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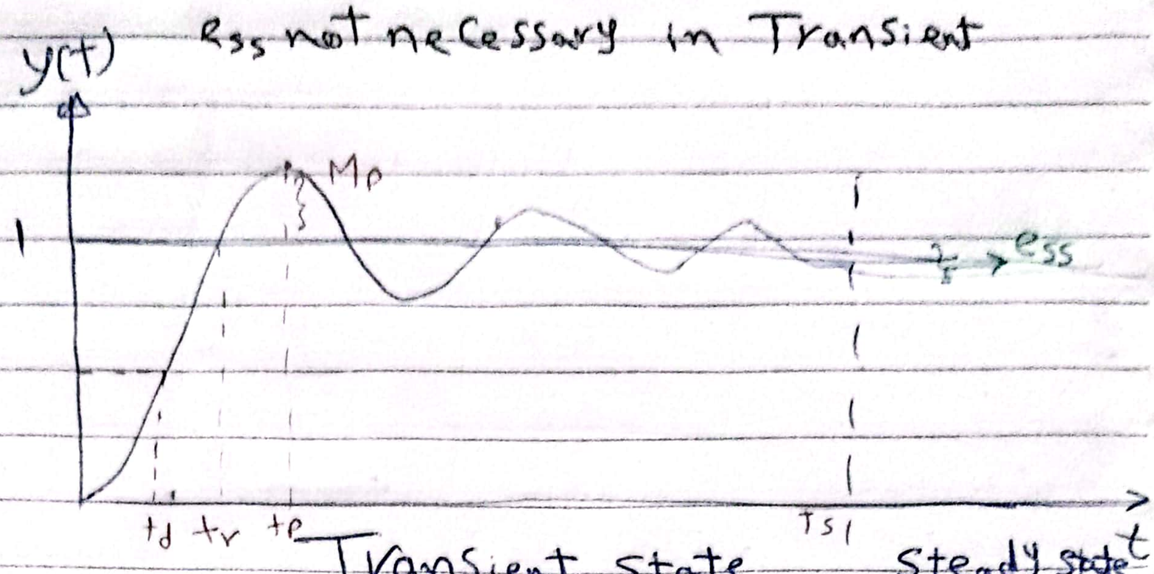
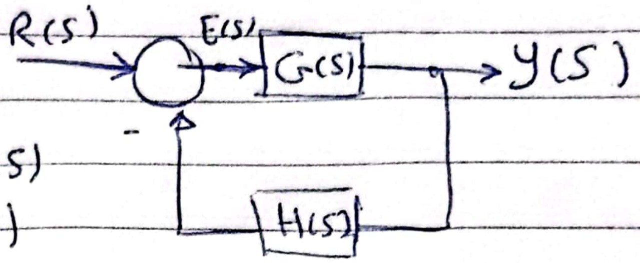
Sec (3)

Steady state error.

e_{ss} not necessary in Transient





$$E(S) = R(S) - Y(S)H(S)$$

$$E(S) = R(S) - G(S)E(S)H(S)$$

$$E(S)(1 + G(S)H(S)) = R(S)$$

$$E(S) = \frac{1}{1 + G(S)H(S)} R(S)$$

This is error but we need stead state error that represents the final value.

$e_{ss} = \lim_{S \to 0} S E(S)$ * depend on the input R(S) & the open loop T.f GH(S)

$$e_{ss} = \lim_{S \to 0} \frac{1}{1 + G(S)H(S)} R(S)$$

Steady State error for Some input Signals

1. unit step input $r(t) = 1$ for $t \geq 0$

$$R(S) = \frac{1}{S}$$

$$e_{ss} = \lim_{S \to 0} S E(S)$$

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$$e_{ss} = \lim_{s \to 0} s \cdot \frac{1}{1+G(s)} * \frac{1}{s}$$

$$= \lim_{s \to 0} \frac{1}{1+G(s)} = \frac{1}{1+\lim_{s \to 0} G(s)} = \frac{1}{1+K_p}$$

K_p :- Position error Constant $\qquad K_p = \lim_{s \to 0} G(s)$

Unit ramp input :-

$r(t) = t \qquad t \geq 0 \qquad \frac{dr}{dt} = 1 \rightarrow$ velocity

$$R(s) = \frac{1}{s^2}$$

$$e_{ss} = \lim_{s \to 0} s \frac{1}{1+G(s)} \cdot \frac{1}{s^2} = \lim_{s \to 0} \frac{1}{s + sG(s)}$$

$$e_{ss} = \frac{1}{\lim_{s \to 0} sG(s)} = \frac{1}{K_v}$$

K_v : Velocity Constant , $K_v = \lim_{s \to 0} sG(s)$

Parabolic input :

$r(t) = \frac{1}{2} t^2 \qquad t \geq 0 \qquad \frac{dr}{dt} = t \quad , \quad \frac{d^2r}{dt^2} = 1 \rightarrow$ acceleration

$$R(s) = \frac{1}{s^3}$$

$$e_{ss} = \lim_{s \to 0} s * \frac{1}{1+G(s)} * \frac{1}{s^3} = \lim_{s \to 0} \frac{1}{s^2 + s^2 G(s)}$$

$$e_{ss} = \frac{1}{\lim_{s \to 0} s^2 G(s)} = \frac{1}{K_a}$$

K_a : acceleration Constant , $K_a = \lim_{s \to 0} s^2 G(s)$

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input / type	unit step	unit ramp	Parabolic
0	$e_{ss} = \frac{1}{1+K_p}$ $K_p = \lim_{s\to 0} G(s)$	∞	∞
1	0	$e_{ss} = \frac{1}{K_v}$ $K_v = \lim_{s\to 0} s\,G(s)$	∞
2	0	0	$e_{ss} = \frac{1}{K_a}$ $K_a = \lim_{s\to 0} s^2 G(s)$

For the following open loop transfer functions

(i) $GH(S) = \frac{4}{S+1}$ (ii) $GH(S) = \frac{6(S+3)}{(S+2)(S-6)}$

(iii) $GH(S) = \frac{5}{S(S^3-3S^2+5S)}$ (iv) $GH(S) = \frac{10}{S^2(S^2+2S+1)}$

a) what is the system type & order

b) find the error constant K_p, K_v, and K_a

c) If input $r(t) = 1+2t+4t^2$ is applied to the unity feedback system, determine e_{ss}.

i) $GH(S) = \frac{4}{S+1}$

order: 1 type: zero

$K_p = \lim_{s\to 0} \frac{4}{s+1} = 4$

$K_v = \lim_{s\to 0} s\,G(s) = \lim_{s\to 0} \frac{4s}{s+1} = 0$

$K_a = \lim_{s\to 0} \frac{4s^2}{s+1} = 0$

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$$e_{ss} = \frac{1}{1+K_p} + \frac{2}{K_v} + \frac{8}{K_a}$$

$$e_{ss} = \frac{1}{5} + \frac{2}{0} + \frac{8}{0} = \infty$$

ii) $$GH(S) = \frac{6(S+3)}{(S+2)(S-6)}$$

order: 2 , type: Zero

$$K_p = \lim_{S \to 0} \frac{6(S+3)}{(S+2)(S-6)} = \frac{24}{-12} = -2$$

$$K_v = \lim_{S \to 0} \frac{6S(S+3)}{(S+2)(S-6)} = 0$$

$$K_a = \lim_{S \to 0} \frac{6S^2(S+3)}{(S+2)(S-6)} = 0$$

$$e_{ss} = \frac{1}{1-2} + \frac{2}{0} + \frac{8}{0} = \infty$$

iii) $$GH(S) = \frac{5}{S(S^3-3S^2+5S)} = \frac{5}{S^2(S^2-3S+5)}$$

order: 4 type: 2

$$K_p = \lim_{S \to 0} \frac{5}{S^2(S^2-3S+5)} = \frac{5}{0} = \infty$$

$$K_v = \lim_{S \to 0} \frac{5}{S(S^2-3S+5)} = \frac{5}{0} = \infty$$

$$K_a = \lim_{S \to 0} \frac{5}{S^2-3S+5} = 1$$

$$e_{ss} = \frac{1}{\infty} + \frac{2}{\infty} + \frac{8}{1} = 8$$

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iv) $GH(s) = \dfrac{10}{s^2(s^2+2s+1)}$

type : 2 order : 4

$K_p = \lim_{s\to 0} \dfrac{10}{s^2(s^2+2s+1)} = \dfrac{10}{0} = \infty$

$K_v = \lim_{s\to 0} \dfrac{10}{s(s^2+2s+1)} = \dfrac{10}{0} = \infty$

$K_a = \lim_{s\to 0} \dfrac{10}{s^2+2s+1} = 10$

$e_{ss} = \dfrac{1}{\infty} + \dfrac{2}{\infty} + \dfrac{8}{10} = .8$

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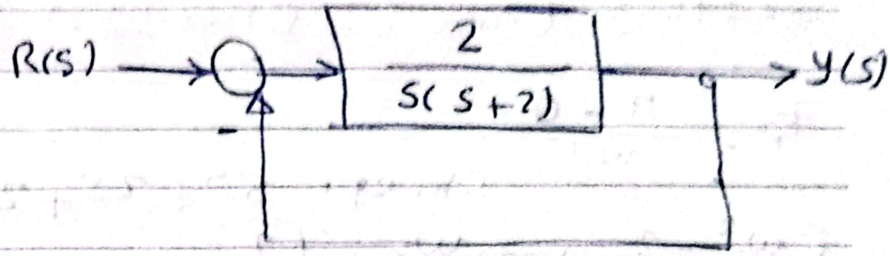
For the following feedback system



Find

1) open loop and closed loop transfer function

$$o.L.T.f = \frac{2}{S(S+2)}$$

Closed looped:
$$\frac{2}{S^2+2S+2}$$

2) type and order of system

type: 1 order: 2

3) error constant K_p, K_v, K_a

$$K_p = \lim_{S\to 0} \frac{2}{S(S+2)} = \frac{2}{0} = \infty$$

$$K_v = \lim_{S\to 0} \frac{2}{S+2} = 1$$

$$K_a = \lim_{S\to 0} \frac{2S}{S+2} = 0$$

4) steady state error to the input

$r(t) = 2 - .5t$

$$e_{ss} = \frac{2}{\infty} - \frac{.5}{1} = -.5$$

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5) rise time, Peak time, maximum overshoot settling time (2% error) for unit step input

$$t_r = \frac{\pi - \theta}{\omega_d}$$

Ch. eq: $S^2 + 2S + 2$

$\omega_n^2 = 2 \rightarrow \omega_n = \sqrt{2}$

$2\zeta\omega_n = 2 \rightarrow \zeta\omega_n = 1 \rightarrow \zeta = \frac{1}{\omega_n} = \frac{1}{\sqrt{2}}$

$\theta = \cos^{-1}\zeta = \cos^{-1}\frac{1}{\sqrt{2}} = \frac{\pi}{4}$

$\omega_d = \omega_n\sqrt{1-\zeta^2} = \sqrt{2}\sqrt{1-\frac{1}{2}} = 1$

$t_r = \frac{\pi - \frac{\pi}{4}}{1} = 2.36$ sec

$t_p = \frac{\pi}{\omega_d} = 3.14$ sec

$t_s = \frac{4}{\zeta\omega_n} = 4$ sec

$M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}} = e^{\frac{-\pi}{\sqrt{2}}/\sqrt{1-\frac{1}{2}}}$

) Closed loop Poles of the system & Check Stability

Ch. equ: $S^2 + 2S + 2$

Poles: $(S + (1-j))(S + (1+j))$

$-1-j, -1+j$

left side $\rightarrow$ stable.

(3) Report

⑥ T.F $= \dfrac{18}{S^2(2S^2+8S+25)}$

find ξ, ω_n, M_p, t_p, t_s (5% error), C(t) for i/p u(t)

→ make T.f in general form

T.f $= \dfrac{9}{12.5} * \dfrac{12.5}{S^2+4S+12.5}$ $\quad \dfrac{9}{12.5}$ يؤثر فقط ع C(t) ومش ع F ال response

$\omega_n^2 = 12.5 \rightsquigarrow \omega_n = 3.54$

$2\xi\omega_n = 4 \rightsquigarrow \xi = .56$

$M_p = e^{-\xi\pi/\sqrt{1-\xi^2}} = .12 = 12\%$

$t_p = \dfrac{\pi}{\omega_d} = \dfrac{\pi}{\omega_n\sqrt{1-\xi^2}} = \dfrac{\pi}{2.92} = 1.07$ Sec

$t_s = \dfrac{3}{\xi\omega_n} = \dfrac{3}{.56 * 3.54} = 1.5$ Sec

$C(t) = \dfrac{9}{12.5}\left(1 - \dfrac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t + \theta)\right)$

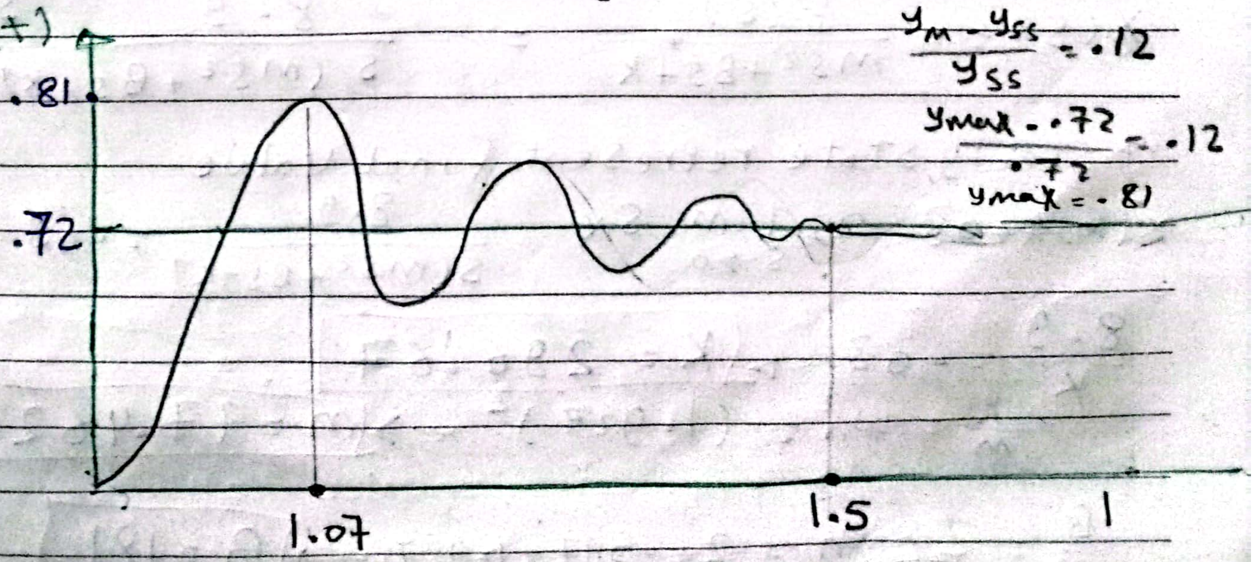
$\emptyset = \cos^{-1}\xi = \cos^{-1} .56 = 56^\circ$

$C(t) = .72 - .864\, e^{-1.97t}\sin(2.92t + 56^\circ)$

$\dfrac{y_m - y_{ss}}{y_{ss}} = .12$

$\dfrac{y_{max} - .72}{.72} = .12$

$y_{max} = .81$



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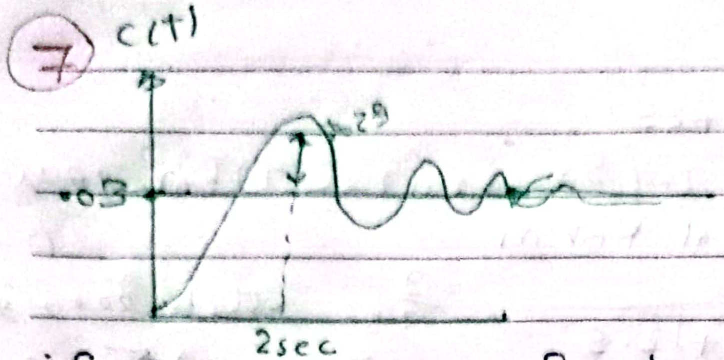

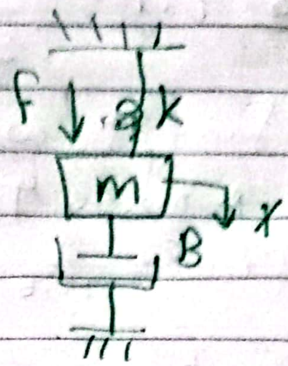

if f(t) = 8.9 N Find M, B, K

→ mechanical system

$\sum F = m\ddot{x} \longrightarrow F - Kx - B\dot{x} = m\ddot{x}$

$F = X(S)(mS^2 + BS + K)$

$$\frac{X(S)}{F(S)} = \frac{1}{mS^2+BS+K} = \frac{1}{K} * \frac{K/M}{S^2+\frac{B}{m}S+\frac{K}{m}}$$

$\omega_n^2 = \frac{K}{m}$, $2\zeta\omega_n = \frac{B}{m}$

$$M_p = \frac{y_{max}-y_{ss}}{y_{ss}} = \frac{.0029}{.03} = .967 = e^{-\zeta\pi/\sqrt{1-\zeta^2}}$$

$\zeta = 5.97$

$t_p = \frac{\pi}{\omega_d} = 2 \rightarrow \frac{\pi}{2} = \omega_d \rightarrow \omega_d = 1.57$

$$\omega_n = \frac{1.57}{\sqrt{1-(0.597)^2}} = 1.957$$

From T.f , $f(t) = 8.9 \rightarrow F(S) = \frac{8.9}{S}$

$$X(S) = \frac{F(S)}{mS^2+BS+K} = \frac{8.9}{S(mS^2+BS+K)}$$

∴ steady state represent final value

$$x(\infty) = .03 \rightarrow \lim_{S\to 0} S * \frac{8.9}{S(mS^2+BS+K)} = .03$$

$\frac{8.9}{K} = .03 \rightarrow \boxed{K = 296.67}$

$\frac{K}{m} = \omega_n^2 = (1.927)^2 \rightarrow \boxed{m = 77.462}$

$\frac{B}{m} = 2\zeta\omega_n = 2 * .597 * 1.957 \rightarrow \boxed{B = 181}$

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