

6.3

8. Consider a tree with n vertices. It has exactly $n - 1$ edges [Lemma 2], so the sum of its of the degrees of its vertices is $2n - 2$.

(a) A tree has two vertices of degree 5, three of degree 3, two of degree 2, and the rest of degree 1. How many vertices are in the graph?

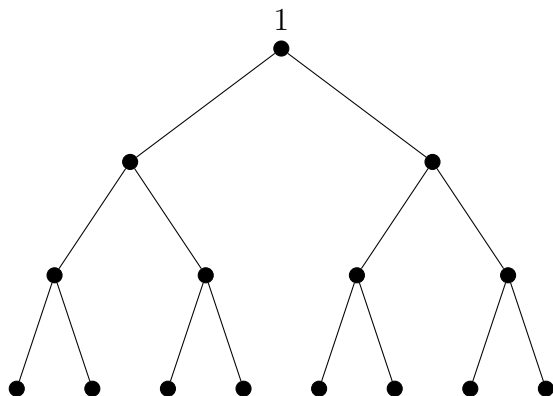
$$2|E(G)| = 2n - 2 = 2(5) + 3(3) + 2(2) + x$$

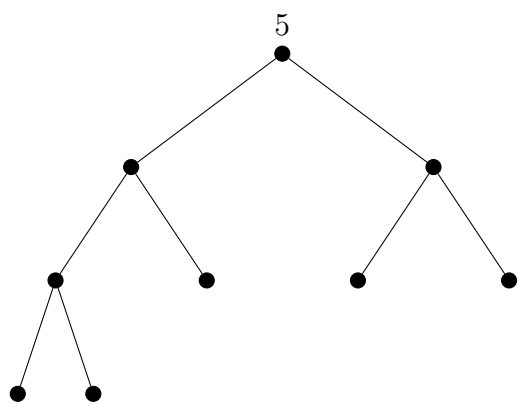
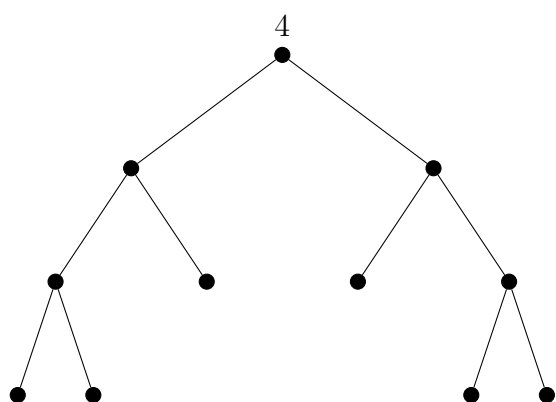
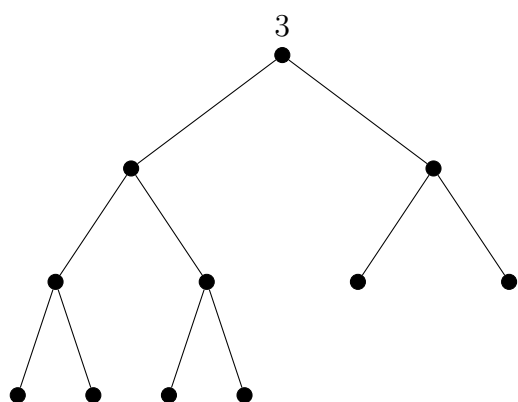
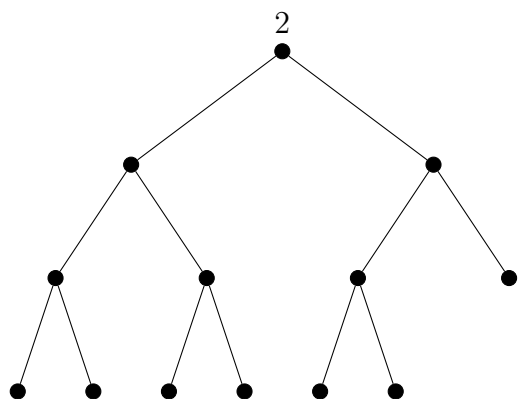
11. (a) Show that a forest with n vertices and m components has $n - m$ edges.

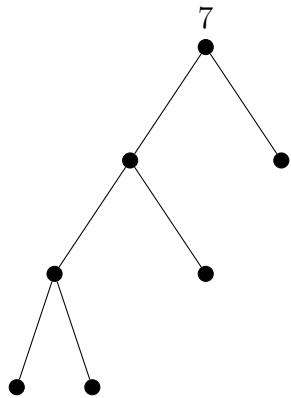
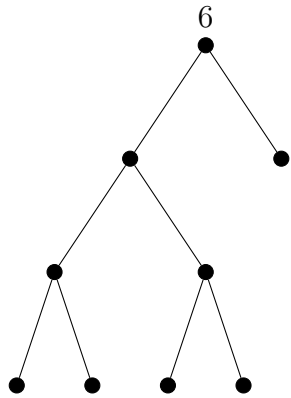
Suppose the components have $n_1, n_2, n_3, \dots, n_m$ vertices. The total vertices of the forest is then $n_1 + n_2 + n_3 + \dots + n_m = n$. The i th component is a tree, so it has $n_i - 1$ edges by Theorem 4. The total amount of edges in the forest is then $(n_1 - 1) + (n_2 - 1) + (n_3 - 1) + \dots + (n_m - 1) = n - m$.

6.4

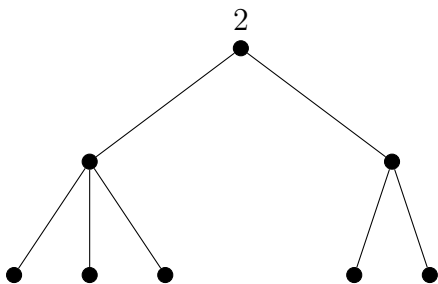
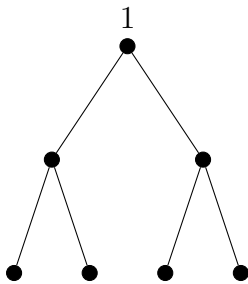
6. (a) Draw each of the seven types of rooted trees of height 3 in which each node that is not a leaf has 2 children.

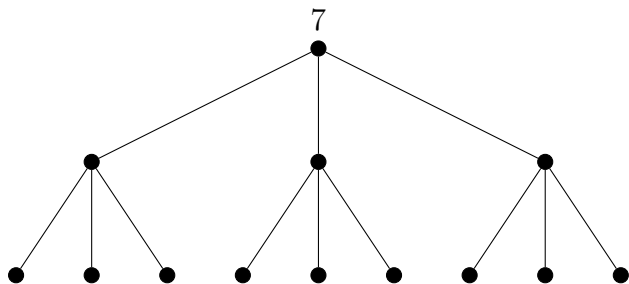
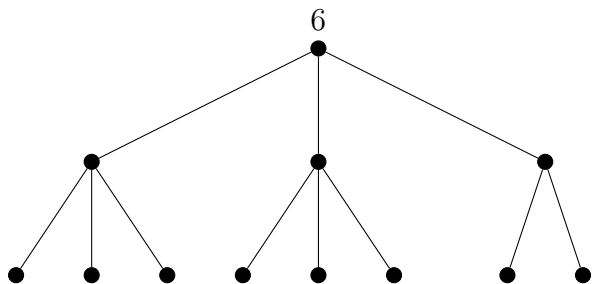
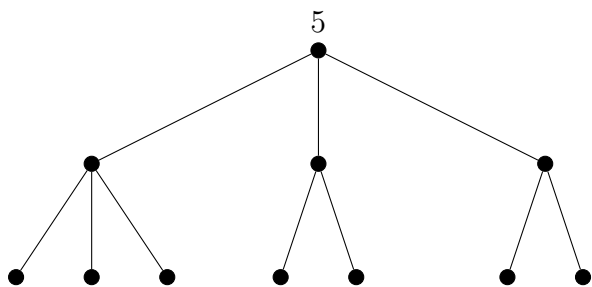
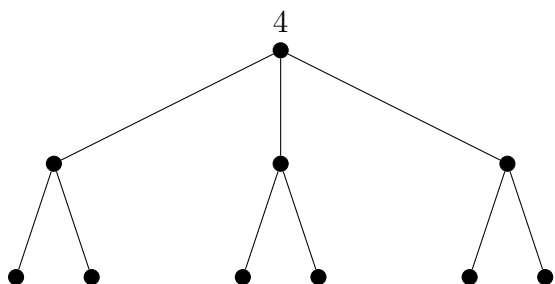
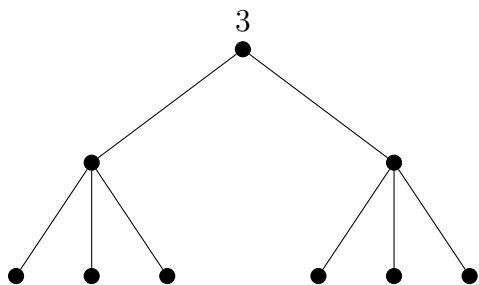






8. A 2-3 tree is a rooted tree such that each interior node, including the root if the height is 2 or more, has either two or three children and all paths from the root to the leaves have the same length. There are seven different types of 2-3 trees of height 2. Draw one tree of each type.





10. Consider a full binary tree T of height h .

(a) How many leaves does T have?

A full binary tree has 2^h leaves.

(b) How many vertices does T have?

A full m -ary tree has $\frac{m^{h+1}-1}{m-1}$ vertices, so

a full binary tree has $\frac{2^{h+1}-1}{2-1} = 2^{h+1} - 1$ vertices.

12. Give some real-life examples of information storage that can be viewed as labeled trees.

A labeled tree can be used to visualize a family tree, and a labeled tree can also be used to visualize the syntax of a language.

1. Additional Problem: Draw a tree with Prufer code $(5, 2, 1, 4, 4, 1, 6, 1, 1)$, with no crossing edges. Show some work.

$$\begin{array}{ll} (5, 2, 1, 4, 4, 1, 6, 1, 1) \rightarrow & (\cancel{5}, 2, 1, 4, 4, 1, 6, 1, 1) \rightarrow \\ \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\} \rightarrow & \{1, 2, \cancel{3}, 4, 5, 6, 7, 8, 9, 10, 11\} \rightarrow \end{array}$$

$$\begin{array}{ll} (\cancel{5}, \cancel{2}, 1, 4, 4, 1, 6, 1, 1) \rightarrow & (\cancel{5}, \cancel{2}, \cancel{1}, 4, 4, 1, 6, 1, 1) \rightarrow \\ \{1, 2, \cancel{3}, 4, \cancel{5}, 6, 7, 8, 9, 10, 11\} \rightarrow & \{1, \cancel{2}, \cancel{3}, 4, \cancel{5}, 6, 7, 8, 9, 10, 11\} \rightarrow \end{array}$$

$$\begin{array}{ll} (\cancel{5}, \cancel{2}, \cancel{1}, \cancel{4}, 4, 1, 6, 1, 1) \rightarrow & (\cancel{5}, \cancel{2}, \cancel{1}, \cancel{4}, \cancel{4}, 1, 6, 1, 1) \rightarrow \\ \{1, \cancel{2}, \cancel{3}, 4, \cancel{5}, 6, \cancel{7}, 8, 9, 10, 11\} \rightarrow & \{1, \cancel{2}, \cancel{3}, 4, \cancel{5}, 6, \cancel{7}, \cancel{8}, 9, 10, 11\} \rightarrow \end{array}$$

$$\begin{array}{ll} (\cancel{5}, \cancel{2}, \cancel{1}, \cancel{4}, \cancel{4}, \cancel{1}, 6, 1, 1) \rightarrow & (\cancel{5}, \cancel{2}, \cancel{1}, \cancel{4}, \cancel{4}, \cancel{1}, \cancel{6}, 1, 1) \rightarrow \\ \{1, \cancel{2}, \cancel{3}, \cancel{4}, \cancel{5}, 6, \cancel{7}, \cancel{8}, 9, 10, 11\} \rightarrow & \{1, \cancel{2}, \cancel{3}, \cancel{4}, \cancel{5}, 6, \cancel{7}, \cancel{8}, \cancel{9}, 10, 11\} \rightarrow \end{array}$$

$$\begin{array}{ll} (\cancel{5}, \cancel{2}, \cancel{1}, \cancel{4}, \cancel{4}, \cancel{1}, \cancel{6}, \cancel{1}, 1) \rightarrow & (\cancel{5}, \cancel{2}, \cancel{1}, \cancel{4}, \cancel{4}, \cancel{1}, \cancel{6}, \cancel{1}, \cancel{1}) \rightarrow \\ \{1, \cancel{2}, \cancel{3}, \cancel{4}, \cancel{5}, \cancel{6}, \cancel{7}, \cancel{8}, \cancel{9}, 10, 11\} \rightarrow & \{1, \cancel{2}, \cancel{3}, \cancel{4}, \cancel{5}, \cancel{6}, \cancel{7}, \cancel{8}, \cancel{9}, \cancel{10}, 11\} \rightarrow \end{array}$$

$$\begin{array}{l} (\cancel{5}, \cancel{2}, \cancel{1}, \cancel{4}, \cancel{4}, \cancel{1}, \cancel{6}, \cancel{1}, \cancel{1}) \rightarrow \\ \{1, \cancel{2}, \cancel{3}, \cancel{4}, \cancel{5}, \cancel{6}, \cancel{7}, \cancel{8}, \cancel{9}, \cancel{10}, \cancel{11}\} \rightarrow \end{array}$$

$$\rightarrow \{(5, 3), (2, 5), (1, 2), (4, 7), (4, 8), (1, 4), (6, 9), (1, 6), (1, 10), (1, 11)\}$$