Frank Cline
Math 307
HW 11

# 5.1

4. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $B = \{2, 3, 5, 7, 11, 13, 17, 19\}$.

   (b) How many subsets of A are there?
   There are 11 possible combinations:
   The empty set, set of length 1, set of length 2, ... , set of length 10
   $= \binom{10}{0} + \binom{10}{1} + \binom{10}{2} + \binom{10}{3} + \binom{10}{4} + \binom{10}{5} + \binom{10}{6} + \binom{10}{7} + \binom{10}{8} + \binom{10}{9} + \binom{10}{10}$
   $= 1 + 10 + 45 + 120 + 210 + 252 + 210 + 120 + 45 + 10 + 1$
   $= 1024$

   (c) How many 4-element subsets of A are there?
   The amount of 4-element subsets of A is equal to $\binom{10}{4} = 210$.

   (d) How many 4-element subsets of A consist of 3 even and 1 odd number?
   Three of the elements must be even, and since there are 5 even numbers total in A, an even number is given by $\binom{10}{5}$. Since there are 3 even numbers, and numbers can't be repeated, that is $\binom{10}{5}\binom{10}{4}\binom{10}{3}$. The remaining element must be odd, and there are 5 elements in A that are odd. Thus the remaining element must be $\binom{10}{5}$. That makes the total to be $\binom{10}{5}\binom{10}{5}\binom{10}{4}\binom{10}{3} = 834$.

6. A certain class consists of 12 men and 16 women. How many committees can be chosen from this class consisting of

   (a) 7 people?
   This is equal to $\binom{28}{7}$ because there are 28 people total, you need to choose 7 of them, and order doesn't matter. $\binom{28}{7} = 1,184,040$

   (b) 3 men and 4 women
   This is equal to $\binom{12}{3}\binom{16}{4}$ because you need to choose 3 out of 12 men and 4 out of 16 women, and order doesn't matter. $\binom{12}{3}\binom{16}{4}$= 400400

   (c) 7 women or 7 men
   This is equal to $\binom{12}{7} + \binom{16}{7}$ because there are two possibilities: choosing 7 men OR choosing 7 women where order doesn't matter. $\binom{12}{7} + \binom{16}{7} = 12,232$

8. How many committees consisting of 4 men and 4 women can be chosen from a group of 8 men and 6 women?
   This is equal to $\binom{8}{4}\binom{6}{4}$ because order doesn't matter and we're choosing 4 of 8 men, and 4 of 6 women. $\binom{8}{4}\binom{6}{4} = 85$.

9. Let $S = \{a, b, c, d\}$ and $T = \{1, 2, 3, 4, 5, 6, 7\}$.

   (a) How many one-to-one functions are there from T into S?
   There are no one-to-one functions from T into S because T has more elements than S does, thus, it's impossible to map each element in T to a unique element in S which is the definition of one-to-one.

   (b) How many one-to-one functions are there from S into T?
   The first element can map to one of 7 elements in T. The second element can then map to one of the 7 elements in T excluding the element one is mapped to, and so on. Thus the number of different one-to-one functions for S into T is $(7)(7-1)(7-2)(7-3) = (7)(6)(5)(4) = 840$.

   (c) How many functions are there from S into T?
   The number of functions from S into is equal to $(7)(7)(7)(7)$ because each of the 4 elements in S can map to any one of the 7 elements in T. $(7)(7)(7)(7) = 2401$

10. Let $P = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $Q = \{A, B, C, D, E\}$.

   (a) How many 4-element subsets of P are there?
   The number of 4-element subsets of P is equal to $\binom{9}{4}$ because you're choosing 4 elements from P and order doesn't matter. $\binom{9}{4} = 126$.

   (b) How many permutations of Q are there?
   There are 5 elements in Q, so the number of permutations of Q is equal to $5! = 120$.

   (c) How many license plates are there consisting of 3 letters from Q followed by 2 numbers from P? Repetition is allowed.
   The cases are:

   - No letter rep: $P(5, 3)$
     Find the permutations of size 3 from the 5 letters.
   - 2 letter rep: $\binom{5}{1}\binom{4}{1}\binom{2}{1}$
     Choose 1 of the 5 letters, then choose another of the remaining 4 letters. The final letter must be the same as one of the first 2 letters.
   - 3 letter rep: $\binom{5}{1}\binom{1}{1}\binom{1}{1}$
     Choose 1 of the 5 letters, and the other 2 must be that letter.

- No num rep: $P(9,2)$

  Find the permutations of size 2 from the 9 numbers.
- 2 num rep: $\binom{9}{1}$

  Choose 1 of the numbers and they will be repeated.

The total number of possibilities
$= [(\text{No letter rep}) + (2 \text{ letter rep}) + (3 \text{ letter rep})] \times [(\text{no num rep}) + (2 \text{ num rep})]$
$= [P(5,3) + (5)(4)(2) + (5)] \times [P(9,2) + 9]$
$= (60 + 40 + 5) \times (72 + 9) = (105) \times (81)$
$= 8505.$

13.