Frank Cline
Math 307
HW 2

3.1

4. The following relations are defined on N.

   (a) Write the relation R_1 defined by $(m, n) \in R_1$ if $m + n = 5$ as a set of ordered pairs.
   $R_1 = \{(0, 5), (1, 4), (2, 3), (3, 2), (4, 1), (5, 0)\}$

   (b) Do the same for R_2 defined by $\max\{m, n\} = 2$.
   $R_2 = \{(0, 2), (1, 2), (2, 2), (2, 1), (2, 0)\}$

   (c) The relations R_3 defined by $\min\{m, n\} = 2$ consists of infinitely many ordered pairs. List five of them.
   {(2,3),(2,4),(2,5),(2,6),(2,7)}

6. Consider the relation R on $\mathbb{Z}$ defined by $(m, n) \in R$ if and only if $m^3 - n^2 \equiv 0 \bmod(5)$. Which of the properties $(R), (AR), (S), (AS)$, and (T) are satisfied by R?

   - R does not satisfy (R) because if $(m, n) = (3, 3)$ then $3^3 - 3^2 = 27 - 9 = 18$ and $5 \nmid 18$, so 3 is not related to itself. Thus it's not reflexive.
   - R does not satisfy (AR) because if $m, n = 5$ then $5^3 - 5^2 = 125 - 25 = 100$ and $5 \mid 100$, so 5 is related to itself. Thus it's not antireflexive.
   - R does not satisfy (S) because if $m = 1, n = 4$ then $1^3 - 4^2 = 1 - 16 = -15$ and $5 \mid -15$, so 1 is related to 4. if $m = 4, n = 1$ then $4^3 - 1^2 = 64 - 1 = 63$ and $5 \nmid 63$, so 4 is not related to 1. Since 1 is related to 4 and 4 is not related to 1, it's not symmetric.
   - R does not satisfy (AS) because if $m = 5, n = 0$ then $5^3 - 0^2 = 125$ and $5 \mid 125$, so 5 is related to 0. if $m = 0, n = 5$ then $0^3 - 5^2 = -25$ and $5 \mid -25$, so 0 is related to 5. Since 0 is related to 5 and 5 is related to 0, but $5 \neq 0$ it's not antisymmetric.
   - R satisfies (T).

7. Define the "divides" relation R on $\mathbb{N}$ by $(m, n) \in R$ if $m \mid n$.

   (a) Which of the properties $(R), (AR), (S), (AS), (T)$ does R satisfy?

   - R satisfies (R).
   - R does not satisfy (AR) because if $m, n = 5$ then $5 \mid 5$ thus $(5, 5) \in R$.
   - R does not satisfy (S) because if $m = 5, n = 10$ then $5 \mid 10$, but if $m = 10, n = 5$ then $10 \nmid 5$ thus $(5, 10) \in R$, but $(10, 5) \notin R$.
   - R satisfies (AS).

- R satisfies (T).

(b) Describe the converse relation $R^{\leftarrow}$.
$R^{\leftarrow}$ is the relation on $\mathbb{N}$ by $(m, n) \in R^{\leftarrow}$ if $n \mid m$.

(c) Which of the properties $(R), (AR), (S), (AS), (T)$ does the converse satisfy?

- $R^{\leftarrow}$ satisfies (R).
- $R^{\leftarrow}$ does not satisfy (AR) because if $m, n = 5$ then $5 \mid 5$ thus $(5, 5) \in R^{\leftarrow}$.
- $R^{\leftarrow}$ does not satisfy (S) because if $m = 10, n = 5$ then $5 \mid 10$, but if $m = 5, n = 10$ then $10 \nmid 5$ thus $(5, 10) \in R^{\leftarrow}$, but $(10, 5) \notin R^{\leftarrow}$.
- $R^{\leftarrow}$ satisfies (AS).
- $R^{\leftarrow}$ satisfies (T).

10. Give an example of the relation that is:

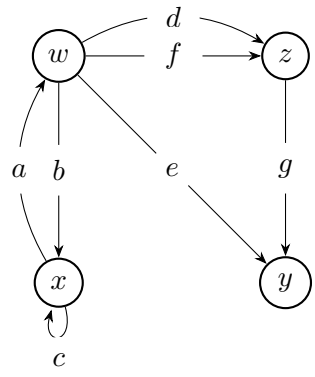
(a) antisymmetric and transitive but not reflexive.
The relation R on $\mathbb{Z}$ by $(m, n) \in R$ if and only if $m < n$.

(b) symmetric but not reflexive or transitive.
The relation R on the set $\{0, 1, 2, 3\}$ by $(m, n) \in R$ if and only if $max(m, n) = 3$.

3.2

2. Draw a picture of the digraph G with vertex set $V(G) = \{w, x, y, z\}$, edge set $E(G) = \{a, b, c, d, e, f, g\}$, and γ given by the following table:

e	a	b	c	d	e	f	g
$\gamma(e)$	(x, w)	(w, x)	(x, x)	(w, z)	(w, y)	(w, z)	(z, y)



3. Which of the following vertex sequences describe paths in the digraph pictured in Figure(7a)?

(a) $zyvwt$
Yes

(b) $xzwt$
Yes

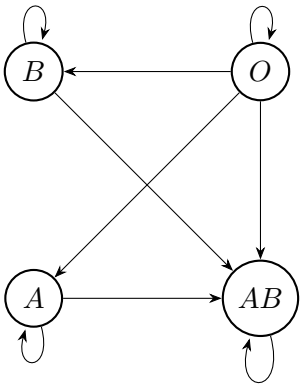
(c) $vstx$
No

(d) $zysu$
No

(e) $xzyvs$
Yes

(f) $suxt$
No

6. There are four basic blood types: A, B, AB, and O. Type O can donate to any of the four types. A and B can donate to AB as well as their own types. AB can only donate to AB. Draw a digraph that presents this information. Is the digraph acyclic?



- Yes the digraph is acyclic because there are no cycles.

8. Determine the reachability relation for the digraps in figures $6(a), (c)$, and (d).

- If R is the reachable relation on $V(G)$, then R is the universal relation for figures $6(a), (c)$, and (d).

16. For the graph in Figure $8(a)$, give an example of each of the following. Be sure to specify the edge sequence and the vertex sequence.

(a) a path of length 2 from w to z.
$w \xrightarrow{d} x \xrightarrow{f} z$

(b) a path of length 4 from z to itself.
$z \xrightarrow{e} x \xrightarrow{f} z \xrightarrow{g} x \xrightarrow{e} z$

(c) a path of length 5 from z to itself.
Impossible.

(d) a path of length 3 from w to x.
$w \xrightarrow{b} y \xrightarrow{b} w \xrightarrow{d} x$

3.3

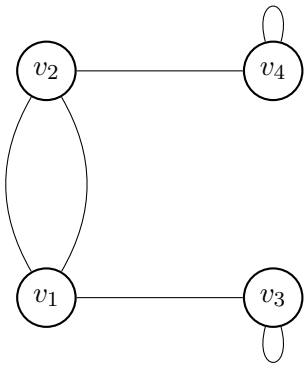
16. Write the matrices for the graphs in Figure 4.

(c) $M = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$

(d) $M = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

18. For each matrix in figure 6, draw a graph having the matrix.

(c) Graph:



(d) Graph: