Background Questionnaire for INF5620

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The purpose of the following test is to map out the students' background in mathematics, numerical methods, and programming when they come to INF5620. Solve the exercises on a piece of paper and give it to the lecturer by Aug 31, 2012. Don't write your name on the solution – the purpose is not to uncover the knowledge of the individual student but the group as a whole.

1 Previous courses and topics

Mark the topics that have been covered in previous courses and indicate which course (if you remember).

- Analytical differentiation and integration
- Numerical differentiation and integration
- Area, volume, line, and surface integrals
- Numerical methods for nonlinear scalar equations
- Numerical methods for systems of nonlinear equations
- Analytical linear algebra
- Numerical linear algebra
- Scalar fields
- Vector fields
- Analytical methods for ordinary differential equations
- Numerical methods for ordinary differential equations
- Analytical methods for partial differential equations
- Numerical methods for partial differential equations
- Particle and rigid body dynamics (Newton's 2nd law)
- The Laplace/Poisson equation

- The time-dependent diffusion equation
- The wave equation
- Maxwell's equations
- The equation for linear elasticity
- The Navier-Stokes equation
- Scaling and dimensionless variables
- Python programming
- Matlab programming
- Java programming
- Fortran programming
- C++ programming
- C programming
- Unix shell programming
- Parallel computing
- Report writing with LaTeX
- Version control systems (svn, hg, git, bzr)

In what subject is your master's or phd thesis? Any need for solving specific PDEs or learning specific tools for scientific computing?

2 Differentiation

1. Differentiate the function

$$u(x,t) = A\exp\left(ik(x-ct)\right)$$

with respect to t. The symbols A, k, and c are constants, and i is the imaginary unit ($i=\sqrt{-1}$).

2. You know the three values $y(0)$, $y(0.25)$, and $y(0.5)$ of some function $y(t)$. How can you compute dy/dt at these three points?

3 Integration

3. Describe how you would compute the integral

$$\int_0^1 e^{-t^2}dt.$$

4 Ordinary differential equations

4. Describe how you would solve the ordinary differential equation problem

$$u^{-1}u' + 1 = 0, \quad u(0) = 1.$$

5 Nonlinear equations

5. Formulate Newton's method for solving

$$w + c\Delta t w^2 = u,$$

with respect to w. The symbols c, Δt, and u are known numbers.

6. Implement the algorithm in Exercise 5 in your favorite programming language (just sketch the program on paper).

6 Scalar and vector fields

7. Compute the divergence of the gradient of the scalar field $u(x, y) = \sin \pi x \sin \pi y$ (i.e., $\nabla \cdot (\nabla u)$).

7 Partial differential equations

8. Formulate a numerical method for solving

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}, \quad x \in (0, 1),\ t > 0,$$

where $k > 0$ is a constant. The initial condition reads $u(x, t) = 1$, while we set $u(0) = u(1) = 0$ as boundary conditions.

9. What is the most complicated partial differential equation problem you have solved, or seen be solved by a lecturer?

8 Physical modeling

10. Give a physical interpretation of u in the equations below.

$$\frac{du}{dt} = -au$$

$$\frac{d^u}{dt^2} + \omega^2 u = 0$$

$$\frac{\partial u}{\partial t} = \alpha\frac{\partial^2 u}{\partial t^2}$$

$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u$$

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial t} = 0$$

$$\frac{\partial^2 u}{\partial t^2} = (\lambda + \mu)\nabla(\nabla \cdot u) + \mu \nabla^2 u$$

$$\frac{\partial u}{\partial t} + u\nabla u = -\nabla p + \frac{1}{\mathrm{Re}}\nabla^2 u, \quad \nabla \cdot u = 0$$

11. Given a vector field $\mathbf{v}(\mathbf{x}, t)$ describing the velocity of some flowing material, what is the physical interpretation of $\int_\Gamma \mathbf{v} \cdot \mathbf{n} ds$? Γ is some closed 3D surface and $\mathbf{n}$ is the unit outward normal vector to this surface.

12. In Exercise 11, transform the surface integral to a volume integral and use the result to give a physical interpretation of the divergence of $\mathbf{v}$.