2/2566 FRA501: Pattern Recognition

Homework 4.1 (Backpropagation)

Instructions

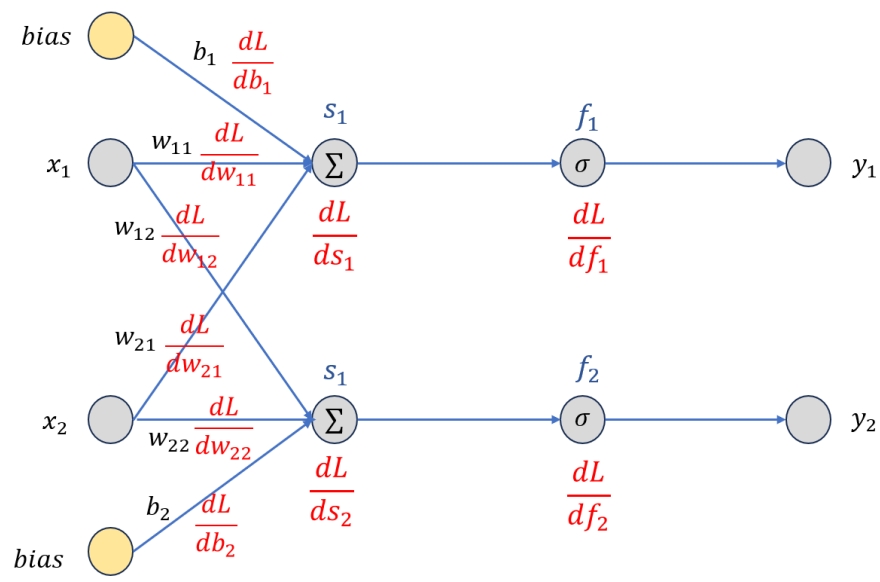
Answer the questions and upload your answers to google drive. Answers can be in Thai or English. Answers can be either typed or handwritten and scanned.

Try Backpropagation by hands!!!

Neural Network Backpropagation Exercise (100 points) (10%)

We are providing you with an opportunity to manually implement backpropagation in a neural network. The conditions are as follows:

A. The neural network has the following structure: 1 input layer consisting of 2 nodes and 1 output layer consisting of 2 nodes.



Note: σ : Sigmoid function, $\sum$: sum function

B. The initial values of weights and biases are as follows: $[w_{11}, w_{12}, w_{21}, w_{22}] = [0.2, 0.3, 0.1, 0.5]$ and $[b_1, b_2] = [0.3, 0.4]$

C. The input values and the target values for each round are as follows:

Round	x_1	x_2	y_1	y_2
1	0.4	0.8	1	0
2	0.5	0.7	1	0
3	0.1	0.3	0	1
4	0.2	0.4	0	1

D. The learning rate used for updating is 1.
E. Loss function is defined as MSE:

$$L = \frac{\sum_n (y_n - f_n)^2}{2}$$

where y_n is the target of node n and f_n is the output of node n.

Please answer the following questions:

1. Ans in term of function in forward pass and the backward pass (Think in perspective of recursive function) **(30 points)**

Forward pass	Backward pass
$s_1 =$ $w_{11} \times x_1 + w_{21} \times x_2 + b_1$	$\frac{dL}{df_1} =$ $f_1 - y_1$
$s_2 =$ $w_{12} \times x_1 + w_{22} \times x_2 + b_2$	$\frac{dL}{df_2} =$ $f_2 - y_2$
$f_1 =$ $\text{sigmoid}(s_1)$	$\frac{dL}{ds_1} =$ $\frac{dL}{df_1} \times f_1 \times (1 - f_1)$
$f_2 =$ $\text{sigmoid}(s_2)$	$\frac{dL}{ds_2} =$ $\frac{dL}{df_2} \times f_2 \times (1 - f_2)$
$L =$ $\frac{(y_1 - f_1)^2}{2} + \frac{(y_2 - f_2)^2}{2}$	$\frac{dL}{db_1} =$ $\frac{dL}{ds_1} \times 1$
	$\frac{dL}{db_2} =$ $\frac{dL}{ds_2} \times 1$
	$\frac{dL}{dw_{11}} =$ $\frac{dL}{ds_1} \times x_1$
	$\frac{dL}{dw_{21}} =$ $\frac{dL}{ds_1} \times x_2$
	$\frac{dL}{dw_{12}} =$ $\frac{dL}{ds_2} \times x_1$
	$\frac{dL}{dw_{22}} =$ $\frac{dL}{ds_2} \times x_2$

Note:

I. **For the forward pass**, the answer to a function must be a function that depends on the previous function.
II. **For the backward pass**, the answer to a function must be a function that depends on the previous function and the variable used to find the derivative.
III. **For the backward pass,** the answer to the target functions ($\frac{dL}{dw_{11}}, \ldots \frac{dL}{dw_{22}}, \frac{dL}{db_1} \frac{dL}{db_2}$) must be the functions that depend on input variables.

2. For the first round (Initialization step), answer the values in the below table (Answer in format with 4 decimal places). **(35 points)**

1st round (Initialization step)			
Parameters	**Forward pass**	**Backward pass**	**Updated Parameters**
$x_1 =$ 0.4	$s_1 =$ 0.46	$\frac{dL}{df_1} =$ -0.386986	$w_{11} =$ 0.236722
$x_2 =$ 0.8	$s_2 =$ 0.92	$\frac{dL}{df_2} =$ 0.715042	$w_{12} =$ 0.241722
$y_1 =$ 1	$f_1 = \frac{1}{1+e^{-0.46}} =$ 0.613014	$\frac{dL}{ds_1} =$ -0.091804	$w_{21} =$ 0.173443
$y_2 =$ 0	$f_2 = \frac{1}{1+e^{-0.92}} =$ 0.715042	$\frac{dL}{ds_2} =$ 0.145695	$w_{22} =$ 0.383444
$w_{11} =$ 0.2	$y_1 - f_1 =$ 0.386986	$\frac{dL}{db_1} =$ -0.091804	$b_1 =$ 0.391804
$w_{12} =$ 0.3	$y_2 - f_2 =$ -0.715042	$\frac{dL}{db_2} =$ 0.145695	$b_2 =$ 0.254305
$w_{21} =$ 0.1	$(y_1 - f_1)^2 =$ 0.149758	$\frac{dL}{dw_{11}} =$ -0.036722	
$w_{22} =$ 0.5	$(y_2 - f_2)^2 =$ 0.511285	$\frac{dL}{dw_{21}} =$ -0.073443	
$b_1 =$ 0.3	$L =$ 0.3305216	$\frac{dL}{dw_{12}} =$ 0.058278	
$b_2 =$ 0.4		$\frac{dL}{dw_{22}} =$ 0.116556	

3. For the second round, answer the values in the below table (Answer in format with 4 decimal places). **(35 points)**

2nd round			
Parameters	**Forward pass**	**Backward pass**	**Updated Parameters**
$x_1 =$ 0.5	$s_1 =$ 0.631575	$\frac{dL}{df_1} =$ -0.347154	$w_{11} =$ 0.276061
$x_2 =$ 0.7	$s_2 =$ 0.643577	$\frac{dL}{df_2} =$ 0.655562	$w_{12} =$ 0.167709
$y_1 =$ 1	$f_1 =$ 0.652846	$\frac{dL}{ds_1} =$ -0.078678	$w_{21} =$ 0.228518
$y_2 =$ 0	$f_2 =$ 0.655562	$\frac{dL}{ds_2} =$ 0.148026	$w_{22} =$ 0.279826

$w_{11} =$ 0.236722	$y_1 - f_1 =$ 0.347154	$\frac{dL}{db_1} =$ -0.078678	$b_1 =$ 0.470482
$w_{12} =$ 0.241722	$y_2 - f_2 =$ -0.655562	$\frac{dL}{db_2} =$ 0.148026	$b_2 =$ 0.106279
$w_{21} =$ 0.173443	$(y_1 - f_1)^2 =$ 0.120516	$\frac{dL}{dw_{11}} =$ -0.039339	
$w_{22} =$ 0.383444	$(y_2 - f_2)^2 =$ 0.429761	$\frac{dL}{dw_{21}} =$ -0.055075	
$b_1 =$ 0.391804	$L =$ 0.275138	$\frac{dL}{dw_{12}} =$ 0.074013	
$b_2 =$ 0.254305		$\frac{dL}{dw_{22}} =$ 0.103618	

For 4 to 6 are Extra!!! (3%)

4. For the third round, answer the values in the below table (Answer in format with 4 decimal places). **(35 points)**

3rd round			
Parameters	**Forward pass**	**Backward pass**	**Updated Parameters**
$x_1 =$ 0.1	$s_1 =$ 0.566643	$\frac{dL}{df_1} =$ 0.637988	$w_{11} =$ 0.261326
$x_2 =$ 0.3	$s_2 =$ 0.206998	$\frac{dL}{df_2} =$ -0.448435	$w_{12} =$ 0.178801
$y_1 =$ 0	$f_1 =$ 0.637988	$\frac{dL}{ds_1} =$ 0.147349	$w_{21} =$ 0.184313
$y_2 =$ 1	$f_2 =$ 0.551654	$\frac{dL}{ds_2} =$ -0.110916	$w_{22} =$ 0.313101
$w_{11} =$ 0.276061	$y_1 - f_1 =$ -0.637988	$\frac{dL}{db_1} =$ 0.147349	$b_1 =$ 0.323133
$w_{12} =$ 0.167709	$y_2 - f_2 =$ 0.448435	$\frac{dL}{db_2} =$ -0.110916	$b_2 =$ 0.217195
$w_{21} =$ 0.228518	$(y_1 - f_1)^2 =$ 0.407029	$\frac{dL}{dw_{11}} =$ 0.014735	
$w_{22} =$ 0.279826	$(y_2 - f_2)^2 =$ 0.201094	$\frac{dL}{dw_{21}} =$ 0.044205	
$b_1 =$ 0.470482	$L =$ 0.304061	$\frac{dL}{dw_{12}} =$ -0.01109	
$b_2 =$ 0.106279		$\frac{dL}{dw_{22}} =$ -0.033275	

5. For the fourth round, answer the values in the below table (Answer in format with 4 decimal places). **(35 points)**

4th round			
Parameters	**Forward pass**	**Backward pass**	**Updated Parameters**
$x_1 =$ 0.2	$s_1 =$ 0.449123	$\frac{dL}{df_1} =$ 0.610431	$w_{11} =$ 0.232293
$x_2 =$ 0.4	$s_2 =$ 0.378196	$\frac{dL}{df_2} =$ -0.406562	$w_{12} =$ 0.198419
$y_1 =$ 0	$f_1 =$ 0.610431	$\frac{dL}{ds_1} =$ 0.145164	$w_{21} =$ 0.126248
$y_2 =$ 1	$f_2 =$ 0.593438	$\frac{dL}{ds_2} =$ -0.098091	$w_{22} =$ 0.352337
$w_{11} =$ 0.261326	$y_1 - f_1 =$ -0.610431	$\frac{dL}{db_1} =$ 0.145164	$b_1 =$ 0.177970
$w_{12} =$ 0.178801	$y_2 - f_2 =$ 0.406562	$\frac{dL}{db_2} =$ -0.098091	$b_2 =$ 0.315286
$w_{21} =$ 0.184313	$(y_1 - f_1)^2 =$ 0.372626	$\frac{dL}{dw_{11}} =$ 0.029033	
$w_{22} =$ 0.313101	$(y_2 - f_2)^2 =$ 0.165293	$\frac{dL}{dw_{21}} =$ 0.058065	
$b_1 =$ 0.323133	$L =$ 0.26896	$\frac{dL}{dw_{12}} =$ -0.019618	
$b_2 =$ 0.217195		$\frac{dL}{dw_{22}} =$ -0.039236	

6. Plot graph between loss (y-axis) and number of round (x-axis). **(5 points)**