# DYNAMICS OF STRUCTURED POPULATIONS

31 January 2020 ECOLOGICAL AND EVOLUTIONARY RESPONSES TO CLIMATIC VARIATION

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### CONCEPTS

population dynamics

population growth rate

lambda, λ

demography

vital rates

population structure

sensitivity

elasticity

fecundity

stochasticity

stable stage distribution

reproductive value

#### OUTLINE

- Population dynamics
- Structured populations
- Modelling the dynamics of structured populations

Case study

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### THE POPULATION

- Spatial disjunction: limited movement between groups
- Genetic: among-group differences in allele frequencies
- Demographic: among-group differences in vital rates





#### POPULATION DYNAMICS

- Change in population structure through time
- Abundance and spatial distribution
- Demographics or genetic structure



#### THREE IMPORTANT CONCEPTS

POPULATION DYNAMICS

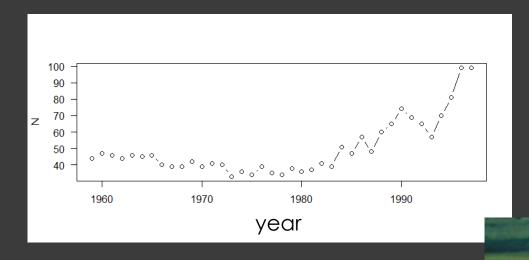
POPULATION GROWTH RATE

**DEMOGRAPHY** 

### POPULATION DYNAMICS

Changes in population size and structure through time

$$N_{(t+1)} = N_{(t)} + B - D (+ I - E)$$

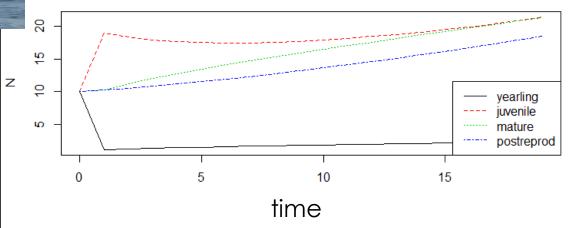


### POPULATION DYNAMICS

Changes in population size and structure through time

$$N_{(t+1)} = N_{(t)} + B - D (+ I - E)$$





#### POPULATION GROWTH RATE

$$\lambda$$
 (lambda) 
$$N_{(t+1)} / N_{(t)} = \lambda$$
 
$$\lambda = e^{r} \quad (r = \ln(\lambda))$$

- $\lambda > 1$ : Population is increasing
- $\lambda$  < 1: Population is decreasing
- $\lambda = 1$ : No change

#### DEMOGRAPHY

#### DEMOGRAPHY

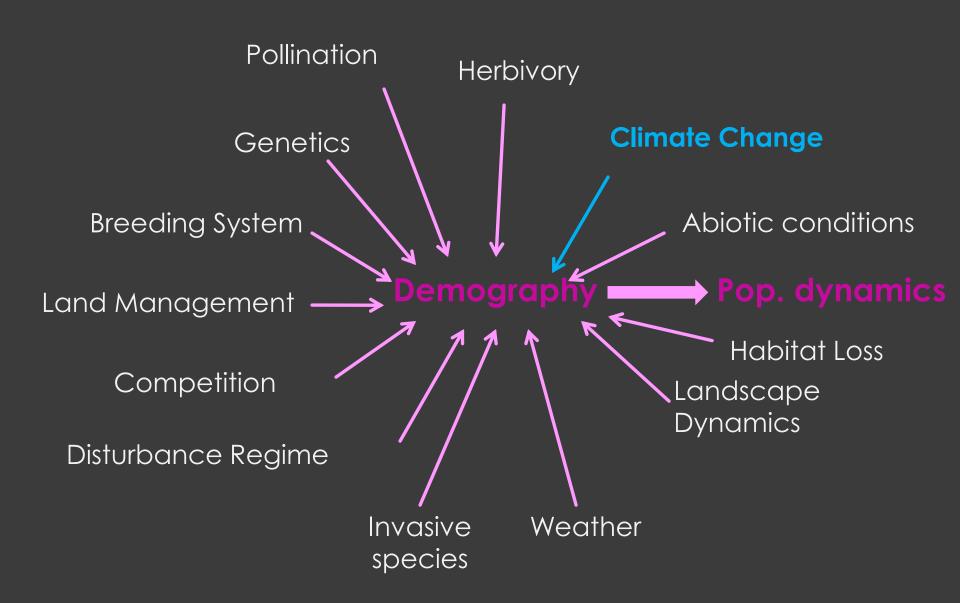
Demo = the people; graphy = measurement Here: study of the processes (vital rates) underlying population dynamics, i.e. reproduction, growth and survival

- Answer questions about population dynamics and conservation
- Understanding natural selection and life-history evolution

#### Important parameters:

- Population viability
- Population growth rate, λ
- Lifetime fitness

#### CONCEPTUALIZING POPULATION DYNAMICS



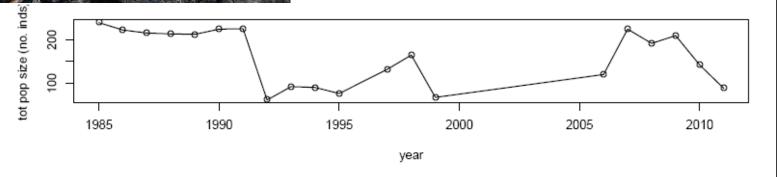
# QUESTIONS WITHIN THE FIELD OF POPULATION DYNAMICS

- Is the population increasing or decreasing?
- What is the risk of extinction?
- What are the causes of population decline/increase?
- How are changes in the environment and climate influencing populations?
- How does hunting, fishing and harvesting influence populations?
- What are the best management options for rare and endangered species?



## Fumana procumbens

 a long-lived dwarf shrub in rocky habitats



What factors are driving changes in population size among years?

# Dracocephalum austriacum

an endangered herbgrowing on exposed cliffsin the Alps

How will population viability be influenced by a warmer climate?



Florence Nicolè, Johan P. Dahlgren, Agnès Vivat, Irène Till-Bottraud, Johan Ehrlén (2011) Journal of Ecology

#### OUTLINE

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Case study

#### STRUCTURED POPULATIONS











#### Individuals differ regarding:

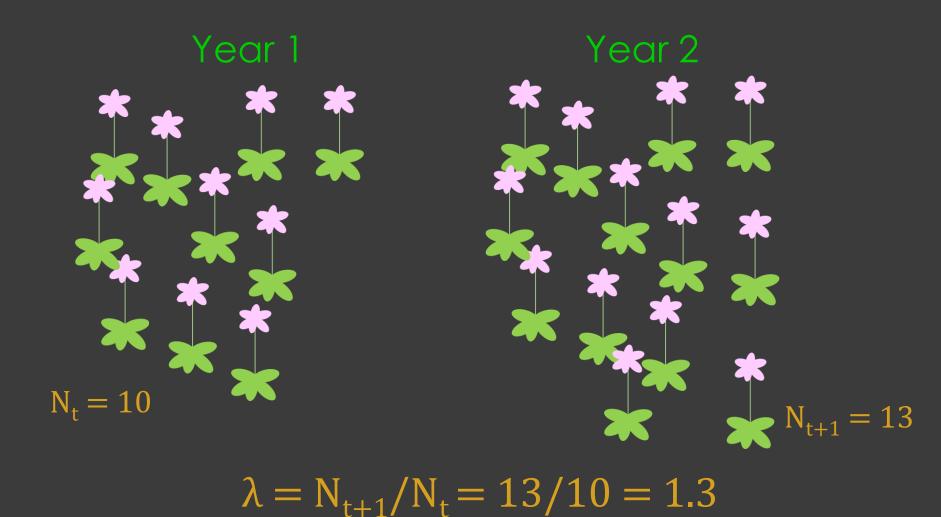
- Size
- Age
- Development stage
- Morphological stage
- Etc.

# Such differences can be related to differences in:

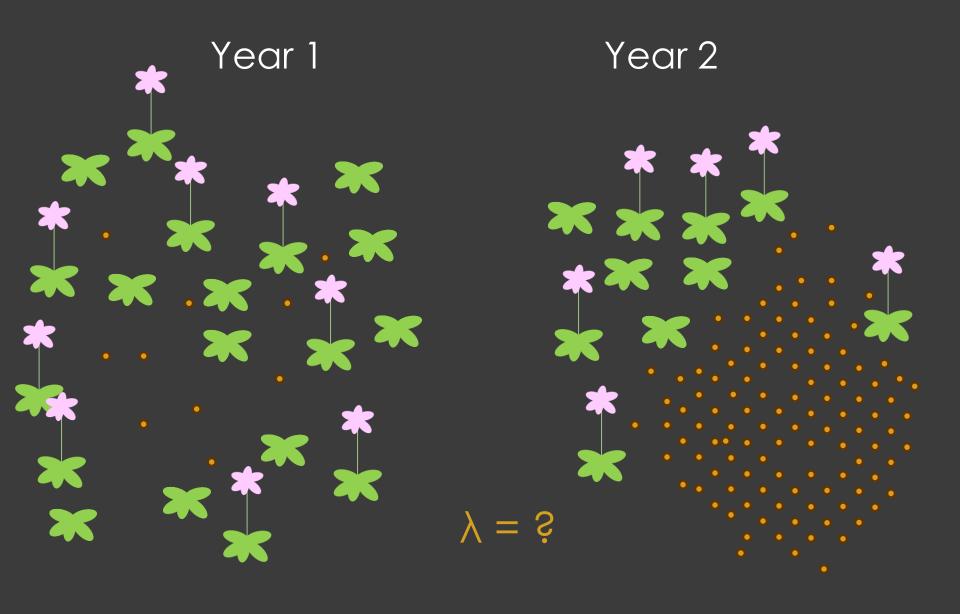
- vital rates (growth, reproduction, mortality)
- impact of stressors
- → differential impact on population growth, and thus more or less important as targets of conservation actions

# HOW DOES POPULATION STRUCTURE INFLUENCE POPULATION DYNAMICS ?

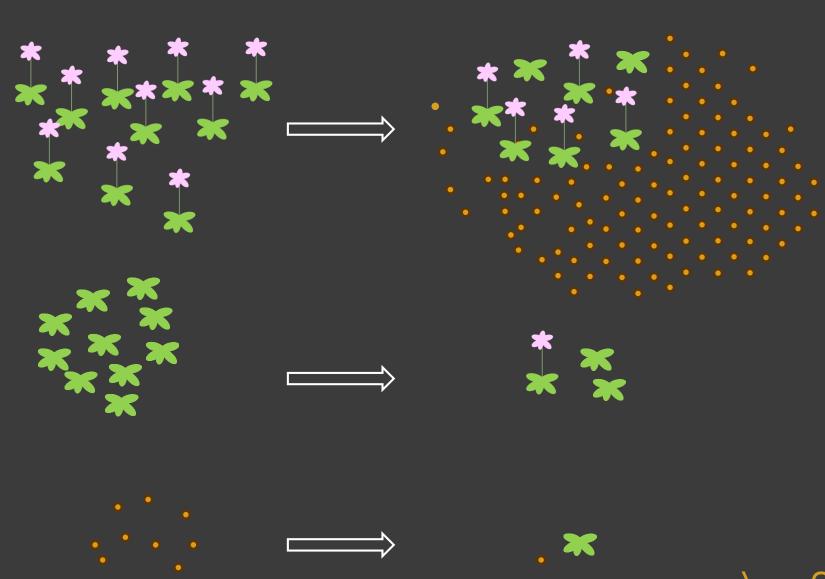
#### **UNSTRUCTURED POPULATION:**



## STRUCTURED POPULATIONS



Year 1 Year 2



 $y = \dot{s}$ 

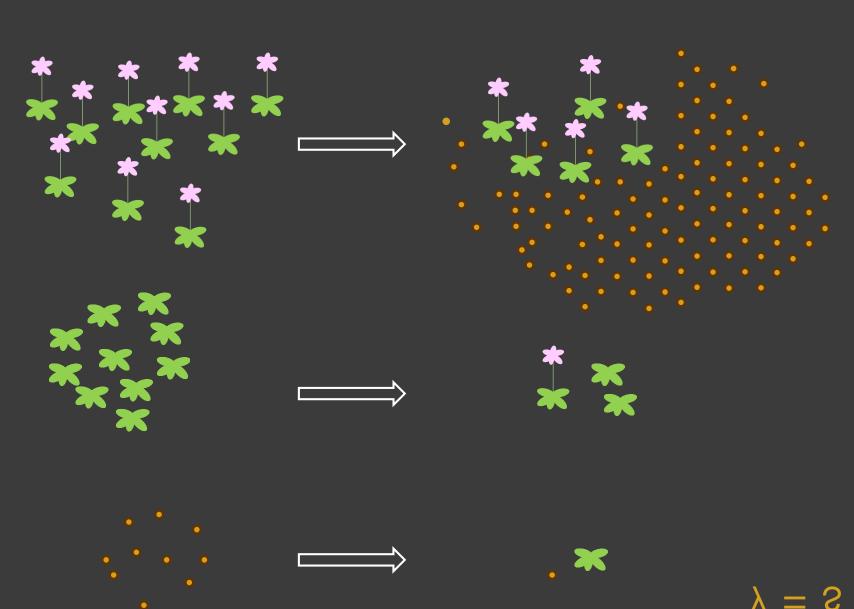
#### OUTLINE

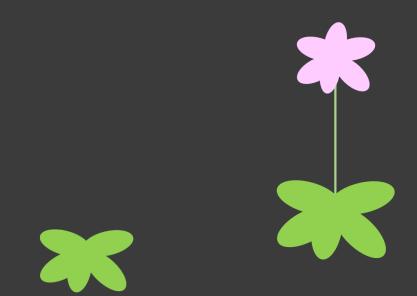
- Population dynamics
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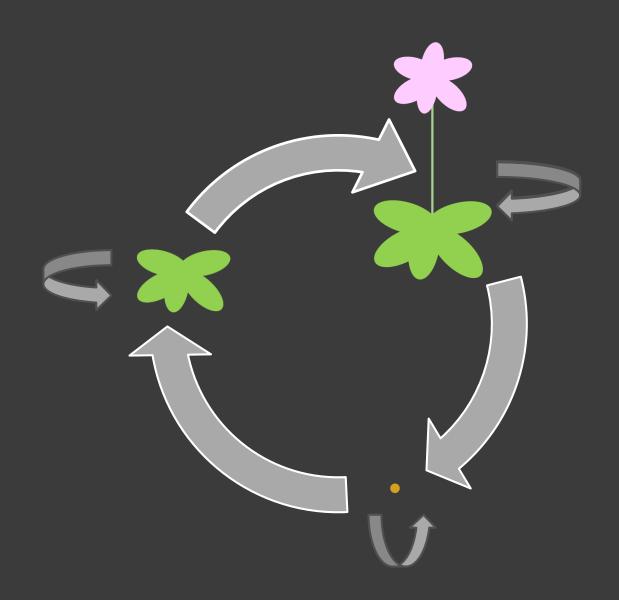
Case study

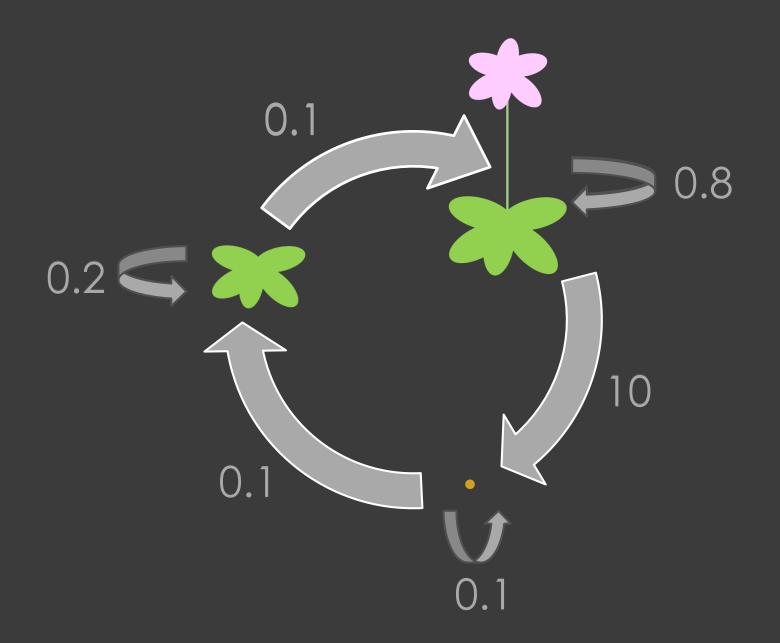
## ORGANISMS ARE THEIR LIFE-CYCLES

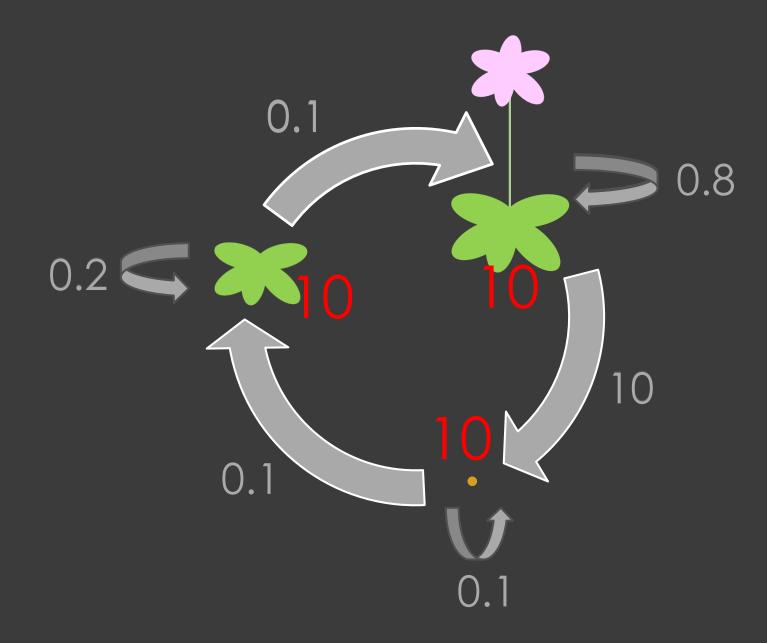
Year 1 Year 2

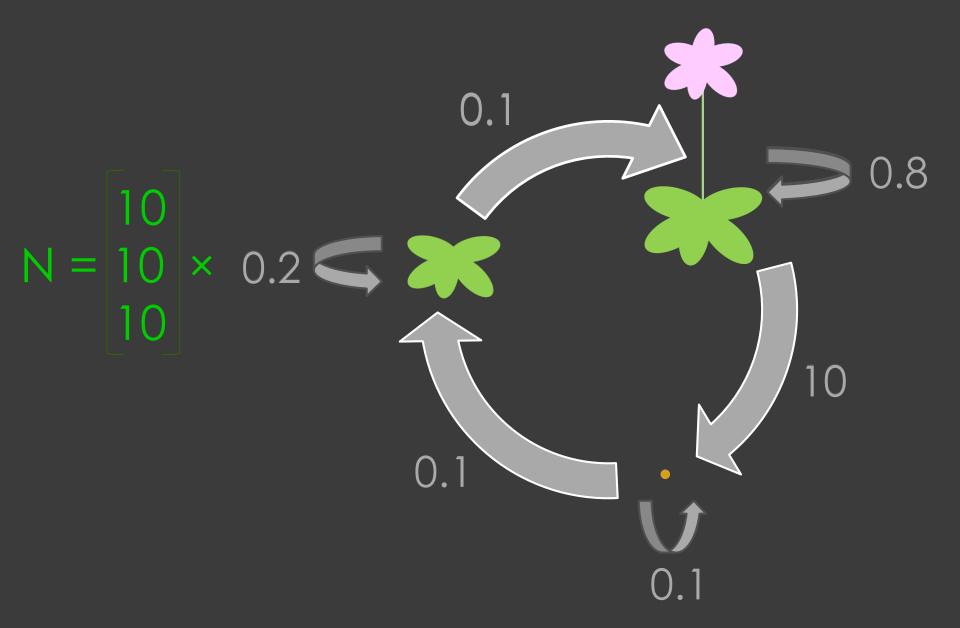












## POPULATION SIMULATION

Year	1	2	3	4	5	6	7	8	9	10	11	12
Seeds	10.0	101.0	100.1	85.0	79.2	76.6	73.6	70.2	67.0	63.9	61.1	58.3
Vegetative	10.0	3.0	10.7	12.2	10.9	10.1	9.7	9.3	8.9	8.5	8.1	7.7
Flowering	10.0	9.0	7.5	7.1	6.9	6.6	6.3	6.0	5.7	5.5	5.2	5.0
	30.0	113.0	118.3	104.2	97.0	93.3	89.5	85.5	81.6	77.9	74.4	71.0
Lambda		3.767	1.047	0.881	0.931	0.962	0.959	0.955	0.954	0.955	0.955	0.955
		01. 07				01/ 02						ution
Stage												
distribution	0.33	0.89	0.85	0.82	0.82	0.82	0.82	0.82	0.82	0.82	0.82	0.82
	0.33	0.03	0.09	0.12	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11
	0.33	0.08	0.06	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07

#### STABLE STAGE DISTRIBUTION

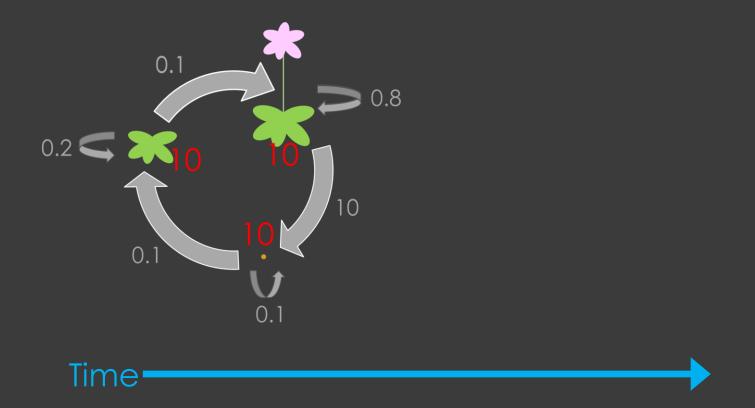
Constant proportion of individuals in each stage. Lambda is stable

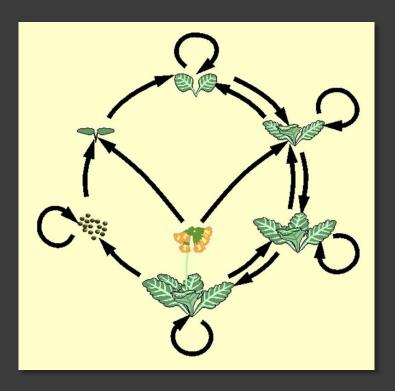
Useful for comparison among populations: individuals from different classes do not contribute equally to population growth rate.

#### REPRODUCTIVE VALUE

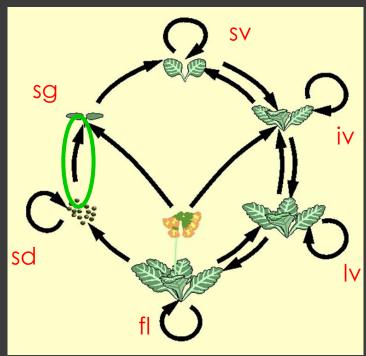
The contribution of individuals from each stage to future population sizes

### POPULATION MODELS ARE PROJECTIONS





# THE LIFE-CYCLE OF PRIMULA VERIS



			veg	veg	veg	
Seed	a <sub>sdsd</sub>					a <sub>sdfl</sub>
Seedling	$a_{sgsd}$					$a_{sgfl}$
Small veg		$a_{svsg}$	a <sub>svsv</sub>	a <sub>sviv</sub>		
Interm veg			a <sub>ivsv</sub>	a <sub>iviv</sub>	a <sub>ivlv</sub>	$a_{ivfl}$
Large veg				$a_{lviv}$	a <sub>lvlv</sub>	a <sub>lvfl</sub>
Flowering					$a_{fllv}$	a <sub>flfl</sub>

Small

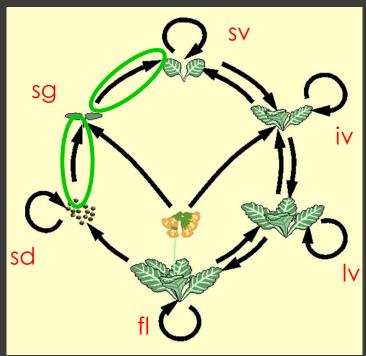
Interm

Large

Flowering

Seed

Seedling



Flowering

			veg	veg	veg	
Seed	a <sub>sdsd</sub>					a <sub>sdfl</sub>
Seedling	a <sub>sgsd</sub>					$a_{sgfl}$
Small veg		a <sub>svsg</sub>	a <sub>svsv</sub>	a <sub>sviv</sub>		
Interm veg			a <sub>ivsv</sub>	a <sub>iviv</sub>	a <sub>ivlv</sub>	a <sub>ivfl</sub>
Large veg				a <sub>lviv</sub>	a <sub>lvlv</sub>	$a_{lvfl}$
Flowering					$a_{fllv}$	a <sub>flfl</sub>

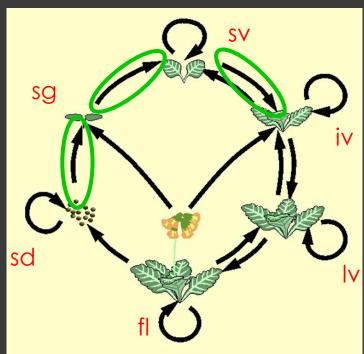
Small

Interm

Large

Seedling

Seed



Flowering

			, <sub>2</sub> 9	, c	, e	
Seed	a <sub>sdsd</sub>					a <sub>sdfl</sub>
Seedling	a <sub>sgsd</sub>					$a_{sgfl}$
Small veg		a <sub>svsg</sub>	$a_{svsv}$	a <sub>sviv</sub>		
Interm ve	g	(	a <sub>ivsv</sub>	a <sub>iviv</sub>	a <sub>ivlv</sub>	a <sub>ivfl</sub>
Large veg				a <sub>lviv</sub>	a <sub>lvlv</sub>	a <sub>lvfl</sub>
Flowering					$a_{fllv}$	$a_{flfl}$

Small

veg

Interm

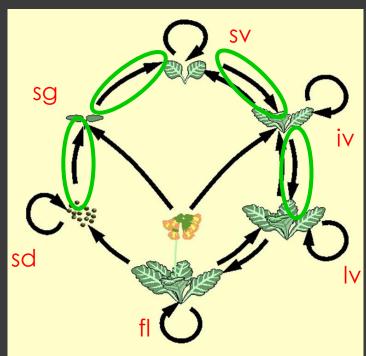
veg

Large

veg

Seedling

Seed



Flowering

			V-8	V - 8	V-8	
Seed	$a_{sdsd}$					a <sub>sdfl</sub>
Seedling	$a_{sgsd}$					$a_{sgfl}$
Small veg		a <sub>svsg</sub>	a <sub>svsv</sub>	a <sub>sviv</sub>		
Interm veg			a <sub>ivsv</sub>	a <sub>iviv</sub>	$a_{ivlv}$	a <sub>ivfl</sub>
Large veg				a <sub>lviv</sub>	$a_{lvlv}$	a <sub>lvfl</sub>
Flowering					$a_{fllv}$	$a_{flfl}$

Small

veg

Interm

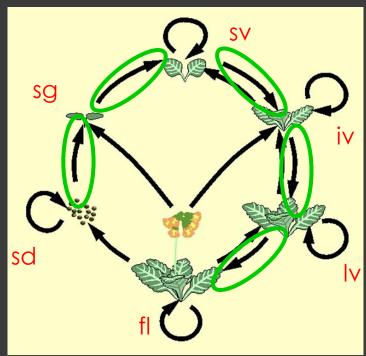
veg

Large

veg

Seedling

Seed



		veg	veg	veg	
Seed $a_{ m sdsd}$					a <sub>sdfl</sub>
Seedling $a_{sgsd}$	)				$a_{sgfl}$
Small veg	a <sub>svsg</sub>	$a_{svsv}$	a <sub>sviv</sub>		
Interm veg	(	a <sub>ivsv</sub>	a <sub>iviv</sub>	a <sub>ivlv</sub>	a <sub>ivfl</sub>
Large veg			alviv	$a_{lvlv}$	a <sub>lvfl</sub>
Flowering				a <sub>fllv</sub>	a <sub>flfl</sub>

Small

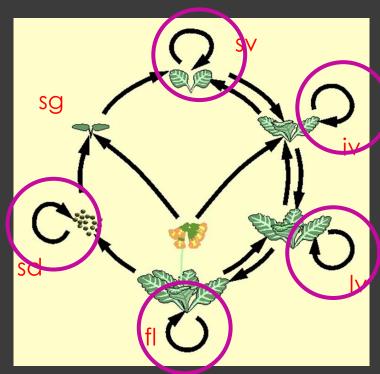
Interm

Large

Flowering

Seed

Seedling



Flowering

		veg	veg	veg	
Seed $a_{sdsd}$					a <sub>sdfl</sub>
Seedling $a_{sgsd}$					$a_{sgfl}$
Small veg	$a_{svsg}$	a <sub>svsv</sub>	$a_{sviv}$		
Interm veg		a <sub>ivsv</sub>	a <sub>iviv</sub>	$a_{ivlv}$	a <sub>ivfl</sub>
Large veg			$a_{lviv}$	a <sub>lvlv</sub>	$a_{lvfl}$
Flowering				$a_{fllv}$	a <sub>flfl</sub>

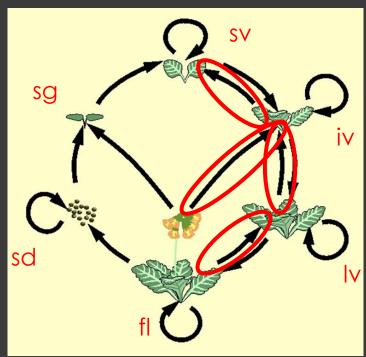
Small

Interm

Large

Seed

Seedling



Flowering

			VCS	veg	veg	
Seed	$a_{\rm sdsd}$					a <sub>sdfl</sub>
Seedling	$a_{sgsd}$					$a_{sgfl}$
Small veg		$a_{svsg}$	a <sub>svsv</sub>	a <sub>sviv</sub>		
Interm veg			a <sub>ivsv</sub>	a <sub>iviv</sub>	a <sub>ivlv</sub>	a <sub>ivfl</sub>
Large veg				$a_{lviv}$	a <sub>lvlv</sub>	a <sub>lvfl</sub>
Flowering					$a_{fllv}$	a <sub>flfl</sub>

Small

Veg

Interm

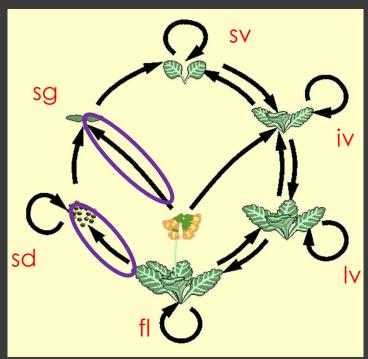
Veg

Large

Veg

Seedling

Seed



Flowering

			VCS	VCB	veg	
Seed	a <sub>sdsd</sub>					a <sub>sdfl</sub>
Seedling	$a_{sgsd}$					a <sub>sgfl</sub>
Small veg		a <sub>svsg</sub>	a <sub>svsv</sub>	a <sub>sviv</sub>		
Interm veg			a <sub>ivsv</sub>	a <sub>iviv</sub>	a <sub>ivlv</sub>	$a_{ivfl}$
Large veg				a <sub>lviv</sub>	a <sub>lvlv</sub>	$a_{lvfl}$
Flowering					$a_{fllv}$	a <sub>flfl</sub>

Small

Veg

Interm

Veg

Large

Veg

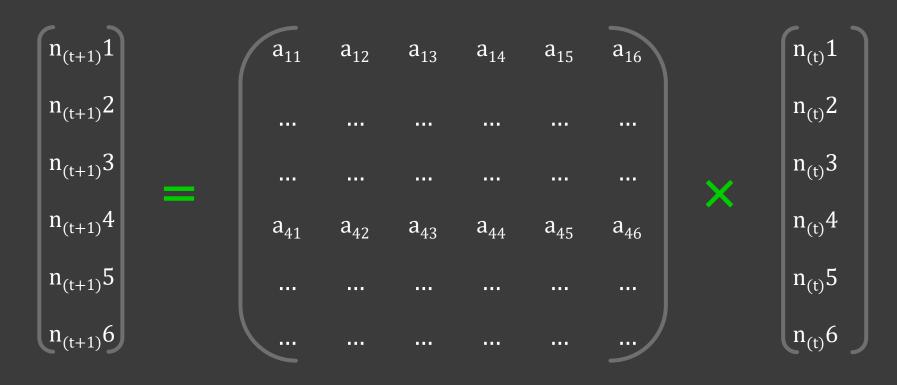
Seedling

Seed

- Right multiplying a vector with a matrix

$$\mathbf{n}_{(t+1)} = \mathbf{A}\mathbf{n}_{(t)}$$

- Right multiplying a vector with a matrix



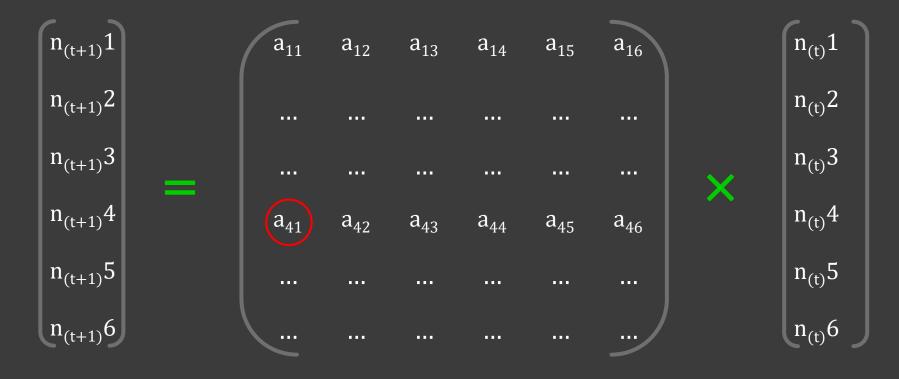
 $\mathbf{n}_{(t+1)}$ 

A

 $\mathbf{n}_{(t)}$ 

## - Right multiplying a vector with a matrix

Entries referred to as matrix elements (a) with subscripted numbers for the row and column.  $a_{41}$  is the matrix element on row 4, column 1

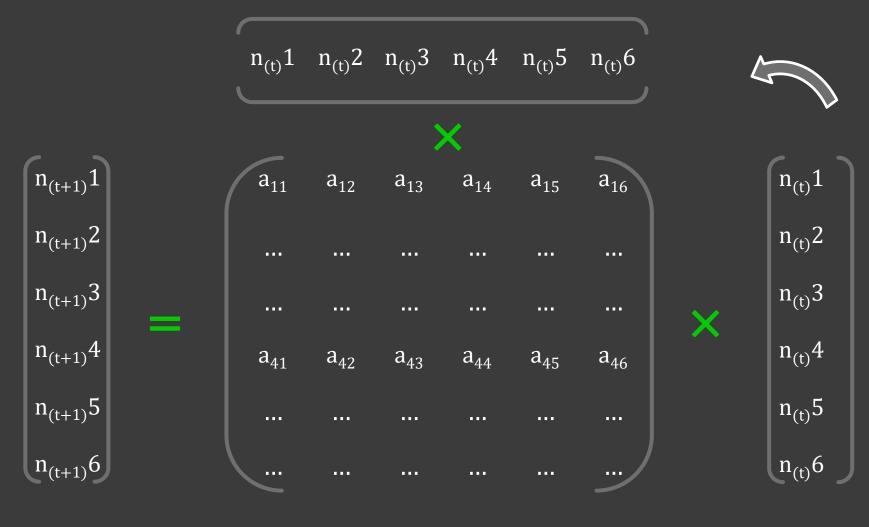


$$\mathbf{n}_{(t+1)}$$

A

n<sub>(t)</sub>

# - Right multiplying a vector with a matrix



 $\mathbf{n}_{(t+1)}$ 

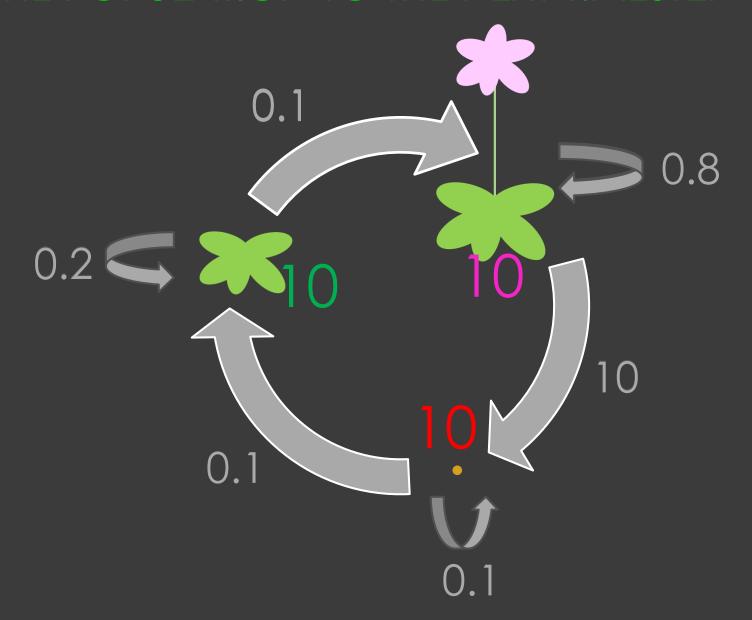
A

 $\mathbf{n}_{(t)}$ 

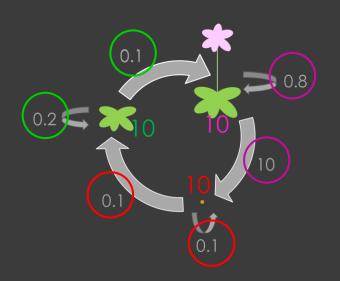
## - Right multiplying a vector with a matrix

$$\mathbf{N}_{(t+1)} = \mathbf{A}\mathbf{n}_{(t)}$$

## PROJECT THE POPULATION TO THE NEXT TIMESTEP



### PROJECT THE POPULATION TO THE NEXT TIMESTEP



How many individuals will be in each size class in the next timestep?

$n_{t+1}$	
101.0	Seeds
3.0	Vegetative
9.0	Flowering

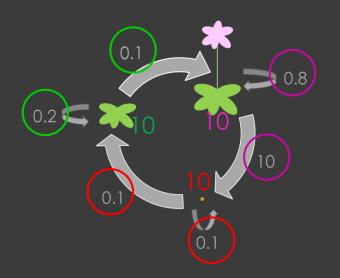
Seeds	Vegetative	Flowering
$(10\times0.1)$	+ 0	$+(10 \times 10)$
$(10\times0.1)$	$+(10 \times 0.2)$	+ 0
0	$+(10 \times 0.1)$	$+(10 \times 0.8)$

10 10 10

Transition matrix with vital rates

Population vector

## PROJECT THE POPULATION TO THE NEXT TIMESTEP



# What is the population growth rate $(\lambda)$ ?

$$\lambda = n_{t+1}/n_t$$

$n_{t+1}$	
101.0	Seeds
3.0	Vegetative
9.0	Flowering

Seeds	Vegetative	Flowering
$(10 \times 0.1)$	+ 0	$+(10 \times 10)$
$(10 \times 0.1)$	$+(10 \times 0.2)$	+ 0
0	$+(10 \times 0.1)$	$+(10 \times 0.8)$

10 10 10

Transition matrix with vital rates

Population vector

#### MATRIX PROPERTIES

- The dominant right eigenvalue of the matrix,  $\lambda_1$ , corresponds to the deterministic population growth rate
- The right eigenvector corresponds to the stablestage distribution
- The left eigenvector corresponds to the reproductive values

## Population projection

Rana temporaria, the common frog

Conservation status: Least concern

Main threat: Habitat loss (wetlands)

IUCN Redlist (www-iucnredlist.org/species/58734/86470817)

## Population projection

**Your task:** Predict future population sizes and the (long term) growth rate ( $\lambda$ ) for a population of the common frog.

**Methods:** Matrix multiplication (manual perturbations) & analytical methods (eigen-analysis)

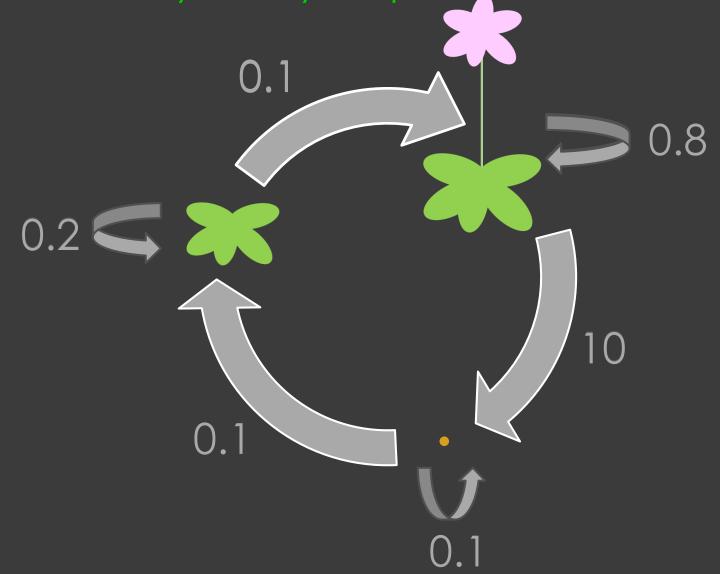
#### **Questions for assignment**

(Temporary) link:

https://fogelstrom.github.io/popDynamicsEERCV/

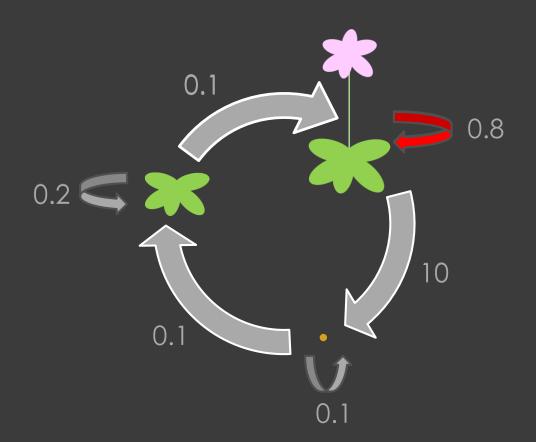
## THE IMPORTANCE OF THE PARTS TO THE WHOLE

- Identification of key life cycle phases



# THE IMPORTANCE OF THE PARTS TO THE WHOLE - Identification of key life cycle phases

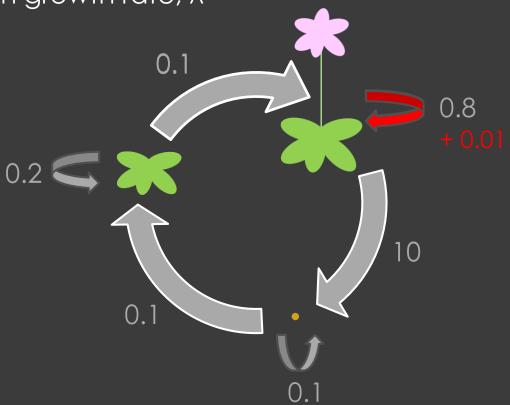
Sensitivity and Elasticity
estimate how "important"
a particular transition is to
population growth by
examining the effects of
small changes in different
parts of the life cycle



# SENSITIVITY, sii

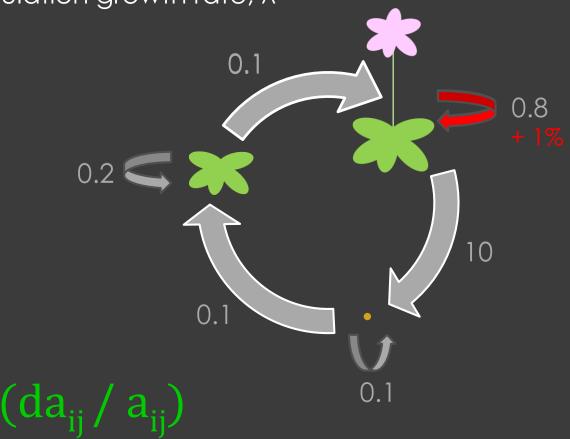
estimates the effects of small absolute changes in a matrix element,  $a_{ii}$ , on population growth rate,  $\lambda$ 

$$s_{ij} = d\lambda / da_{ij}$$

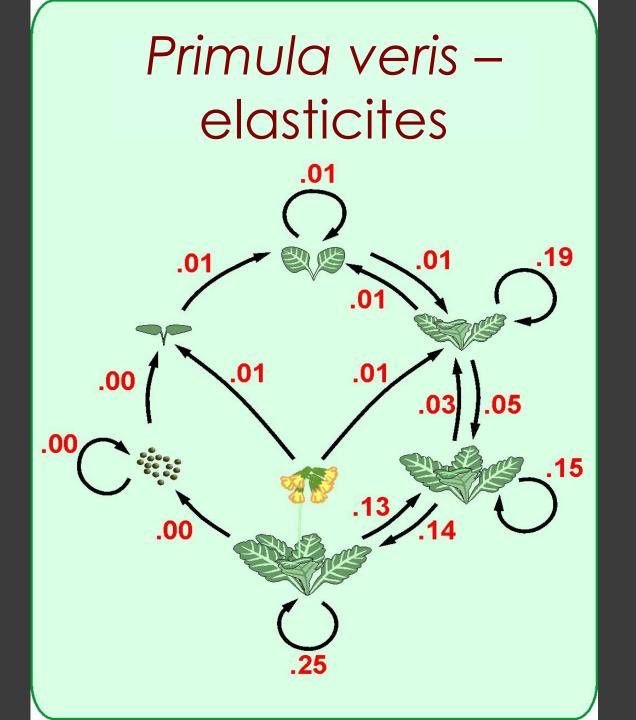


# ELASTICITY, e<sub>ii</sub>

estimates the effects of small relative changes in a matrix element,  $\alpha_{\parallel}$ , on population growth rate,  $\lambda$ 



$$e_{ij} = (d\lambda / \lambda) / (da_{ij} / a_{ij})$$



### Estimate sensitivities and elasticities

Hudsonia montana The mountain golden heather endemic to North Carolina, USA fragmented populations on mountain slopes. only seven populations are known

Conservation status: threatened

#### Main threats:

disturbance from hikers and mountain climbers. Altered fire regimes

#### Estimate sensitivities and elasticities

**Your task:** find out which vital rates have the strongest influence on population growth rate.

Use your results to choose a management regime that

- 1) increases seed survival in the seed bank, or
- 2) increases seed production in the smallest size class (tiny)

**Methods**: Sensitivity and elasticity analysis (manual and analytical)

#### **Questions for assignment**

(Temporary) link:

https://fogelstrom.github.io/popDynamicsEERCV/

### STOCHASTICITY

Environmental: Random climate fluctuations

Demographic: Random fluctuations in vital rates, or in population age or size structure

## STOCHASTICITY

The stochastic growth rate  $(\lambda_S)$  is the geometric mean of  $\lambda$ 

$$\lambda_S = e^{\bar{r}}$$

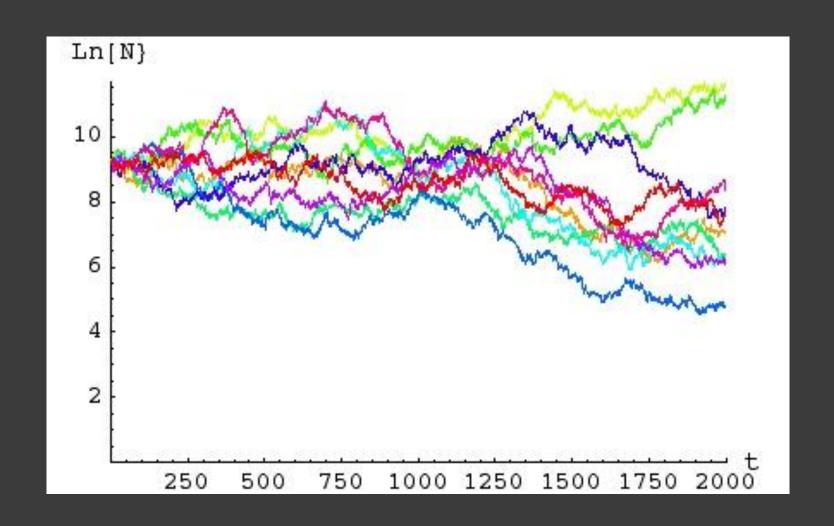
### STOCHASTICITY

Stochasticity can be taken into account by

 Incorporating environmental stochasticity into demographic models: Among-year variation

Incorporating demographic stochasticity into demogrphic models

## STOCHASTIC MATRIX SIMULATIONS



# LINKING VARIATION IN POPULATION DYNAMICS TO ENVIRONMENTAL VARIATION

# Effects of the environment or climate on individual organisms

 Direct effects – organisms experience changes in vital rates due to changed environmental conditions, e.g. in terms of altered light availability, temperature, humidity etc.

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 Indirect effects – organisms use the environment as a cue

# Effects of the environment or climate on individual organisms

 Direct effects – organisms experience changes in vital rates due to changed environmental conditions, e.g. in terms of altered light availability, temperature, humidity etc.

 Indirect effects – organisms use the environment as a cue

 Effects mediated by species interactions – changes in the environment affect interactions among species

# Responses of vital rates of individuals to the environment translates into effects on:

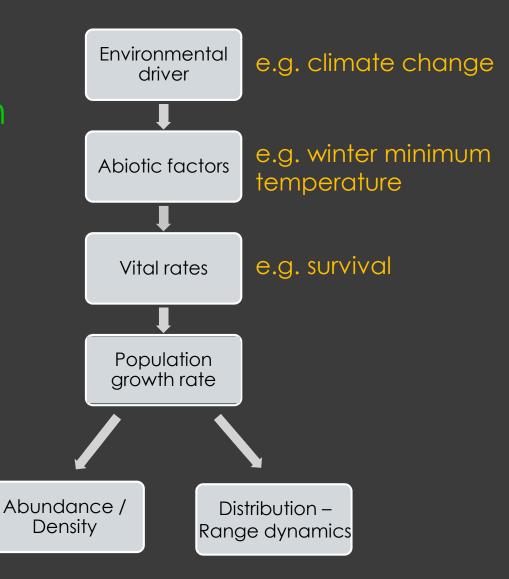
- Population growth rate and extinction risk
- Abundance
- Distribution

# Linking variation in vital rates and population growth rates to environmental variation

#### Necessary to ...

- ... Identify the drivers of temporal and spatial variation in population dynamics to understand the causes of population decline and increase
- ... Predict population dynamics if the environment changes
- ... Frame populations dynamics in terms of factors that are manageable, and provide tools for efficient management actions

Linking variation in vital rates and population growth rates to environmental variation



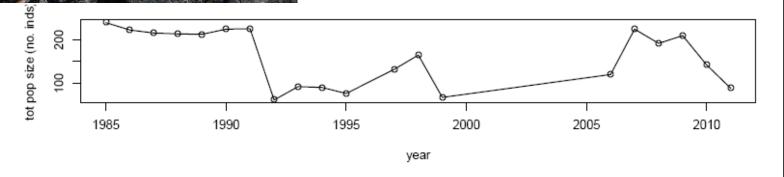
# What tools do we need to do to link population dynamics to environmental variation?

Environmental Predict future changes in drivers drivers Link environmental factors to drivers Abiotic factors Link vital rates to environmental factors Vital rates Link population dynamics to vital rates **Population** arowth rate



# Fumana procumbens

 a long-lived dwarf shrub in rocky habitats

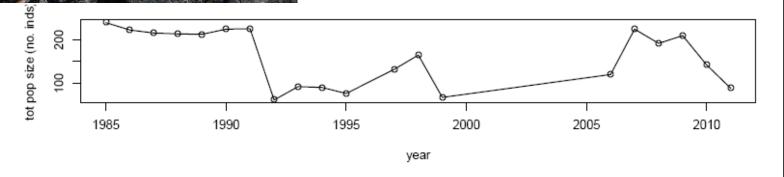


What factors are driving changes in population size among years?



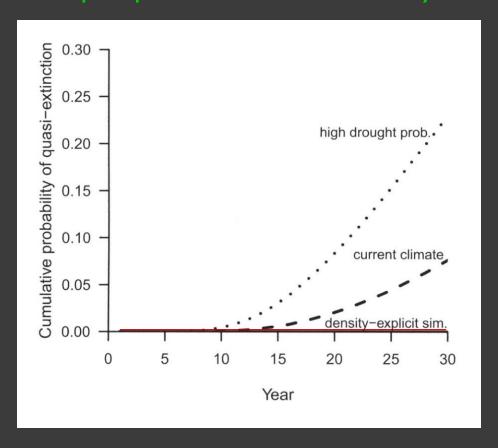
# Fumana procumbens

 a long-lived dwarf shrub in rocky habitats



What factors are driving changes in population size among years?

# Linking population dynamics to environmental variation – and population density



Bad year (climate) -> reduction in pop.size -> reduced density -> release -> increased growth rate

### POPULATION VIABILITY ANALYSIS

Hudsonia montana The mountain golden heather

#### Your task:

- 1. Calculate stochastic growth rate ( $\lambda_s$ )
- 2. Evaluate potential effects of increased disturbance
- 3. Estimate probability of quasi-extinction

(Temporary) link:

https://fogelstrom.github.io/popDynamics19/

### OUTLINE

- Population dynamics
- Structured populations
- Modelling the dynamics of structured populations

Case study

# Dracocephalum austriacum

an endangered herb
growing on exposed cliffs in
the Alps

#### Questions:

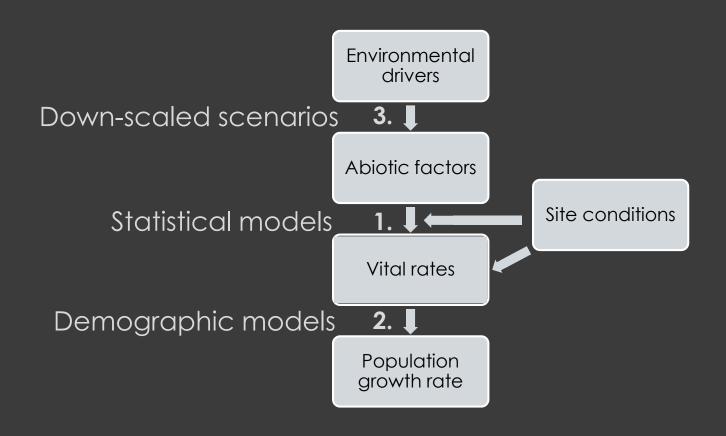
Will the population viability of this species be influenced by a warmer climate?

Will climate effects depend on local habitat conditions?



Florence Nicolè, Johan P. Dahlgren, Agnès Vivat, Irène Till-Bottraud, Johan Ehrlén (2011) Journal of Ecology

Model of effects of climate change on population dynamics, and of how such effects vary with site conditions



#### DESIGN

- ~1500 individuals in 7 populations followed 7 years
- Measurements of 5 vital rates
- Measurement of 12 environmental factors
- Data on temperature and precipitation from local weather stations
- Climate change scenarios Down-scaled



## BUILDING A DEMOGRAPHIC MODEL – FIELD DATA

- Recordings over several years
- Permanently marked individuals
- All life cycle stages
- Measuring and recording vegetative and reproductive traits
- Time of year



# BUILDING AN INTEGRAL PROJECTION MODEL

- Continuous, fine-grained matrix model
- Models vital rates as functions of variables.
- Parameterized using relationships from statistical models
- Possible to include multiple state variables, including environmental parameters



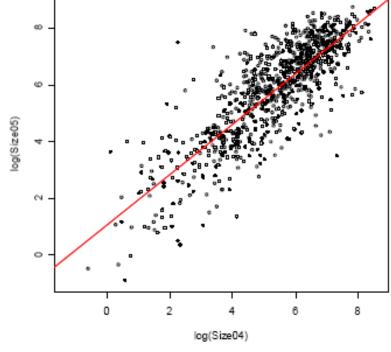
# BUILDING AN INTEGRAL PROJECTION MODEL

- Continuous, fine-grained matrix model
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```
lm(formula = log(Size05) ~ log(Size04))
Residuals:
           10 Median
    Min
                               30
                                       Max
-4.09389 -0.51484 0.03605 0.54020 4.36272
Coefficients:
           Estimate Std. Error t value Pr(>|t|)
(Intercept) 1.0711 0.1108 9.667 <2e-16 ***
log(Size04) 0.8865 0.0199 44.540 <2e-16 ***
               0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '
Signif. codes:
Residual standard error: 0.9554 on 890 degrees of freedom
  (110 observations deleted due to missingness)
Multiple R-Squared: 0.6903, Adjusted R-squared: 0.69
F-statistic: 1984 on 1 and 890 DF, p-value: < 2.2e-16
> abline(mod1,col="red",lwd=2)
```

> summary(mod1<-lm(log(Size05)~log(Size04)))</pre>

Call:



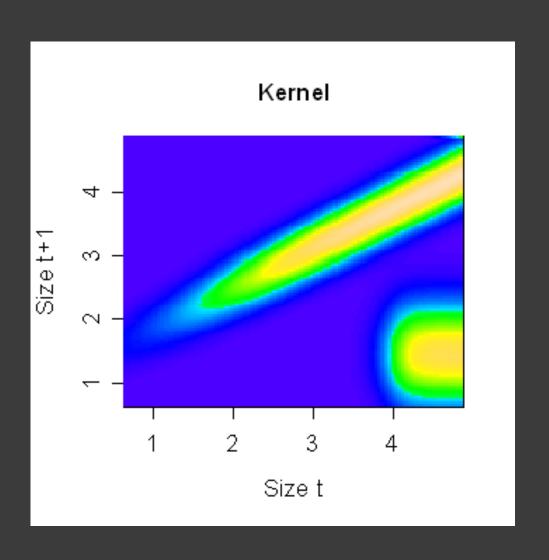
```
> summary(mod2<-glm(FlProb05~log(Size04+0.1),binomial))
Call:
glm(formula = FlProb05 ~ log(Size04 + 0.1), family =
binomial)
Deviance Residuals:
              10
                  Median
                                 3Q
                                         Max
    Min
-2.1420 -0.8220
                 -0.2534
                             0.8465
                                      3.0473
Coefficients:
                  Estimate Std. Error z value Pr(>|z|)
                               0.41847
(Intercept)
                  -6.02128
                                       -14.39
                                                  <2e-16 ***
log(Size04 + 0.1) 1.02360
                               0.07179
                                        14.26
                                                <2e-16 ***
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '
(Dispersion parameter for binomial family taken to be 1)
    Null deviance: 1348.06 on 1001 degrees of freedom
Residual deviance: 984.72 on 1000 degrees of freedom
AIC: 988.72
Number of Fisher Scoring iterations: 5
> plot(log(Size04),FlProb05)
> points (log(Size04), predict(mod2, type="response")
+ ,col="green")
                                                         9.0
                                                      FIProb05
                                                         4.0
                                                         0.2
                                                                 0
                                                                         2
                                                                                               8
                                                                            log(Size04)
```



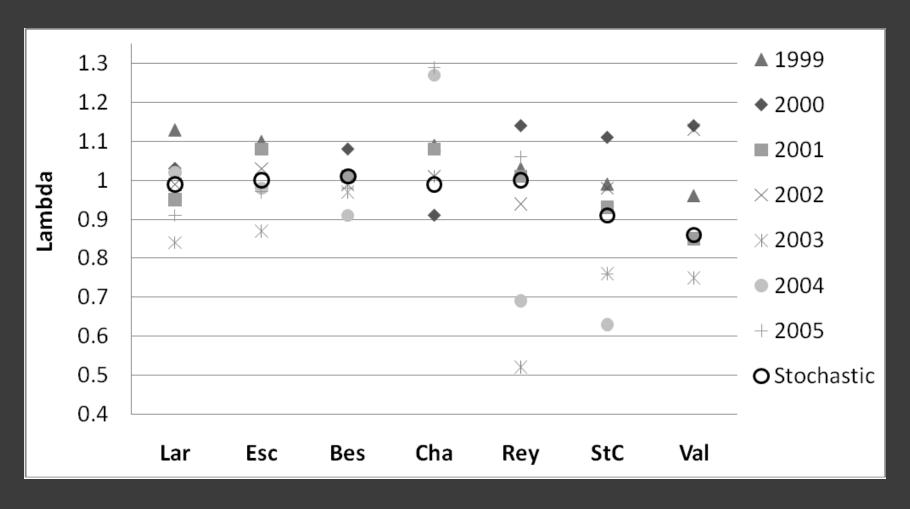
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### INTEGRAL PROJECTION MODELS



Stochastic and yearly deterministic growth rate for each population of *Dracocephalum* austriacum over the census period (1999-2006).



# 1 & 2 Relationship between climatic and environmental factors and population growth rates

#### Among populations:

# Among years:

$$\lambda_{\rm S} = 1.13$$

$$\lambda_{\rm D} = 1.21$$

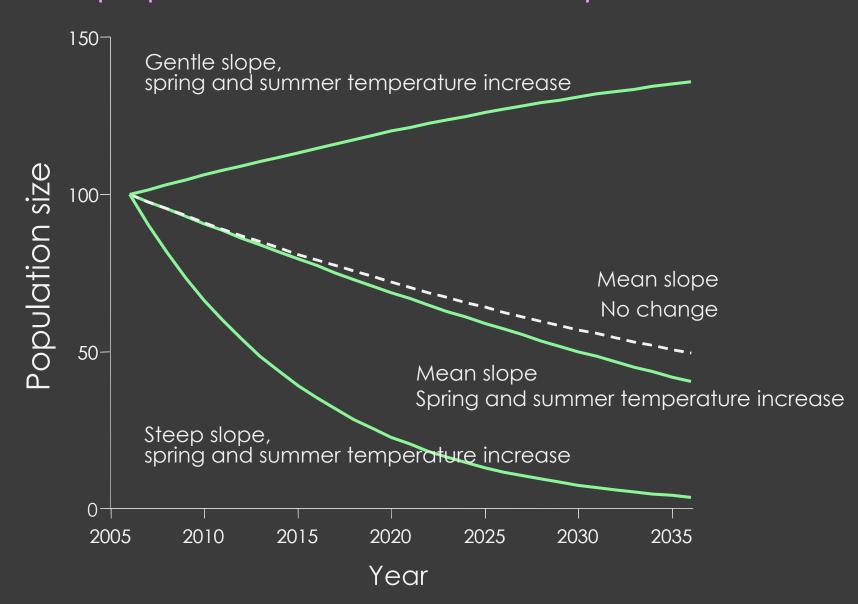
- (0.085 × Slope inclination)

 $-(0.014 \times Summer temp)$ 

+ (0.016 × Spring temp)

 $-(0.007 \times Summer temp \times Slope)$ 

# 3. Effects of climate change scenario A1B (IPCC) on population size of Dracocephalum



#### CONCLUSIONS

- Different aspects of a warmer climate may have opposing effects on population viability
- Climatic effects may depend on local habitat quality.
- Such interactive effects should be accounted for when down-scaling effects of large-scale environmental changes on viability of local populations

### **OVERALL CONCLUSIONS**

 Demography is fundamental to understand variation in population growth rates, abundances and distributions of species, to link this variation to environmental variation, and to identify and halt decline in threatened species

 For organisms with structured populations, life cycle approaches and demographic models, such as matrix models, are important and useful tools to examine these questions