Frank Cline
Math 307
HW 4

3.4

1. Which of the following describe equivalence relations? For those that are not equivalence relations, specify which of $(R), (S), (T)$ fail and illustrate the failures with examples.

   (a) $L_1 || L_2$ for straight lines in the plane if L_1 and L_2 are the same or are parallel.
   Yes.

   (b) $L_1 \perp L_2$ for straight lines in the plane if L_1 and L_2 are perpendicular.
   No.
   Not (R) because a straight line can never be perpendicular to itself. $L_1 \not\perp L_1$.
   Not (T) because if $L_1 \perp L_2$ and $L_2 \perp L_3$, then $L_1 \not\perp L_3$. $L_1 || L_3$.

   (c) $p_1 \sim p_2$ for Americans if p_1 and p_2 live in the same state.
   No. There are some Americans that don't live in a state like in Washington D.C.

   (d) $p_1 \approx p_2$ for Americans if p_1 and p_2 live in the same state or in neighboring states.
   No. There are some Americans that don't live in a state like in Washington D.C.

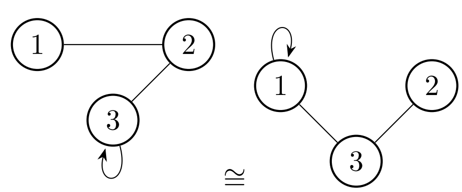
   (e) $p_1 \approx p_2$ for people if p_1 and p_2 have a parent in common.
   No.
   Not (T) because if $p_1 \approx p_2$ and $p_2 \approx p_3$, it doesn't mean $p_1 \approx p_3$ because they could have different parents.

   (f) $p_1 \cong p_2$ for people if p_1 and p_2 have the same mother.
   Yes.

2. For each example of an equivalence relation in Exercise 1, describe the members of some equivalence class.

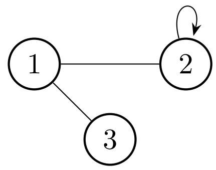
   (a) DO WORK HERE

5. If G and H are both graphs with vertex set $\{1, 2, ..., n\}$, we say that G is isomorphic to H, and write $G \cong H$, in case there is a way to label the vertices of G so that it becomes H.



$\cong$

(a) Give a picture of another graph isomorphic to these two.



(b) Find a graph with vertex set $\{1, 2, 3\}$ that is not isomorphic to the graphs yet has three edges and exactly one is a loop.

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(c) Find another example as in part(b) that isn't isomorphic to the answer of part(b) and the other two graphs.

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(d) Show that $\cong$ is an equivalence relation on the set of all graphs with the vertex set $\{1, 2, ..., n\}$.

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8. (a) For $m, n \in \mathbb{Z}$, define $m \sim n$ in case $m - n$ is odd. Is the relation reflexive? symmetric? transitive? Is it an equivalence relation?

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(b) For a and b in $\mathbb{R}$, define $a \sim b$ in case $a - b \leq 1$. One could say that $a \sim b$ in case a and b are close enough or approximately equal. Answer the question in part (a).

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17. (a) Verify that the relation $\cong$ defined in Example 5b (the reachable Relation R on $V(G)$ by $(v, w) \in R$) is an equivalence relation on $V(G)$.

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(b) Given a vertex v in $V(G)$, describe in words the equivalence class containing v.

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3.5

2. Find $nDIVm$ and $nMODm$ for the following values of n and m.

(a) $n = 20, m = 3$

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(b) $n = 20, m = 4$

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(c) $n = -20, m = 3$

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(d) $n = -20, m = 4$

.

(e) $n = 371,246$, $m = 265$

.

(f) $n = -371,246$, $m = 265$

.

4. (a) List all equvalence classes of $\mathbb{Z}$ for the equivalence relation congruence mod 4.

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   (b) How many different equivalence classes of $\mathbb{Z}$ are there with respect to congruence mod 73.

   .

5. For each of the following integers m, find the unique integer r in $\{0, 1, 2, 3\}$ such that $m \equiv n$ mod(4).

   (a) 17

   .

   (b) 7

   .

   (c) -7

   .

   (d) 2

   .

   (e) -88

   .

8. (a) List the elements in the sets A_0, A_1, A_2 defined by
   $A_k = \{m \in \mathbb{Z} : -10 \leq m \leq 10 \text{ and } m \equiv k \text{ mod}(3)\}$.

   .

   (b) What is A_3, A_4, A_5?

   .

12. For $m, n \in \mathbb{N}$, define $m \sim n$ if $m^2 - n^2$ is a multiple of 3.

    (a) Show that $\sim$ is an equivalence relation on $\mathbb{N}$.

    .

    (b) List four elements in the equivalence class [0].

    .

    (c) List four elements in the equivalence class [1].

    .

    (d) Do you think there are any more equivalence classes?

    .

11.1

1. Draw Hasse diagrams for the following posets.

   (a) ({1,2,3,4,6,8,12,24},|) where $m \mid n$ means m divides n. .

   (b) The set of subsets of {3,7} with $\subseteq$ as a partial order.

   .

15. Define the relations $<, \leq, \preceq$ on the plane $\mathbb{R} \times \mathbb{R}$ by
    $(x, y) < (z, w)$ if $x^2 + y^2 < z^2 + w^2$,
    $(x, y) \leq (z, w)$ if $(x, y) < (z, w)$ or $(x, y) = (z, w)$,
    $(x, y) \preceq (z, w)$ if $x^2 + y^2 \preceq z^2 + w^2$.

    (a) Which of these relations are partial orders? Explain.

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    (c) Draw a sketch of $\{(x, y) : (x, y) \leq (3, 4)\}$.

    .

    (d) Draw a sketch of $\{(x, y) : (x, y) \preceq (3, 4)\}$.

    .

1. Draw a Hasse diagram for $S = \{4, 6, 8, 12\}$ with the partial order $|$.
   DO WORK HERE.

11.2

1. Draw a Hasse diagram for the given order on $S \times T$, where $S = \{4, 6, 8, 12\}$ with the partial order $|$, and $T = \{2, 3, 5\}$ with the partial order $\leq$.

   (a) the product order

   .

   (b) the lexicographic order

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