Root Locus

→ graphical method to determine the roots of ch. equ. roots varying from zero to infinity.
→ depends on dynamic variable K
→ Poles change according to K

* General Steps for drawing the root locus of system

1. Determine the open loop Poles and zeros
2. Determine root locus existance & non existance regions
   exist if the sum of no. of Poles & zeros to the right is odd
3. Find asymptotic Lines to determine direction and Plot the root locus
   - no. of Lines $(N) = n_{Poles} - n_{Zeros}$
   - Centroid $(\sigma) = \dfrac{\sum \text{o.L. Poles} - \sum \text{o.L. zeros}}{N}$
   - Angle $(\phi) = \dfrac{(2q+1)180}{N}$, $q = 0, 1, 2, \ldots$

   $n_p - n_z = 1$ $\quad \phi = 180°$
   if $n_p - n_z = 2$ $\quad \phi = 90°, -90°$ angles Counter clockwise
   $n_p - n_z = 3$ $\quad \phi = 60°, -60°, 180°$ عكس عقارب الساعة
   $n_p - n_z = 4$ $\quad \phi = 45°, -45°, 135°, -135°$
4. Determine the Break Points : Break away between two successive Poles & break in between two successive zeros.
   - Construct ch. equ : $1 + GH(s) = 0$
   - write K in terms of S
   - Find $\dfrac{dK}{dS} = 0$ → roots represents Break away & in.

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⑤ intersection Points with imaginary axis ($J\omega$)

Because Points are on $J\omega$ So we Put $s = J\omega$ on ch. equ.

$J = \sqrt{-1}$, $J^2 = -1$, $J^3 = -J$, $J^4 = 1$

→ after solving equ There are value of ω which represent intersection Point. if $\omega = 0$ neglect.

→ then Calculate K_{cr}

→ Stable $0 < K < K_{cr}$

→ Critical Stable $K = K_{cr}$

→ unstable $K_{cr} < K$

* Find Value of K at any Point on the root Locus

① when value is ζ exist

- Find angle $\theta = \cos^{-1}\zeta$, draw θ with clockwise (مع عقارب الساعة)
- The Line intersect with the root Locus. Calculate
- Length from intersection Point to both Poles & Zeros

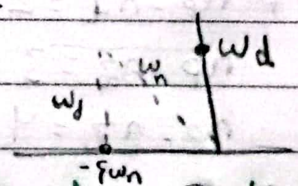
$$K = \frac{\prod \text{Length of Poles}}{\prod \text{length of Zeros}} \quad , \text{if no Zeros } \frac{\prod \text{Poles}}{1}$$

حاصل ضرب ← $\prod$

② Mp exist → Calculate ζ then repeat Previous steps.

③ given ω_d → أقيس بعد رأسي كدا $J\omega$

وأوصل خط يقطع الـ root locus ونقطة تقاطع أكد أول



④ K when the system is critically damped.

→ at break in & break away the Poles are real, equal So system is critical. from these Points Calculate length then (K)

if Poles not equal, real → over damped

Complex → under damped.

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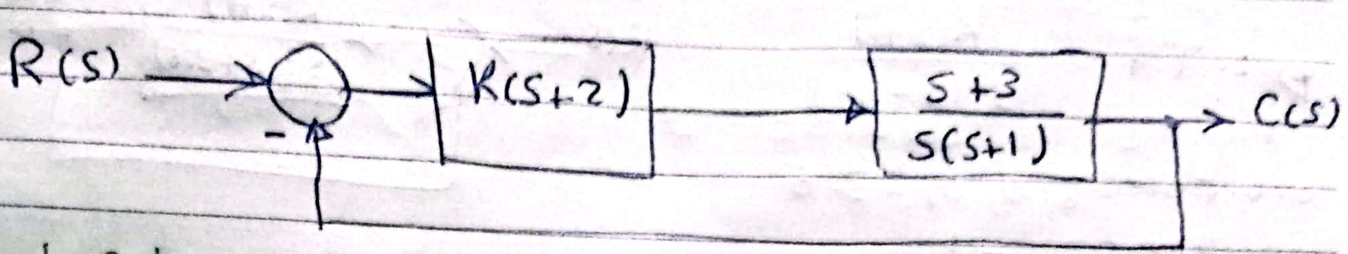
Sketch the root loci for the system shown in figure.



1- locate o.L. Poles & zeros for $\frac{K(S+2)(S+3)}{S(S+1)}$

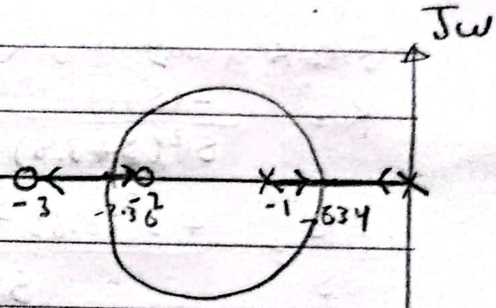
o.L. Poles : 0 , -1

o.L. zeros : -2 , -3

No. of asymptotes = 0

2- Determine break in & break away Points:

$$1+\frac{K(S+2)(S+3)}{S(S+1)}=0$$



$$K=\frac{-S^2-S}{S^2+5S+6}$$

$$\frac{dK}{dS}=\frac{(-2S-1)(S^2+5S+6)-(2S+5)(-S^2-S)}{(S^2+5S+6)^2}$$

$$\cancel{-2S^3}-S^2-10S^2-\cancel{5S}-12S-6+\cancel{2S^3}+10S^2+5S^2+\cancel{5S}=0$$

$$-4S^2-12S-6=0$$

$$S=-.634 \quad , \quad S=-2.366$$

Root locus involve a circle with center $\frac{-.634-2.366}{2}=-1.5$

$$K=\frac{-S(S+1)}{(S+2)(S+3)} \quad \text{at } S=-.634 \rightarrow K=.0718$$

$$S=-2.366 \rightarrow K=14$$

$0<K<.0718$ over $.0718<K<14$ under $14<K$ over

$K=.0718$, $K=14$ critical

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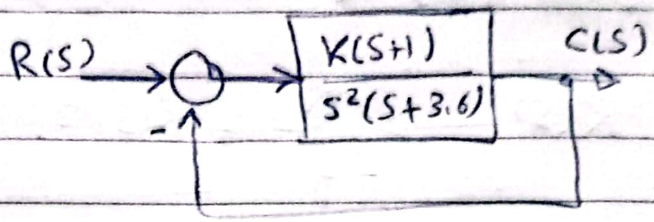
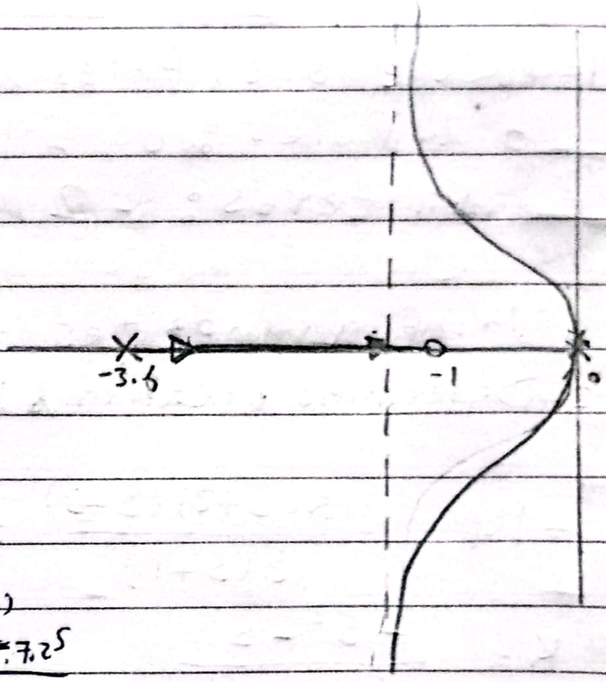
② Sketch the root Loci for the system.

O.L.T.F $= \dfrac{K(S+1)}{S^2(S+3.6)}$



o.L. Poles: 0, 0, −3.6

o.L. Zeros: −1



asymtotic Lines

$N = 3 - 1 = 2$

Center $= \dfrac{0+0-3.6-(-1)}{2} = -1.3$

angle $= 90°, -90°$

Break away Point

$K = \dfrac{S^2(S+3.6)}{(S+1)} = \dfrac{S^3+3.6S^2}{(S+1)}$

$\dfrac{dK}{dS} = \dfrac{(S+1)(3S^2+7.2S) + S^3+3.6S^2 - 3S^3 - 3S^2 - 7.2S^2 - 7.2S}{(S+1)^2}$

$-2S^3 - 6.6S^2 - 7.2S = 0$

$-S(2S^2 + 6.6S + 7.2) = 0$

$S = 0$, img مرفوض

matlab code

```
Num = [1 1];
DEN = [1 3.6 0 0]
SYS= tf (NUM, DEN')
rlocus (SYS)
```

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③ $GH(S) = \dfrac{K}{(S+1)(S+4)(S+7)}$

1- Sketch the root locus

Find the value of K at f=.5

determine the range of K for unstability.

O.L. Poles: $-1, -4, -7$

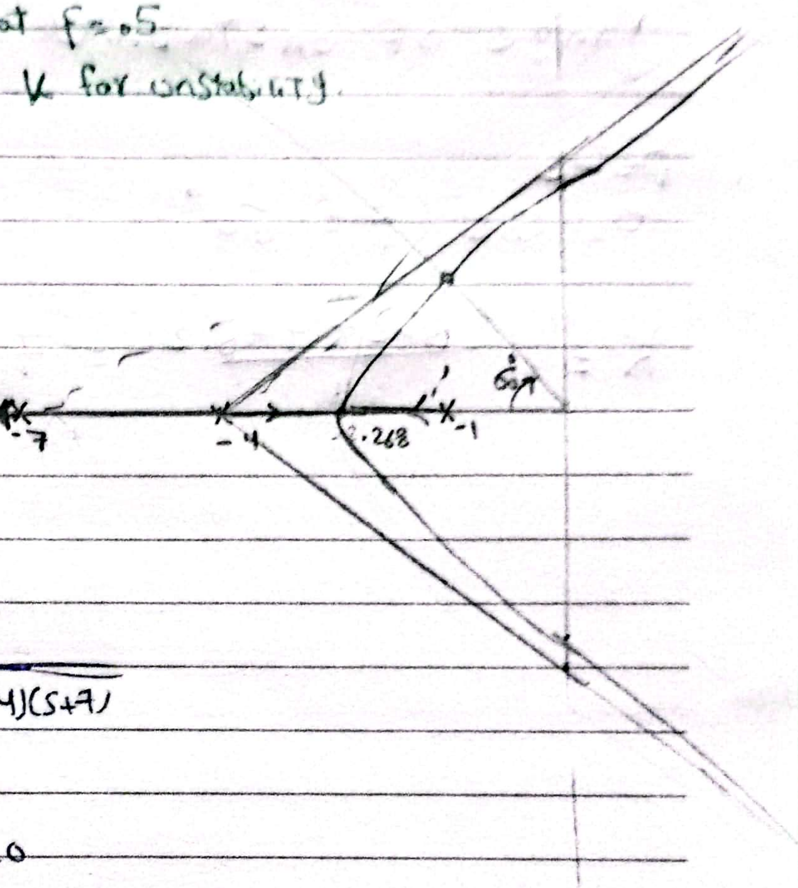
O.L. Zeros: None

asymptotic Lines:

$N = 3$

Center $= \dfrac{-1-4-7}{3} = -4$

angle $= 60°, -60°, 180°$



*Break Points:

$1 + GH(S) = 1 + \dfrac{K}{(S+1)(S+4)(S+7)}$

$K = -(S+1)(S+4)(S+7)$

$(S^2+5S+4)(S+7)+K=0$

$S^3+12S^2+39S+28+K=0$

$K = -(S^3+12S^2+39S+28)$

$\dfrac{dK}{dS} = -(3S^2+24S+39)=0$

$S_1 = -2.268$ ✓ ∈ root locus.

$S_2 = -5.7320$ X

* intersection with Jω ch. eqn. $S^3+12S^2+39S+28+K$

$(J\omega)^3+12(J\omega)^2+39J\omega+28+K=0$

$J^3\omega^3+12J^2\omega^2+39J\omega+28+K=0$

$-J\omega^3-12\omega^2+39J\omega+28+K=0$

$(-12\omega^2+28+K)+J(-\omega^3+39\omega)=0$

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$-\omega^3 + 39\omega = 0 \Rightarrow \omega = \cancel{0}, \omega = \pm\sqrt{39} = \pm 6.244$

$-12\omega^2 + 28 + K = 0 \Rightarrow K = 440 = K_{cr}$

range of unstability at $K > 440$

$\xi = .5$

$\theta = \cos^{-1}\xi = 60^\circ$

$K = \frac{3.2 * 4 * 6.2}{1} = 79.36$

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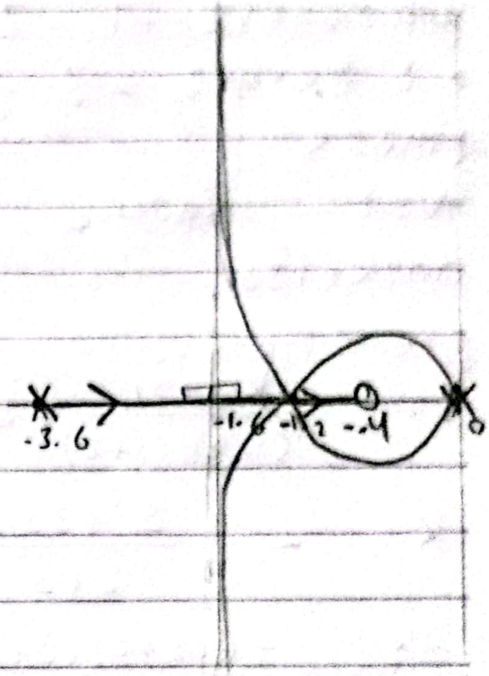
4) G.H.s $= \dfrac{K(S+.4)}{S^2(S+3.6)}$ Sketch the root loci

- o.l. Poles: $0, 0, -3.6$
- o.l. Zeros: $-.4$
- Asymptotic lines

$N = n_p - n_z = 3 - 1 = 2$

Center $= \dfrac{-3.6 + .4}{2} = -1.6$

angle $= 90, -90$



*Break Points

ch. equ: $1 + \dfrac{K(S+.4)}{S^2(S+3.6)}$

$$K = \frac{-(S^3 + 3.6S^2)}{S+.4}$$

$$\frac{dK}{ds} = \frac{-(3S^2 + 7.2S)(S+.4) + S^3 + 3.6S^2}{(S+.4)^2}$$

$$S^3 + 2.4S^2 + 1.44S = 0$$

$S(S+1.2)^2 = 0 \longrightarrow S = 0, -1.2$ → in, away   2 break Points

if break Points $0, -1, -1.4$