

# Background Questionnaire for INF5620

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The purpose of the following test is to map out the students' background in mathematics, numerical methods, and programming when they come to INF5620. Solve the exercises on a piece of paper and give it to the lecturer by Aug 31, 2012. Don't write your name on the solution – the purpose is not to uncover the knowledge of the individual student but the group as a whole.

## 1 Previous courses and topics

Mark the topics that have been covered in previous courses and indicate which course (if you remember).

- Analytical differentiation and integration
- Numerical differentiation and integration
- Area, volume, line, and surface integrals
- Numerical methods for nonlinear scalar equations
- Numerical methods for systems of nonlinear equations
- Analytical linear algebra
- Numerical linear algebra
- Scalar fields
- Vector fields
- Analytical methods for ordinary differential equations
- Numerical methods for ordinary differential equations
- Analytical methods for partial differential equations
- Numerical methods for partial differential equations
- Particle and rigid body dynamics (Newton's 2nd law)
- The Laplace/Poisson equation

- The time-dependent diffusion equation
- The wave equation
- Maxwell's equations
- The equation for linear elasticity
- The Navier-Stokes equation
- Scaling and dimensionless variables
- Python programming
- Matlab programming
- Java programming
- Fortran programming
- C++ programming
- C programming
- Unix shell programming
- Parallel computing
- Report writing with L<sup>A</sup>T<sub>E</sub>X
- Version control systems (svn, hg, git, bzt)

In what subject is your master's or phd thesis? Any need for solving specific PDEs or learning specific tools for scientific computing?

## 2 Differentiation

1. Differentiate the function

$$u(x, t) = A \exp(ik(x - ct))$$

with respect to  $t$ . The symbols  $A$ ,  $k$ , and  $c$  are constants, and  $i$  is the imaginary unit ( $i = \sqrt{-1}$ ).

2. You know the three values  $y(0)$ ,  $y(0.25)$ , and  $y(0.5)$  of some function  $y(t)$ . How can you compute  $dy/dt$  at these three points?

## 3 Integration

3. Describe how you would compute the integral

$$\int_0^1 e^{-t^2} dt.$$

## 4 Ordinary differential equations

4. Describe how you would solve the ordinary differential equation problem

$$u^{-1}u' + 1 = 0, \quad u(0) = 1.$$

## 5 Nonlinear equations

5. Formulate Newton's method for solving

$$w + c\Delta tw^2 = u,$$

with respect to  $w$ . The symbols  $c$ ,  $\Delta t$ , and  $u$  are known numbers.

6. Implement the algorithm in Exercise 5 in your favorite programming language (just sketch the program on paper).

## 6 Scalar and vector fields

7. Compute the divergence of the gradient of the scalar field  $u(x, y) = \sin \pi x \sin \pi y$  (i.e.,  $\nabla \cdot (\nabla u)$ ).

## 7 Partial differential equations

8. Formulate a numerical method for solving

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}, \quad x \in (0, 1), \quad t > 0,$$

where  $k > 0$  is a constant. The initial condition reads  $u(x, t) = 1$ , while we set  $u(0) = u(1) = 0$  as boundary conditions.

9. What is the most complicated partial differential equation problem you have solved, or seen be solved by a lecturer?

## 8 Physical modeling

10. Give a physical interpretation of  $u$  in the equations below.

$$\frac{du}{dt} = -au$$

$$\frac{d^2 u}{dt^2} + \omega^2 u = 0$$

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial t^2}$$

$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u$$

$$\frac{\partial u}{\partial t} + c \frac{\partial u}{\partial t} = 0$$

$$\frac{\partial^2 u}{\partial t^2} = (\lambda + \mu) \nabla(\nabla \cdot u) + \mu \nabla^2 u$$

$$\frac{\partial u}{\partial t} + u \nabla u = -\nabla p + \frac{1}{\text{Re}} \nabla^2 u, \quad \nabla \cdot u = 0$$

**11.** Given a vector field  $\mathbf{v}(\mathbf{x}, t)$  describing the velocity of some flowing material, what is the physical interpretation of  $\int_{\Gamma} \mathbf{v} \cdot \mathbf{n} ds$ ?  $\Gamma$  is some closed 3D surface and  $\mathbf{n}$  is the unit outward normal vector to this surface.

**12.** In Exercise 11, transform the surface integral to a volume integral and use the result to give a physical interpretation of the divergence of  $\mathbf{v}$ .