Menu Costs and Phillips Curves

Lionel CHAMBON   Tanfei LI   Tianxiang YUAN

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Introduction

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So far, we have worked on New Keynesian models using **Calvo (1983) pricing assumptions.** But are these realistic? Can you think of potential limits?

Golosov and Lucas (2007) depart from Calvo pricing by introducing a **real menu cost**.

Outline

Problems with Calvo Pricing

Add here the intuition why the Calvo assumptions have limits.

Deriving Monetary Equilibrium Equations - Setup

We will use a New Keynesian framework very similar to the one we already know, with a few modifications. The economy is subject to **stochastic monetary shocks and firm-specific supply shocks.**

Log money supply follows a **Brownian motion:**

$$dlog(m_t) = \mu dt + \sigma_m dZ_m \tag{1}$$

What is a Brownian Motion?

A Brownian Motion is a way to model random walks. μ is a drift parameter indicating the deterministic, expected incremental change over time. Z_m is the random, Brownian motion term and σ_m a volatility parameter.

Deriving Monetary Equilibrium Equations - Setup

Technology shocks are independent across firms and follow a mean-reverting process:

$$dv = -\eta log(v_t)dt + \sigma_v dZ_v, \quad \eta > 0 \tag{2}$$

In equilibrium, **nominal wages** also follow a Brownian Motion:

$$dlog(w_t) = \mu dt + \sigma_m dZ_m \tag{3}$$

Deriving Monetary Equilibrium Equations - Setup

Claims to the monetary unit are traded in a capital market, where

$$E\left[\int_0^\infty Q_t Y_t dt\right] \tag{4}$$

reflects the present value of a dollar earning stream at time 0.

Thus, the **state at time t** is determined by:

- m_t and w_t
- The joint distribution $\phi_t(p, v)$: At time t, the situation of an individual firm depends also on the price p that it carries into t from earlier dates and its idiosyncratic productivity shock. Thus, **each seller is characterized by a pair (p_t, v_t), distributed according to the measure $\phi_t(p, v)$.**

Deriving Monetary Equilibrium Equations - Households

Households **choose a consumption strategy, money holdings** m_t **and a labor supply** l_t**.** By Dixit-Stiglitz aggregation, consumption is given by:

$$c_t = \left[\int C_t(p)^{1-(1/\epsilon)} \phi_t(dp, dv) \right]^{\epsilon/(\epsilon-1)} \tag{5}$$

Households maximize

$$E \left[\int_0^\infty e^{-\rho t} \left[\frac{1}{1-\gamma} c_t^{1-\gamma} - \alpha l_t + \log \left(\frac{\hat{m}_t}{P_t} \right) \right] dt \right] \tag{6}$$

subject to

$$E \left[\int_0^\infty Q_t \left[\int p C_t(p) \phi_t(dp, dv) + R_t \hat{m}_t - W_t l_t - \Pi_t \right] dt \right] \leq m_0 \tag{7}$$

Deriving Monetary Equilibrium Equations - Household FOCs

For money holdings:

$$e^{-\rho t}\frac{1}{m_t} = \lambda Q_t R_t \tag{7}$$

For consumption choices and labor supply:

$$e^{-\rho t} c_t^{-\gamma} c_t^{1/\epsilon} C_t(p)^{-1/\epsilon} = \lambda Q_t p \tag{8}$$

$$e^{-\rho t}\alpha = \lambda Q_t w_t \tag{9}$$

Deriving Monetary Equilibrium Equations - Household FOCs

There is an equilibrium in which the nominal rate is constant at the level

$$R_t = R = \rho + \mu \tag{10}$$

In such an equilibrium, (7), (9), and (10) imply

$$w_t = \alpha R m_t \tag{11}$$

Deriving Monetary Equilibrium Equations - Firms

At a current price level p, a firm profit is

$$C_t(p)\left(p - \frac{w_t}{v_t}\right) \tag{12}$$

At any price $q \neq p$, profits are

$$C_t(q)\left(q - \frac{w_t}{v_t}\right) - kw_t \tag{13}$$

where k is the hours of labor needed to change the price.

Deriving Monetary Equilibrium Equations - Firms

This firm chooses a shock-contingent repricing time T and a shock-contingent price q to be chosen at t+T to solve the following **Bellman equation**:

$$\begin{aligned}\varphi\left(p, v, w, \phi_t\right) = & \max_T E_t \left[\int_t^{t+T} Q_s C_s(p)\left(p - \frac{w_s}{v_s}\right) ds \right. \\ & \left. + Q_T \cdot \max_q \left[\varphi\left(q, v_{t+T}, w_{t+T}, \phi_{t+T}\right) - k w_{t+T}\right]\right]\end{aligned} \tag{14}$$

What is a Bellman equation?

$$\mathrm{V}(\mathrm{state}_t) = \max_{control_t}\{u(state_t, control_t) + \beta V(state_{t+1})\}$$

Instead of solving the optimal sequence $\{control_t\}_{t=0}^{T}$, it looks for the time-invariant value function and policy function $control_t = g(state_t)$ that solve the dynamic problem.

Deriving Monetary Equilibrium Equations - Firms

The demand function for each good is:

$$C_t(p) = c_t^{1-\epsilon\gamma} \left(\frac{\alpha p}{w_t} \right)^{-\epsilon} \tag{15}$$

Applying the natural normalization $Q_0 = 1$ to (9), we obtain:

$$Q_t = e^{-\rho t} \frac{w_0}{w_t} \tag{16}$$

Then we can express the Bellman as:

$$\begin{aligned} \varphi\left(p, v, w, \phi_t\right) = \max_T E_t \Bigg[& \int_t^{t+T} e^{-\rho(s-t)} \frac{w}{w_s} c_s^{1-\epsilon\gamma} \left(\frac{\alpha p}{w_s} \right)^{-\epsilon} \left(p - \frac{w_s}{v_s} \right) ds \\ & + e^{-\rho T} \frac{w}{w_T} \cdot \max_q \left[\varphi\left(q, v_{t+T}, w_{t+T}, \phi_{t+T}\right) - k w_{t+T} \right] \Bigg] \end{aligned} \tag{17}$$

Deriving Monetary Equilibrium Equations - Firms

Labor market clearing implies:

$$l_t = \int \frac{C_t(p)}{v} \phi_t(dp, dv) + k\Upsilon_t \tag{18}$$

Let's recap:

- Two kinds of shocks v_t, m_t and a family of measure $\phi_t(p, v)$ shape the states.
- The household chooses goods demand, labor supply, and money-holding strategies according to the states of the world.
- A firm's pricing strategy is now defined as choices of stopping times T and prices q.

Equilibrium under Constant Wage Inflation

(17) is hard to analyze because of the presence of $\phi_t(p, v)$ as a state variable. Either we can provide or construct a law of motion for it, **OR** we can conjecture an equilibrium in which the distributions are all equal to an invariant measure, $\phi(p, v)$

Why $\phi(p, v)$?

$\phi_t(p, v)$ enters (17) only as a determinant of the consumption aggregate. If we use $\phi(p, v)$ instead, we can get a constant corresponding consumption aggregate facing the firms.

Equilibrium under Constant Wage Inflation

Consumption aggregate:

$$c_t = \left[\int \left(\frac{\alpha p}{w_t} \right)^{1-\epsilon} \phi_t(dp, dv) \right]^{1/[\gamma(\epsilon-1)]} \tag{18}$$

Define real price:

$$x = p/w_t \tag{19}$$

Restate(18) as:

$$c_t = \left[\alpha^{1-\epsilon} \int x^{1-\epsilon} \tilde{\phi}_t(dx, dv) \right]^{1/[\gamma(\epsilon-1)]} \tag{20}$$

Here, the process of nominal wage growth is deterministic, thus we can construct a joint distribution$\tilde{\phi}_t$ in the similar form of ϕ_t.

Equilibrium under Constant Wage Inflation

With an invariant measure $\tilde{\phi}$ and a corresponding constant consumption aggregate $\bar{c}$, rewrite the Bellman as:

$$\begin{aligned}\varphi(p, v, w) = \max_T E \Bigg[& \int_0^T e^{-\rho s} \frac{w}{w_s} \bar{c}^{1-\epsilon\gamma} \left(\frac{\alpha p}{w_s} \right)^{-\epsilon} \left(p - \frac{w_s}{v_s} \right) ds \\ & + e^{-\rho T} \frac{w}{w_T} \cdot \max_q \left[\varphi\left(q, v_T, w_T\right) - k w_T \right] \Bigg]\end{aligned} \tag{21}$$

Use x, instead of p/w_t to restate:

$$\begin{aligned}\frac{1}{w}\varphi(wx, v, w) = \max_T E \Bigg[& \int_0^T e^{-\rho s} \bar{c}^{1-\epsilon\gamma} \left(\alpha x_s \right)^{-\epsilon} \left(x_s - \frac{1}{v_s} \right) ds \\ & + e^{-\rho T} \frac{1}{w_T} \cdot \max_{x'} \left[\varphi\left(w_T x', v_T, w_T\right) - k w_T \right] \Bigg]\end{aligned} \tag{22}$$

Equilibrium under Constant Wage Inflation

Finally, we seek a solution to (22) of the form:

$$\varphi(p, v, w) = w\psi(x, v) \tag{23}$$

Where

$$\psi(x, v) = \max_T E\left[\int_0^T e^{-\rho t}\bar{c}^{-1-\epsilon}(\alpha x_t)^{-\epsilon}\left(x_t - \frac{1}{v_t}\right)dt + e^{-\rho T}\cdot\max_{x'}\left[\psi\left(x', v(T)\right) - k\right]\right] \tag{24}$$

Find the value of $\bar{c}$ by solving the fixed-point problem:

$$\bar{c} = \left[\alpha^{1-\epsilon}\int x^{1-\epsilon}\tilde{\phi}_t(dx, dv; \bar{c})\right]^{1/[\gamma(\epsilon-1)]} \tag{25}$$

Equilibrium under Constant Wage Inflation

Now study the Bellman equation, using a discrete-time and state approximation—a Markov chain:

$$\psi(x,v) = \max\left\{\Pi(x,v)\Delta t + e^{-r\Delta t}\sum_{x',v'}\pi\left(x',v' \mid x,v\right)\psi\left(x',v'\right),\right.$$
$$\left.\max_{\xi}\left[\Pi(\xi,v)\Delta t + e^{-r\Delta t}\sum_{x',v'}\pi\left(x',v' \mid \xi,v\right)\psi\left(x',v'\right)\right] - k\right\} \tag{26}$$

Under the assumption that:

$$\Pi(x,v) = \bar{c}^{-1-\epsilon}(\alpha x)^{-\epsilon}\left(x - \frac{1}{v}\right) \tag{27}$$

Equilibrium under Constant Wage Inflation

Define function:

$$\Omega(v) = \max_x [\psi(x, v)] \tag{28}$$

So $\Omega(v)$ is the value the firm would have if it could move costlessly to a new price when the wage is w and the productivity level is v.

$$D(v) = \{x > 0 : \psi(x, v) > \Omega(v) - k\}, \tag{29}$$

So $D(v)$ is the set at which a firm does not pay to re-price, the **Inaction Region**. The policy function for (26) thus can be defined by:

$$f(x, v) = x \text{ if } x \in D(v)$$
$$f(x, v) = g(v) \text{ if } x \notin D(v)$$

Figure 1

FIG1_p181.png

Results

Testing the Model

We know have a set of equations characterizing equilibrium. We will show you that the model works remarkably well to predict empirical evidence.

Data (Klenow and Kryvstov, 2005)

- Bureau of Labor Statistics (BLS) survey from the US
- Ca. 80.000 time series on price quotes in 88 locations from 1988 to 1997 (monthly or bimonthly frequency)
- First, we use **real data to calibrate** the model. Then, we use the model to **predict what happens when parameters change.**

- **Results:** Frequency of price changes is
  - Insensitive to μ
  - Increasing in ${\sigma^2}_v$
  - Decreasing in k

Calibrating the Model

TAB1_p183.png

Simulating the Model

FIG3_p186.png

Simulating the model

The model predicts data very well - *better* than conventional models that do not include firm-specific shocks.

Specifically, the model works well **in low inflation economies,** where conventional models fail. **Why?**

Simulating the model

The model predicts data very well - *better* than conventional models that do not include firm-specific shocks.

Specifically, the model works well **in low inflation economies,** where conventional models fail. **Why?**

Because in **low inflation environments,** it matters more *who* changes their price - which is exactly what this model captures.

Impulse Response Functions

FIG4a_p188.png

(a) Response to a transient monetary shock of real GDP and employment

FIG4b_p188.png

(b) Response to a transient monetary shock of the repricing and inflation rate

What's Going on Here?

Back to Calvo

FIG5_p189.png

Results

Back to Calvo

FIG6_p190.png

Results

Results

- Under Calvo assumptions, the effects of a positive monetary shock are **larger and more persistent:** price changes are random and smaller on average
- Figure 6a shows us that while Calvo price adjustment are near evenly distributed, under menu costs, **large price adjustments occur** when a firm is **far** off its desirable price
- During an aggregate shock, a firm will want to raise its price, and the distribution shifts to the left
  - Firms far to the left will increase prices
  - Others will wait since inflation offsets negative v shock

Overall: Price adjustments per firm are larger and no longer random.

Figure 1 Figure 5 Figure 6

Discussion

- What if expectations across firms about the nature of the shock are different?

Concluding Remarks

References