Probabilidade

Distribuição contínua II

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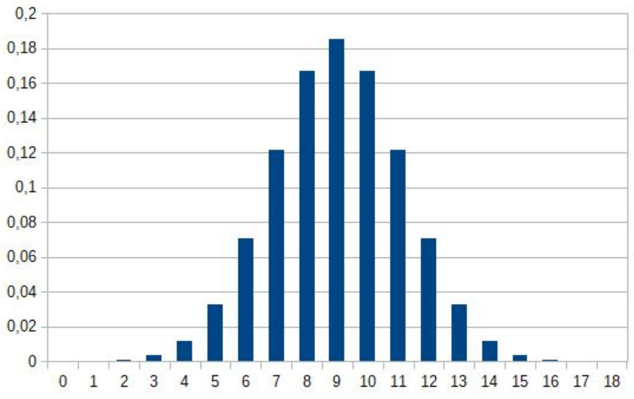
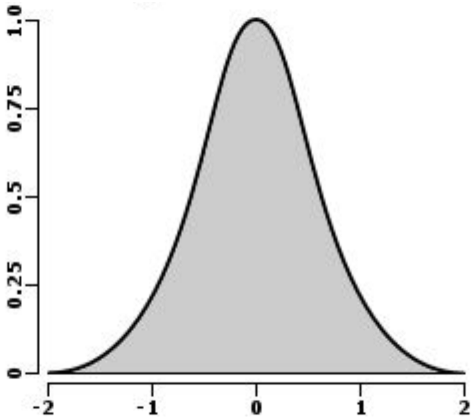
Slides e notebook em:

github.com/tetsufmbio/IMD0033/

Na aula passada





Função massa de probabilidade P(x)

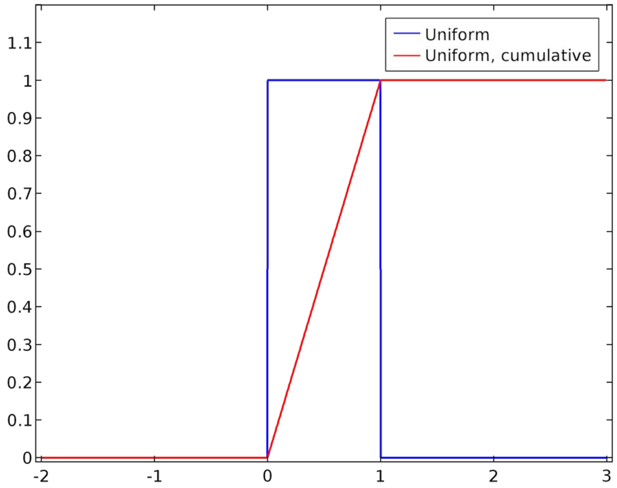
Função densidade de probabilidade f(x)

- $P(x) \geq 0$;
- $\sum P(x_i) = 1$;

- $f(x) \geq 0$;
- Área sob a curva = 1;

Função de distribuição acumulada

$$F(x) = P(X < x) = \int_{-\infty}^{x} f(x)dx$$



$$f(x) \begin{cases} 1 \text{ se } 0 < x < 1 \\ 0 \end{cases}$$

F(x) = x

$$f(x) = F(x)'$$

Exemplo

Suponha que o tempo de vida de um componente eletrônico em meses seja uma variável aleatória contínua que com $f(x) = 10/x^2$, $x > 10$.

1. Encontre a função de distribuição acumulada;
2. Determine $P(X > 20)$;
3. Determine a probabilidade de entre 6 desses componentes, 2 deles funcionarem por mais que 20 meses;

Exemplo

$f(x) = 10/x^2$, $x > 10$

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Exemplo

$f(x) = 10/x^2$, $x > 10$

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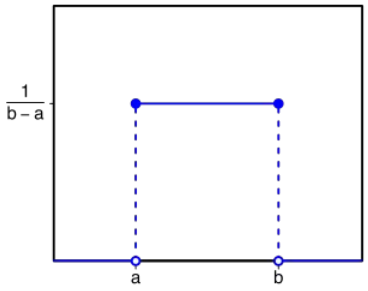
Exemplo

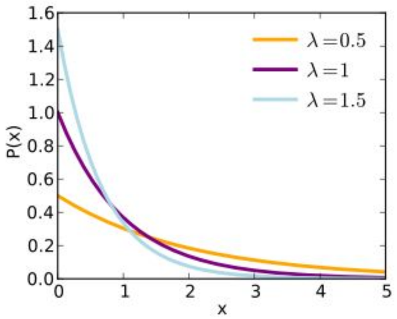
$f(x) = 10/x^2$, $x > 10$

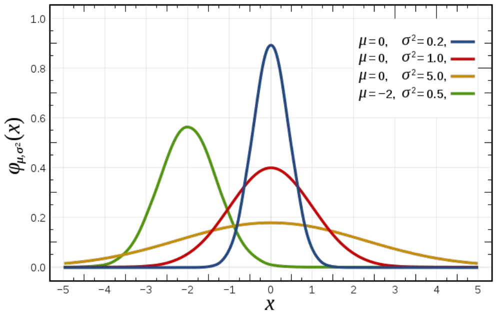
Determine a probabilidade de entre 6 desses componentes, 2 deles funcionarem por mais de 20 meses;

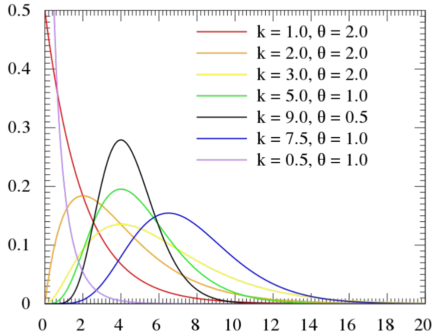
Famílias de distribuição de variáveis aleatórias contínuas

- Uniforme;
- Normal (Gaussiana);
- Exponencial;
- Gama;
- etc...

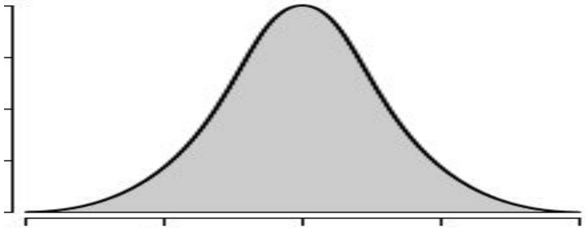




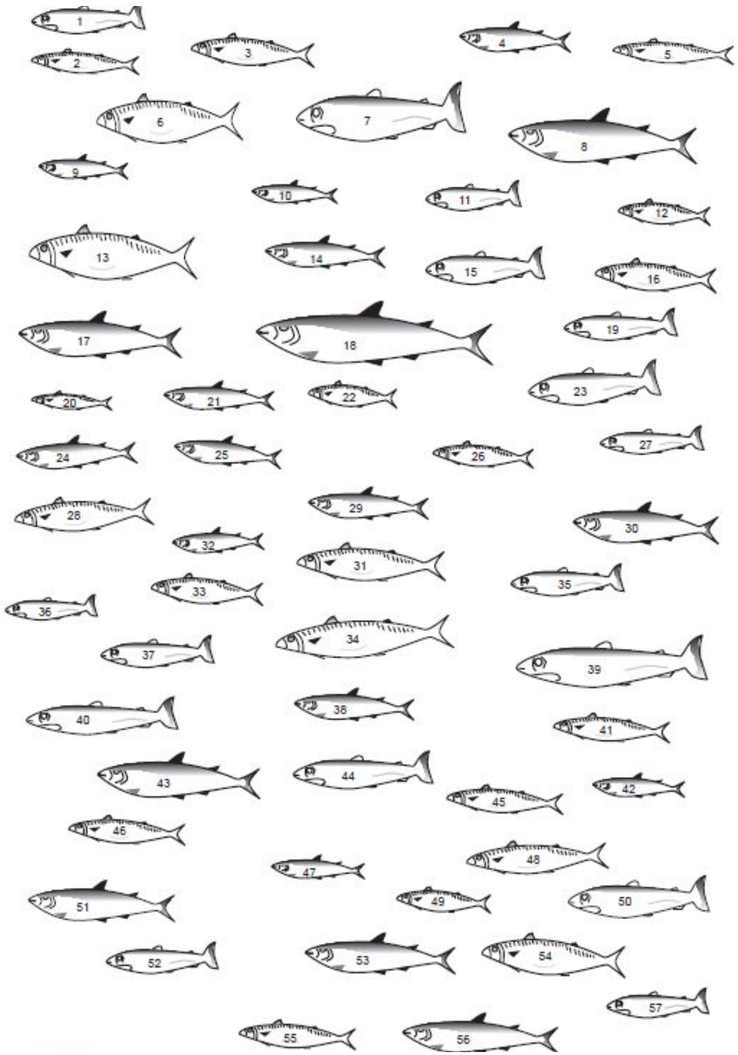




Peixes na rede



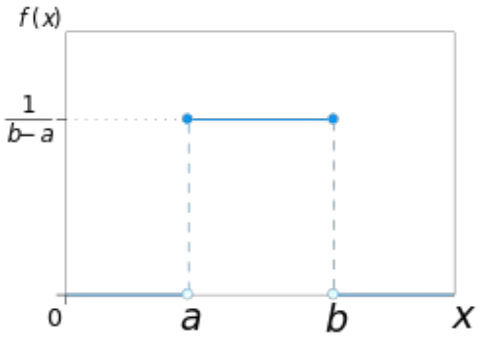
X ~ N(μ = 30, σ^2 = 10)

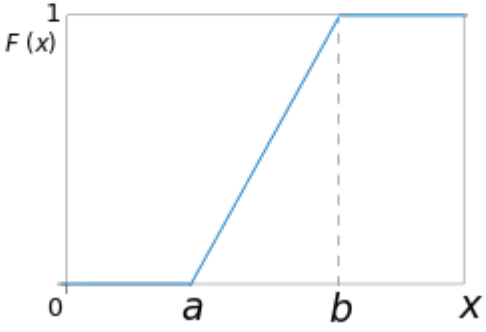


Distribuição Uniforme

f(x) = $\begin{cases} \frac{1}{b-a} & \text{for } x \in [a,b] \\ 0 & \text{otherwise} \end{cases}$

F(x)= $\begin{cases} 0 & \text{for } x < a \\ \frac{x-a}{b-a} & \text{for } x \in [a,b) \\ 1 & \text{for } x \geq b \end{cases}$





E(X) = $\frac{1}{2}(a+b)$

V(X) = $\frac{1}{12}(b-a)^2$

Distribuição exponencial

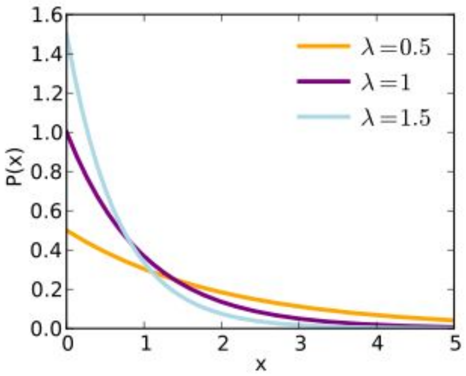
Análogo contínuo da distribuição geométrica;

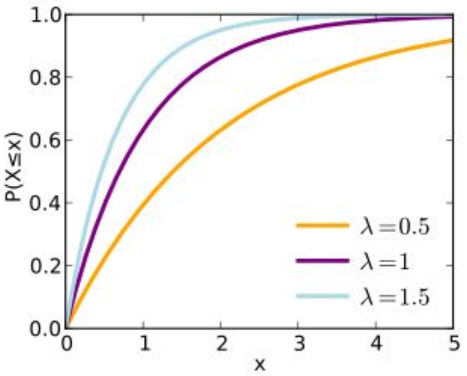
$$f(x) = \lambda e^{-\lambda x}, \lambda > 0$$

$$F(x) = 1 - e^{-\lambda x}$$

$$E(X) = \frac{1}{\lambda}$$

$$V(X) = \frac{1}{\lambda^2}$$





Distribuição normal (gaussiana)

Uma das distribuições mais importantes na estatística (**Teorema Central do Limite**).

Formato de sino;

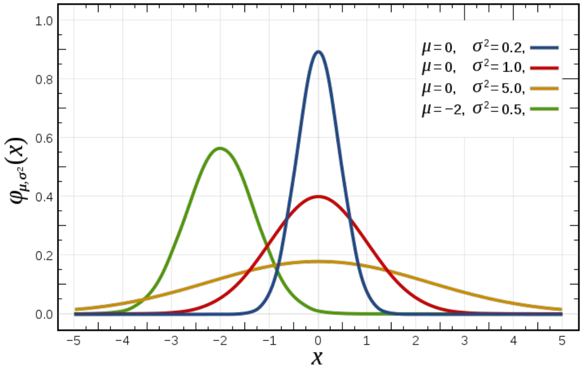
X ~ N(μ, σ^2)

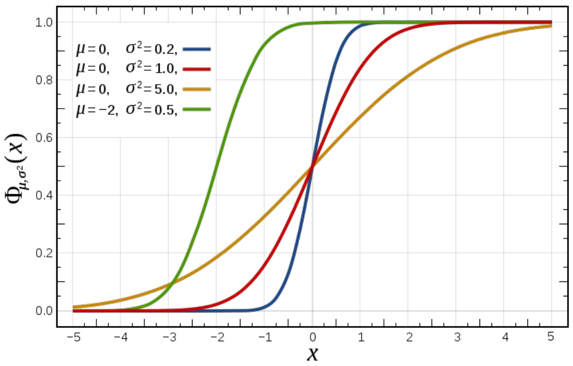
$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$F(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{x} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\, dx.$$

$$E(X) = \mu$$

$$V(X) = \sigma^2$$





Distribuição normal (gaussiana)

A forma mais simples de uma distribuição normal é quando: $\mu = 0, \sigma^2 = 1$

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$$F(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{x^2}{2}}\, dx.$$

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Distribuição normal padrão

Distribuição normal (gaussiana)

Transformação linear da distribuição normal:

$$X \sim N(\mu, \sigma^2)$$

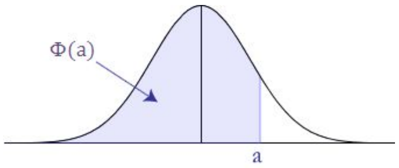
$$Y = aX + b$$

Y também terá uma distribuição normal!

$$\mu_Y = a\mu_X + b$$

$$\sigma_Y = a\sigma_X$$

$$Z = \frac{X - \mu}{\sigma}$$



Valores de probabilidades para dist. normal padrão ($\mu = 0$, $\sigma^2 = 1$)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Teorema Central do Limite

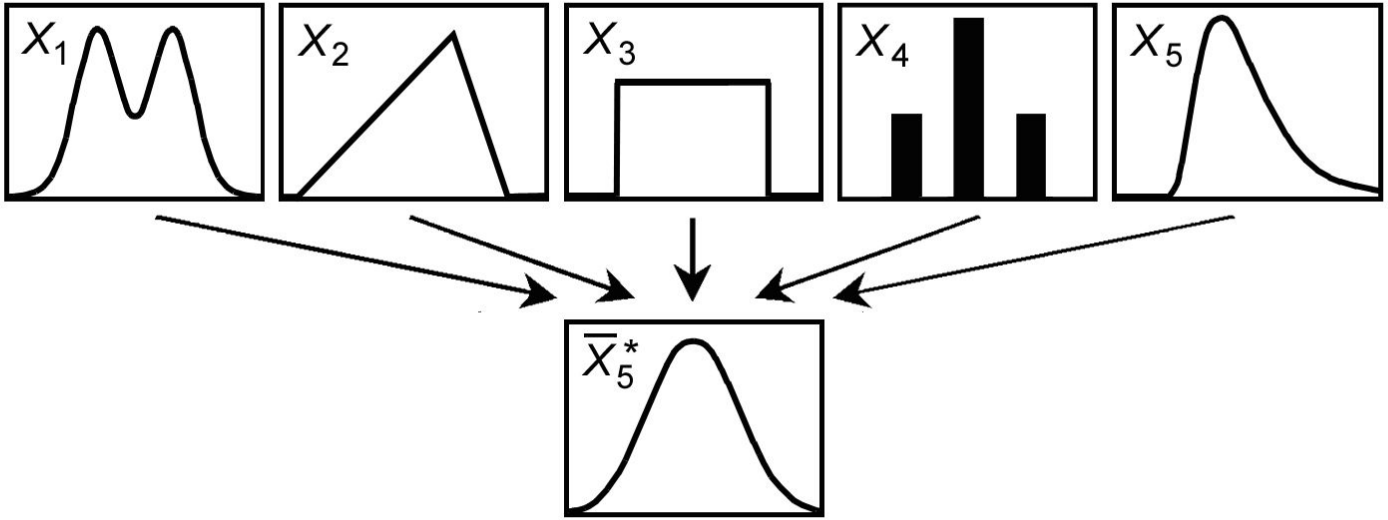
Quando variáveis aleatórios independentes são somados muitas vezes, a variável aleatória resultante tende a distribuir de forma normal.

$$Y = X_1 + X_2 + X_3 + \ldots \sim N(\mu_Y, \sigma_Y^2)$$

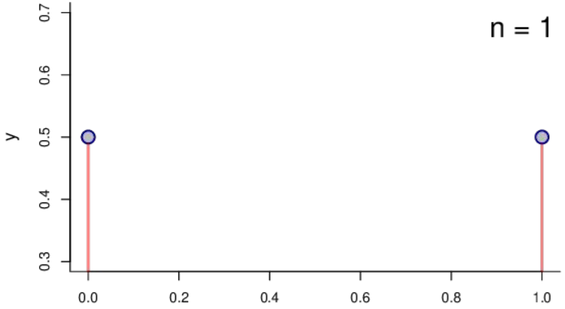
Teorema Central do Limite

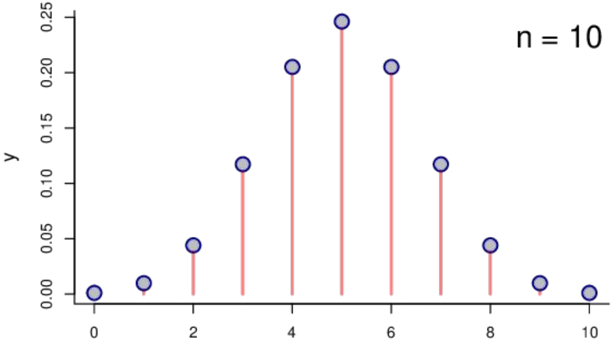
Quando variáveis aleatórios independentes são somados muitas vezes, a variável aleatória resultante tende a distribuir de forma normal.

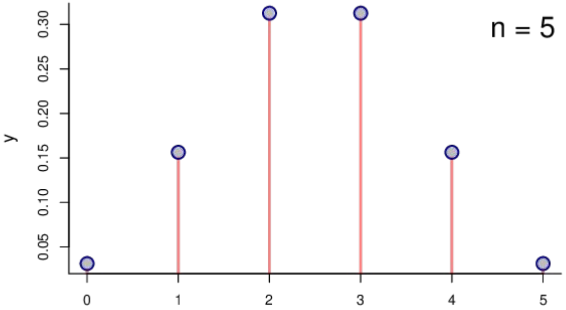
Independente da distribuição de X.

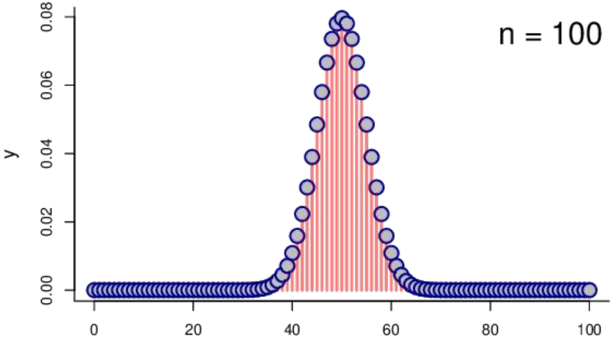


Teorema Central do Limite









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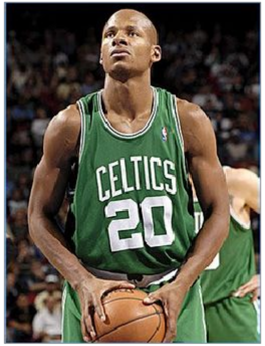
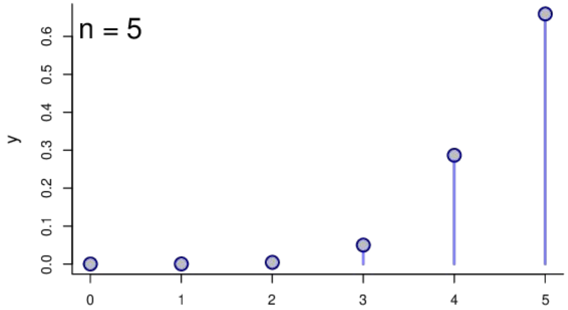
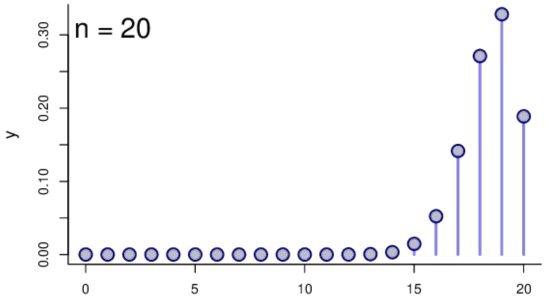
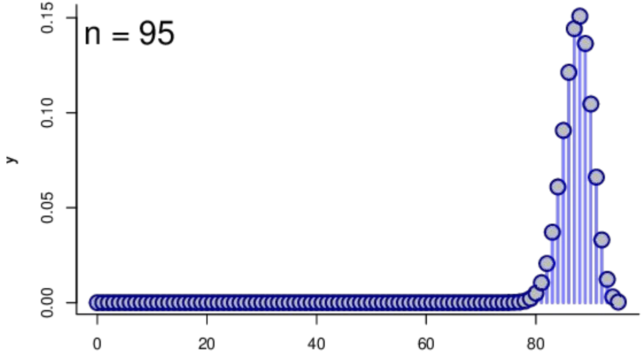


Ray Allen, (p=0.92)







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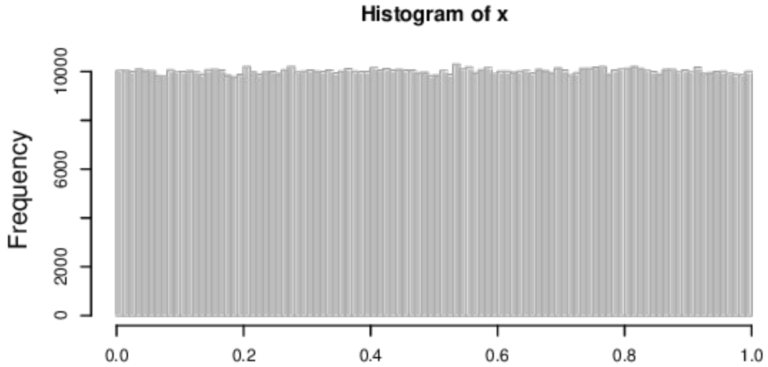

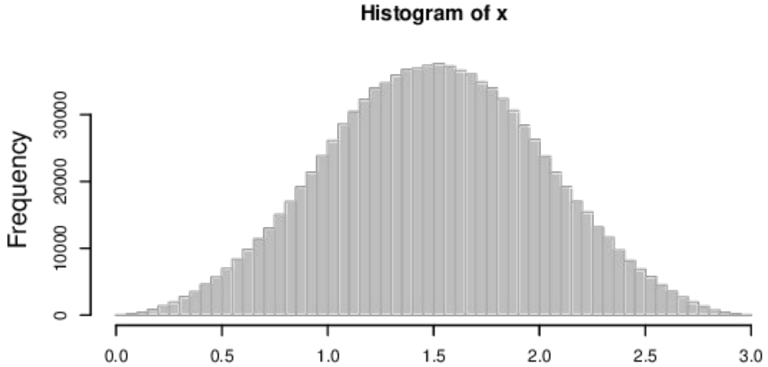

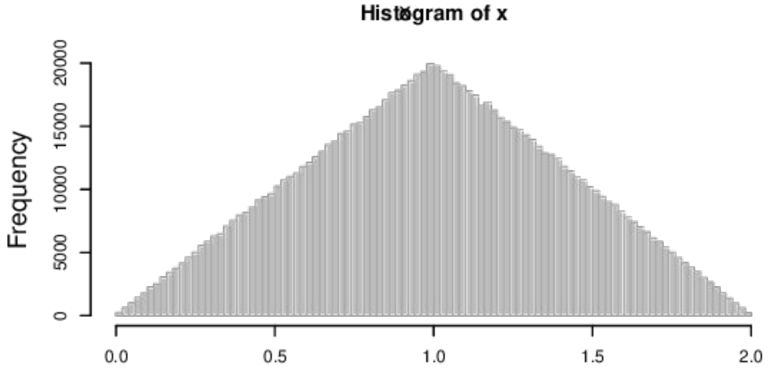

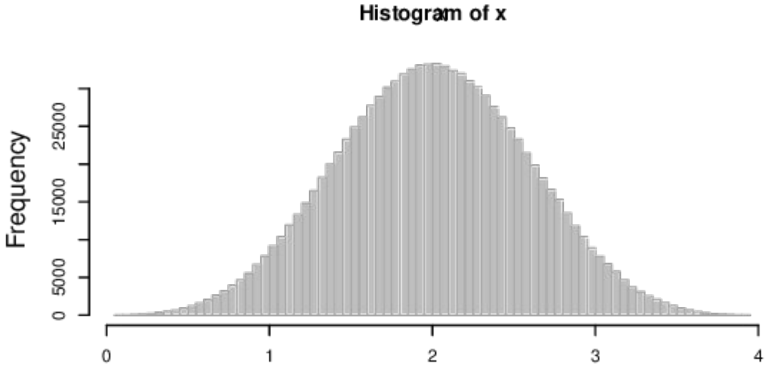

Teorema Central do Limite