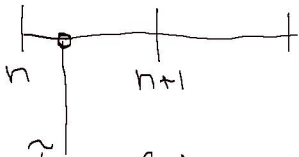


θ -rule :



$$\tilde{t} = \theta t_{n+1} + (1-\theta)t_n \quad \text{weighted average}$$

$$\theta \in [0, 1]$$

$\rightarrow \tilde{t} = t_{n+\theta}$ Sample the ODE at $t_{n+\theta}$

$$U'(t_{n+\theta}) \approx \frac{U^{n+1} - U^n}{\Delta t}$$

$$-a U(t_{n+\theta}) \approx -a (\theta U^{n+1} + (1-\theta) U^n) \quad \text{weighted average}$$

$$\Rightarrow \frac{U^{n+1} - U^n}{\Delta t} = -a \theta U^{n+1} - a(1-\theta) U^n$$

solve wrt U^{n+1} :

$$U^{n+1} = \frac{1 - (1-\theta)a\Delta t}{1 + \theta a\Delta t} U^n$$

Valid for any $\theta \in [0, 1]$

$\theta = 0$: Forward Euler

$\theta = \frac{1}{2}$: Crank-Nicolson

$\theta = 1$: Backward Euler