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# Sec 5
## ( Root locus )

~~* For Complex roots~~

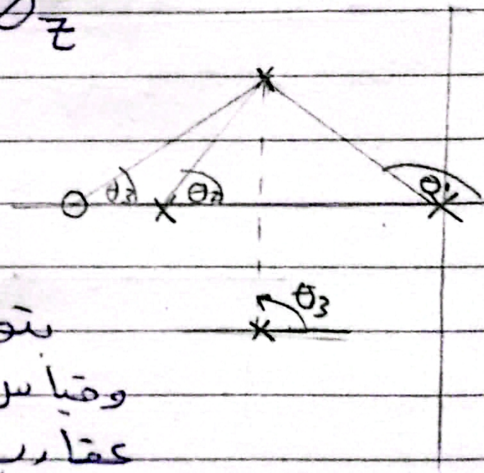
→ For zeros, we obtain arrival angle

$$\emptyset_A = 180 - \sum \emptyset_z + \sum \emptyset_p$$

→ For Poles, we obtain departure Angle

$$\emptyset_D = 180 - \sum \emptyset_p + \sum \emptyset_z$$

$$\sum \emptyset_z - \sum \emptyset_p = 180^\circ$$




فى حالة ظهور Complex Pole نقوم
بتوصيله بجميع الـ Poles, zeros
وقياس الزاوية لكل خط متصل عكس
عقارب الساعة. والتعويض بقيم الزوايا
فى قانون $\emptyset_D$ .

نتيجه وجود تماثل بين 2 Complex Poles تكون
الزوايا للجذر الآخر بعكس الإشارة.

* بعد الحصول على الزاوية إذا كانت بإشارة موجبة
يتم قياسها عكس عقارب الساعة.
- إشارة سالبه تقاس مع عقارب الساعة.

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① $GH(S) = \dfrac{K}{S(S+1)(S^2+4S+13)}$

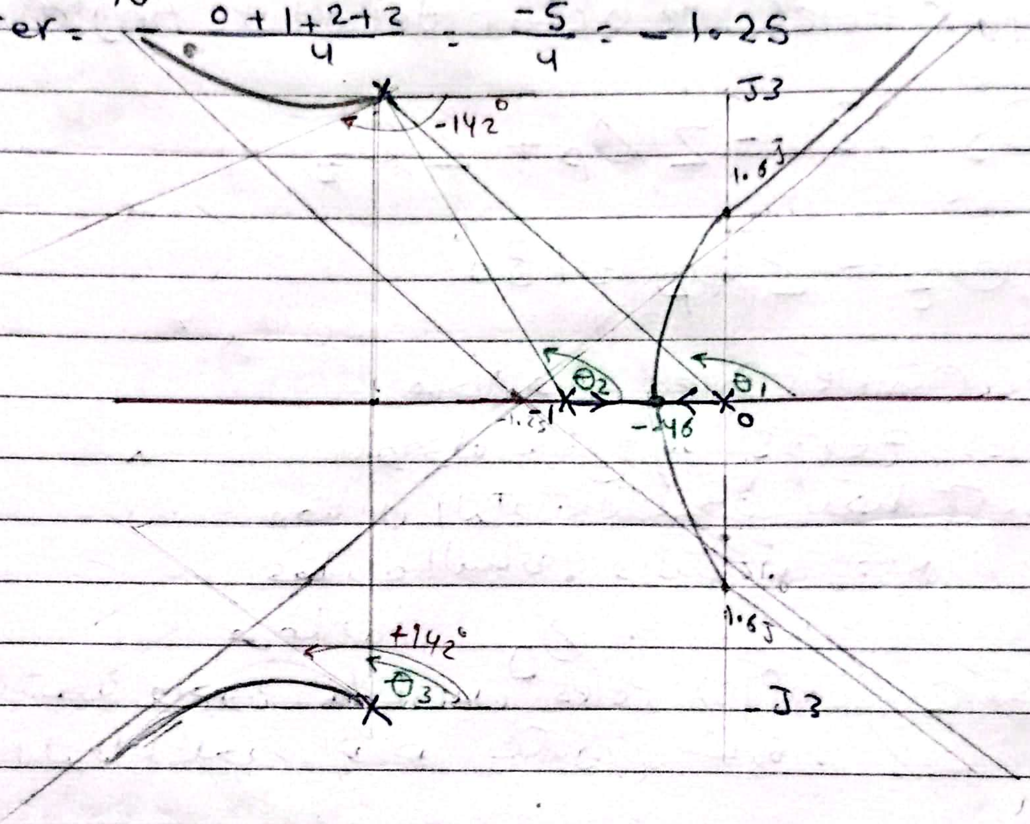
→ o.l. Poles: $0, -1, -2+3J, -2-3J$

o.l. Zeros: None

→ Lines: $N = 4$

$\phi = \dfrac{(2q+1)180}{N} = 45°, -45°, 135°, -135°$

Center $= \dfrac{0+1+2+2}{4} = \dfrac{-5}{4} = -1.25$




Break Points

Ch. equ: $1 + \dfrac{K}{S(S+1)(S^2+4S+13)}$

$K = -S(S^3+4S^2+13S+S^2+4S+13)$

$K = -(S^4+5S^3+17S^2+13S)$

$\dfrac{dK}{dS} = -(4S^3+15S^2+35S+13) = 0$

$S_1 = -.46$ Breakaway $S_{2,3} = -1.6 \pm J\,2.1$

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→ real Poles :-

Intersection with $J\omega$

Ch. equ: $K_{cr} + S^4 + 5S^3 + 17S^2 + 13S$

$K_{cr} + J^4\omega^4 + 5J^3\omega^3 + 17J^2\omega^2 + 13J\omega = 0$

$K_{cr} + \omega^4 - 5J\omega^3 - 17\omega^2 + 13J\omega = 0$

$(K_{cr} + \omega^4 - 17\omega^2) + J(-5\omega^3 + 13\omega) = 0$

$-5\omega^3 + 13\omega = 0$

$\omega(-5\omega^2 + 13) = 0$

$\omega = 0 \qquad \omega = \pm\sqrt{\frac{13}{5}} = \boxed{\pm 1.6}$ intersection

$K_{cr} + \left(\frac{\sqrt{13}}{5}\right)^4 - 17\left(\frac{13}{5}\right) = 0 \Rightarrow K_{cr} = 37.44$

→ Complex Poles

Departure Angle

$\emptyset_0 = 180 - (\theta_1 + \theta_2 + \theta_3)$

$\theta_1 = 123.69^\circ \quad , \theta_2 = 108.44^\circ \quad , \theta_3 = 90^\circ$

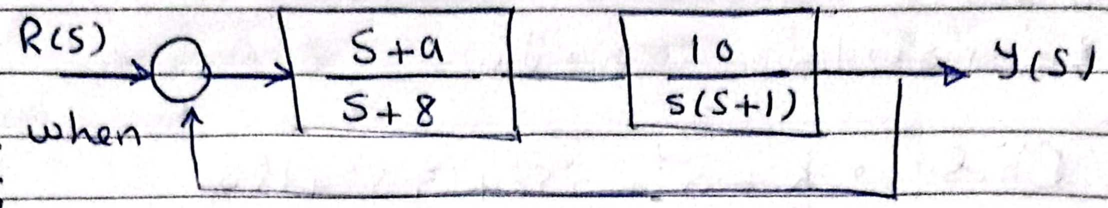
$\emptyset_0 = 180 - (123.69 + 108.44 + 90) = -142.13^\circ$

ارسم الزاوية مع عقارب الساعة

* ارسمها عند الـ Pole السالب ( $142.13^\circ$ ) عكس عقارب الساعة.

((ALAQSA))

(2)




Find a when $\zeta = .5$

$$GH(S) = \frac{10(S+a)}{S(S+8)(S+1)}$$

Ch. equ: $1 + GH(S)$

$$\frac{S(S+8)(S+1) + 10S + 10a}{S(S+8)(S+1)} = 0$$

اقسم ع جميع الحدود ماعدا 10a

$$1 + \frac{10a}{S(S+8)(S+1) + 10S}$$

$$1 + \frac{10a}{S^3 + 9S^2 + 8S + 10S} = 1 + \frac{10a}{S(S^2 + 9S + 18)}$$

$$GH(S) = \frac{10a}{S(S+3)(S+6)} = \frac{a'}{S(S+3)(S+6)}$$

$\boxed{a' = 10a}$

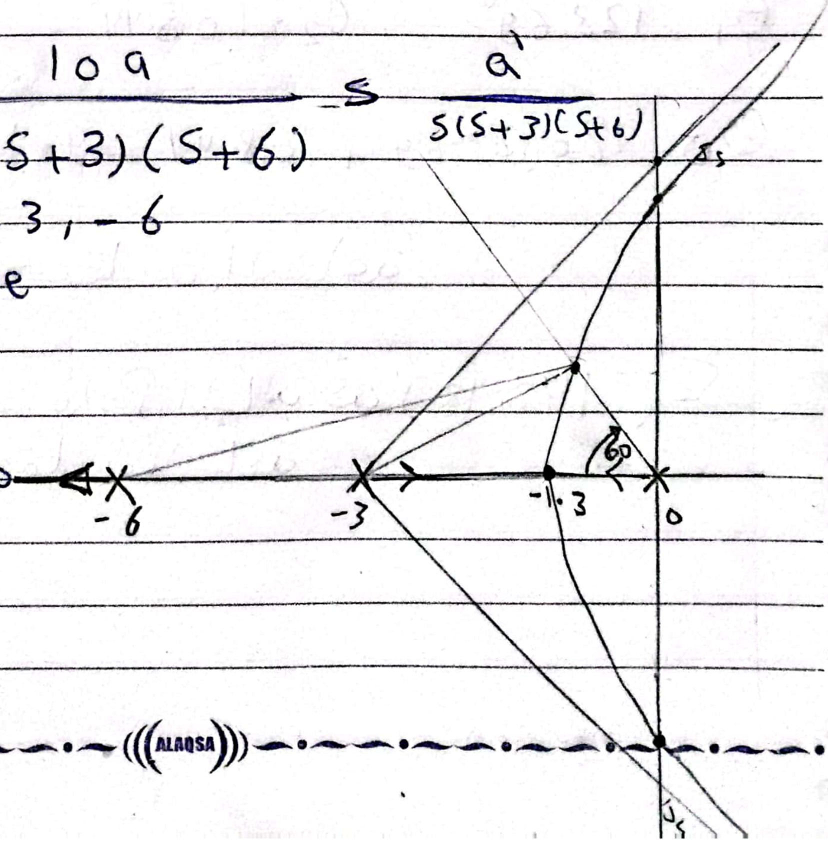
o.l. Poles: $0, -3, -6$

o.l. zeros None

Lines: $N = 3$

Center $= \frac{-9}{3} = -3$

$\theta = 60, -60, 180$



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Break Points:

$$1 + \frac{10a'}{S(S+3)(S+6)}$$

$$a' = (S^3 + 9S^2 + 18S)$$

$$\frac{da'}{ds} = (3S^2 + 18S + 18) = 0$$

$S_1 = -1.3$ ✓ , $S_2 = -4.7 \notin$ root locus

Intersection with Jw

$$a' + S^3 + 9S^2 + 18S = 0$$

$$a' + J^3\omega^3 + 9J^2\omega^2 + 18J\omega = 0$$

$$a' - J\omega^3 - 9\omega^2 + 18J\omega = 0$$

$$(a' - 9\omega^2) + J(-\omega^3 + 18\omega) = 0$$

$$\omega(-\omega^2 + 18) = 0$$

$\omega = 0$ ✗ $\quad \omega = \pm\sqrt{18} = \pm 4.24$ intersection

$a'_{cr} = 9 * 18 \rightarrow a'_{cr} = 162 \Rightarrow a_{cr} = \frac{162}{10} = 16.2$

a at $\zeta = .5$

$$\theta = \cos^{-1} .5 = 60^\circ$$

$$a' = \frac{L_{P1} * L_{P2} * L_{P3}}{1} = 280$$

$$a = \frac{a'}{10} = 28$$

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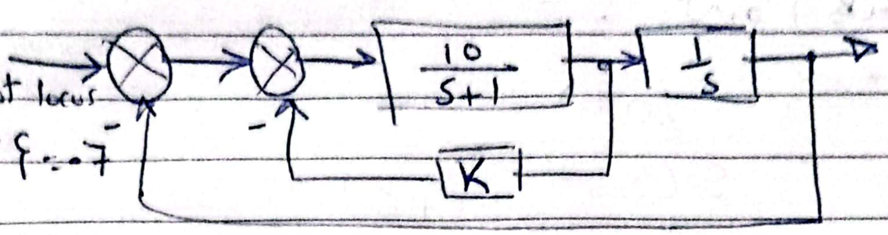
(3)

→ Sketch root locus

→ Find K at $\zeta = .7$




$$\frac{\dfrac{10}{s+1}}{1+\dfrac{10K}{s+1}} * \frac{1}{s} = \frac{10}{s(s+1+10K)}$$

$$GH(s) = \frac{10}{s^2+s+10Ks}$$

$$\text{Ch. equ: } 1 + \frac{10}{s^2+s+10Ks} = 0$$

$$s^2+s+10Ks+10 = 0 \quad \div$$

$$1 + \frac{10Ks}{s^2+s+10}$$

$$GH(s) = \frac{10Ks}{s^2+s+10} \xrightarrow{K' = 10K} \frac{K's}{s^2+s+10}$$

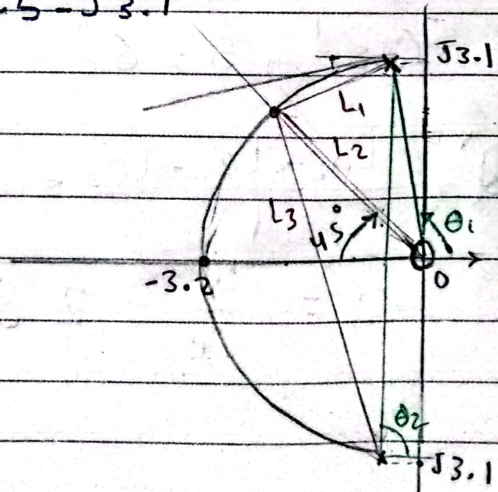
→ O.L. Poles: $-.5+J3.1$, $-.5-J3.1$

O.L. Zeros: $0$

→ Lines: $N = 2-1 = 1$

$\theta = 180°$

Center $= -1$



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Break Points

$$1 + \frac{\acute{K} s}{s^2 + s + 10}$$

$$K = -\frac{s^2 + s + 10}{s}$$

$$\frac{dk}{ds} = \frac{-s(2s+1) + (s^2+s+10)}{s^2} = 0$$

$$-2s^2 - s + s^2 + s + 10 = 0$$

$$-s^2 + 10 = 0$$

$$s^2 = 10 \Rightarrow s = \pm\sqrt{10} = \pm 3.2$$

Break in Point $= -3.2$

* Departure angle

$$\phi_0 = 180 - (90) + (99) = 189^\circ$$

$\acute{K}$ at $\xi = .7$

$$\theta = \cos^{-1} .7 = 45.57^\circ$$

$$\acute{K} = \frac{L_1 L_3}{L_2} = 34.27$$

$$K = \frac{\acute{K}}{10} = 3.427$$

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(4) $GH(s) = \frac{K(s^2+.5s+5)}{s^3(s+5)}$

O.L. Poles: 0, 0, 0, -5   $n_p = 4$

O.L. Zeros: $-.25+J2.22$, $-.25-J2.22$   $n_z = 2$

Lines: $N = 4-2 = 2$

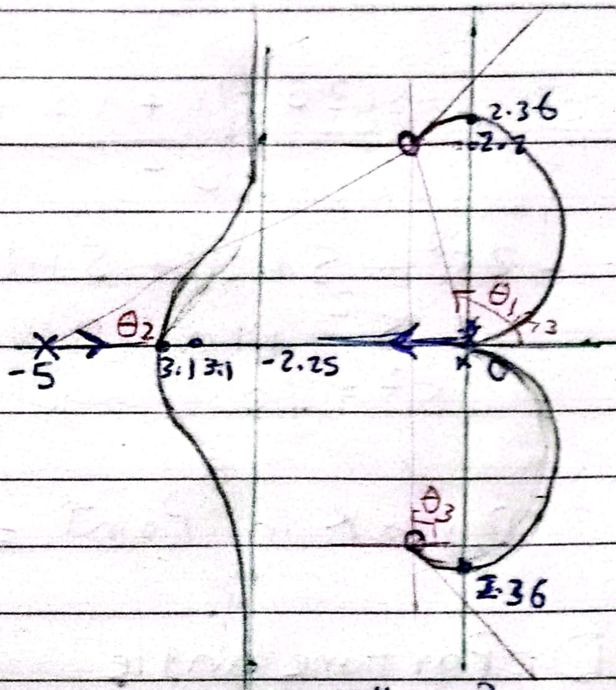
$\sigma_0 = \frac{-5+.5}{2} = -2.25$

$\phi = \frac{(2q+1)180^\circ}{2} = 90^\circ, -90^\circ$

2 Poles → ∞

2 Poles → 2 Zeros.

Break Points




$1+GH(s)$

$1+\frac{K(s^2+.5s+5)}{s^3(s+5)}$

$K = \frac{-s^3(s+5)}{s^2+.5s+5}$

$K = \frac{-s^4-5s^3}{s^2+.5s+5}$

$\frac{dK}{ds} = \frac{-(4s^3+15s^2)(s^2+.5s+5)+(2s+.5)(s^4+5s^3)}{(s^2+.5s+5)^2} = 0$

$\to -4s^5-2s^4-20s^3-15s^4-7.5s^3-75s^2+2s^5+10s^4+.5s^4+2.5s^3=0$

$\to -2s^5-6.5s^4-25s^3-75s^2=0$

$\to -s^2(2s^3+6.5s^2+25s+75)=0$

$s_1=0$, $s_2=0$, $s_3=-3.17$, $s_{4,5} = -.07 \pm J3.5$

✓ Break away

Arrival angle:-

$$\phi_A = 180 - \sum \phi_Z + \sum \phi_P$$

$\theta_{1,2,3} = 96.43$ , $\theta_2 = 25^\circ$ $\theta_3 = 90^\circ$

$$\phi_A = 180 - 90 + 3(96.43) + 25 = 404.33^\circ$$

$404.33 - 360 = 44.33^\circ$ +ve نقيسها عكس عقارب الساعة

عند الخروج من 0 أو دخل ال zero سنحني بعكس الزاوية 44 نقطع Jw في نقطتين

Intersection with Jw

$1 + GH(s) = 0$ , $S = JW$

$S^4 + 5S^3 + KS^2 + .5KS + 5K = 0$

$J^4 w^4 + 5J^3 w^3 + KJ^2 w^2 + .5KJW + 5K = 0$

$w^4 - J5w^3 - Kw^2 + .5K\,JW + 5K = 0$

$(w^4 - Kw^2 + 5K) + J(-5w^3 + .5Kw) = 0$

$w^4 - Kw^2 + 5K = 0 \longrightarrow 1$

$w(.5K - 5w^2) = 0 \Longrightarrow w^2 = \dfrac{K}{10}$

بالتعويض في 1

$$\frac{K^2}{100} - \frac{K^2}{.10} + 5K = 0$$

$$\frac{K^2 - 10K^2}{100} + 5K = 0 \Longrightarrow -9K^2 + 500K = 0$$

$K(-9K + 500) = 0 \Longrightarrow K_{cr} = 0$ , $K_{cr} = \dfrac{500}{9}$

$$w^2 = \frac{K_{cr}}{10} = \frac{5.00}{90} \Longrightarrow w = \pm\sqrt{\frac{500}{90}}$$

$w = \pm 2.357$ intersection Points with Jw