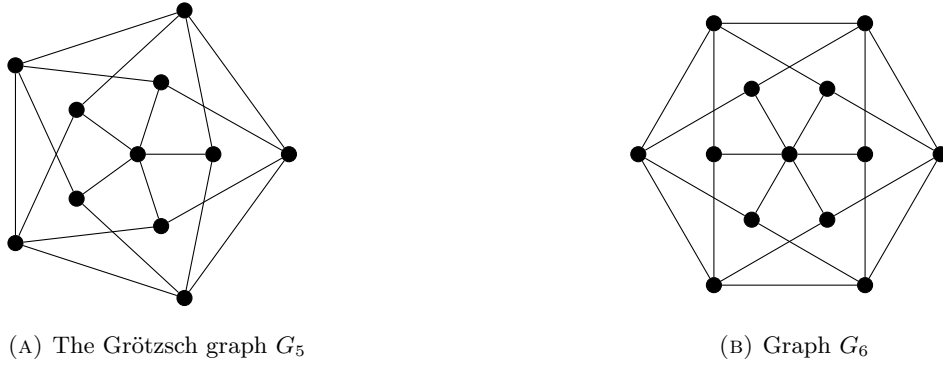FIGURE 1. Determine the chromatic number of the following graphs.

FIGURE 2. A graph

(A) The Grötzsch graph G_5

(B) Graph G_6

FIGURE 3. The Grötzsch graph and a generalization