Frank Cline
Math 307
HW 6

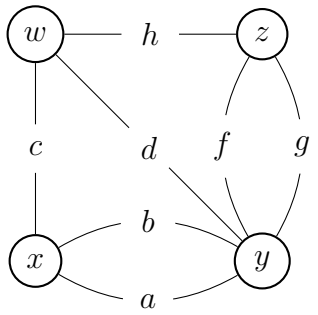
6.1

8. Can a graph have an odd number of vertices of odd degree? Explain.

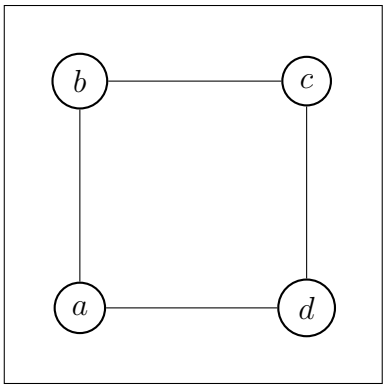
   No.

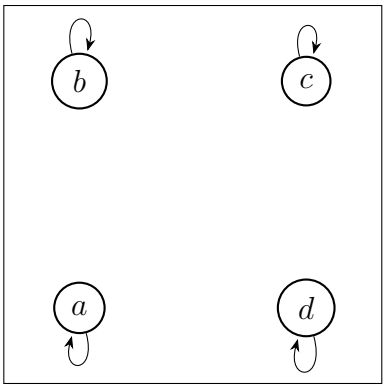
10. Draw a picture of the graph G with $V(G) = \{x, y, z, w\}, E(G) = \{a, b, c, d, f, g, h\}$ and γ as given by the table

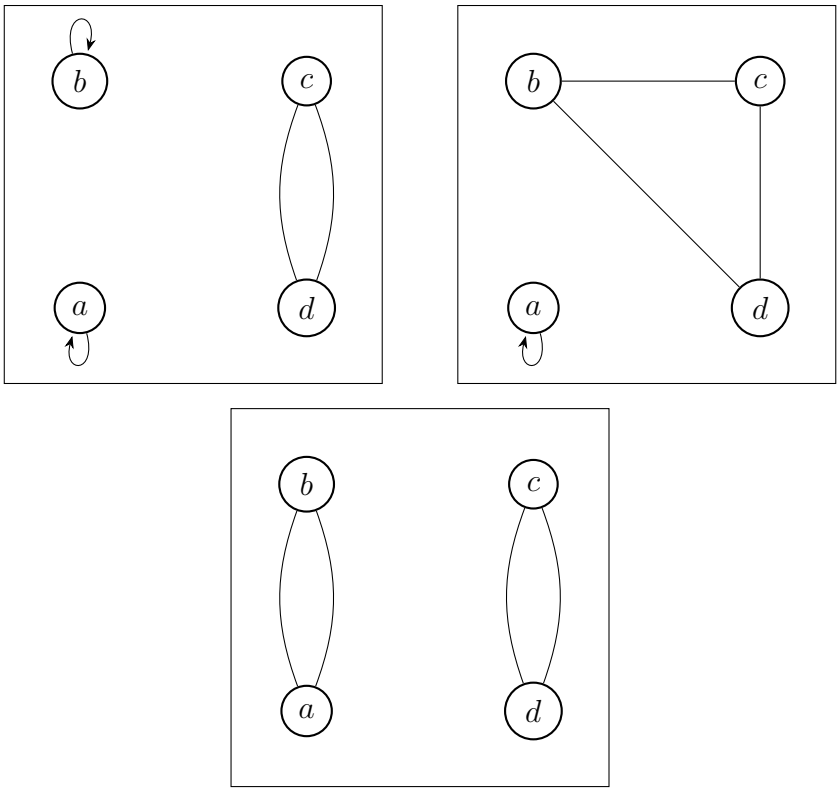
e	a	b	c	d	f	g	h
$\gamma(e)$	$\{x, y\}$	$\{x, y\}$	$\{w, x\}$	$\{w, y\}$	$\{y, z\}$	$\{y, z\}$	$\{w, z\}$



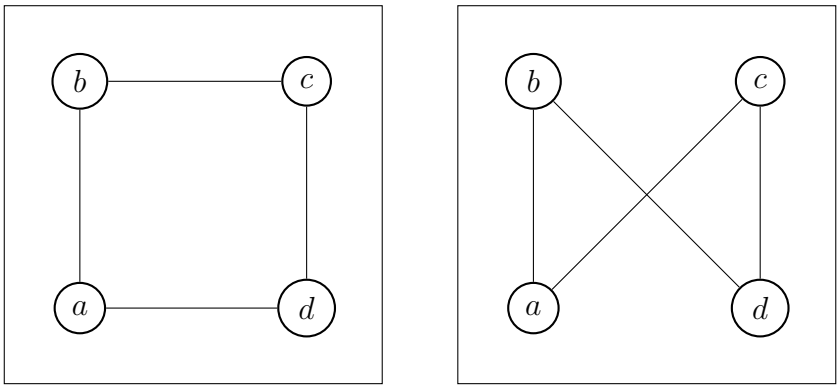
13. (a) Draw pictures of all five of the regular graphs that have four vertices each vertex of degree 2. "All" here means that every regular graph with four vertices and each vertex of degree 2 is isomorphic to one of the five, and no two of the five are isomorphic to each other.





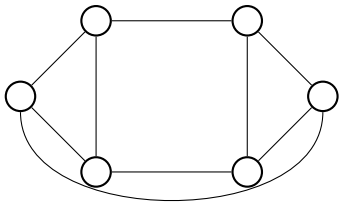


14. (b) Draw pictures of the two graphs with four vertices and four edges that have no loops or parallel edges.

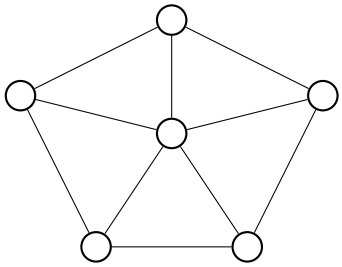


15. Which, if any, of the pairs of graphs shown are isomorphic? Justify your answer by describing an isomorphism or explaining why one does not exist.

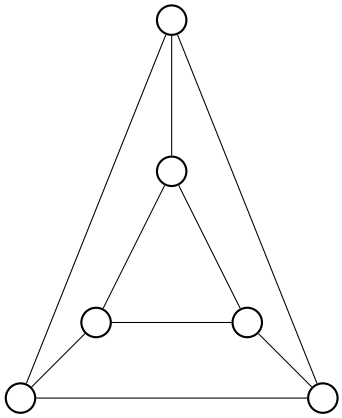
    (a) .



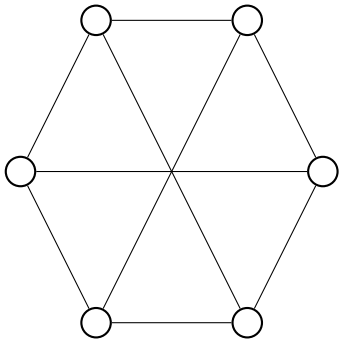
(b) .



(c) .

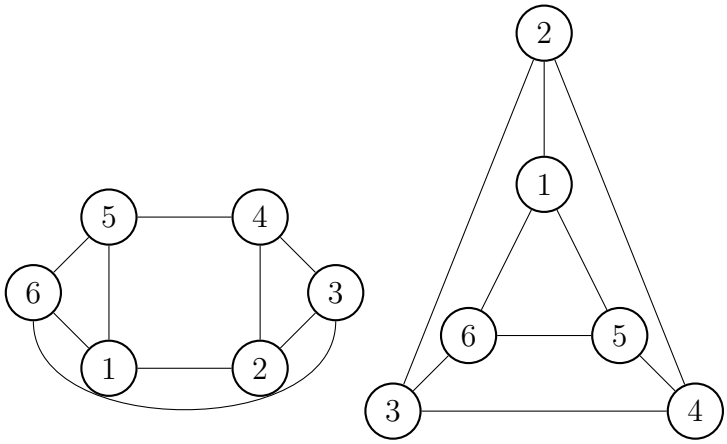


(d) .



(a $\ncong$ b) Not isomorphic because the maximum degree of a vertex in a is 3, while the maximum degree of a vertex in b is 5.

(a $\cong$ c) Isomorphic by labeling each graph:



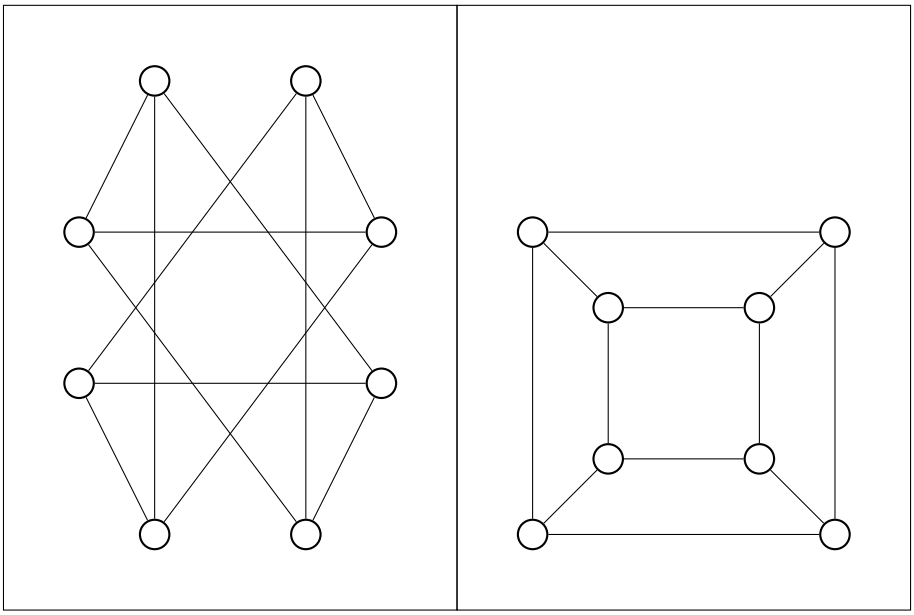
(a $\ncong$ d) Not isomorphic because a has a cycle of length 3 while d does not.

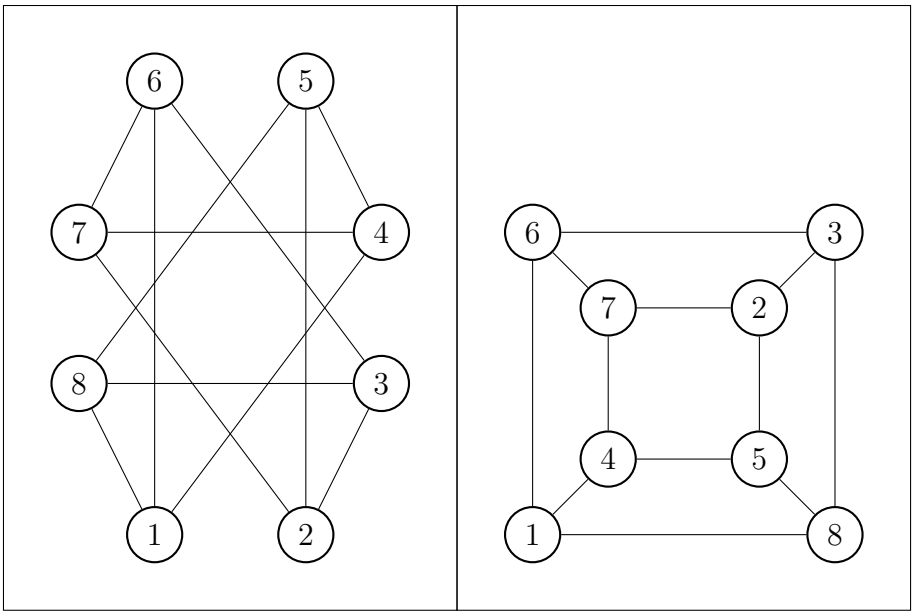
(b $\ncong$ c) Not isomorphic because the maximum degree of a vertex in c is 3, while the maximum degree of a vertex in b is 5.

(b $\ncong$ d) Not isomorphic because the maximum degree of a vertex in d is 3, while the maximum degree of a vertex in b is 5.

(c $\ncong$ d) Not Isomorphic because c has a cycle of length 3 while d does not.

16. Describe an isomorphism between the graphs:



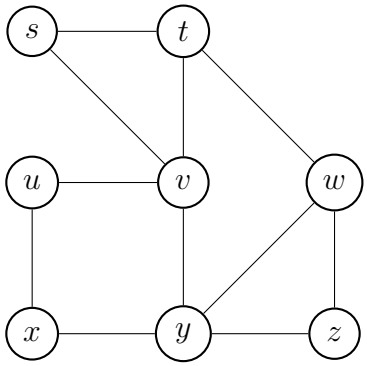


20. (a) A graph with 21 edges has seven vertices of degree 1, three of degree 2, seven of degree 3 and the rest of degree 4. How many vertices does it have?
$2 \cdot |E(G)|$ = The sum of the degree of all $v \in V(G)$.
$2 \cdot 21 = 7(1) + 3(2) + 7(3) + x(4) = 7 + 6 + 21 + 4x = 34 + 4x$
$4x = 42 - 34 = 8$, so $x = 2$.
$7 + 3 + 7 + 2 = 19$ vertices.

(b) How would your answer to part (a) change if the graph also had six vertices of degree 0?
The answer would include 6 extra vertices, so it would be 25 not 19 vertices.

6.2

2. (a) Give the vertex sequence of a simple closed path of largest possible length in the graph: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