Hm 13

Example p. 4

$\cancel{x=y}$ $x^2 - 2z = 0 \Rightarrow z = \frac{x^2}{2}$

$x = y\ ;\ y = 0\ ;\ x = \sqrt{3}$

3 $\cancel{d\sigma = \left|\frac{\nabla \vec{G}}{\nabla G \cdot \hat{k}}\right| dx\,dy = \left|\frac{x\hat{i}}{x\hat{i}\cdot\hat{k}}\right| dx\,dy} =$

3 $\nabla \vec{G} = \langle x, 0 \rangle$

$= d\sigma = |\vec{r}_x \times \vec{r}_y| = |2x\hat{i} \times 0| = \hat{i}\ \hat{j}\ \hat{k}\ 2x\ 0$

let's parameterize

$x(u,v) = u \qquad \vec{r}(u,v) = \langle u, v, \frac{u^2}{2} \rangle$

$y(u,v) = v$

$z(u,v) = \frac{u^2}{2}$

$\frac{\partial \vec{r}}{\partial u} = \langle 1, 0, u \rangle \qquad \frac{\partial \vec{r}}{\partial v} = \langle 0, 1, 0 \rangle$

$$\frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & u \\ 0 & 1 & 0 \end{vmatrix} = -u\hat{i} + \hat{k}$$

$$\left|\frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v}\right| = u^2 + 1 \Rightarrow d\sigma = (u^2+1)\,du\,dv$$

$$\int_0^{\sqrt{3}}\int_0^{u} (u^2+1)\,dv\,du = \int_0^{\sqrt{3}} \left[u^2 v + v\right]_0^u du =$$

$$= \int_0^{\sqrt{3}} (u^3+u)\,du = \left[\frac{u^4}{4}+\frac{u^2}{2}\right]_0^{\sqrt{3}} = \frac{9}{4}+\frac{3}{2} = \frac{18}{4} = \frac{9}{2}$$

Example p. 7

$$\vec{r}(r,\theta) = r\cos\theta\,\hat{i} + r\sin\theta\,\hat{j} + r\hat{k}$$

$$(x,y,z) = (r\cos\theta,\ r\sin\theta,\ r)$$

$$\vec{r}_r = \cancel{\cos\theta\,\hat{i} + \sin\theta\,\hat{i} + \cos\theta\,\hat{j} + \hat{k}}\ \cos\theta\,\hat{i} + \sin\theta\,\hat{j} + \hat{k}$$

$$\vec{r}_\theta = -r\sin\theta\,\hat{i} + r\cos\theta\,\hat{j}$$

$$\vec{r}_r \times \vec{r}_\theta = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos\theta & \sin\theta & 1 \\ -r\sin\theta & r\cos\theta & 0 \end{vmatrix} = -r\cos\theta\,\hat{i} - r\sin\theta\,\hat{j} + (r\cos^2\theta + r\sin^2\theta)\,\hat{k} = -r\cos\theta\,\hat{i} - r\sin\theta\,\hat{j} + r\hat{k}$$

$$|\vec{r}_r \times \vec{r}_\theta| = \sqrt{r^2+r^2} = r\sqrt{2}$$

$$\iint r^2\cos^2\theta \cdot r\sqrt{2}\, dr\, d\theta =$$

$$\cancel{\sqrt{2}\int_0^1\int_0^{2\pi} r^3\cos\theta\, d\theta\, dr = \sqrt{2}\int_0^1 r^3\left[\sin\theta\right]_0^{2\pi}}$$

$$= \sqrt{2}\int_0^{2\pi}\int_0^1 r^3\cos\theta\, dr\, d\theta = \sqrt{2}\int_0^{2\pi}\left[\frac{r^4}{4}\right]_0^1 \cos\theta \cdot d\theta =$$

$$= \frac{\sqrt{2}}{4}\left[\sin\theta\right]_0^{2\pi} = 0$$

Example 9.9

$\vec{F} = \langle 0, y^2, z^2 \rangle$

$y^2 + z^2 = 1$, $z \geq 0$

$x = 1$, $x = 0$

~~$r(u,v) =$~~

$r(x,y,z) = \cancel{y^2+z^2=1} = y^2+z^2-1 = \langle x(u,v), y(u,v), z(u,v) \rangle$

$x(u,v) =$

$y(u,v) = \cos(u)$

$z(u,v) = \sin(u)$ $\Rightarrow$ $r(u,v) = \langle 1, \cos u, \sin u \rangle$

$\vec{r}_u = \langle 0, -\sin u, \cos u \rangle$

$\vec{r}_v = \langle 0, 0, 0 \rangle$

$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & -\sin u & \cos u \\ 0 & 0 & 0 \end{vmatrix} = \langle 0, 0, 0 \rangle$

$|\vec{r}_u \times \vec{r}_v| = 0$

$x(r,\theta) = r$ $\quad \vec{H}(r,\theta) = \langle r, \cos\theta, \sin\theta \rangle$

$y(r,\theta) = \cos\theta$ $\quad \vec{r}_r = \langle 1, 0, 0 \rangle$

$z(r,\theta) = \sin\theta$ $\quad \vec{r}_\theta(r,\theta) = \langle 0, -\sin\theta, \cos\theta \rangle$

$$\vec{r}_r \times \vec{r}_\theta = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 0 \\ 0 & -\sin\theta & \cos\theta \end{vmatrix} = \cancel{\sin\theta\hat{k}} - \cos\theta\,\hat{i} - \sin\theta\,\hat{j}$$

$$|\vec{r}_r \times \vec{r}_\theta| = 1$$

$$\vec{n} = \vec{r}_r \times \vec{r}_\theta = -\cos\theta\,\hat{i} - \sin\theta\,\hat{j} \quad ; \quad \hat{n} = \frac{\vec{r}_r \times \vec{r}_\theta}{|\vec{r}_r \times \vec{r}_\theta|} = -\cos\theta\,\hat{i} - \sin\theta\,\hat{j}$$

$$\iint_S \vec{F}\,\hat{n}\,d\sigma =$$

$$= \iint_S \langle 0, yz, z^2 \rangle \cdot \langle -\cos\theta, -\sin\theta, 0 \rangle \cdot dr \cdot d\theta =$$

$$= \iint_S -\cos\theta \sin^2\theta \, dr\, d\theta = -\int_0^{\pi} \cos\theta \cdot \sin^2\theta \left[r \right]_0^1 d\theta = -\int_0^{\pi} \cos\theta \sin^2\theta \, d\theta$$

$$= \left| \begin{matrix} t = \sin\theta \\ \cancel{\frac{d(\cos t)}{dt} = \sin t} \end{matrix} \Rightarrow \frac{dt}{d\theta} = +\cos\theta \Rightarrow \cancel{d\theta =} d\theta = \frac{dt}{\cos\theta} \quad d\theta = \frac{dt}{\cos\theta} \right| =$$

$$= -\int_0^{\pi} t^2\,dt = \left| \begin{matrix} a = \sin\pi = 0 \\ b = \sin 0 = 0 \end{matrix} \right| = -\int_0^0 t^2\,dt = \underline{0}$$

~~P~~ Example p. 15

$\vec{F} \langle (x^2 - y), 4z, x^2 \rangle$

$\cancel{\iint}$

$$\oint_C \vec{F}\hat{T}\,ds = \oint \vec{F}\,d\vec{r} = \oint_C M\,dx + N\,dy + P\,dz = \iint_S (\nabla \times \vec{F}) \cdot \hat{n}\,d\sigma$$

$$\vec{r}(\theta) = r\cos\theta\,\hat{i} + r\sin\theta\,\hat{j} + r\,\hat{k}$$

$$\vec{r}'(\theta) = -r\sin\theta\,\hat{i} + r\cos\theta\,\hat{j} + 0$$

$$|\vec{r}'(\theta)| = \sqrt{r^2} = r$$

$$\oint_C \vec{F}\hat{T}\,ds = \int_0^{2\pi} \left(r^2\cos^2\theta - r\sin\theta)\,i + 4r\,j + r\cos^2\theta\,\hat{k}\right)$$

$$\cdot(-r\sin\theta\,\hat{i} + r\cos\theta\,\hat{j})\,d\theta = \int_0^{2\pi} \left(-r^3\sin\theta\cos^2\theta + r^2\sin^2\theta + 4r^2\cos\theta\,\hat{j}\right)d\theta = r^2\int_0^{2\pi} (-r\sin\theta\cos^2\theta + \sin^2\theta + 4\cos\theta)\,d\theta$$

$$\frac{du}{d\theta} = -\sin\theta \Rightarrow d\theta = \frac{du}{-\sin\theta}$$

$$= r^2\int_0^{2\pi} r\,u^2\,du = \underline{0}$$

$$\nabla\times\vec{F} = \left(\frac{\partial(r^2)}{\partial y} - \frac{\partial(4z)}{\partial z}\right)\hat{i} + \left(\frac{\partial(x^2-y)}{\partial z} - \frac{\partial x^2}{\partial x}\right)\hat{j} +$$

$$\left(\frac{\partial(4z)}{\partial x} - \frac{\partial(x^2-y)}{\partial y}\right)\hat{k} = -4\hat{i} - 2x\hat{j} - \hat{k}$$

$$\vec{r}_r = \hat{k}$$

$$\vec{r}_\theta = -r\sin\theta\,\hat{i} + r\cos\theta\,\hat{j}$$

$$\vec{r}_r \times \vec{r}_\theta = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ -r\sin\theta & r\cos\theta & 0 \end{vmatrix} = -r\cos\theta\,\hat{i} - r\sin\theta\,\hat{j}$$

$|\vec{r}_r \times \vec{r}_\theta| = r$

$$\iint_S \left(-u\hat{i} - 2r\cos\theta\hat{j} - \hat{k}\right) \cdot \left(-\cos\theta\hat{i} - \sin\theta\hat{j}\right) r\,dr\,d\theta =$$

$$= \iint_S \left(u\cos\theta + 2r\cos\theta\sin\theta\right) r\,dr\,d\theta = \int_0^{2\pi}\int_0^{r} \left(u\cos\theta\, r + 2r^2\sin\theta\cos\theta\right)$$

$$d\theta = \int_0^{2\pi} \left[2\cos\theta\, r^2 + \frac{2}{3}r^3\sin\theta\cos\theta\right]_0^r d\theta = r^2\int_0^{2\pi} 2\cos\theta + \frac{2}{3}r\sin\theta\cos\theta$$

$$= r^2\left[2\underbrace{\sin\theta}_{0} - \frac{1}{3}\underbrace{\cos^2\theta}_{1}\right]_0^{2\pi} = \underline{0}$$

Example. p17

$x^2 - 2z = 0$

$x(u,v) = u$

$y(u,v) = v$

$z(u,v) = \frac{1}{2}u^2$

$r(u,v) = \langle u, v, \frac{1}{2}u^2 \rangle$

$\nabla \times \vec{F} = (x - 3z)\hat{j} \quad y\hat{k}$

$$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & u \\ 0 & 1 & 0 \end{vmatrix} = -u\hat{i} + \hat{k}$$

$|\vec{r}_u \times \vec{r}_v| = u^2+1$

$\hat{n} = \frac{-u\hat{i}+\hat{k}}{u^2+1} = -\frac{u}{u^2+1}\hat{i} + \frac{1}{u^2+1}\hat{k}$

$\iint_S \left(\left(u-\frac{3}{2}u^2\right)\hat{j} + v\hat{k}\right)\cdot\left(-\frac{u}{u^2+1}\hat{i} + \frac{1}{u^2+1}\hat{k}\right)(u^2+1)\,du\,dv =$

$= \iint_D v\,du\,dv = \int_0^{\sqrt{3}}\int_0^{v} v\,du\,dv = \int_0^{\sqrt{3}} v^2\,dv = \left[\frac{v^3}{3}\right]_0^{\sqrt{3}} = \underline{1}$

example p. 18

$\vec{F} = \langle y, -2z, xz, xz^2\rangle \qquad z = x^2+4y^2$

$\nabla\times\vec{F} = -x\hat{i} + z^2\hat{j} + (-z+1)\hat{k}$

$x(u,v) = u$

$y(u,v) = v$

$z(u,v) = u^2+4v^2$

$r(u,v) = \langle u, v, u^2+4v^2\rangle$

$\vec{r}_u = \langle 1, 0, 2u\rangle$

$\vec{r}_v = \langle 0, 1, 8v\rangle$

$\vec{r}_u\times\vec{r}_v = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 0 & 2u\\ 0 & 1 & 8v\end{vmatrix} = -2u\hat{i} - 8v\hat{j} + \hat{k}$

7

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$$|r_u \times r_v| = 4u^2 + 64v^2 + 1$$

$$\iint_S \langle -u\hat{i}, (u^2+4v^2)^2\hat{j}, -u^2-4v^2-1 \rangle \cdot (-2u\hat{i} - 8v\hat{j} + \hat{k})$$

$$= \iint_S \left(2u^2 - 8v(u^2+4v^2)^2 + (-u^2-4v^2-1)\right) du\, dv =$$

$$= \iint_S \left(2u^2 - 8v(u^4 + 8u^2v^2 + 16v^4) - u^2 - 4v^2 - 1\right) du\, dv =$$

$$= \iint_S \left(u^2 - 8vu^4 - 64v^3u^2 - 128v^5 - 4v^2 - 1\right) du\, dv =$$

$$=$$