5. FTCS Stability Analysis

$$T_x^{\wedge} = G^{\wedge} e^{iKx} e^{ily dy} \qquad e^{-iK\times dx} + e^{ik\times dy} = 2\cos(K\cdot dx)$$

$$T_{i,j}^{n+1} = C\left[T_{i+1,j}^{\wedge} + T_{i-1,j}^{\wedge} + T_{i,j+1}^{\wedge} + T_{i,j-1}^{\wedge} - 4T_{i,j}^{\wedge}\right] + T_{i,j}^{\wedge}$$

$$G^{n+1} e^{ihxdx} e^{iLydy} = G^{\wedge} e^{ihx} e^{iky} + C\left[G^{\wedge} e^{(x-1)dx\, ildy} + G^{\wedge} e^{(x+1)\cdot dx\, ildy} + G^{\wedge} e^{xdx\, i(y+1)dy} + G^{\wedge} e^{xdx\, i(y-1)dy} - 4G^{\wedge} e^{iKxdx} e^{iLydy}\right]$$

$$G^{n+1} = (1-4C)G^{\wedge} + C\,G^{\wedge}(2\cdot\cos((dx)) + C\,G^{\wedge}(2\cdot\cos(dy))$$

$$\frac{G^{n+1}}{G^{\wedge}} = (1-4C) + C(2\cdot\cos(dx)) + C(2\cdot\cos(dy))$$

$$(1-4C) + 2C(2\cdot\cos(u\cdot dx))$$

$$= 1-4C+4C(-1)$$

$$= 1-4C-4C = 1-8C$$

$$1 > 1-8C \qquad -1 < 1-8C$$

$$0 > -8C \qquad -2 < -8C \rightarrow$$

2D-FTCS

$$\boxed{0 < C < 1/4}$$

$$T_{i,j,k}^{n+1} = C\left[T_{i+1,j,k}^{\wedge} + T_{i-1,j,k}^{\wedge} + T_{i,j+1,k}^{\wedge} + T_{i,j-1,k}^{\wedge} + T_{i,j,k+1}^{\wedge} + T_{i,j,k-1}^{\wedge} - 6T_{i,j,k}^{\wedge}\right] + T_{i,j,k}^{\wedge}$$

$$G^{n+1} e^{ikxdx} e^{iLdy} = G^{\wedge} e^{ikx} e^{iky} + C\left[G^{\wedge} e^{(k-1)dx\, ildy} - 6G^{\wedge} e^{ikxdx\, ildy}\right]$$

$$G^{n+1} = (1-6C)G^{\wedge} + C\cdot G^{\wedge}(2\cdot\cos(dx)) + C\,G^{\wedge}(2\cdot\cos(dy)) +$$

$$\frac{G^{n+1}}{G^{\wedge}} = (1-6C) + C(2\cdot\cos(dx)) + C(2\cdot\cos(dy)) + C(2\cdot\cos(dz))$$

$$= 1-6C+3C(2\cdot\cos(u))$$

$$= 1-6C+3C(2\cdot(-1))$$

$$= 1-6C-6C = 1-12C$$

$$1 > 1-12C \qquad -1 < 1-12C$$

$$0 > -12C \qquad -2 < -12C$$

3D-FTCS

$$\boxed{0 < C < 1/6}$$