Frank Cline
Math 307
HW 7

6.3

8. Consider a tree with n vertices. It has exactly $n-1$ edges [Lemma 2], so the sum of its of the degrees of its vertices is $2n-2$.

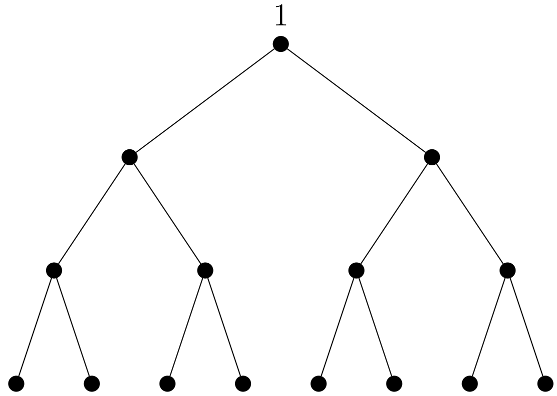
   (a) A tree has two vertices of degree 5, three of degree 3, two of degree 2, and the rest of degree 1. How many vertices are in the graph?
   $2|E(G)| = 2n - 2 = 2(5) + 3(3) + 2(2) + x$

11. (a) Show that a forest with n vertices and m components has $n - m$ edges.
    Suppose the components have $n_1, n_2, n_3, ..., n_m$ vertices. The total vertices of the forest is then $n_1 + n_2 + n_3 + ... + n_m = n$. The ith component is a tree, so it has $n_i - 1$ edges by Theorem 4. The total amount of edges in the forest is then $(n_1 - 1) + (n_2 - 1) + (n_3 - 1) + ... + (n_m - 1) = n - m$.

6.4

6. (a) Draw each of the seven types of rooted trees of height 3 in which each node that is not a leaf has 2 children.

1



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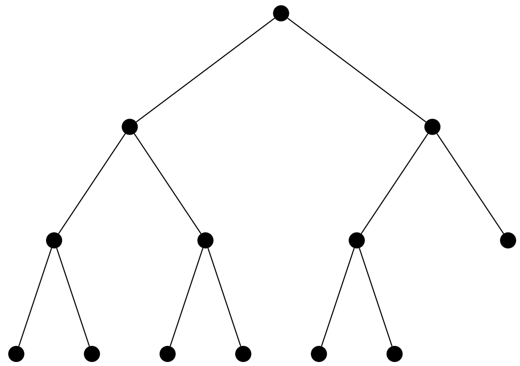

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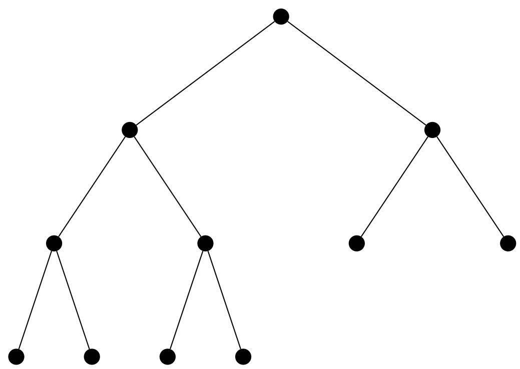

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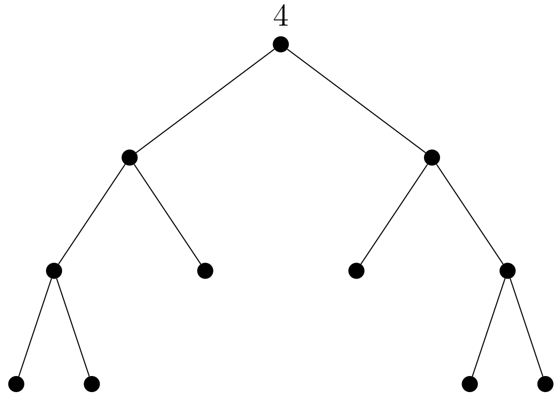

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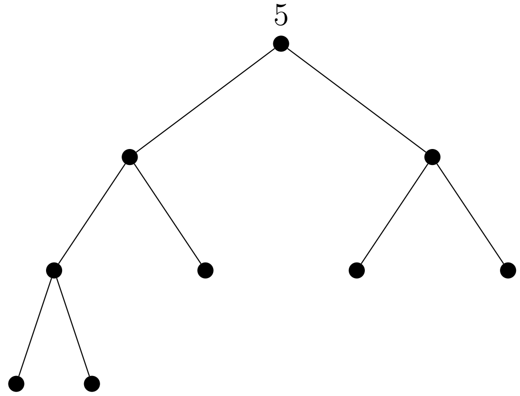

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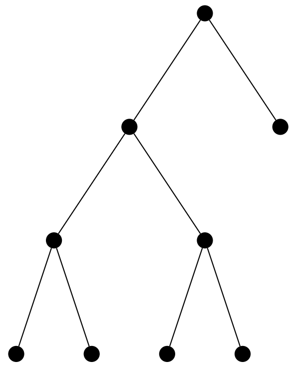

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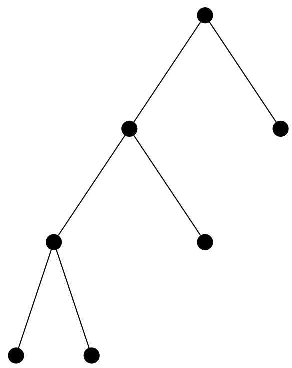
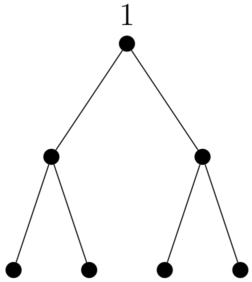


8. A 2-3 tree is a rooted tree such that each interior node, including the root if the height is 2 or more, has either two or three children and all paths from the root to the leaves habe the same length. There are seven different types of 2-3 trees of height 2. Draw one tree of each type.

1



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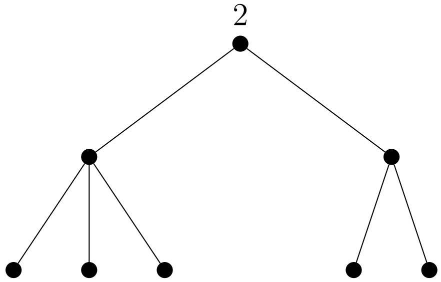

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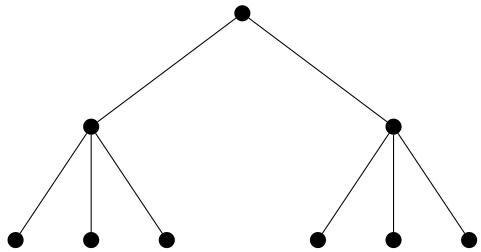

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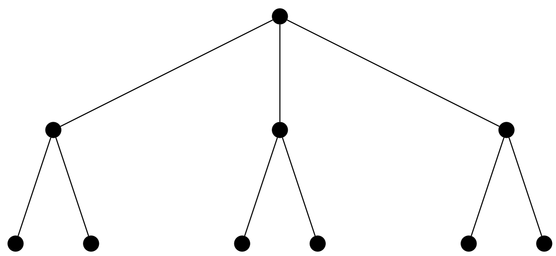

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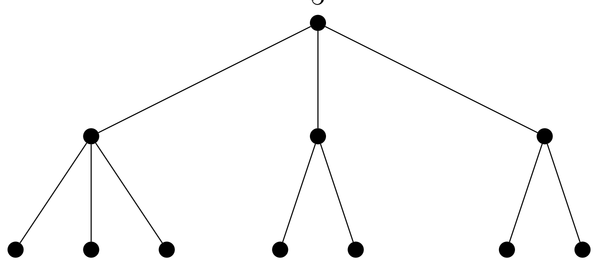

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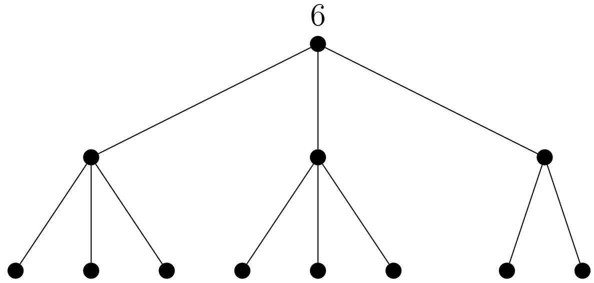

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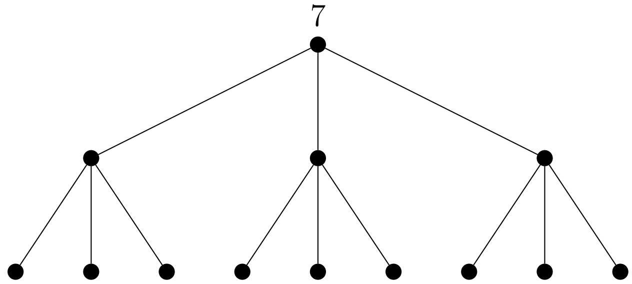

10. Consider a full binary tree T of height h.

    (a) How many leaves does T have?
    A full binary tree has 2^h leaves.

(b) How many vertices does T have?

A full m-ary tree has $\frac{m^{h+1}-1}{m-1}$ vertices, so

a full binary tree has $\frac{2^{h+1}-1}{2-1} = 2^{h+1} - 1$ vertices.

12. Give some real-life examples of information storage that can be viewed as labeled trees.

A labeled tree can be used to visualize a family tree, and a labeled tree can also be used to visualize the syntax of a language.

1. Additional Problem: Draw a tree with Prufer code (5,2,1,4,4,1,6,1,1), with no crossing edges. Show some work.

$$(5,2,1,4,4,1,6,1,1) \to \quad (\cancel{5},2,1,4,4,1,6,1,1) \to \quad (\cancel{5},\cancel{2},1,4,4,1,6,1,1) \to$$
$$\{1,2,3,4,5,6,7,8,9,10\} \to \quad \{1,2,\cancel{3},4,5,6,7,8,9,10\} \to \quad \{1,2,\cancel{3},4,\cancel{5},6,7,8,9,10\} \to$$

$$(\cancel{5},\cancel{2},\cancel{1},4,4,1,6,1,1) \to \quad (\cancel{5},\cancel{2},\cancel{1},\cancel{4},4,1,6,1,1) \to$$
$$\{1,\cancel{2},\cancel{3},4,\cancel{5},6,7,8,9,10\} \to \quad \{1,\cancel{2},\cancel{3},4,\cancel{5},6,\cancel{7},8,9,10\} \to$$