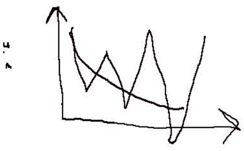
Aug. 30: FE:  large Δt

BE:  numerical   Why?

Analytical solution of the discrete equations!

$$u^n = A^n \cdot I, \quad A = \frac{1-(1-\theta)\Delta t\, a}{1+\theta \Delta t\, a}$$

$A > 0$: no oscillations

$A < 1$: decay (no growth)

$u_e = A_e^n \cdot I, \quad A_e = e^{-\Delta t \cdot a}$

$A_e - A = ?$ $A_e \sim$ Taylor series in Δt (expect small Δt)

$A \sim$ —— " ——

Summary:

FE, BE : $A_e - A = \mathcal{O}(\Delta t^2)$ $\Delta t \ll 1$

CN : $A_e - A = \mathcal{O}(\Delta t^3)$ $\Delta t^2 \gg \Delta t^3 \gg \Delta t^4$

A: error in advancing one time step.

True error: $u_e(t_n) - u^n = I e^{-a n \Delta t} - A^n = ?$

FE, BE : $u_e - u^n = \mathcal{O}(\Delta t)$ first-order schemes

CN : $u_e - u^n = \mathcal{O}(\Delta t^2)$ second —— " ——

Use symbolic software tools for computing these asymptotic ($\Delta t \to 0$) expressions. See file for sympy example.

Application of convergence rates (i.e., r in $|u_e - u| \sim \Delta t^r$):

* Verification *

Method: Choose some u_e, insert in differential eq., add extra term such that u_e fulfills the eq.

Compute error measures for some Δt values and estimate r. If r approaches the expected value for small Δt, this provides considerable evidence that the program works. See example in the notes.

Course overview:

applications →

↓ tools: decay vib wave diff advect elast N-S ...

finite differences
finite elements
(finite volumes)
programming
verification
software engineering
testing
experiments
numerical artifacts
theoretical analysis
truncation error
convergence tests
scaling
physics & modeling