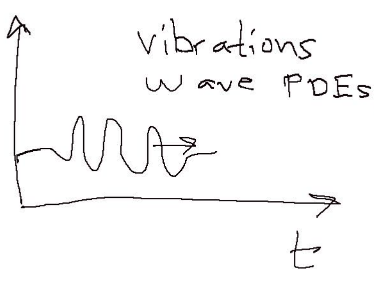


$u' = -au \ (+b)$ $\qquad$ $u'' + \omega^2 u = 0$ "vib"

$u'' + \omega^2 u = 0, \quad u(0) = I, \ u'(0) = 0, \quad t \in (0, T]$

1. time mesh: $t_n = n\Delta t, \quad n = 0, \ldots, N$
2. $u''(t_n) + \omega^2 u(t_n) = 0, \quad n = 0, \ldots, N$
3. $u''(t_n) \approx \frac{u^{n+1} - 2u^n + u^{n-1}}{\Delta t^2}$
4. Algorithm:

$$u^{n+1} = 2u^n - \underline{u^{n-1}} - \Delta t^2 \omega^2 u^n$$

Problem: $u^1 = 2\underset{\overset{\shortparallel}{I}}{u^0} - u^{-1}$???

Solution: $u'(0) = 0 \qquad \frac{u^1 - u^{-1}}{2\Delta t} = 0 \Rightarrow u^1 = u^{-1}$

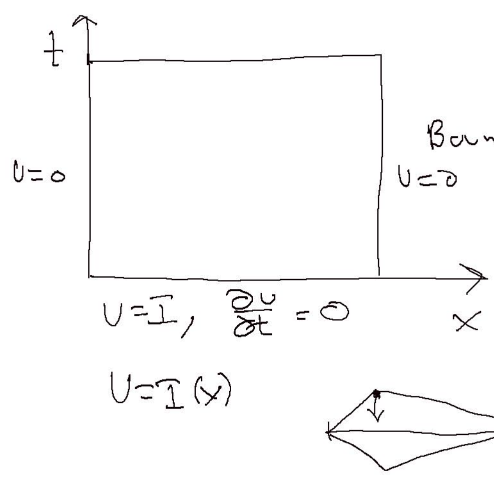
Scheme

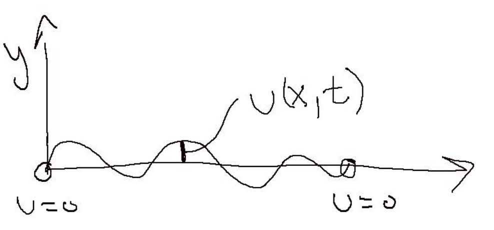
$\Rightarrow \underline{u^1} = 2I - \underline{u^1} - \omega^2 I$, can solve wrt u^1

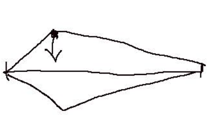
Alg: $u^0 = I$
$u^1 = \ldots\ldots$
for $n = 1, 2, \ldots N-1$
$u^{n+1} = \ldots\ldots$

Extend ideas to wave PDE:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

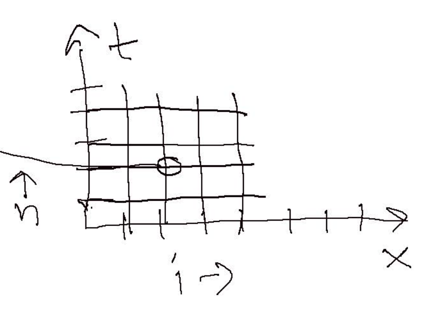






$$u''(t_n) \approx [D_t D_t u]^n$$

Step 2: PDE holds at each mesh point



$$\frac{\partial^2 u(x_i, t_n)}{\partial t^2} = c^2 \frac{\partial}{\partial x^2} u(x_i, t_n), \qquad i = 0, \ldots, \ n = 1, 2, \ldots$$

$$\downarrow \qquad\qquad \downarrow$$

$$[D_t D_t u]^n_i = c^2 [D_x D_x u]^n_i$$

$$\downarrow \qquad\qquad \downarrow$$

$$\frac{u^{n+1}_i - 2u^n_i + u^{n-1}_i}{\Delta t^2} = c^2 \frac{u^n_{i+1} - 2u^n_i + u^n_{i-1}}{\Delta x^2}$$

Assume: u^{n-1}_i, u^n_i known for all i

Only unknown: u^{n+1}_i

$$u^{n+1}_i = 2u^n_i - u^{n-1}_i + \frac{c^2 \Delta t^2}{\Delta x^2}\left(u^n_{i+1} - 2u^n_i + u^n_{i-1}\right)$$

$u^1_i = ?$ $\qquad \frac{\partial u}{\partial t} = 0, t = 0$