Frank Cline
Math 307
HW 6

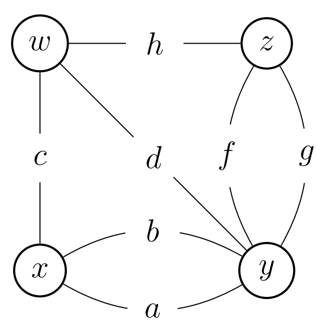
6.1

8. Can a graph have an odd number of vertices of odd degree? Explain.

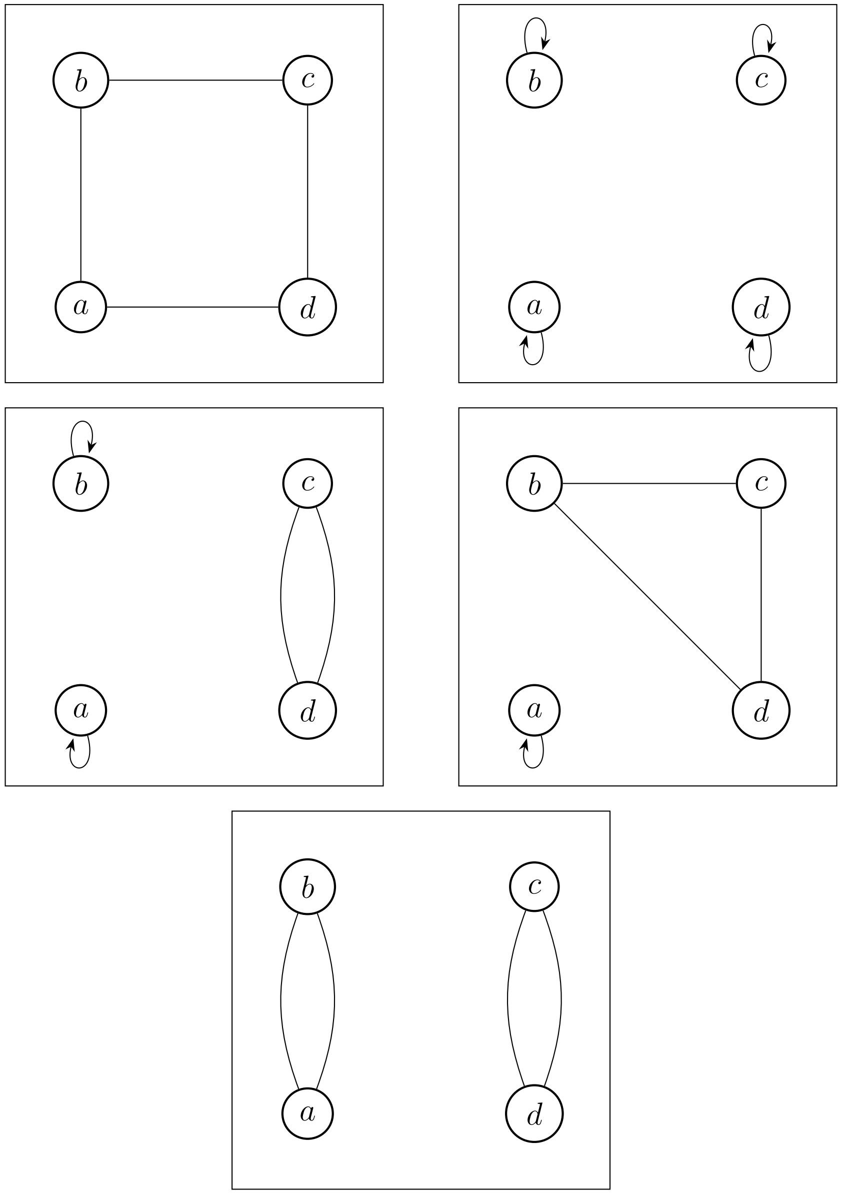
   No. The sum of the degree of vertices is equal to $2 \cdot |E(G)|$. If there is an odd number of odd degree vertices, their sum is odd. However, $2 \cdot |E(G)|$ is always even, so the sum of the degree of vertices must always be even.

10. Draw a picture of the graph G with $V(G) = \{x, y, z, w\}, E(G) = \{a, b, c, d, f, g, h\}$ and γ as given by the table

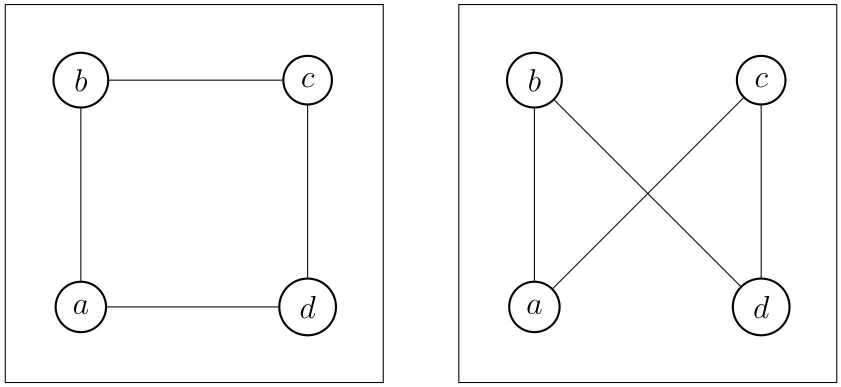
e	a	b	c	d	f	g	h
$\gamma(e)$	$\{x, y\}$	$\{x, y\}$	$\{w, x\}$	$\{w, y\}$	$\{y, z\}$	$\{y, z\}$	$\{w, z\}$



13. (a) Draw pictures of all five of the regular graphs that have four vertices each vertex of degree 2. "All" here means that every regular graph with four vertices and each vertex of degree 2 is isomorphic to one of the five, and no two of the five are isomorphic to each other.

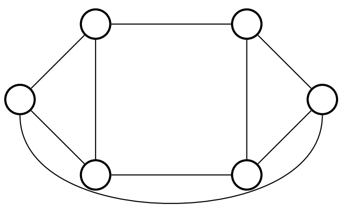


14. (b) Draw pictures of the two graphs with four vertices and four edges that have no loops or parallel edges.

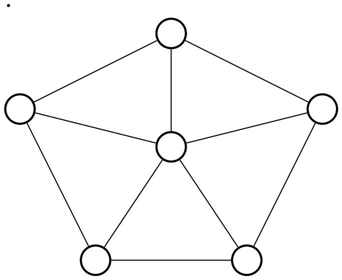


15. Which, if any, of the pairs of graphs shown are isomorphic? Justify your answer by describing an isomorphism or explaining why one does not exist.

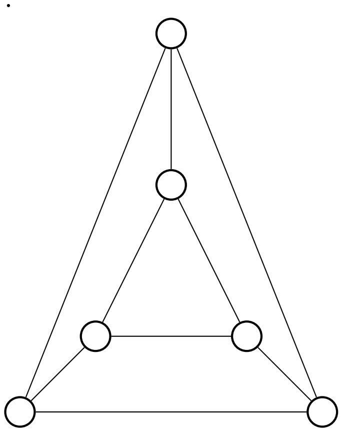
    (a) .



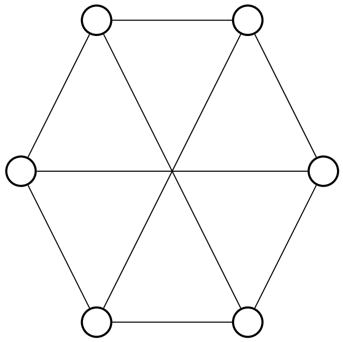
    (b) .



    (c) .

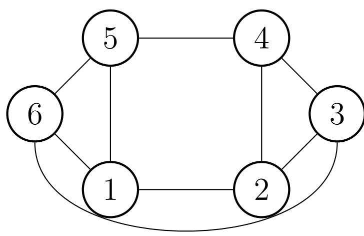


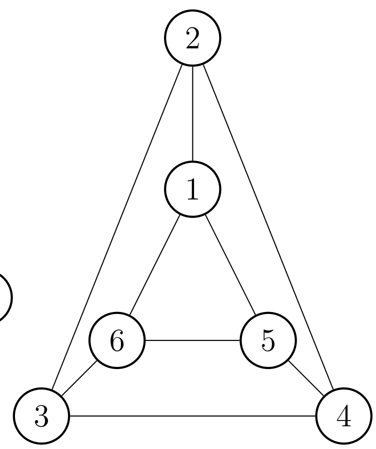
    (d) .



(a $\ncong$ b) Not isomorphic because the maximum degree of a vertex in a is 3, while the maximum degree of a vertex in b is 5.

(a $\cong$ c) Isomorphic by labeling each graph:





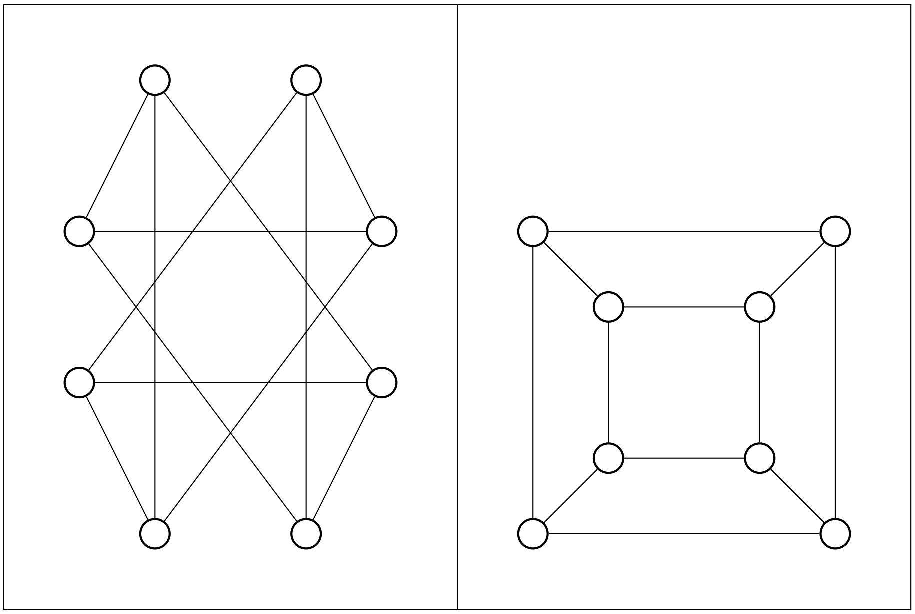
(a $\ncong$ d) Not isomorphic because a has a cycle of length 3 while d does not.

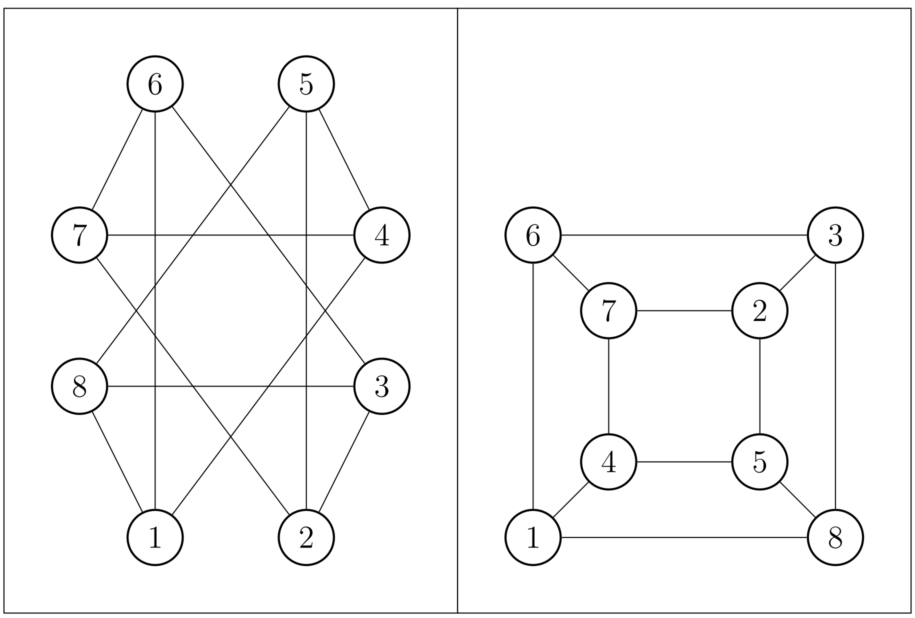
(b $\ncong$ c) Not isomorphic because the maximum degree of a vertex in c is 3, while the maximum degree of a vertex in b is 5.

(b $\ncong$ d) Not isomorphic because the maximum degree of a vertex in d is 3, while the maximum degree of a vertex in b is 5.

(c $\ncong$ d) Not Isomorphic because c has a cycle of length 3 while d does not.

16. Describe an isomorphism between the graphs:





20. (a) A graph with 21 edges has seven vertices of degree 1, three of degree 2, seven of degree 3 and the rest of degree 4. How many vertices does it have?

    $2 \cdot |E(G)| =$ The sum of the degree of all $v \in V(G)$.

    $2 \cdot 21 = 7(1) + 3(2) + 7(3) + x(4) = 7 + 6 + 21 + 4x = 34 + 4x$

    $4x = 42 - 34 = 8$, so $x = 2$.

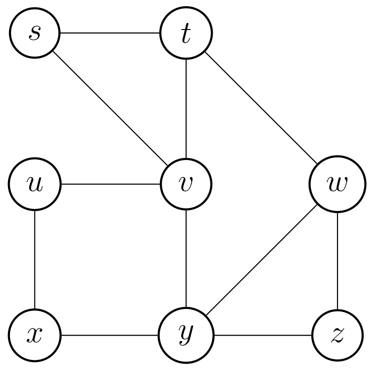
    $7 + 3 + 7 + 2 = 19$ vertices.

    (b) How would your answer to part (a) change if the graph also had six vertices of degree 0?

The answer would include 6 extra vertices, so it would be 25 not 19 vertices.

6.2

2. (a) Give the vertex sequence of a simple closed path of largest possible length in the graph:

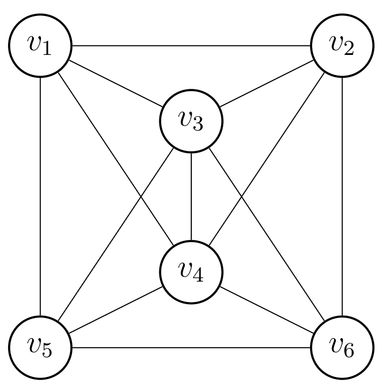


$y - v - t - s - v - u - x - y - w - z - y$ which has a length of 10.

(b) Is your answer to part (a) an Euler Circuit for the graph?

No because there is an edge that is not used. Also a Euler Circuit doesn't exist because there are 2 odd degree vertices (t and w).

7. Consider the graph shown:



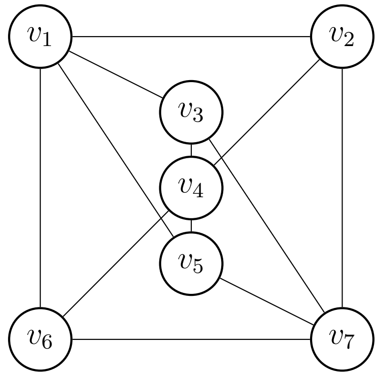
(a) Describe an Euler path for this graph or explain why there isn't one.

$v_4 - v_3 - v_5 - v_4 - v_6 - v_3 - v_2 - v_4 - v_1 - v_2 - v_6 - v_5 - v_1 - v_3$

(b) Describe an Euler Circuit for this graph or explain why there isn't one.

There isn't a Euler Circuit because a Euler Circuit means all vertices must be even degree, and the degrees of v_3 and v_4 are odd. Thus there is no Euler Circuit.

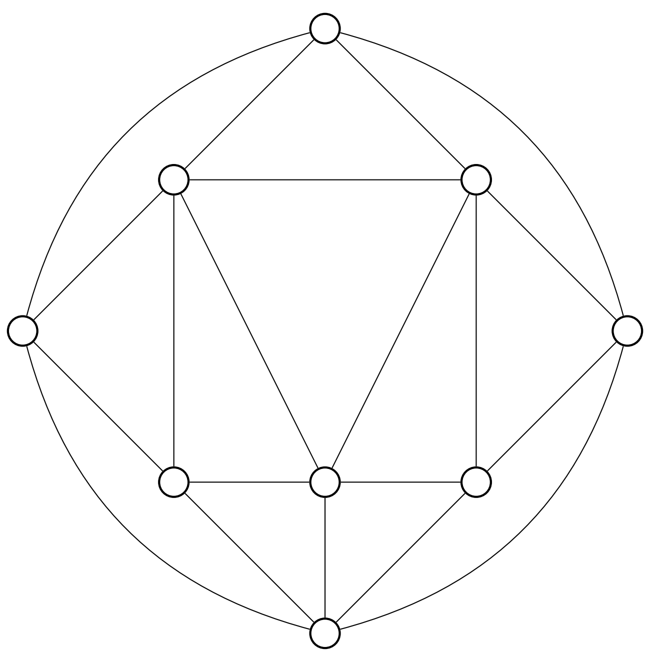
8. Consider the graph shown:



(a) Describe an Euler path for this graph or explain why there isn't one.
There is no Euler Path for this graph because there are more than 2 odd degree vertices (v_3, v_2, v_5, v_6).

(b) Describe an Euler Circuit for this graph or explain why there isn't one.
There isn't a Euler Circuit because a Euler Circuit means all vertices must be even degree, and the degrees of $v_3, v_2, v_5,$ and v_6 are odd. Thus there is no Euler Circuit.

15. An old puzzle presents a house with 5 rooms and 16 doors. The problem is to figure out how to walk around and through the house so as to go through each door exactly once.



(a) Is such a walk possible? Explain.

No because there are more than two odd degree vertices. Thus there is no Euler Path.

(b) How does your answer change if the door adjoining the two larger rooms is sealed shut?

If the door adjoining the two larger rooms is sealed shut, then there exists a Euler Path because there are exactly 2 odd degree vertices, and the degree of the rest of the vertices is even.