Proof of Expected Noise for Non-Uniform Image Noise Algorithm

Nikita Kosarev

18.10.2024

We aim to prove that the proposed algorithm for non-uniform noise application ensures that the expected noise percentage over the image matches the specified noise percentage parameter. Let the following symbols be defined:

- $N \times N$: dimensions of the image.
- (x_c, y_c): coordinates of the noise epicenter.
- α: decay parameter of noise probability with distance.
- $p(x, y)$: unnormalized probability of noise for point (x, y).
- $P_{\text{norm}}(x, y)$: normalized probability of noise for point (x, y).
- noisePercentage: the desired noise percentage (0-100).

The unnormalized noise probability $p(x, y)$ for a point (x, y) is given by:

$$p(x, y) = \exp(-\alpha \cdot d(x, y)), \quad \text{where } d(x, y) = \sqrt{(x - x_c)^2 + (y - y_c)^2}$$

The probability $p(x, y)$ decreases exponentially with the distance $d(x, y)$ from the epicenter.

The raw expected value of noise before normalization is given by:

$$E_{\text{raw}} = \frac{1}{N^2} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} p(x, y)$$

To ensure that the expected noise percentage matches the specified value, we introduce a normalization factor:

$$P_{\text{norm}}(x, y) = p(x, y) \times \text{normFactor}$$

The normalization factor is computed as:

$$\text{normFactor} = \frac{\text{noisePercentage}}{100 \times E_{\text{raw}}}$$

After applying this normalization, the new expected value becomes:

$$E_{\text{norm}} = \frac{1}{N^2} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} P_{\text{norm}}(x, y)$$

Substituting $P_{\text{norm}}(x, y)$ gives:

$$E_{\text{norm}} = \frac{1}{N^2} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} p(x, y) \times \text{normFactor}$$

Since normFactor is constant, we can factor it out:

$$E_{\text{norm}} = \text{normFactor} \times \frac{1}{N^2} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} p(x, y) = \text{normFactor} \times E_{\text{raw}}$$

Substituting the expression for normFactor:

$$E_{\text{norm}} = \frac{\text{noisePercentage}}{100 \times E_{\text{raw}}} \times E_{\text{raw}} = \frac{\text{noisePercentage}}{100}$$

Thus, the expected value of noise across the image is equal to the specified noise percentage.