Background Questionnaire for INF5620

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The purpose of the following test is to map out the students' background in mathematics, numerical methods, and programming when they come to INF5620. Solve the exercises on a piece of paper and give it to the lecturer by Feb 3, 2011. Don't write your name on the solution – the purpose is not to uncover the knowledge of the individual student but the group as a whole.

1 Previous courses and topics

Mark the topics that have been covered in previous courses and indicate which course (if you remember).

- Analytical differentiation and integration
- Numerical differentiation and integration
- Area, volume, line, and surface integrals
- Numerical methods for nonlinear scalar equations
- Numerical methods for systems of nonlinear equations
- Analytical linear algebra
- Numerical linear algebra
- Scalar fields
- Vector fields
- Analytical methods for ordinary differential equations
- Numerical methods for ordinary differential equations
- Analytical methods for partial differential equations
- Numerical methods for partial differential equations
- Physical modeling with differential equations (Newton's 2nd law, equation of continuity, equation of heat transfer, etc.)
- Python programming

- Matlab programming
- Java programming
- Fortran programming
- C++ programming
- C programming
- Unix shell programming
- Report writing with LaTeX

2 Differentiation

1. Differentiate the function

$$u(x,t) = A\exp\left(ik(x-ct)\right)$$

with respect to t. The symbols A, k, and c are constants, and i is the imaginary unit ($i=\sqrt{-1}$).

2. Compute $y(0)$ and $y(\frac{1}{2})$ where

$$y(x) = A\sin\left(\frac{\pi}{6}x\right).$$

Use the two values to approximate the derivative of $y(x)$ for $x=0$, $x=0.25$, and $x=0.5$.

3 Integration

3. How would you compute the integral

$$\int_0^1 Ae^{-at^2}\,dt,$$

where A and a are constants?

4 Ordinary differential equations

4. How can you find the exact solution to the ordinary differential equation

$$u^{-1}u' + c = 0, \quad u(0) = 1,$$

where c is a constant?

5. Describe a simple numerical method for solving the problem in Exercise 4. Formulate the method as an algorthm.

6. Implement the algorithm in Exercise 5 in your favorite programming language (just sketch the program on paper).

5 Scalar and vector fields

7. Compute the divergence of the gradient of the scalar field

$$u(x, y) = \sin \pi x \sin \pi y$$

$(\nabla \cdot (\nabla u))$

6 Nonlinear equations

8. Formulate Newton's method for solving

$$w + c\Delta t w^2 = u_k,$$

with respect to w. The symbols c, Δt, and u_k are known numbers.

7 Partial differential equations

9. Formulate a numerical method for solving

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}, \quad x \in (0, 1),\ t > 0,$$

where $k > 0$ is a constant. The initial condition reads $u(x, t) = 1$, while we set $u(0) = u(1) = 0$ as boundary conditions.

8 Physical modeling

10. Set up the equations governing the motion of a ball through the air (treat the ball as a mathematical point). Two forces are acting on the ball: 1) gravity and 2) air resistance. The latter is modeled as $b|\mathbf{v}|\mathbf{v}$, where b is some constant and $\mathbf{v}$ is the velocity of the ball.

11. Given a vector field $\mathbf{v}(\mathbf{x}, t)$ describing the velocity of some flowing material, what is the physical interpretation of $\int_\Gamma \mathbf{v} \cdot \mathbf{n} ds$? Γ is some closed 3D surface and $\mathbf{n}$ is the unit outward normal vector to this surface.

12. In Exercise 11, transform the surface integral to a volume integral and use the result to give a physical interpretation of the divergence of $\mathbf{v}$.