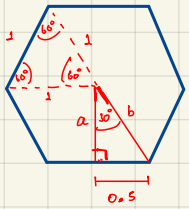
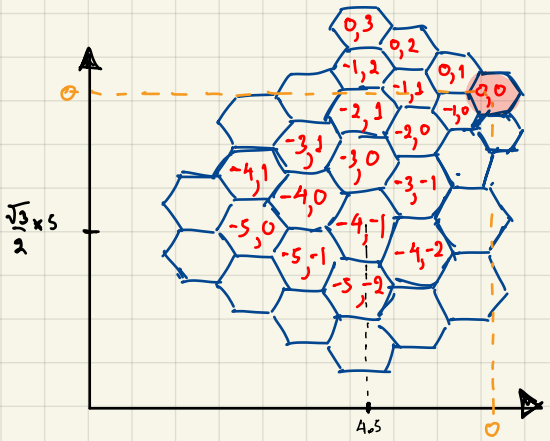


$\tan 30^\circ = \frac{\frac{1}{2}}{a}$, $a = \frac{1}{2\tan 30^\circ} = 2\,\frac{\cos 30^\circ}{\sin 30^\circ} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{2}$

$\cos 30^\circ = \frac{a}{1}$, $a = \cos 30^\circ = \frac{\sqrt{3}}{2}$

$\sin 30^\circ = \frac{1}{2}b$, $b = 2\sin 30^\circ = 2\times\frac{1}{2} = 1$



0 = ลงล่าง
1 = forward
2 = back ward
3 = left 60°
4 = right 60°

Rotation Matrix

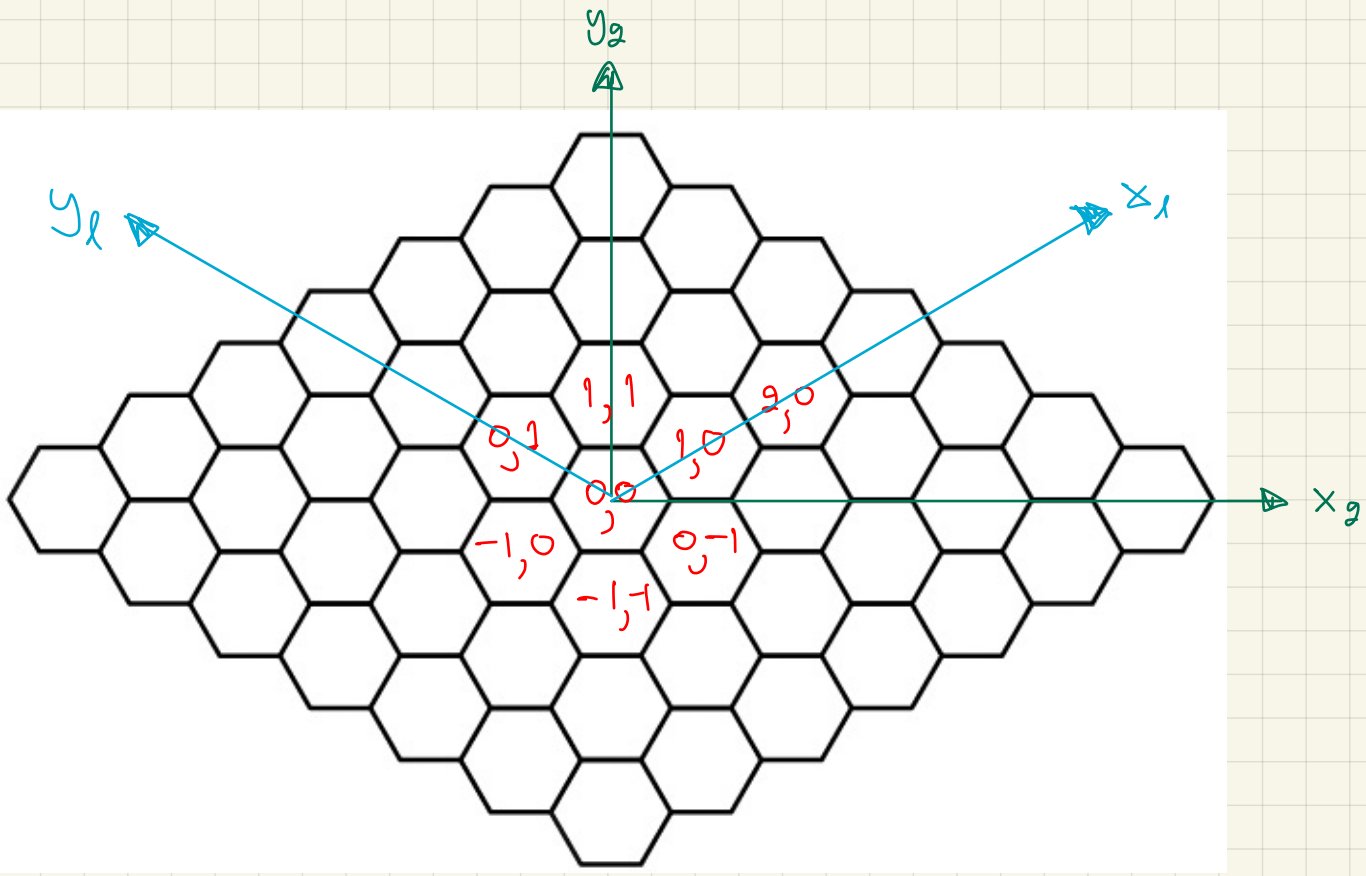
$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

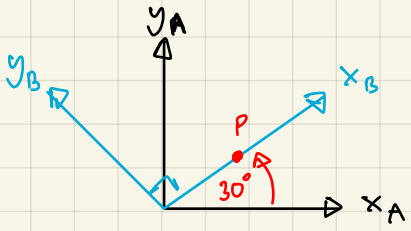
$x' = x\cos\theta - y\sin\theta$
$y' = x\sin\theta + y\cos\theta$
$z' = z$

Translation Matrix

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$x' = x + t_x$
$y' = y + t_y$





"Pure Rotation"

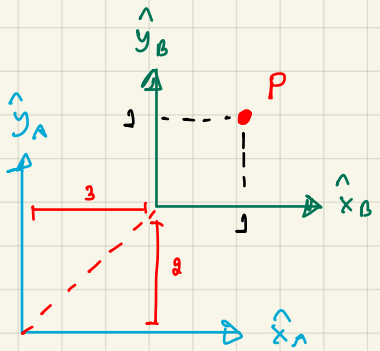
$P_{AP}^{A} = R_{B}^{A} P_{BP}^{B}$ ← แปลง B กลับมา A

$= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_B \\ y_B \end{bmatrix}$ ← $\sqrt{3}$

$= \begin{bmatrix} \frac{\sqrt{3}}{2} \cdot \sqrt{3} & -\frac{1}{2}(0) \\ \frac{1}{2}\sqrt{3} & + \frac{\sqrt{3}}{2}(0) \end{bmatrix}$

$= \begin{bmatrix} \frac{3}{2} \\ \frac{\sqrt{3}}{2} \end{bmatrix}$

"Translation Matrix"



$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_o \\ y_o \\ 1 \end{bmatrix}$$

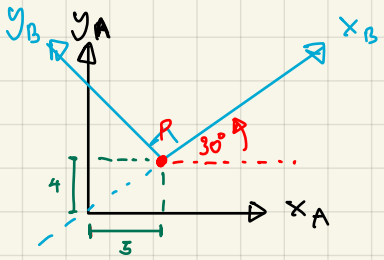
$$P_{OP}^{A} = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+3 \\ 1+2 \\ 1 \end{bmatrix}$$

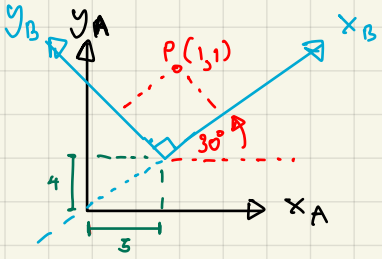
$$P_{OP}^{A} = \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}$$ "คือ P อยู่เทียบกับ Frame A

"Rotation and Translation"

$$P_{AP}^{A} = \begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_o \\ y_o \\ 1 \end{bmatrix}$$

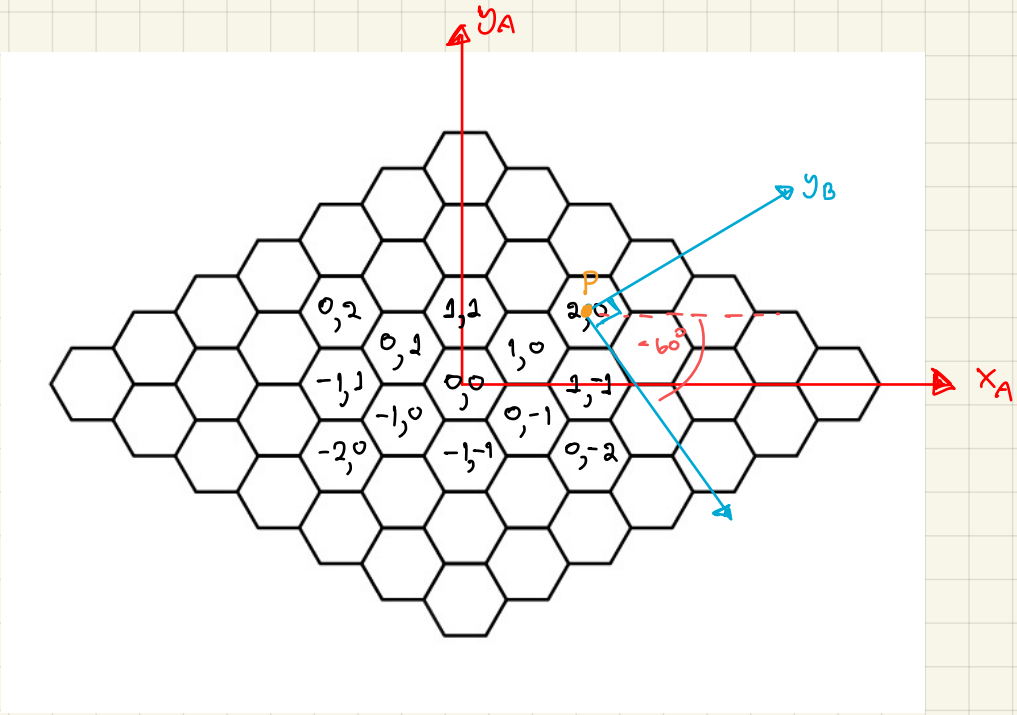


$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos 30^\circ & -\sin 30^\circ & 5 \\ \sin 30^\circ & \cos 30^\circ & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \\ 1 \end{bmatrix}$$



$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \cos 30^\circ & -\sin 30^\circ & 5 \\ \sin 30^\circ & \cos 30^\circ & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} - \frac{1}{2} + 5 \\ \frac{1}{2} + \frac{\sqrt{3}}{2} + 4 \\ 1 \end{bmatrix}$$

Test 1



$$P_{OP}^{A} = \begin{bmatrix} \cos(-60^\circ) & -\sin(-60^\circ) & 3 \\ \sin(-60^\circ) & \cos(60^\circ) & \sqrt{3} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

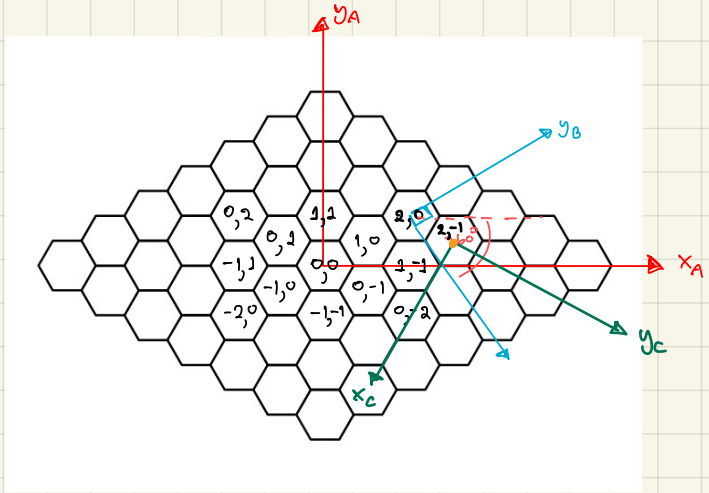
$$= \begin{bmatrix} 3 \\ \sqrt{3} \\ 1 \end{bmatrix} \Rightarrow \begin{aligned} x_P^A &= 3 \\ y_P^A &= \sqrt{3} \end{aligned}$$

$$R_C^A = R_B^A R_C^B$$

$$R_C^A = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{4} - \frac{3}{4} & -\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} \\ -\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} & -\frac{3}{4} + \frac{1}{4} \end{bmatrix}$$

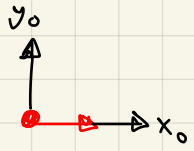
$$= \begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & -\frac{1}{2} \end{bmatrix}$$



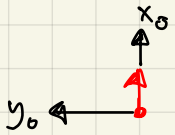
$$P_{OC}^{B} = \begin{bmatrix} \cos(-60^\circ) & -\sin(-60^\circ) & 1.5 \\ \sin(-60^\circ) & \cos(-60^\circ) & \frac{\sqrt{3}}{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1.5 \\ \frac{\sqrt{3}}{2} \\ 1 \end{bmatrix} \Rightarrow \begin{aligned} x_P^B &= 1.5 \\ y_P^B &= \frac{\sqrt{3}}{2} \end{aligned}$$

Test 2



$$\text{Init} = \begin{bmatrix} \cos 90^\circ & -\sin 90^\circ & 0 \\ \sin 90^\circ & \cos 90^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

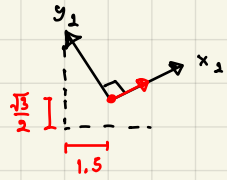


$$\text{Rotate } -60^\circ = \begin{bmatrix} \cos 90^\circ & -\sin 90^\circ & 0 \\ \sin 90^\circ & \cos 90^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-60^\circ) & -\sin(-60^\circ) & 0 \\ \sin(-60^\circ) & \cos(-60^\circ) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Trans} \begin{bmatrix} \sqrt{3} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & \sqrt{3} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} & \frac{3}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

ตำแหน่งก่อนหน้า



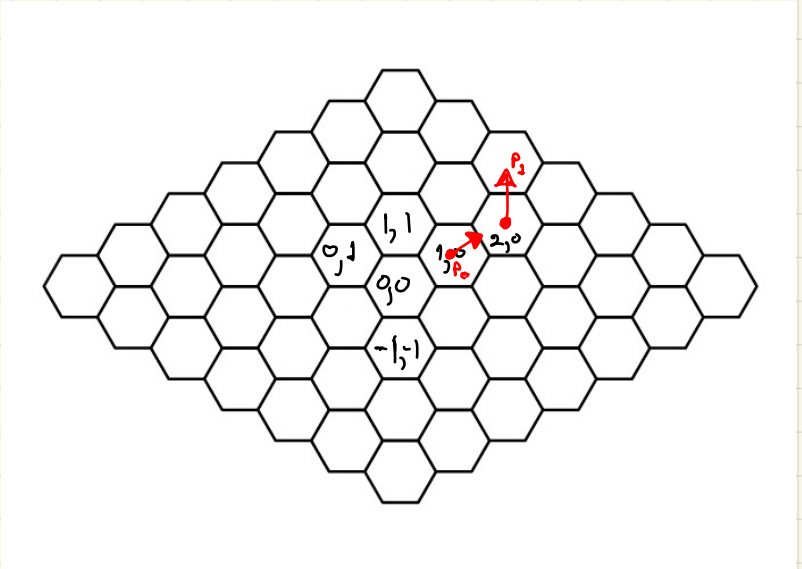
$$\text{Rotate } 60^\circ \text{ \& Trans} \begin{bmatrix} \sqrt{3} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} & \frac{3}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 60^\circ & -\sin 60^\circ & \sqrt{3} \\ \sin 60^\circ & \cos 60^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

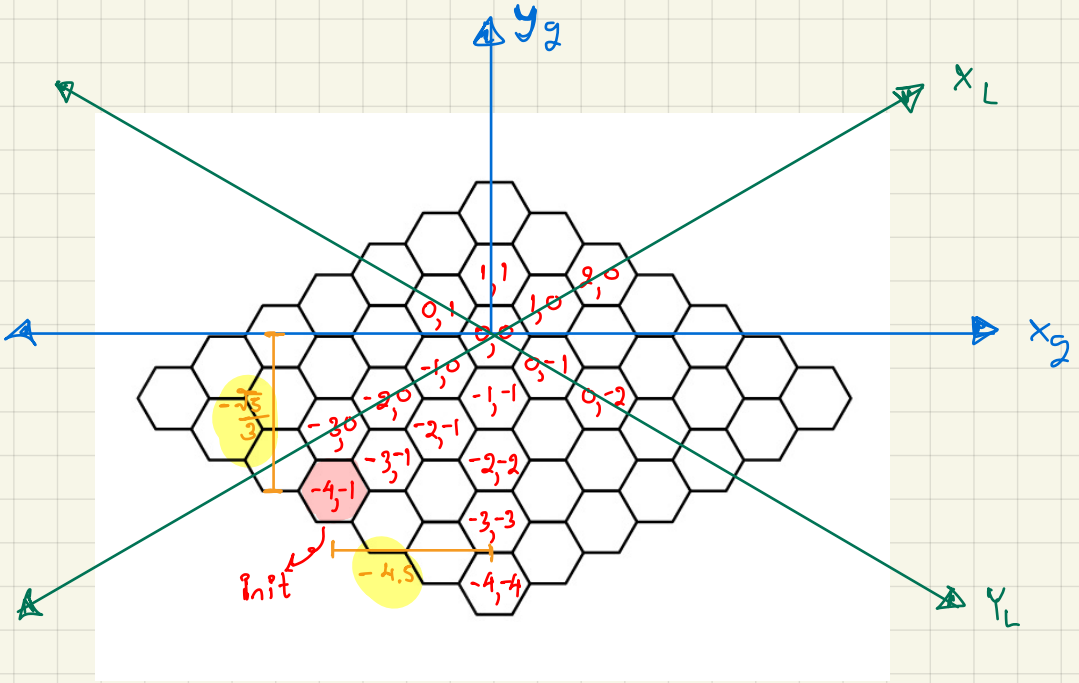
$$= \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} & \frac{3}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & \sqrt{3} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} & -\frac{3}{4} - \frac{1}{4} & \frac{3}{2} + \frac{3}{2} \\ \frac{1}{4} + \frac{3}{4} & -\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} & \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 3 \\ 1 & 0 & \sqrt{3} \\ 0 & 0 & 1 \end{bmatrix}$$

$\theta = 90^\circ$

ขยับเลื่อนก่อน แล้วค่อยหมุน





Index to position

From, $a = \begin{bmatrix} \frac{3}{2} \\ \frac{\sqrt{3}}{2} \end{bmatrix} \times (i-1)$

$b = \begin{bmatrix} -\frac{3}{2} \\ \frac{\sqrt{3}}{2} \end{bmatrix} \times (j-1)$

$$\Rightarrow \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \frac{3}{2}(i-1) \\ \frac{\sqrt{3}}{2}(i-1) \end{bmatrix} + \begin{bmatrix} -\frac{3}{2}(j-1) \\ \frac{\sqrt{3}}{2}(j-1) \end{bmatrix} + \begin{bmatrix} 0 \\ \sqrt{3} \end{bmatrix}$$

ex. $[-4,-1]$ $\quad a = \begin{bmatrix} \frac{3}{2} \\ \frac{\sqrt{3}}{2} \end{bmatrix} \times -5$, $\quad b = \begin{bmatrix} -\frac{3}{2} \\ \frac{\sqrt{3}}{2} \end{bmatrix} \times -2$

$a = \begin{bmatrix} -\frac{15}{2} \\ -\frac{5\sqrt{3}}{2} \end{bmatrix}$ $\qquad b = \begin{bmatrix} 3 \\ -\sqrt{3} \end{bmatrix}$

return $a + b + \begin{bmatrix} 0 \\ \sqrt{3} \end{bmatrix} = \begin{bmatrix} -\frac{15}{2} \\ -\frac{5\sqrt{3}}{2} \end{bmatrix} + \begin{bmatrix} 3 \\ -\sqrt{3} \end{bmatrix} + \begin{bmatrix} 0 \\ -\sqrt{3} \end{bmatrix}$

$$= \begin{bmatrix} -4.5 \\ -4.33012\ldots \end{bmatrix}$$

Post to Index ⇔ Inverse and Index to Post

idx2pos

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{3}{2}(i-1) \\ \frac{\sqrt{3}}{2}(i-1) \end{bmatrix} + \begin{bmatrix} -\frac{3}{2}(j-1) \\ \frac{\sqrt{3}}{2}(j-1) \end{bmatrix} + \begin{bmatrix} 0 \\ \sqrt{3} \end{bmatrix}$$

$$x = \frac{3}{2}i - \frac{3}{2} - \frac{3}{2}j + \frac{3}{2} + 0 \quad \Rightarrow \quad \frac{3}{2}(i-j)$$

$$y = \frac{\sqrt{3}}{2}i - \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}j - \frac{\sqrt{3}}{2} + \sqrt{3} \quad \Rightarrow \quad \frac{\sqrt{3}}{2}(i+j)$$

∴ $x = \frac{3}{2}i - \frac{3}{2}j$ $\qquad y = \frac{\sqrt{3}}{2}i + \frac{\sqrt{3}}{2}j$

$$i = \left(x + \frac{3}{2}j\right)\frac{2}{3} \qquad j = \left(y - \frac{\sqrt{3}}{2}i\right)\frac{2}{\sqrt{3}}$$

$$i = \frac{2}{3}x + j \quad -(1) \qquad j = \frac{2y}{\sqrt{3}} - i \quad -(2)$$

แทน j จาก (1) ; $i = \frac{2}{3}x + \frac{2y}{\sqrt{3}} - i$

$$2i = \frac{2x}{3} + \frac{2y}{\sqrt{3}}$$

$$i = \frac{x}{3} + \frac{y}{\sqrt{3}}$$

แทน i จาก (2) ; $j = \frac{2y}{\sqrt{3}} - \frac{2}{3}x - j$

$$2j = \frac{2y}{\sqrt{3}} - \frac{2}{3}x$$

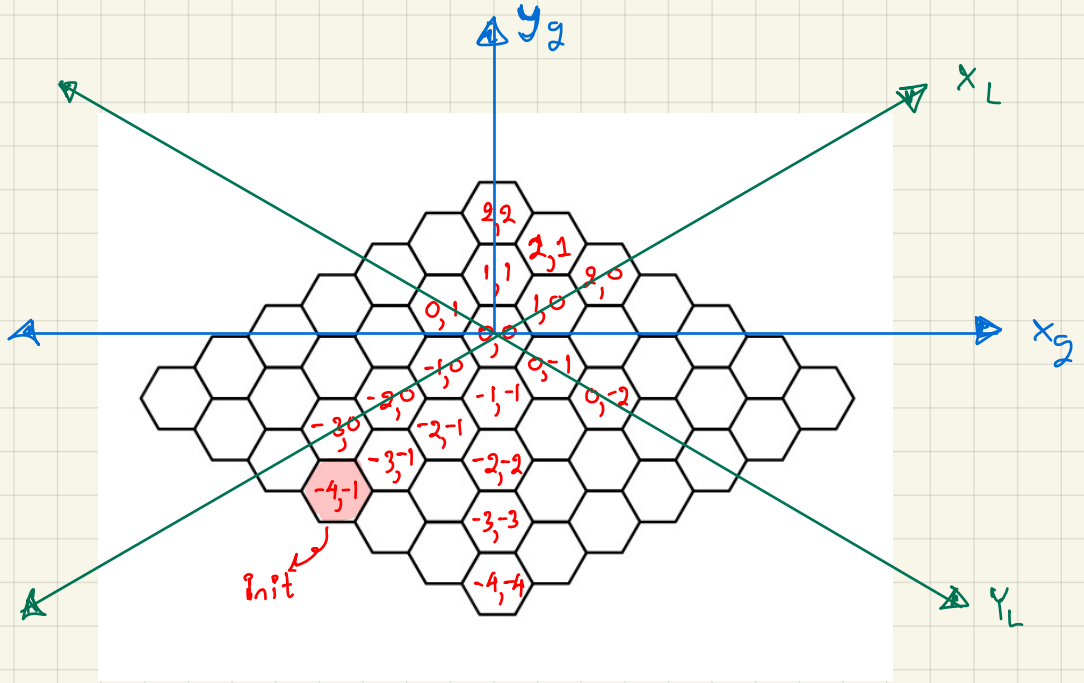
$$j = \frac{y}{\sqrt{3}} - \frac{x}{3}$$

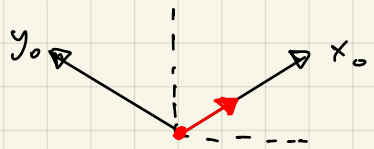
ex. $[-4, -1]$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4.5 \\ -\frac{5\sqrt{3}}{2} \end{bmatrix} \Rightarrow \begin{bmatrix} i \\ j \end{bmatrix} = \begin{bmatrix} -\frac{4.5}{3} - \frac{3\sqrt{3}}{2\sqrt{3}} \\ -\frac{5\sqrt{3}}{2\sqrt{3}} + \frac{4.5}{3} \end{bmatrix} = \begin{bmatrix} -1.5 - 2.5 \\ -2.5 + 1.5 \end{bmatrix} = \begin{bmatrix} -4 \\ -1 \end{bmatrix}$$

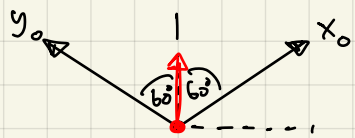
∴

$$\begin{bmatrix} i \\ j \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{1}{\sqrt{3}} \\ -\frac{1}{3} & \frac{1}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$





$$\begin{bmatrix} \cos 60^\circ & -\sin 60^\circ & 0 \\ \sin 60^\circ & \cos 60^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$\begin{bmatrix} \cos 60^\circ & -\sin 60^\circ & 0 \\ \sin 60^\circ & \cos 60^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-60^\circ) & -\sin(-60^\circ) & 0 \\ \sin(60^\circ) & \cos(-60^\circ) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

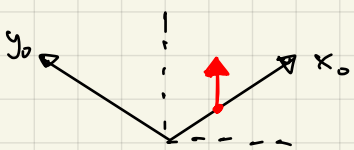
$$= \begin{bmatrix} \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \frac{\sqrt{3}}{2}\left(\frac{\sqrt{3}}{2}\right) & \left(\frac{1}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) - \frac{\sqrt{3}}{2}\left(\frac{1}{2}\right) & 0 \\ \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) & \left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Trans

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

RM.

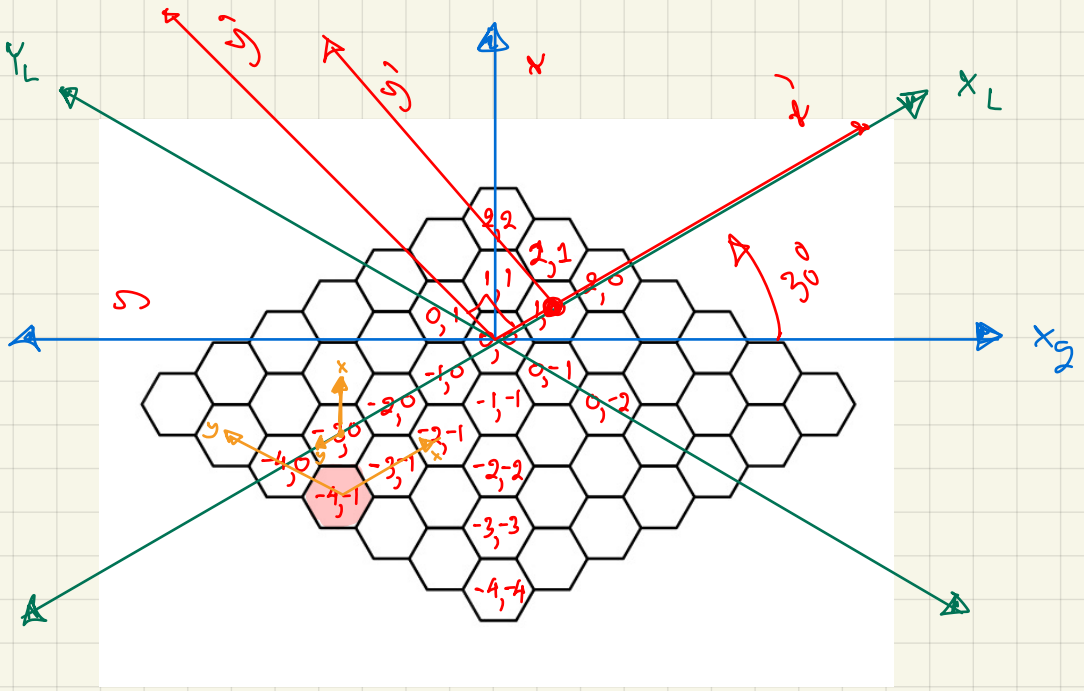


$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 60^\circ & -\sin 60^\circ & 0 \\ \sin 60^\circ & \cos 60^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 1 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

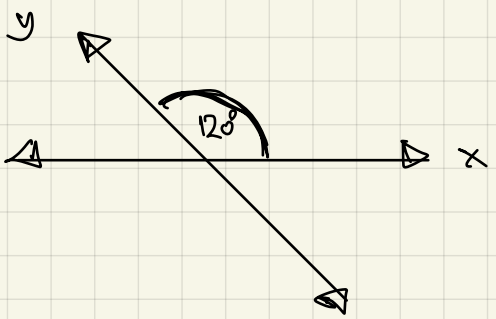
RTM.

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 60^\circ & -\sin 60^\circ & 1 \\ \sin 60^\circ & \cos 60^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 2 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

not same



$$\begin{bmatrix} x_1^0 \\ y_1^0 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos 60^\circ & -\sin 60^\circ & -4 \\ \sin 60^\circ & \cos 60^\circ & -1 \\ 0 & 0 & 1 \end{bmatrix}$$



$$\begin{bmatrix} x_2^1 \\ y_2^1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 60^\circ & -\sin 60^\circ & 0 \\ \sin 60^\circ & \cos 60^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x_2^0 \\ y_2^0 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos 60^\circ & -\sin 60^\circ & -4 \\ \sin 60^\circ & \cos 60^\circ & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & -3.5 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & -0.133975\ldots \\ 0 & 0 & 1 \end{bmatrix}$$