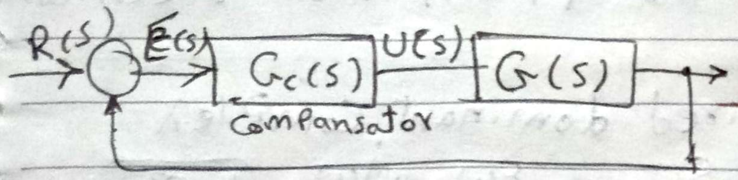
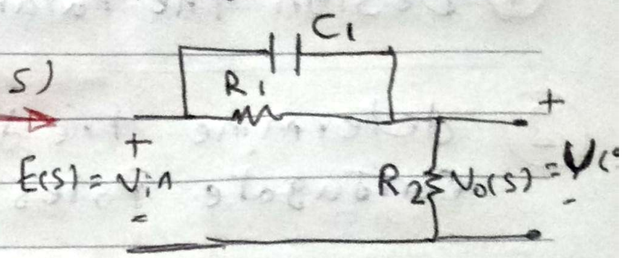
Sec 7 — Phase Lead Compansator

Lead Compansator Control the Transient State (t_r, t_p, t_s, M_p), Control Poles location.



Compansated system



$G_c(S)$

$$G_c(S) = \frac{U(S)}{E(S)} = \frac{V_o(S)}{V_i(S)} = \alpha\left(\frac{1+T_1 S}{1+\alpha T_1 S}\right)$$

$$T_1 = R_1 C_1 \quad , \quad \alpha = \frac{R_2}{R_1+R_2} \quad \& \quad \alpha < 1$$

$$G_c(S) = \frac{S+\frac{1}{T_1}}{S+\frac{1}{\alpha T_1}} = \frac{S-Z_c}{S-P_c}$$

(Z_c → zero, P_c → Pole)

$$Z_c = \frac{-1}{T_1} \quad , \quad P_c = \frac{-1}{\alpha T_1}$$

$Z_c \longrightarrow T_1 \longrightarrow R_1$ & $P_c \longrightarrow \alpha = \frac{Z_c}{P_c} \longrightarrow R$

as from experiment C_1 is given

$$C_1 = 10\ \mu F$$

→ given G(S), Parameter specification (t_r, t_p, t_s, M_p)

or given ξ, ω_n or S_1, S_2 direct.

DATE: SUBJECT:

Phase Lead Compansator Steps:-

① Design the Parameters ζ, ω_n from Specification

② determine the desired dominant Complex Conjugate Poles $S_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$

③ Check if lead Compansator is needed or not

if $S_{1,2}$ located on root locus → no need to lead
if $S_{1,2}$ not on root locus → lead CompA- is needed.

the Simplest → Connect Complex Pole to all Poles, Zeros, → measure angles if

$-\sum\phi_p + \sum\phi_z = -180°$ $S_{1,2}$ located on root locus, Poles of system

good response(no lead Compansator)

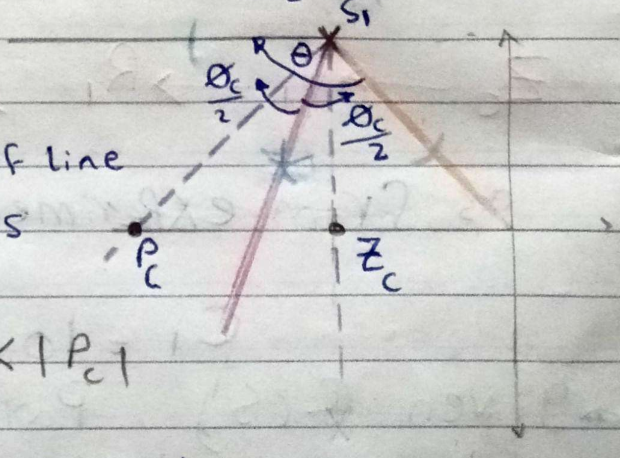
lead Compansator needed $\neq -180$ $S_{1,2}$ not on root locu

< -180

④ design Phase lead Compansator

$\phi_c - \sum\phi_p + \sum\phi_z = -180°$ → ϕ_c = lead angle

- Connect S_1 with origine
- angle θ is halfed (Bisector)
- measure $\frac{\phi_c}{2}$ in both direction of line
- intersection with real axis give Z_c & P_c



$Z_c = \frac{-1}{T_1}$, $P_c = \frac{-1}{\alpha T_1}$ $|Z_c| < |P_c|$

from Z_c, P_c → α, T_1 obtained. , given C_1

R_1, R_2 are obtained next.

DATE: SUBJECT:

5) From Controlled System

$G_t(S) = G_c(S) \cdot G(S)$

$$G_t(S) = \frac{S - Z_c}{S - P_c} \cdot \frac{K(\ )(\ )\ldots}{(\ )(\ )\ldots}$$

use the o.L.T.F to obtain gain K.

- S_1 after lead Compansator $\in$ root locus
- Connect S_1 to all Poles, zeros including P_c, Z_c
- Calculate length of Lines.

$$K = \frac{\prod \text{Length with Poles.}}{\prod \text{length of zeros.}}$$

Notes:-

→ if $+\phi_c < 65^\circ \to 70^\circ$ there is one Compansator

→ $\phi_c > 70^\circ \to$ 2 Compansator $(\phi_{c_1} + \phi_{c_2}) = \phi_c$

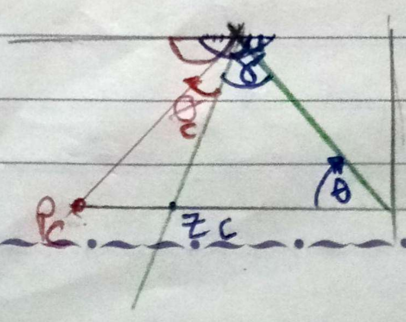
$$T.F = G_c(S) = \left(\frac{S - Z_c}{S - P_c}\right)^2 = \left(\alpha \frac{1 + T_1 s}{1 + \alpha T s}\right)^2$$

$\phi_{c_{new}} = \dfrac{\phi_c}{2}$ To find Z_c, P_c draw angle $\dfrac{\phi_{c_{new}}}{2} = \dfrac{\phi_c}{4}$

To Find $K \to 2 Z_c$, $2 P_c$ used with Poles, zeros.

Another method For P_c, Z_c

$\gamma = \frac{1}{2}(180 - \phi_c - \theta)$



DATE: ______________ SUBJECT: ______________

(3) Consider a unity feedback control system that has open loop transfer function of

$$G(s) = \frac{K}{s(s+1)(s+4)}$$

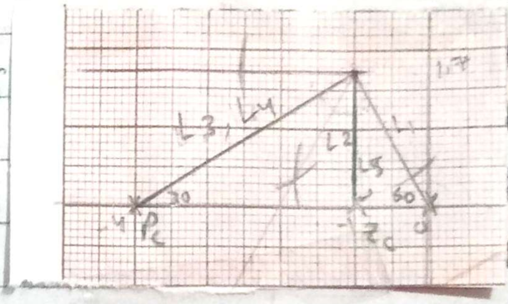
The system is to be compensated such that it meets the following specification:

damping ratio = .5 & undamped natural frequency = 2 rad/sec

→ Determine which type of compensator is to be designed??

as there are only desired specifications of transient state, Lead compensator is to be designed.

$\zeta = .5$, $\omega_n = 2$ rad/s



1) $S_{1,2} = -\zeta\omega_n \pm J\omega_n\sqrt{1-\zeta^2}$

$S_{1,2} = -1 \pm J\sqrt{3} = -1 \pm J\,1.73$

2) $-\sum \phi_p = -(\theta_1 + \theta_2 + \theta_3) = -(120 + 90 + 30) = -240$

$-\sum \phi_p \neq -180 \rightarrow$ we need compensator

$\phi_c - \sum \phi_p = -180 \rightarrow \phi_c - 240 = -180$

$\phi_c = 60^\circ$

DATE: SUBJECT:

Plot $\frac{\phi_c}{2}$, $30°$ after Bisector

$\boxed{Z_c = -1}$

$\boxed{P_c = -4}$

$Z_c = \frac{-1}{T_1} = -1 \rightarrow \boxed{T_1 = 1}$

$\frac{Z_c}{P_c} = \alpha \rightarrow \boxed{\frac{1}{4} = \alpha}$

$T_1 = R_1 C_1$, $C_1 = 10 \mu f$

$1 = R_1 * 10 * 10^{-6}$

$R_1 = 10^5 \Omega$

$\alpha = \frac{R_2}{R_1 + R_2} \rightarrow .25 = \frac{R_2}{10^5 + R_2} = \frac{1}{4}$

$4R_2 = R_2 + 10^5 \rightarrow 3R_2 = 10^5$

$\boxed{R_2 = 3.3 * 10^4 \ \Omega}$

$G_t(S) = \frac{K(S+1)}{S(S+1)(S+4)^2}$

$K = \frac{L_1 L_2 L_3 L_4}{L_5} = \frac{2 * \sqrt{3} * 2\sqrt{3} * 2\sqrt{3}}{\sqrt{3}} = 24$

$\boxed{K = 24}$

CamScanner

DATE: ____________ SUBJECT: ____________

A type-2 system with an open loop transfer function of
$G(S) = \frac{K}{S^2(S+1.5)}$ is to be compansated to meet the following design specifications:

i) $M_p \leq 20\%$ ii) $t_s \leq 4$ sec

Design a lead compansator that satisfies the design requirements.

(1) $e^{-\xi\pi/\sqrt{1-\xi^2}} = .2 \Rightarrow \frac{-\xi\pi}{\sqrt{1-\xi^2}} = \ln .2 = -1.6$

$\xi^2\pi^2 = 2.59 - 2.59\xi^2$

$\xi^2 = .2079 \Rightarrow \boxed{\xi = .456}$

$t_s = \frac{4}{\xi\omega_n} = 4 \Rightarrow \xi\omega_n = 1 \Rightarrow \boxed{\omega_n = 2.19}$

(2) $S_{1,2} = -\xi\omega_n \pm J\omega_n\sqrt{1-\xi^2}$

$-1 \pm 2.19 J * .89$

$S_{1,2} = -1 \pm 1.95 J$

(3) $-\sum\emptyset_p = \theta_1 + \theta_2 + \theta_3 = (117.1 + 117.1 + 75.6) = -309.8^\circ$

(4) $\emptyset_c - \sum\emptyset_p = -180$

$\emptyset_c - 309.8 = -180 \rightarrow \emptyset_c = 129.8^\circ > 70$

There are 2 compansator

$\emptyset_{c_{new}} = \frac{\emptyset_c}{2} = 64.9^\circ$ $\frac{\emptyset_{new}}{2} = 32.45^\circ$

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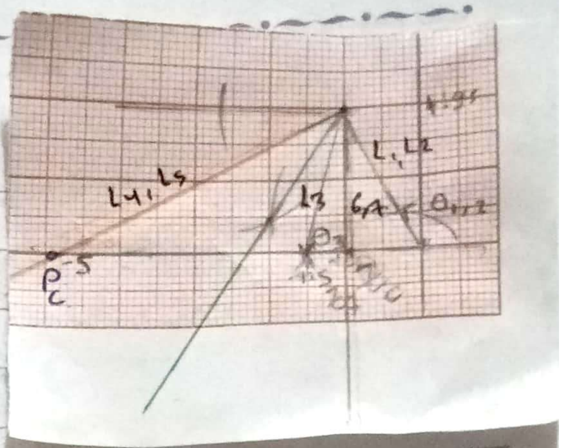


5 $Z_c = -1$

$P_c = -5$

$Z_c = \frac{-1}{T_1} = -1 \rightsquigarrow T_1 = 1$

$P_c = \frac{-1}{\alpha T_1} \rightsquigarrow \frac{Z_c}{P_c} = \alpha \rightsquigarrow \alpha = .2$

$T_1 = R_1 C_1$, $C_1 = 10\,\mu f$

$R_1 = \frac{1}{10^{-5}} = 10^5\,\Omega$

$\alpha = \frac{R_2}{R_1 + R_2} = \frac{R_2}{10^5 + R_2} = .2$

$R_2 = 2 * 10^4 + .2 R_2$

$R_2 = 2.5 * 10^4\,\Omega$

$G_t(S) = \left(\frac{S+1}{S+5}\right)^2 \cdot \frac{K}{S^2(S+1.5)}$

* $K = \frac{L_1 L_2 L_3 L_4 L_5}{L_6 L_7}$

$K = \frac{2.2 * 2.2 * 2.01 * 4.45 * 4.45}{1.95 * 1.95} = 50.7$

DATE: SUBJECT:

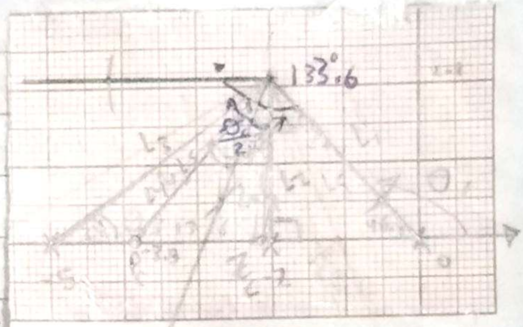
2) $G(S) = \dfrac{K}{S(S+2)(S+5)}$

Design a Phase lead Compansator that is introduced in the Forward Path to achieve the desired Performance Specification

i) $M_p < 5\%$, $t_s < 2$ Sec.

Solution

1) $M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}} = .05$



$$\frac{\zeta^2\pi^2}{1-\zeta^2} = (\ln .05)^2$$

$$\zeta^2\pi^2 = 8.974 - 8.974\zeta^2$$

$$18.84\zeta^2 = 8.974 \rightarrow \zeta^2 = .476$$

$$\boxed{\zeta = .69}$$

$$t_s = \frac{4}{\zeta\omega_n} = 2 \rightarrow \zeta\omega_n = 2$$

$$.69\,\omega_n = 2 \rightarrow \boxed{\omega_n = 2.898}$$

2) $S_{1,2} = -\zeta\omega_n \pm J\omega_n\sqrt{1-\zeta^2}$

$$S_{1,2} = -2 \pm J2.097$$

3) $-\sum \phi_p = -(\theta_1+\theta_2+\theta_3) = -(133.6+90+34.9)$

$$= -258.5 \neq -180^\circ$$

or $(180 - \tan^{-1}\frac{2.1}{2}) + 90 + \tan^{-1}\frac{2.1}{3} = -258.5$

DATE: SUBJECT:

$\because -\sum \phi_p \neq 180 \longrightarrow$ we need lead Compansator

4) Design a lead Compansator

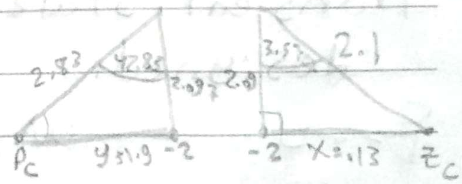
$$\phi_c - \sum \phi_p = -180^\circ$$

$$\phi_c = 258.5 - 180 = 78.5^\circ > 70^\circ$$

$\boxed{\text{we need 2 Compansator}}$

$$\phi_{c\,new} = \frac{78.5}{2} = 39.25^\circ \rightsquigarrow \frac{\phi_{c\,new}}{2} = 19.63$$

Plot $\frac{\phi_{c\,new}}{2}$, Then find Z_c, P_c



$\boxed{Z_c = -2.13}$, $\boxed{P_c = -3.9}$

$$Z_c = \frac{-1}{T_1} = -2.13 \rightsquigarrow T_1 = .469$$

$$P_c = \frac{-1}{\alpha T_1} \rightsquigarrow \frac{Z_c}{P_c} = \alpha \rightsquigarrow \alpha = \frac{-2.13}{-3.9} = .546$$

at $C_1 = 10\,\mu F$, $T_1 = R_1 C_1 \rightsquigarrow .469 = 10*10^{-6} R_1$

$\boxed{R_1 = 46900\ \Omega}$

$$\alpha = \frac{R_2}{R_1 + R_2} \rightsquigarrow \frac{R_2}{46900 + R_2} = .546$$

$R_2 = 25607.4 + .546 R_2 \rightsquigarrow \boxed{R_2 = 56404\ \Omega}$

$$K = \frac{L_1 L_2 L_3 L_4 L_5}{L_6 L_7} = \frac{2.89 * 2.097 * 3.66 * 2.83 * 2.83}{2.1 * 2.1} = 40.5$$