16BIT0133

Digital Assignment 1

Mat 3004 - Applied Linear Algebra

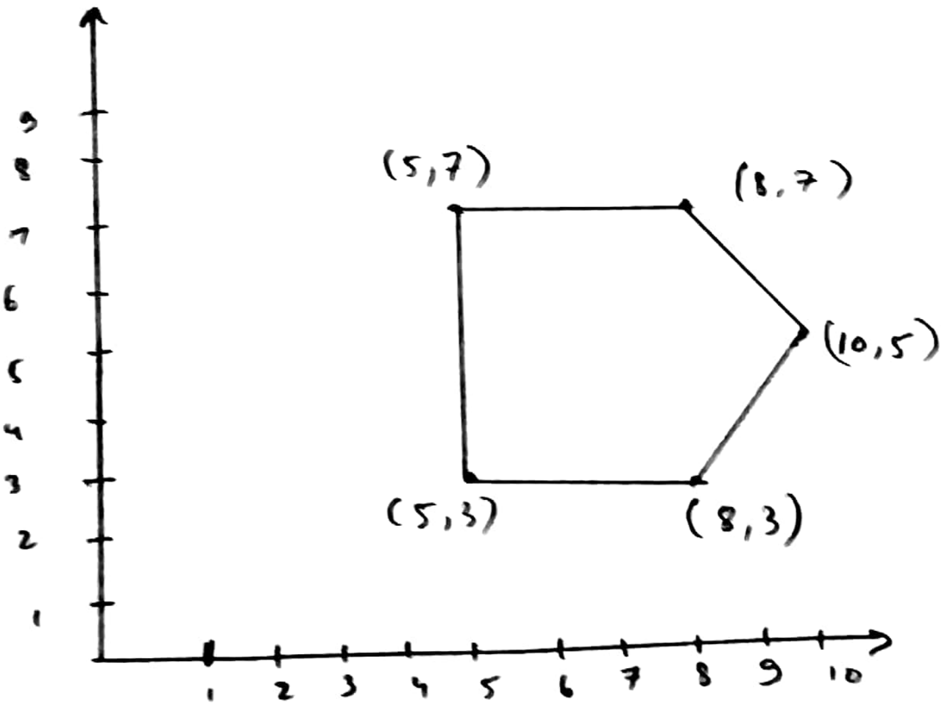
→ **Translation**: Shifting all points of a figure along a fixed vector.

Rotation: Rotating all points of a figure about a given centre point through a given angle θ. We will assume that all rotation are counter clockwise direction in the plane unless otherwise specified.

Reflection: reflecting all points of a figure about a given line.

Scaling: dialating / contracting the distance of all the points in the figure from a given center point.

Q1



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⇒ The cordinate vector is

$$\begin{bmatrix} 5 & 5 & 8 & 10 & 8 \\ 3 & 7 & 7 & 5 & 3 \end{bmatrix}$$

its homoganeous representation

$$\begin{bmatrix} 5 & 5 & 8 & 10 & 8 \\ 3 & 7 & 7 & 5 & 3 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

1. (a) Transalation along vector [4, -2]

→ Transalation of (x, y) along vector [a, b]
we first convert (x, y) to homogenous coordinats
$(x, y) \rightarrow [x, y, 1]$

$$\begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x+a \\ y+b \\ 1 \end{bmatrix}$$

∴ equivalent to the two dimentional point $(x+a, y+b)$

for $\begin{bmatrix} 5 & 5 & 8 & 10 & 8 \\ 3 & 7 & 7 & 5 & 3 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$ $a = 4$, $b = -2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 5 & 8 & 10 & 8 \\ 3 & 7 & 7 & 5 & 3 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 9 & 9 & 12 & 14 & 12 \\ 1 & 5 & 5 & 3 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

∴ new transformed coordinats are :-

[(9, 1), (9, 5), (12, 5), (14, 3), (12, 1)] ans

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1. (b) rotation about the origin through $\theta = 30°$

$\theta = \frac{\pi}{6}$

rotated vectors ~~are~~ = $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 \\ 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 5 & 8 & 10 & 8 \\ 3 & 7 & 7 & 5 & 3 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & 1 & 3 & 6 & 5 \\ 5 & 9 & 10 & 9 & 7 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$ ans rounded values

∴ new rotated ~~and~~ coordinates are :-

[(3,5), (1,9), (3,10), (6,9), (5,7)] ans

[16BT70133]

1.(c) reflection about the line $y = 3x$

$m = 3$

reflected vectors = $\frac{1}{1+m^2}\begin{bmatrix} 1-m^2 & 2m & 0 \\ 2m & m^2-1 & 0 \\ 0 & 0 & 1+m^2 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} -0.8 & 0.6 & 0 \\ 0.6 & 0.8 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & 1 & 3 & 6 & 6 \\ 5 & 9 & 10 & 9 & 7 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -2 & 0 & -2 & -5 & -5 \\ 5 & 9 & 10 & 10 & 7 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$ ans rounded value

new reflected coordinates are

$[(-2,5), (0,9), (-2,10), (-5,10), (-5,7)]$ ans

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1. (d) scaling about the origin with scale factors of 4 in the x-direction and 2 in the y direction.

$$\text{Scaled vector} = \begin{bmatrix} c & 0 & 0 \\ 0 & d & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \quad \text{where } c = 4,\ d = 2$$

$$= \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 3 & 6 & 5 \\ 5 & 9 & 10 & 9 & 7 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 20 & 20 & 32 & 40 & 32 \\ 6 & 14 & 14 & 10 & 6 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

new scaled coordinates are.

[(20,6), (20,14), (32,14), (40,10), (32,6)] ans

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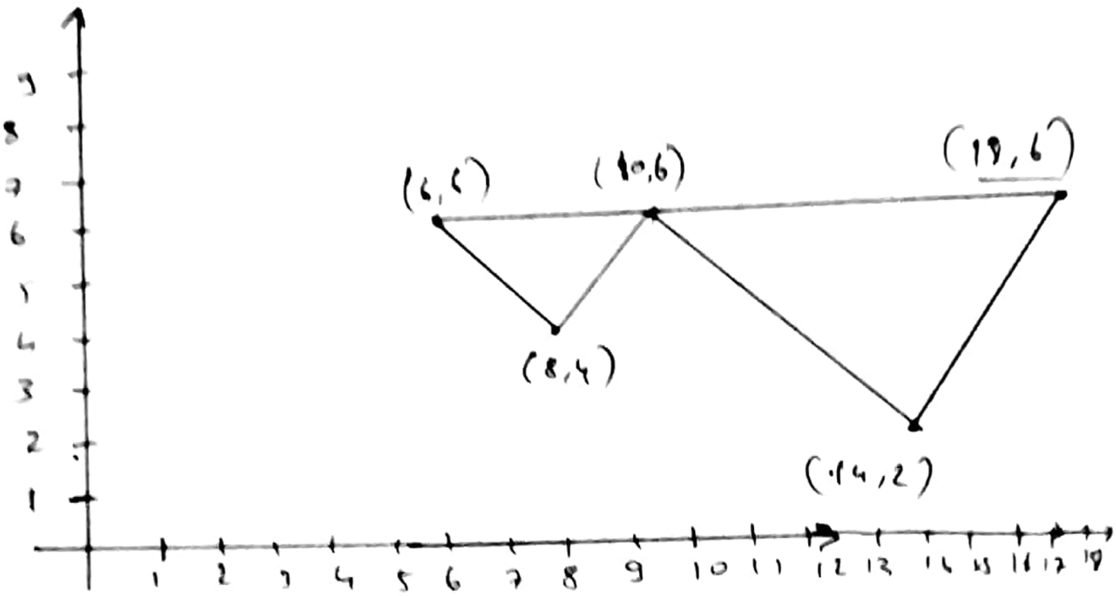
Q2



2. (a) Translation along vector [3, 5]

→ Translation of (x, y) along vector [a, b]

we first convert (x, y) to homogeneous coordinates

(x, y) → [x, y, 1]

$$\begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x+a \\ y+b \\ 1 \end{bmatrix}$$

∴ equivalent to the two dimensional point (x+a, y+b)

for $\begin{bmatrix} 6 & 10 & 18 & 14 & 10 & 8 \\ 6 & 6 & 6 & 2 & 6 & 4 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$ $a = 3$, $b = 5$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 6 & 10 & 18 & 14 & 10 & 8 \\ 6 & 6 & 6 & 2 & 6 & 4 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 9 & 13 & 21 & 17 & 13 & 11 \\ 11 & 11 & 11 & 7 & 11 & 9 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

∴ new transformed coordinate are :-

[(9, 11), (13, 11), (21, 11), (17, 7), (13, 11), (11, 9)] ans

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2 (b) rotation about the origin through $\theta = 120°$

$$\theta = \frac{2\pi}{3}$$

rotated vectors = $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 6 & 10 & 18 & 14 & 10 & 8 \\ 6 & 6 & 6 & 2 & 6 & 4 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -8 & -10 & -14 & -9 & -10 & -7 \\ 2 & 6 & 13 & 11 & 6 & 5 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

rounded

new rotated coordinates :-

$$\boxed{(-8,2), (-10,6), (-14,13), (-9,11), (-10,6), (-7,5)}$$

2 (c) reflection about the line $y = (1/2)x$

$m = 1/2$

reflected vectors = $\frac{1}{1+m^2} \begin{bmatrix} 1-m^2 & 2m & 0 \\ 2m & m^2-1 & 0 \\ 0 & 0 & 1+m^2 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 0.6 & 0.8 & 0 \\ 0.8 & -0.6 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 6 & 10 & 18 & 14 & 10 & 8 \\ 6 & 6 & 6 & 2 & 6 & 4 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 11 & 16 & 10 & 11 & 8 \\ 1 & 4 & 11 & 10 & 4 & 4 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

rounded

new reflected coordinates are

$$\boxed{(8,1), (11,4), (16,11), (10,10), (11,4), (8,4)}$$ ans

2(d) Scaling about the origin with scale factor of 1/2 in the x dimension and 3 in the y-direction.

$$\text{Scaled vector} = \begin{bmatrix} c & 0 & 0 \\ 0 & d & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \qquad c = \frac{1}{2},\; d = 3$$

$$= \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 6 & 10 & 18 & 14 & 10 & 8 \\ 6 & 6 & 6 & 2 & 6 & 4 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & 5 & 9 & 7 & 5 & 4 \\ 18 & 18 & 18 & 6 & 18 & 12 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

new scaled coordinates are

[(3,18), (5,18), (9,18), (7,6), (5,18), (4,12)]

ans

all the Questions are also coded

X —— O —— X