Frank Cline

Math 307

HW 7

6.3

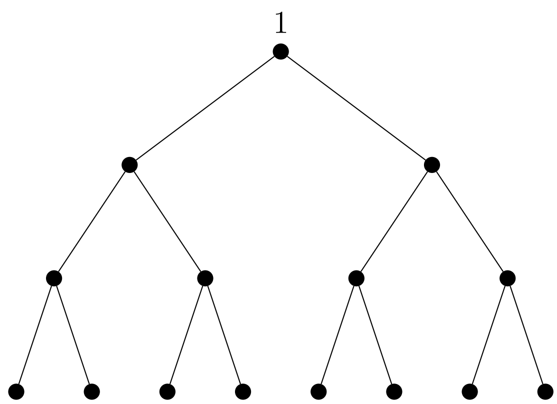
8. Consider a tree with n vertices. It has exactly $n-1$ edges [Lemma 2], so the sum of its of the degrees of its vertices is $2n-2$.

   (a) A tree has two vertices of degree 5, three of degree 3, two of degree 2, and the rest of degree 1. How many vertices are in the graph?
   $2|E(G)| = 2n-2 = 2(5)+3(3)+2(2)+x$

11. (a) Show that a forest with n vertices and m components has $n-m$ edges.
    Suppose the components have $n_1, n_2, n_3, ..., n_m$ vertices. The total vertices of the forest is then $n_1 + n_2 + n_3 + ... + n_m = n$. The ith component is a tree, so it has $n_i - 1$ edges by Theorem 4. The total amount of edges in the forest is then $(n_1-1)+(n_2-1)+(n_3-1)+...+(n_m-1) = n-m$.

6.4

6. (a) Draw each of the seven types of rooted trees of height 3 in which each node that is not a leaf has 2 children.



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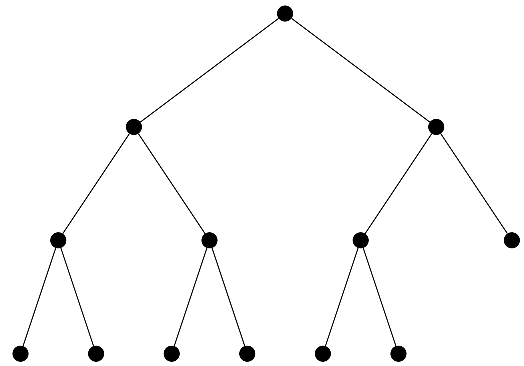

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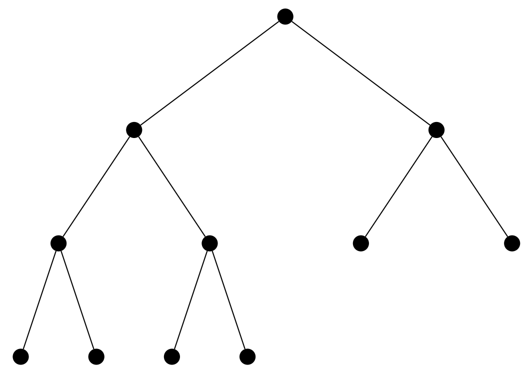

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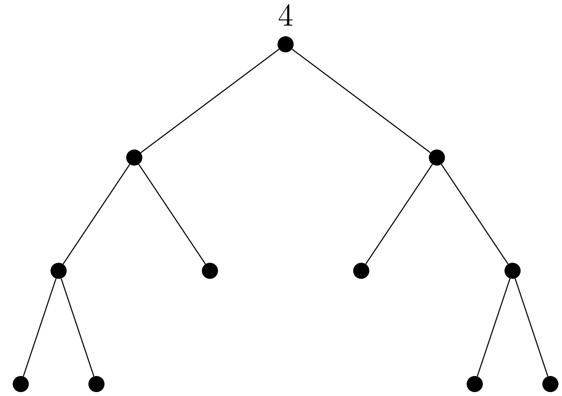

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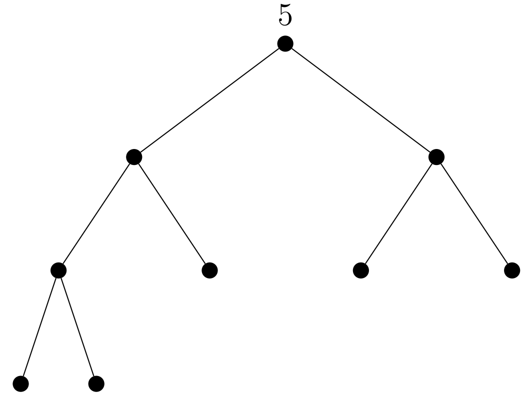

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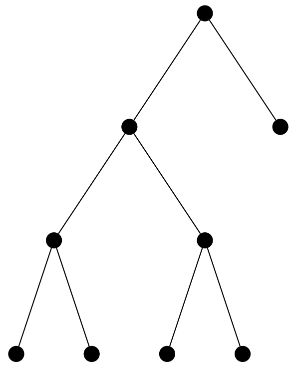

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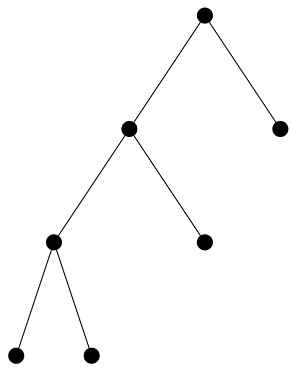
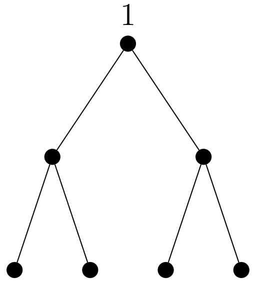


8. A 2-3 tree is a rooted tree such that each interior node, including the root if the height is 2 or more, has either two or three children and all paths from the root to the leaves habe the same length. There are seven different types of 2-3 trees of height 2. Draw one tree of each type.

1



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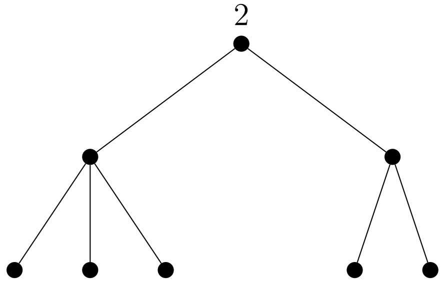

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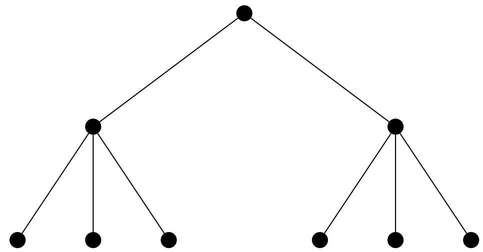

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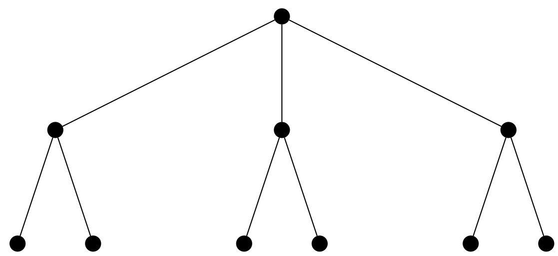

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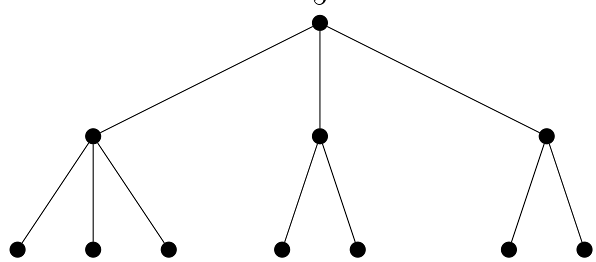

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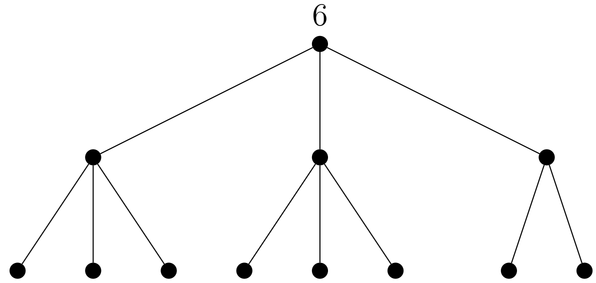

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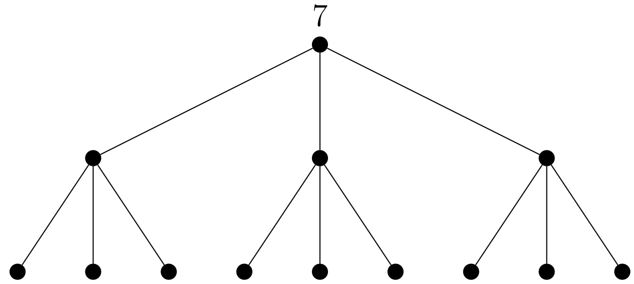


10. Consider a full binary tree T of height h.

    (a) How many leaves does T have?
    A full binary tree has 2^h leaves.

(b) How many vertices does T have?
A full m-ary tree has $\frac{m^{h+1}-1}{m-1}$ vertices, so
a full binary tree has $\frac{2^{h+1}-1}{2-1} = 2^{h+1} - 1$ vertices.

12. Give some real-life examples of information storage that can be viewed as labeled trees. A labeled tree can be used to visualize a family tree, and a labeled tree can also be used to visualize the syntax of a language.

1. Additional Problem: Draw a tree with Prufer code (5,2,1,4,4,1,6,1,1), with no crossing edges. Show some work.

$$(5, 2, 1, 4, 4, 1, 6, 1, 1) \to \quad (\cancel{5}, 2, 1, 4, 4, 1, 6, 1, 1) \to$$
$$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\} \to \quad \{1, 2, \cancel{3}, 4, 5, 6, 7, 8, 9, 10, 11\} \to$$

$$(\cancel{5}, \cancel{2}, 1, 4, 4, 1, 6, 1, 1) \to \quad (\cancel{5}, \cancel{2}, \cancel{1}, 4, 4, 1, 6, 1, 1) \to$$
$$\{1, 2, \cancel{3}, 4, \cancel{5}, 6, 7, 8, 9, 10, 11\} \to \quad \{1, \cancel{2}, \cancel{3}, 4, \cancel{5}, 6, 7, 8, 9, 10, 11\} \to$$

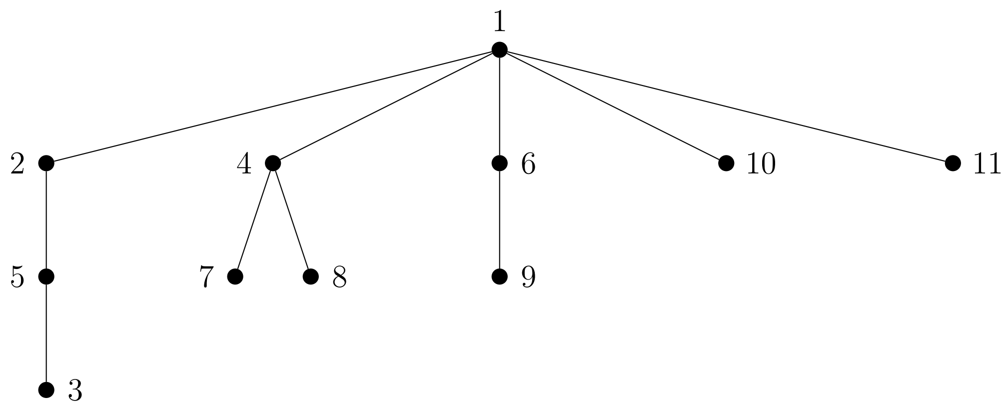
$$(\cancel{5}, \cancel{2}, \cancel{1}, \cancel{4}, 4, 1, 6, 1, 1) \to \quad (\cancel{5}, \cancel{2}, \cancel{1}, \cancel{4}, \cancel{4}, 1, 6, 1, 1) \to$$
$$\{1, \cancel{2}, \cancel{3}, 4, \cancel{5}, 6, \cancel{7}, 8, 9, 10, 11\} \to \quad \{1, \cancel{2}, \cancel{3}, 4, \cancel{5}, 6, \cancel{7}, \cancel{8}, 9, 10, 11\} \to$$

$$(\cancel{5}, \cancel{2}, \cancel{1}, \cancel{4}, \cancel{4}, \cancel{1}, 6, 1, 1) \to \quad (\cancel{5}, \cancel{2}, \cancel{1}, \cancel{4}, \cancel{4}, \cancel{1}, \cancel{6}, 1, 1) \to$$
$$\{1, \cancel{2}, \cancel{3}, \cancel{4}, \cancel{5}, 6, \cancel{7}, \cancel{8}, 9, 10, 11\} \to \quad \{1, \cancel{2}, \cancel{3}, \cancel{4}, \cancel{5}, 6, \cancel{7}, \cancel{8}, \cancel{9}, 10, 11\} \to$$

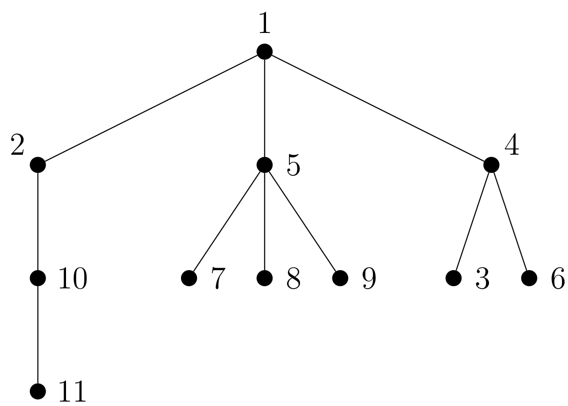
$$(\cancel{5}, \cancel{2}, \cancel{1}, \cancel{4}, \cancel{4}, \cancel{1}, \cancel{6}, \cancel{1}, 1) \to \quad (\cancel{5}, \cancel{2}, \cancel{1}, \cancel{4}, \cancel{4}, \cancel{1}, \cancel{6}, \cancel{1}, \cancel{1}) \to$$
$$\{1, \cancel{2}, \cancel{3}, \cancel{4}, \cancel{5}, \cancel{6}, \cancel{7}, \cancel{8}, \cancel{9}, 10, 11\} \to \quad \{1, \cancel{2}, \cancel{3}, \cancel{4}, \cancel{5}, \cancel{6}, \cancel{7}, \cancel{8}, \cancel{9}, \cancel{10}, 11\} \to$$

$$(\cancel{5}, \cancel{2}, \cancel{1}, \cancel{4}, \cancel{4}, \cancel{1}, \cancel{6}, \cancel{1}, \cancel{1}) \to$$
$$\{\cancel{1}, \cancel{2}, \cancel{3}, \cancel{4}, \cancel{5}, \cancel{6}, \cancel{7}, \cancel{8}, \cancel{9}, \cancel{10}, \cancel{11}\} \to$$

$$\to \{(5,3), (2,5), (1,2), (4,7), (4,8), (1,4), (6,9), (1,6), (1,10), (1,11)\}$$

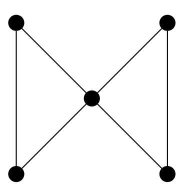


2. Find the Prüfer code of the following tree. Show some work.

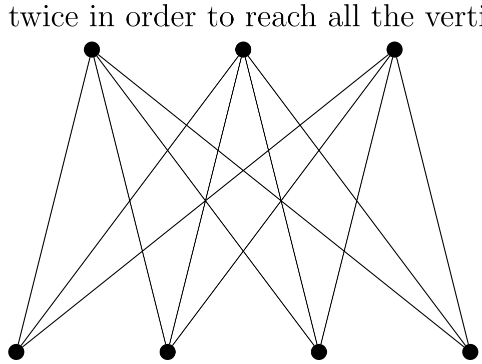


{Leaf (Neighbor),} $\rightarrow$ {3 (4), 6 (4), 4 (1), 7 (5), 8 (5), 9 (5), 5 (1), 1 (2), 2 (10)}
$\rightarrow (4, 4, 1, 5, 5, 5, 1, 2, 10)$

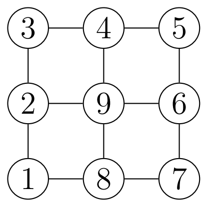
2. For each graph, give a Hamilton circuit or explain why none exists.

(a)


A Hamilton circuit does not exist because the center vertex must always be passed twice in order to reach all the vertices.

(b)


Observe that the graph is bipartite because there exists 2 subsets V_1 and V_2 where every edge in the graph connects a vertex in V_1 to V_2. Suppose The vertices of V_1 are the top 3 and the vertices of V_2 are the bottom 4: $|V_1| = 3$ and $|V_2| = 4$. Notice that $|V_1| \neq |V_2|$ so there can't be a Hamilton circuit by Theorem 4.

(c) 

Observe that the graph is bipartite by splitting the the graph into subgraphs $V_1 = \{1, 3, 5, 7, 9\}$ and $V_2 = \{2, 4, 6, 8\}$. Notice that $|V_1| = 5$ and $|V_2| = 4$. $|V_1| \neq |V_2|$ so by Theorem 4 there can't be a Hamilton circuit.