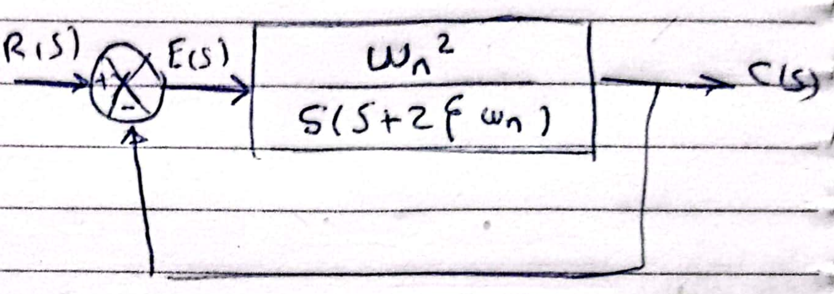
Sheet (2)

DATE: SUBJECT:

1 * Consider the system in figure, where $\zeta = .6$ $\omega_n = 5$ rad/sec. Let us obtain the rise time t_r, Peak time t_p, maximum overshoot M_p, and settling time t_s when the system is subjected to a unit step input



$$t_r = \frac{\pi - \theta}{\omega_d}$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$\theta = \cos^{-1}\zeta$$

$$\omega_d = 5\sqrt{1-(.6)^2} = 4$$

$$\theta = \cos^{-1} .6 = 53.13^\circ * \frac{\pi}{180} = .93 \text{ rad}$$

$$t_r = \frac{\pi - .93}{4} = .554 \text{ sec}$$

$$t_p = \frac{\pi}{\omega_d}$$

$$t_p = \frac{\pi}{4} = .79 \text{ sec}$$

$$t_s = \frac{4}{\zeta\omega_n}$$

$$t_s = \frac{4}{.6 * 5} = 1.33 \text{ sec}$$

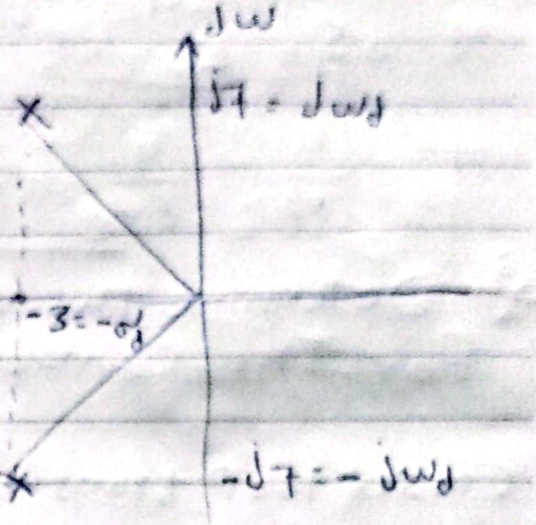
$$M_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} \times 100$$

$$M_p = e^{\frac{-.6\pi}{\sqrt{1-(.6)^2}}} = 9.48\,\%$$

DATE: ____________ SUBJECT: ____________

2 * Consider the Control system whose Closed Loop Poles are given in Figure.

Find ζ, ω_n, T_p, % OS, and T_r



σ_d (damping factor) = $\zeta\omega_n$

$\zeta\omega_n = 3$

$\omega_d = \omega_n\sqrt{1-\zeta^2} = 7$

$\omega_n = \dfrac{3}{\zeta}$

$\omega_d = \dfrac{3}{\zeta}\sqrt{1-\zeta^2} = 7$

$49 = \dfrac{9}{\zeta^2}(1-\zeta^2)$

$49\zeta^2 = 9 - 9\zeta^2$

$\zeta = \dfrac{3}{\sqrt{58}} = .394$

$.394\,\omega_n = \dfrac{3}{\zeta} = 7.62$

$\boxed{\text{or}}$

Find Ch. eq $(s + (3 - j7))(s + (3 + j7))$

$s^2 + 6s + 58 = s^2 + 2\zeta\omega_n s + \omega_n^2$

$\omega_n = \sqrt{58}$, $2\zeta\omega_n = 6 \rightarrow \zeta = .394$

DATE: ____________ SUBJECT: ____________

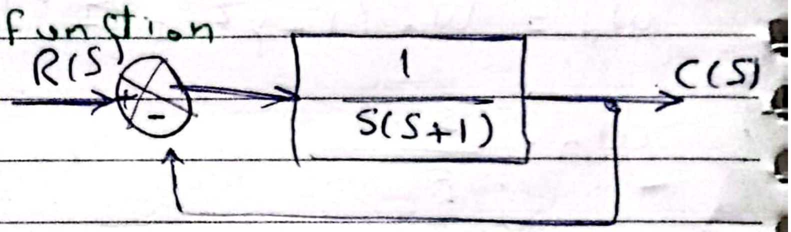
$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{7} = .449 \text{ Sec}$

$\% OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100 = 26.02\%$

$T_s = \frac{4}{\sigma_d} = \frac{4}{\zeta\omega_n} = \frac{4}{3} = 1.33 \text{ sec}$

3 Determine the value of T_d, T_r, T_p and T_s for the control system

Closed loop transfer function



$$\frac{C(S)}{R(S)} = \frac{1/S(S+1)}{1 + \frac{1}{S(S+1)}}$$

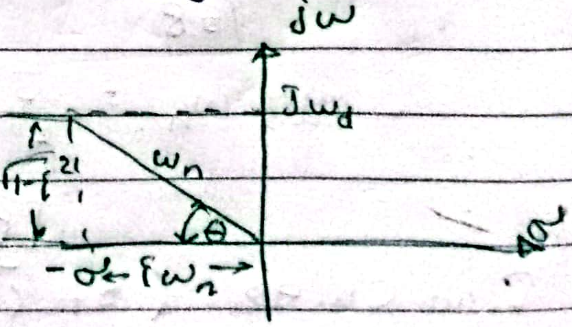
$$\frac{C(S)}{R(S)} = \frac{1}{S^2+S+1} = \frac{\omega_n^2}{S^2+2\zeta\omega_n S+\omega_n^2}$$

$\omega_n^2 = 1 \longrightarrow \omega_n = 1$

$2\zeta\omega_n = 1 \longrightarrow \zeta = .5$

$\omega_d = \omega_n\sqrt{1-\zeta^2} = \sqrt{1-.5^2} = .866$

$T_r = \frac{\pi-\theta}{\omega_d}$



$\theta = \cos^{-1}\zeta = \cos^{-1} .5 = 60^\circ$

$T_r = \frac{\pi - \frac{60 * \pi}{180}}{.866} = 2.41 \text{ Sec}$

$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{.866} = 3.63 \text{ Sec}$

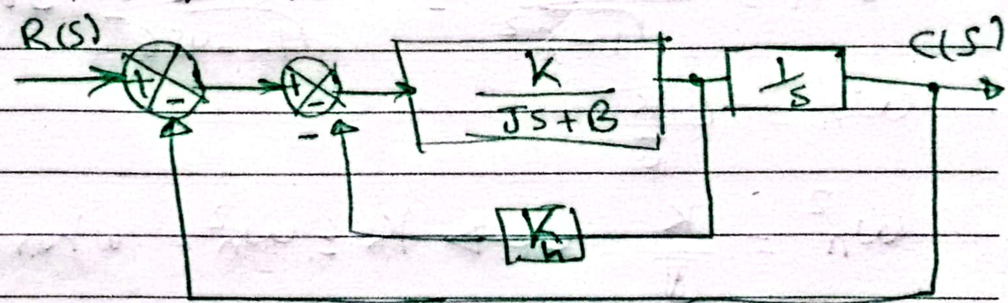
DATE: SUBJECT:

$$T_d = \frac{1+.7\zeta}{\omega_n} = \frac{1+.7(.5)}{1} = 1.35 \text{ sec.}$$

$$M_p = e^{-.5\pi/\sqrt{1-\zeta^2}} = 16.3\%$$

$$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{.5} = 8 \text{ Sec.}$$

* For the system shown in figure, determine the value of gain K and velocity feedback constant K_h so that the M_p in unit step response is .2 and the peak time is 1 sec. With these values of K and K_h obtain the rise time and settling time. assume that $J = 1$ kg-m² and $B = 1$ N-m/rad/sec.



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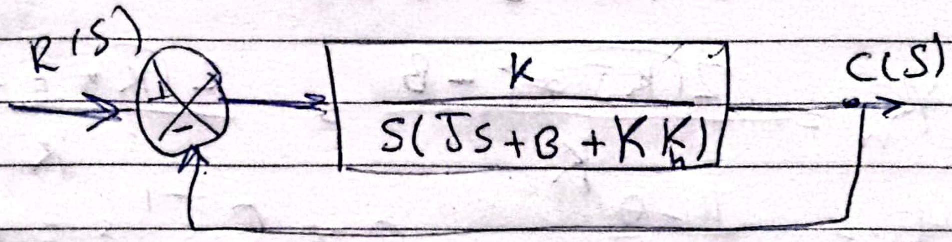


$$\frac{C(S)}{R(S)} = \frac{K}{JS^2 + (B+KK_h)S + K} \quad \div J$$

DATE: SUBJECT:

$\rightarrow \omega_n = \sqrt{\frac{K}{J}}$

$\xi = \frac{B + K K_h}{2\sqrt{KJ}}$

$M_p = .2 = e^{-\xi\pi / \sqrt{1-\xi^2}}$

$\frac{-\xi\pi}{\sqrt{1-\xi^2}} = \ln .2 = 1.61$

$\xi^2\pi^2 = (1.61)^2(1-\xi^2) \rightarrow \xi = .456$

$T_p = 1 = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{\pi}{\omega_d} \longrightarrow \omega_d = 3.14$

$\omega_n = \frac{\omega_d}{\sqrt{1-\xi^2}} = \frac{3.14}{\sqrt{1-.456}} = 3.53$

$\omega_n = \sqrt{\frac{K}{J}}$

$\omega_n^2 = \frac{K}{J} \Rightarrow K = \omega_n^2$ where $J = 1$

$K = 12.5 \; N.m$

$K_h = \frac{2\sqrt{KJ} * \xi - B}{K} = \frac{2\sqrt{K} * \xi - 1}{K} = 1.178$

$t_r = \frac{\pi - \theta}{\omega_d}$, $\theta = \cos^{-1}\xi = 62.87'$

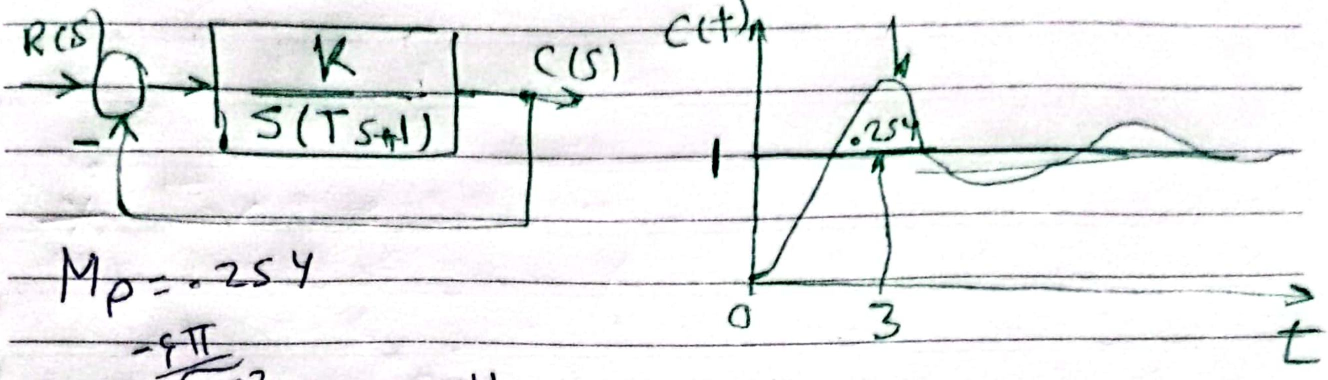
$t_r = \frac{\pi - 62.87 * \pi/180}{3.14} = .65 \; sec$

$t_s = \frac{4}{\xi\omega_n} = 2.48 \; sec$

DATE:
SUBJECT:

5

When system shown in figure (a) is subjected to a unit step input, the system output response as fig (b). Determine K & T



$M_p = .254$

$e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} = .254$

$Ln(.254) = -\zeta\pi / \sqrt{1-\zeta^2}$

$\zeta = .4$

$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 3$

$\frac{\pi}{\omega_n\sqrt{1-(.4)^2}} = 3 \rightarrow \omega_n = 1.14$

From Block digram

$\frac{C(S)}{R(S)} = \frac{K}{TS^2+S+K} = \frac{K/T}{S^2+\frac{1}{T}S+\frac{K}{T}}$

$\omega_n^2 = \frac{K}{T}$, $2\zeta\omega_n = \frac{1}{T}$

$T = \frac{1}{2\zeta\omega_n} = \frac{1}{2*.4*1.14} = 1.09$

$K = \omega_n^2 T = (1.14)^2 * 1.09 = 1.42$