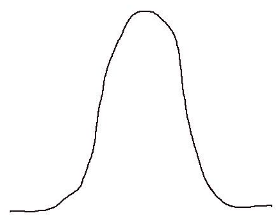
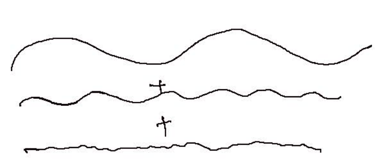
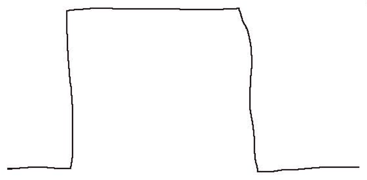


Smooth:

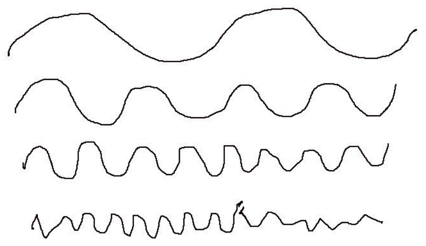


Why is a relatively steep Gaussian function transported accurately from an area with C=1 to an area with a different wave velocity, C=0.25, while a plug-shaped wave develops significant noise?

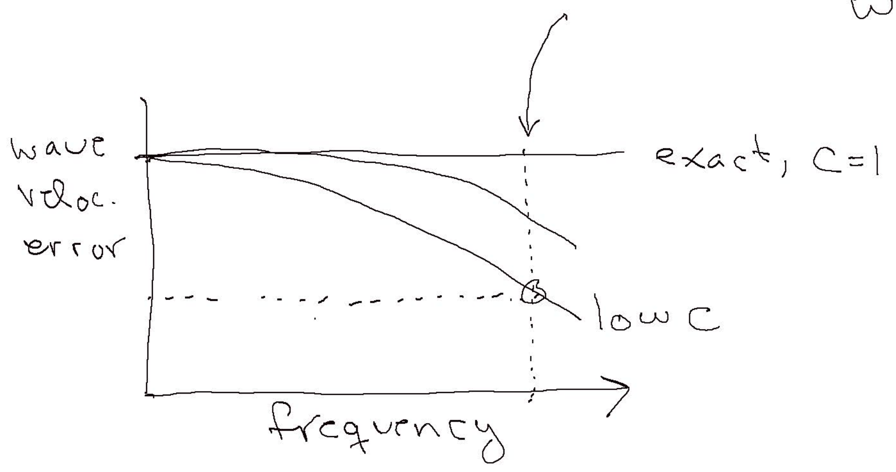
We build up the initial condition from sine components (as in the analysis). The plug has short wave components with significant amplitude and these move with wrong wave velocity for C=0.25.

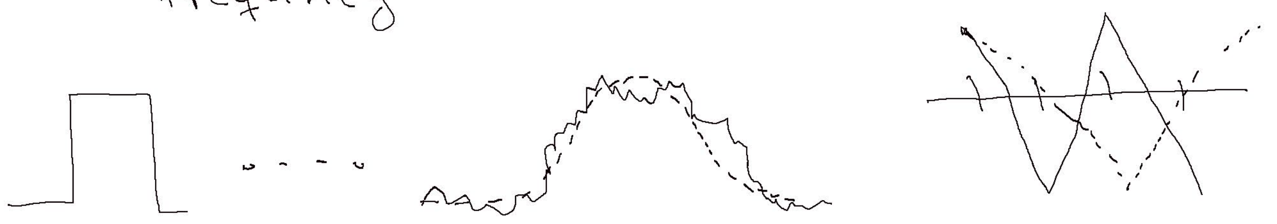


much less smooth!



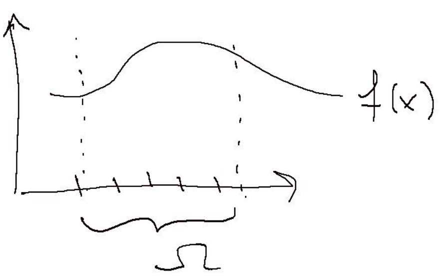
more short waves with significant amplitude





very short waves fall behind and are visible

Approximation



$$v = \sum \underset{\substack{\uparrow \\ \text{unknowns}}}{c_j}\, \underbrace{\varphi_j(x)}_{\text{prescribed}}$$

Least squares: minimize $\| f - v \|$
$c_0, \dots c_N$

$\Updownarrow$

Galerkin (projection) error $\perp V = \text{span}\{\varphi_0, \varphi_1, \dots \varphi_N\}$

Examples: $\varphi_i \sim \sin ix$, $\varphi_i \sim x^i$

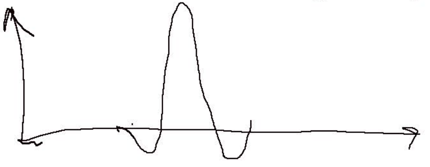
Result: linear system $Ac = b$

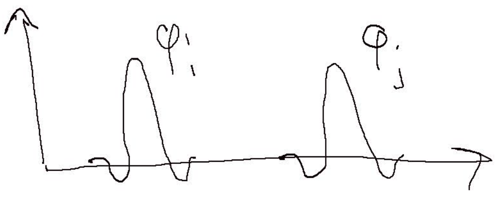
$$A_{ij} = \int_\Omega \varphi_i \varphi_j \, dx, \quad b_i = \int_\Omega f \varphi_i \, dx$$

Finite elements:

localized functions

φ_i :





most likely: $\varphi_i \varphi_j = 0$