Frank Cline
Math 307
HW 4

3.4

1. Which of the following describe equivalence relations? For those that are not equivalence relations, specify which of $(R), (S), (T)$ fail and illustrate the failures with examples.

   (a) $L_1 || L_2$ for straight lines in the plane if L_1 and L_2 are the same or are parallel.

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   (b) $L_1 \perp L_2$ for straight lines in the plane if L_1 and L_2 are perpendicular.

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   (c) $p_1 \sim p_2$ for Americans if p_1 and p_2 live in the same state.

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   (d) $p_1 \approx p_2$ for Americans if p_1 and p_2 live in the same state or in neighboring states.

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   (e) $p_1 \approx p_2$ for people if p_1 and p_2 have a parent in common.

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   (f) $p_1 \cong p_2$ for people if p_1 and p_2 have the same mother.

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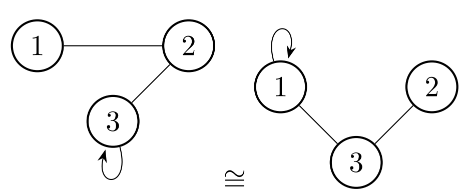
2. For each example of an equivalence relation in Exercise 1, describe the members of some equivalence class.

   (a) DO WORK HERE

5. If G and H are both graphs with vertex set $\{1, 2, ..., n\}$, we say that G is isomorphic to H, and write $G \cong H$, in case there is a way to label the vertices of G so that it becomes H.



   (a) Give a picture of another graph isomorphic to these two.

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   (b) Find a graph with vertex set $\{1, 2, 3\}$ that is not isomorphic to the graphs yet has three edges and exactly one is a loop.

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(c) Find another example as in part(b) that isn't isomorphic to the answer of part(b) and the other two graphs.

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(d) Show that $\cong$ is an equivalence relation on the set of all graphs with the vertex set $\{1, 2, ..., n\}$.

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8. (a) For $m, n \in \mathbb{Z}$, define $m \sim n$ in case $m - n$ is odd. Is the relation reflexive? symmetric? transitive? Is it an equivalence relation?

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(b) For a and b in $\mathbb{R}$, define $a \sim b$ in case $a - b \leq 1$. One could say that $a \sim b$ in case a and b are close enough or approximately equal. Answer the question in part (a).

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17. (a) Verify that the relation $\cong$ defined in Example 5b (the reachable Relation R on $V(G)$ by $(v, w) \in R$) is an equivalence relation on $V(G)$.

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(b) Given a vertex v in $V(G)$, describe in words the equivalence class containing v.

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