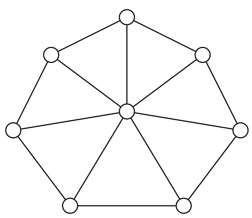
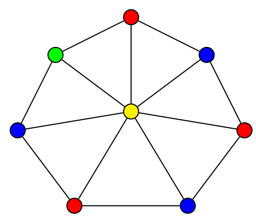
Frank Cline
Math 307
HW 9

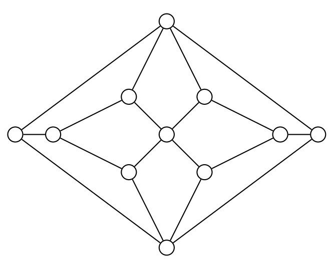
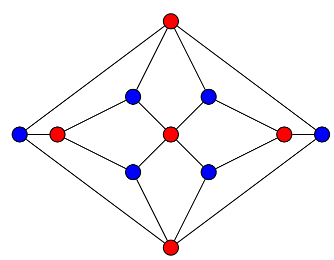
Problems 1-7

1. Find the chromatic number $\chi(G)$ of each of the graphs shown in Figure 1. To do so: (1) provide a coloring (with actual colors) for each of the graphs using $\chi(G)$ colors, and (2) explain why you cannot use fewer colors.

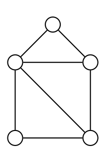
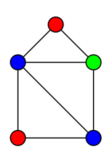
   (a) $\chi(G) = 4$. The outer vertices creates an odd cycle, so there must be at least 3 colorings, and the center vertices connects to all the vertices, so it must be colored differently. That gives us $3 + 1$ colorings which is 4.

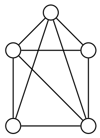
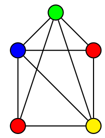
   (b) $\chi(G) = 2$. It's two because that is the smallest possible chromatic numbering for a connected graph.

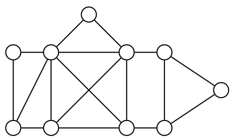
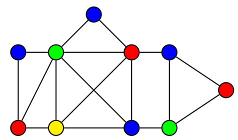
   (c) $\chi(G) = 3$. It's three because there contains an odd cycle, so the chromatic number must be at least 3.

   (d) $\chi(G) = 4$. It's four because there contains an odd cycle, so the chromatic number must be at least 3. There is also a vertex that connects to all other vertices, so for it's color to be different there must be $3 + 1$ colorings which is 4.

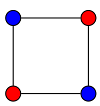
(e) $\chi(G) = 4$. WRITE DOWN WHY HERE!!!

2. Give examples of a graph (that are not examples from class or the worksheet) for which

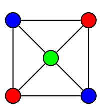
   (a) $\chi(G) = \Delta(G)$

   $\chi(G) = 2$ and $\Delta(G) = 2$ and $2 = 2$.

   (b) $\chi(G) < \Delta(G)$

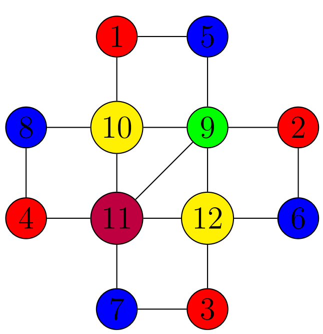
   $\chi(G) = 3$ and $\Delta(G) = 4$ and $3 < 4$.

3. Suppose a graph G has chromatic number 1. What can you say about G?

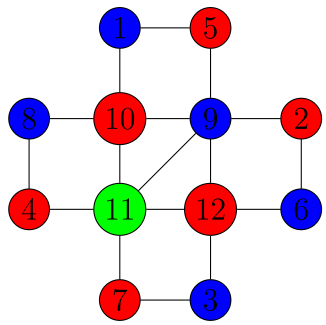
   There are no edges in G.

4. (a) Use the greedy algorithm to color the graph G in Figure 2. How many colors did you use? I used **5** colors.

   (b) Determine the chromatic number of G. Justify that the number you find really is the number of colors needed.

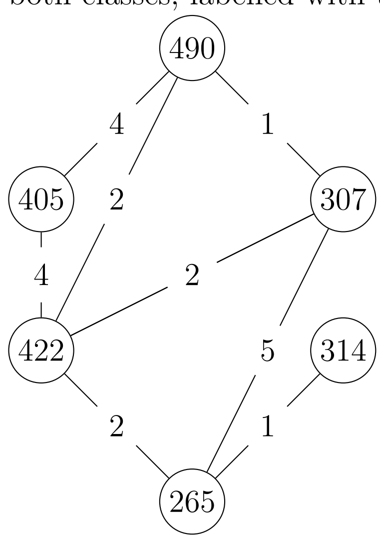
   $\chi(G) = 3$. This is the smallest possible chromatic number because there is an odd cycle.



5. The math department is trying to schedule focus group interviews with students for certain classes outside of the ordinary class time. The courses have the following (entirely made up) student overlaps:

	Math 265	Math 314	Math 307	Math 490	Math 405	Math 422
Math 265		1	5	0	0	2
Math 314	1		0	0	0	0
Math 307	5	0		1	0	2
Math 490	0	0	1		4	2
Math 405	0	0	0	4		4
Math 422	2	0	2	2	4	

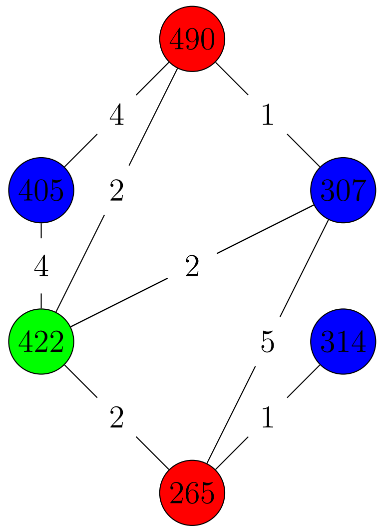
(a) Construct a graph representing the student overlaps (that is, assign the vertices to be the classes, and connect the vertices with an edge if there are students in both classes, labelled with the number of students in both classes).



(b) How many meeting times are needed? Explain, briefly.

The number of meeting times is equal to $\chi(G)$. $\chi(G)$ of the graph below is 3, and

$\chi(G) \geq 3$, so there needs to be at least 3 meetings.

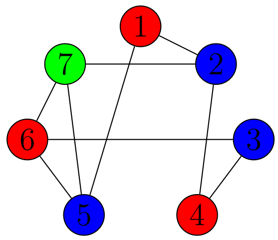


(c) Suppose only three meeting times are available, at 9AM, 10AM and 11AM, and furthermore, suppose that only one class is allowed to meet at 9AM. Is it possible to schedule the focus groups with this restriction? If so, give a possible schedule. If not, explain why not.

- 9 AM: 422
- 10 AM: 405, 307, 314
- 11 AM: 490, 265

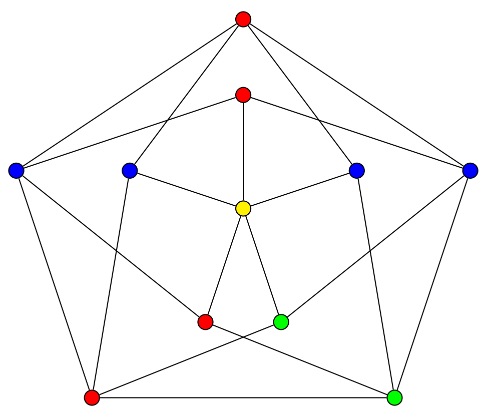
6. There are seven tour bus companies in the Los Angeles Area. During a particular day, each visits at most three locations from among Hollywood, Beverly Hills, Disneyland, and Universal Studios. The same location cannot be visited by more than one company on the same day. The first tour company visits only Hollywood, the second only Hollywood and Disneyland, the third only Universal Studios, the fourth only Disneyland and Universal Studios, the fifth Hollywood and Beverly Hills, the sixth Beverly Hills and Universal Studios, and the seventh Disneyland and Beverly Hills. Can these tours be scheduled only on Monday, Wednesday and Friday? Support/explain your answer.

- Monday: 7th company
- Wednesday: 2nd, 3rd, 5th companies
- Friday: 1st, 6th, 4th companies



7. Prove that the Grötzsch graph G_5, shown in Figure 3a, has $\chi(G) = 4$ by doing the following:

   (a) Find a 4-coloring of G.

   (b) Suppose that G has a 3-coloring, say using red, green, blue. Without loss of generality, we may suppose the center vertex is colored red. Explain why this forces a contradiction.

   The vertices around the center vertex create an odd number cycle, so they must be colored with at least 3 different colors. The center vertex connects to all of these vertices, so it has to be a different color. That makes the chromatic number at least 4. Thus G cannot have a 3-coloring.

   (c) The Grötzsch graph is a member of an infinite family of triangle-free graphs, . The graph G_6 is shown in Figures 3b. What is $\chi(G_6)$?

   $\chi(G) = 3$