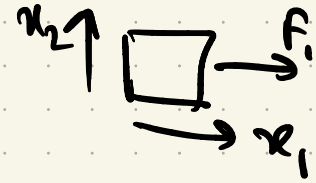
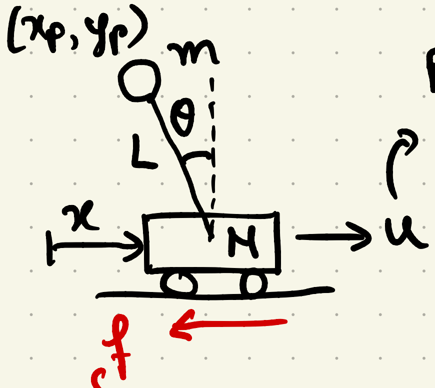
Inverted Pendulum on a Cart





Force on the cart

friction $f = b\dot{x}$

T : K.E.

V : P.E.

$\mathcal{L} = T - V$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = Q_i$$

$\cancel{W = Fq}$

$\cancel{D = \frac{1}{2} b \dot{q}^2}$

Pendulum : $x_p = x - L\sin\theta$; $\dot{x}_p = \dot{x} - L\cos\theta\,\dot{\theta}$

$y_p = L\cos\theta$; $\dot{y}_p = -L\sin\theta\,\dot{\theta}$

$$\text{K.E.} : T = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m\left(\dot{x}_p^2 + \dot{y}_p^2\right)$$

$$= \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m\left(\dot{x}^2 - 2\dot{x}L\cos\theta\,\dot{\theta} + L^2\cos^2\theta\,\dot{\theta}^2 + L^2\sin^2\theta\,\dot{\theta}^2\right)$$

$$\boxed{T = \frac{1}{2}(M+m)\dot{x}^2 + \frac{1}{2}mL^2\dot{\theta}^2 - m\dot{x}L\cos\theta\,\dot{\theta}}$$

$$\text{P.E.} : V = mgy_p = \boxed{mgL\cos\theta}$$

$$\mathcal{L} = T - V$$

$$= \frac{1}{2}(M+m)\dot{x}^2 + \frac{1}{2}mL^2\dot{\theta}^2 - m\dot{x}L\cos\theta\,\dot{\theta} - mgL\cos\theta$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = Q_i \qquad \begin{aligned} q_1 &= x, \quad Q_1 = u - b\dot{x} \\ q_2 &= \theta, \quad Q_2 = 0 \end{aligned}$$

$$\mathcal{L} = \frac{1}{2}(M+m)\dot{x}^2 + \frac{1}{2}mL^2\dot{\theta}^2 - mL\cos\theta\,\dot{\theta}\dot{x} - mgL\cos\theta$$

$$\frac{\partial \mathcal{L}}{\partial \dot{x}} = (M+m)\dot{x} - mL\cos\theta\,\dot{\theta}$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) = (M+m)\ddot{x} - mL\cos\theta\,\ddot{\theta} + mL\sin\theta\,\dot{\theta}^2$$

$$\frac{\partial \mathcal{L}}{\partial x} = 0 \quad ; \quad Q_1 = u - b\dot{x}$$

$$(M+m)\ddot{x} - mL\cos\theta\,\ddot{\theta} + mL\sin\theta\,\dot{\theta}^2 = u - b\dot{x}$$

$$\underline{q_2 = \theta}$$

$$\frac{\partial \mathcal{L}}{\partial \dot{\theta}} = mL^2\dot{\theta} - mL\cos\theta\,\dot{x}$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}}\right) = mL^2\ddot{\theta} - mL\cos\theta\,\ddot{x} + \cancel{mL\sin\theta\,\dot{\theta}\dot{x}}$$

$$\frac{\partial \mathcal{L}}{\partial \theta} = \cancel{mL\sin\theta\,\dot{\theta}\dot{x}} + mgL\sin\theta \qquad Q_2 = 0$$

$$\cancel{m}L^{\cancel{2}}\ddot{\theta} - \cancel{m}\cancel{L}\cos\theta\,\ddot{x} - \cancel{m}g\cancel{L}\sin\theta = 0$$

$$L\ddot{\theta} - L\cos\theta\,\ddot{x} - g\sin\theta = 0$$

$$X = \begin{Bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{Bmatrix} \qquad \dot{X} = f(X) = \qquad \begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_3 &= x_4 \end{aligned}$$

$$\dot{x}_2 = ? \qquad \dot{x}_4 = ?$$

$$\ddot{x} = ? \qquad \ddot{\theta} = ?$$

$$(M+m)\ddot{x} - mL\cos\theta\,\ddot{\theta} + mL\sin\theta\,\dot{\theta}^2 = u - b\dot{x}$$

$$L\ddot{\theta} - L\cos\theta\,\ddot{x} - g\sin\theta = 0$$

$$X_0 = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0.1 \end{Bmatrix}$$

$u = 0$

$m = 1$

$M = 5$

$L = 2$

$g = 9.8$

$b = 1$

tspan $= [0, 100]$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{mg}{\bar{M}} \sin x_3 \cos x_3 - \frac{mL}{\bar{M}} \sin x_3\, x_4^2 - \frac{b x_2}{\bar{M}}$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = \frac{\cos x_3}{L} \left\{ \frac{mg}{\bar{M}} \sin x_3 \cos x_3 - \frac{mL}{\bar{M}} \sin x_3\, x_4^2 - \frac{b x_2}{\bar{M}} \right\} + \frac{g}{L} \sin x_3$$

$$\bar{M} = M + m\sin^2 x_3$$

$$X = [x, \dot{x}, \theta, \dot{\theta}]^T$$