UNIVERSITÁ COMMERCIALE LUIGI BOCCONI

Specialized Master in Quantitative Finance and Risk Management

Derivatives Assignment
CERTIFICATE EVALUATION

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1 Introduction

The assignment goal is to analyse the certificate relating to the Commerzbank Efficiency Growth Index, with ISIN DE000CZ37TL7. Commerzbank began selling $30,000$ Certificates based on the Commerzbank Efficiency Growth Index on April 14, 2014, for an initial price of EUR 90 per Certificate. The analysis will be split into two primary perspectives. The first one is the quantitative analysis, where we will go through all the methods and assumptions used to evaluate the Investment Certificate, based on the Commerzbank Efficiency Growth Index as underlying, which is based on a mutual fund. We used the Black-Scholes model to examine this Certificate since it can be reduced to a basic European call option on the underlying index with no dividends. We also include the Greeks analysis to better understand the option's sensitivity with respect to its parameters. The second one is the qualitative analysis, in which we examine the convenience and other interesting qualities from the perspectives of both the investors and the issuer, as well as the logical progression that was maintained throughout the process.

2 Quantitative Analysis

2.1 Methodology

In order to understand the building blocks of the certificate, we take a step-by-step approach. First, we must compute the realized volatility, which is derived using a strictly rule-based technique with the following formula:

$$\mathrm{RV}_{\text{t-lag}} = \sqrt{\frac{d}{m}} \times \sqrt{\sum_{k=1}^{n} \left(\ln\left(\frac{\mathrm{Basket}_{t-n+k-lag}}{\mathrm{Basket}_{t-n+k-1-lag}} \right) \right)^2 - \frac{\gamma}{n} \left(\sum_{k=1}^{n} \ln\left(\frac{\mathrm{Basket}_{t-n+k-lag}}{\mathrm{Basket}_{t-n+k-1-lag}} \right) \right)^2}$$

where:

t is the reference to the relevant Index Calculation Date,

n is the volatility window which corresponds to the number of days used to calculate the Realized Volatility: 20, d is the annualising factor which represents the expected number of Index Calculation Dates in each calendar year: 252,

γ is the factor of 1,

m is the scaling factor of 20,

lag is the Lag factor equal to 2.

These data are now used to calculate the Weightings, which are the relative proportions of each Fund invested in the Basket versus the Reference Interest Rate provided in the Basket specification. The Index Calculation Agent will determine the Weighting in relation to the Index Calculation Date_t using the following formula:

$$W_t = \text{Min}\left(\text{MaxW}, \frac{\text{TargetVol}}{\text{RealizedVol}_{t-lag}}\right)$$

With TargetVol = 6% (Risk Control Level), MaxW = 150% (Max Exposure) and a lag value equal to 2. In this way, when the volatility of the asset goes below 6%, it is possible to virtually borrow at the reference interest rate and invest more money, with a leverage of maximum 150%. On the Index Commencement Date the Index Value is equal to 100 points expressed in the Index Currency. We calculated it on each subsequent Index Calculation Date (t) in accordance with the following formula:

$$\begin{aligned}\text{Index}_t = \text{Index}_{t-1} \times \Bigg(1 + \left[W_{t-1} \times \left(\frac{\text{Basket}_t}{\text{Basket}_{t-1}} - 1\right)\right] + \\ + \left[(1 - W_{t-1}) \times \text{Rate}_{t-1} \times \frac{\text{Act}_{t,t-1}}{\text{conv}}\right] - \left[\text{PF} \times \frac{\text{Act}_{t,t-1}}{\text{conv}}\right]\Bigg)\end{aligned}$$

where

Index_t is the Index Value on the current Index Calculation Date,

Index_{t-1} is the Index Value with respect to the immediately preceding Index Calculation Date,

W_{t-1} is the Weighting with respect to the immediately preceding Index Calculation Date,

Basket_t is the Basket Value with respect to the current Index Calculation Date,

Basket_{t-1} is the Basket Value with respect to the immediately preceding Index Calculation Date,

Rate_{t-1} is the Reference Interest Rate as determined by the Index Calculation Agent with respect to the immediately preceding Calculation Date. When the number of Payment Business Days between Calculation Dates is more than one, an alternative commercially reasonable rate may be used, as determined by the Index Calculation Agent,

$\text{Act}_{t,t-1}$ is the number of calendar days from, but excluding, the immediately preceding Index Calculation Date to, and including, the relevant Index Calculation Date,

conv is set at 360,

PF is the Protection Fee, which in this case is equal to zero because there is no principal repayment guarantee.

We can see the evolution of the Index Value over time in **Figure 1**, starting from inception in April 2014 to the expiration date in April 2018. We can notice an upward trend from

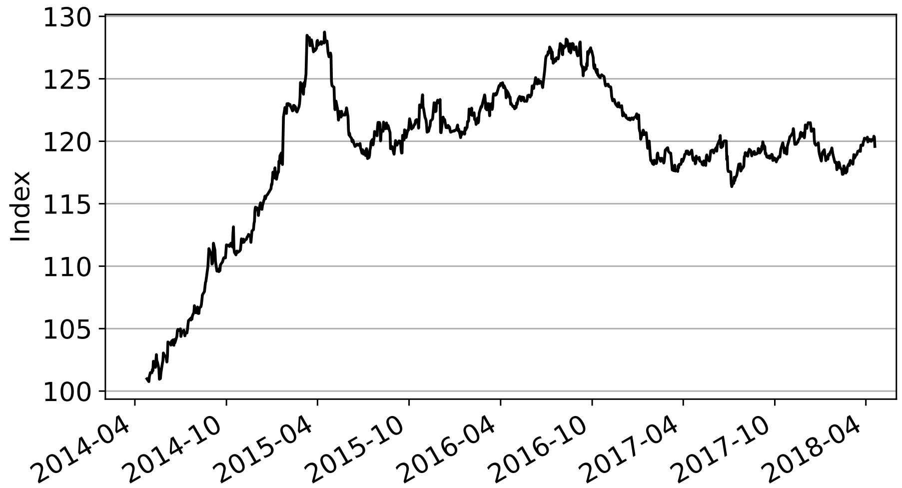

Figure 1: Index Value

the beginning to April 2015, followed by a slightly falling pattern marked by reduced volatility for the rest of the life, when the EURIBOR 1M rate started to reach negative values. It is now possible to compute the Settlement Amount using the following formula:

$$\text{SA} = \text{CA} \times \text{Max}\left(0; \frac{\text{Underlying}_{(\text{Final})}}{\text{Underlying}_{(\text{Strike})}} \times (1 - 1.1\%)^4 - 1\right)$$

where CA is the Calculation Amount that is equal to 1000, and the Underlying Strike is the arithmetic mean of the Reference Prices of the Index during the 22, 23 and 25 April 2014, as it is written in the Certificate. Finally, if we set

$$S = \text{CA} \times \left(\frac{\text{Underlying}_{(\text{Final})}}{\text{Underlying}_{(\text{Strike})}} \times (1 - 1.1\%)^4\right) \quad \text{and} \quad K = \text{CA},$$

we can see the Settlement Amount as a normal payoff at maturity of a Call as follows:

$$\text{SA} = \text{Max}\left(0, S - K\right).$$

Under these circumstances we can apply the usual Black-Scholes formula for a European Call option on a non-dividend-paying underlying:

$$C = SN(d_1) - Ke^{-r\Delta t}N(d_2)$$

with

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)\Delta t}{\sigma\sqrt{\Delta t}} \qquad \text{and} \qquad d_2 = d_1 - \sigma\sqrt{\Delta t}.$$

where $N(\cdot)$ is the cumulative probability function of a standard normal distribution.

2.2 Parameters setup

In this section, we will discuss assumptions that were made about the model parameters. We calculate the time to maturity (Δt) by dividing the number of days to maturity by the amount of typical working days, which are 252 each year. In the data-set provided there were some missing values due to the Luxembourg national holidays. We opted to delete the missing values to solve this issue because there was no reason to replace a missing value with interpolation as the market was closed those days. To have a coherent risk-free rate over time we use linear interpolation between the different curves of EURIBOR and also the Swaps with maturities longer than a year. We can see in **Figure 2** that from 2014 to 2018 we have a decreasing trend, which ends closer to -0.4%. For the volatility used

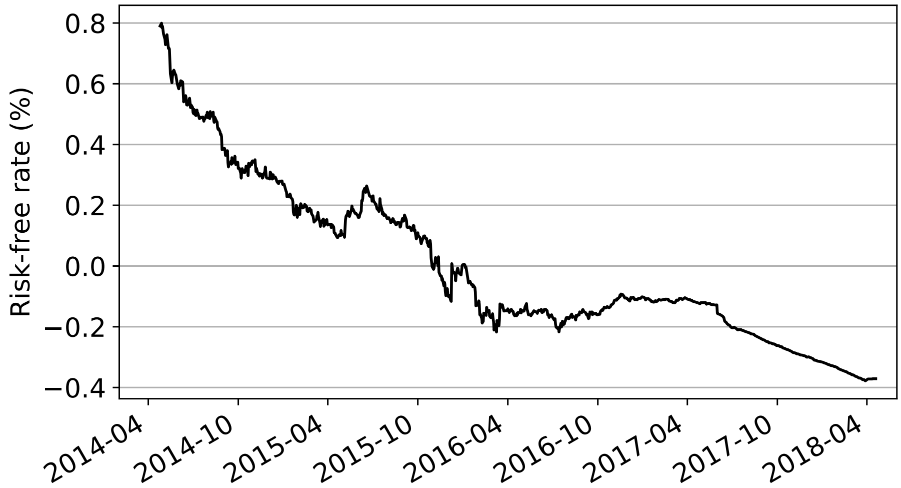

Figure 2: Risk-free rate

in the Black-Scholes model (σ) we used the standard deviation estimated from historical data. To do so we computed the log returns of the underlying (S) over time, named as (u_i), and then we estimate the standard deviation with its natural estimator as follows:

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(u_i - \bar{u})}$$

and then we obtain the annualized volatility just multiplying s by the square root of the working days, so

$$\sigma = s \times \sqrt{252}.$$

The difficult choice was deciding how many previous observations to consider. In general, the more data available, the greater the accuracy. Furthermore, σ changes with time, making earlier data meaningless for predicting future values. The practice is to take between 90 and 180 days, and since our data-set is not very large in terms of previous observations, we decided to use a window of 90 days. Unfortunately, one disadvantage of this strategy is that we have a standard error equal to $\sigma\sqrt{2 \times 90}$, which might explain the presence of volatility values over the 6% barrier.

2.3 Certificate Valuation

Using the Black-Scholes formula for an European Call, with the parameters settled as mentioned before, we obtained an estimation for the fair market value of the Certificate of EUR 33.05 on 25^{th} April of 2014, which is much lower than the initial issue value of EUR 90 per Certificate. The change of the fair value over time is shown in **Figure 3**.

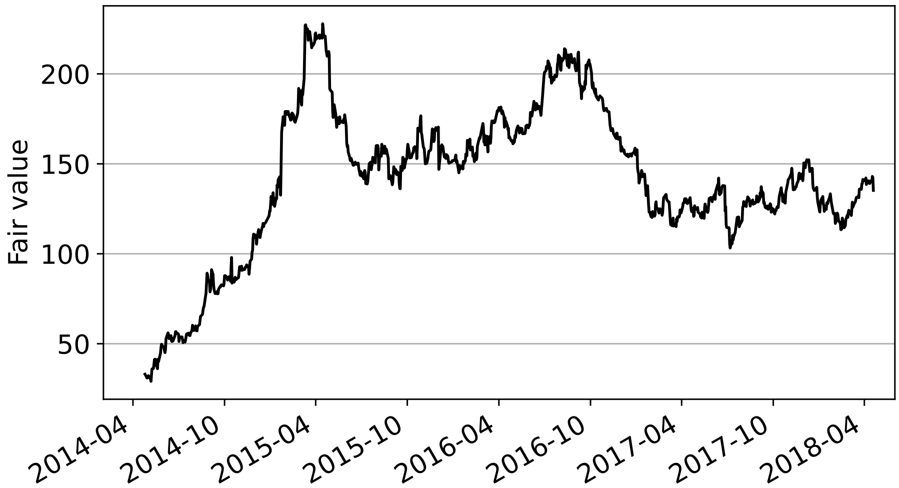

Figure 3: Certificate fair value

2.4 Sensitivities analysis

We conducted a sensitivity analysis of the call price in relation to the many parameters on which it is dependent. To do this, we computed the partial derivatives of C with respect to S, t, σ, r. Because these partial derivatives are designated by Greek letters, they are generally referred to as "the Greeks".

2.4.1 Delta

The delta formula is the following:

$$\Delta = \frac{\partial C}{\partial S} = N(d_1)$$

It follows that $0 < \Delta < 1$, which proves that the function $C(S)$ is strictly increasing and its derivative is always less than 1. The behaviour of delta can be seen in **Figure 4**. This

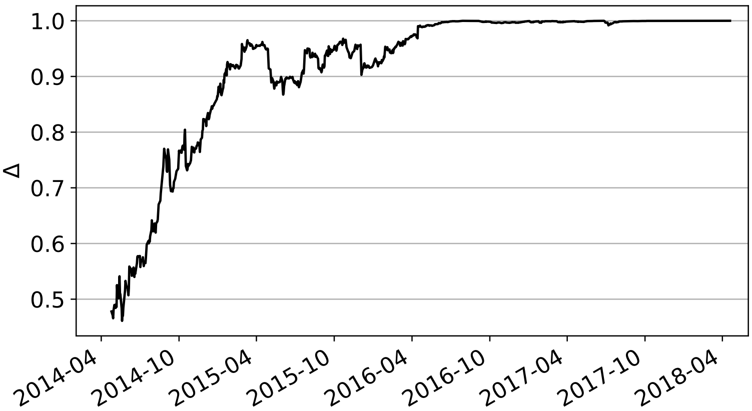

Figure 4: Delta of the Certificate

is a normal In-The-Money (ITM) call pattern, and because we entered deep ITM after June 2016, the value of the delta is close to 1.

2.4.2 Gamma

The gamma of a call option is defined as

$$\Gamma = \frac{\partial^2 C}{\partial S^2} = \frac{N'(d_1)}{S\sigma\sqrt{\Delta t}}$$

It follows that $\Gamma > 0$ and hence the function $C(S)$ is convex. From **Figure 5**, we can see

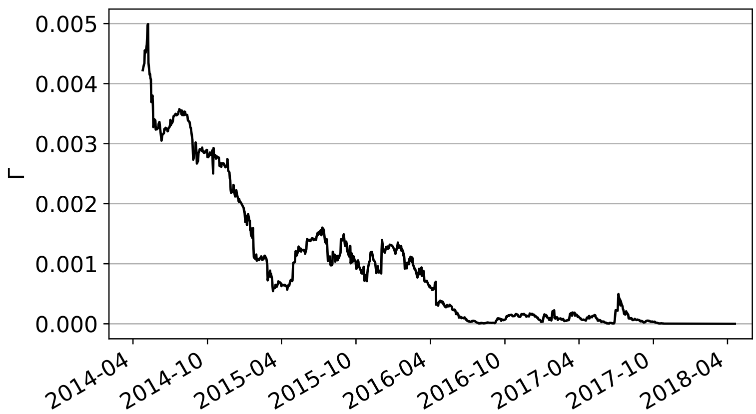

Figure 5: Gamma of the Certificate

that, as for the Delta, it's the typical path of a deep ITM call, where the value of the Greek is still positive but close to zero.

2.4.3 Theta

The theta of an option is defined as

$$\theta = \frac{\partial C}{\partial t} = -Kre^{-r(\Delta t)}N(d_2) - \frac{\sigma S N'(d_1)}{2\sqrt{\Delta t}}$$

Notice that T represents the maturity, which is fixed, while $\Delta t = T - t$ represents the time to maturity. We can see the value of theta over time in figure **Figure 6**. Options are

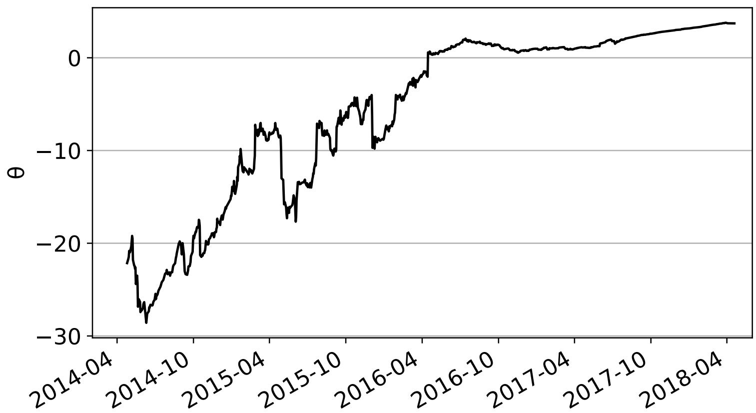

Figure 6: Theta of the Certificate

"perishable goods", in the sense that the mere passage of time tends to lower their value, a phenomenon known as time decay, which corresponds to the fact that θ is negative. The chart shows that theta became positive after the year 2016. This behaviour can be related to the risk-free rate's negative and declining tendency, which is no longer positive following 2016.

2.4.4 Vega

The vega of an option is the derivative of its value with respect to the volatility σ, and it is defined as:

$$\nu = \frac{\partial C}{\partial \sigma} = SN'(d_1)\sqrt{\Delta t}$$

Figure 7 shows the vega of the option over time. An increase in volatility raises option prices from a financial point of view. The reason for this is that a rise in volatility increases

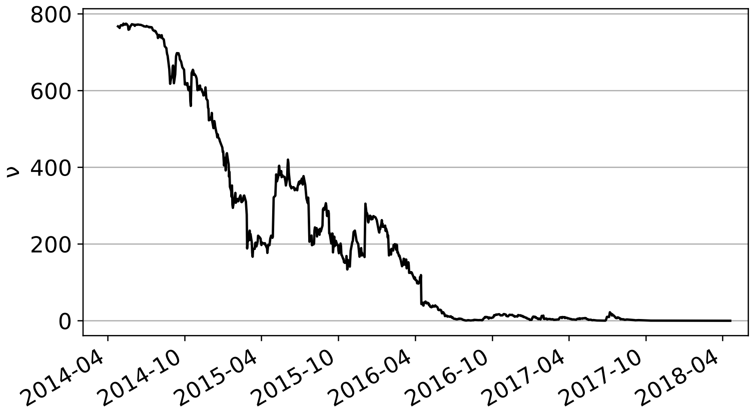

Figure 7: Vega of the Certificate

the likelihood of either extremely high or extremely low underlying values. The owner of a call option, on the other hand, is unaffected by extremely low values but positively affected by extremely high values. This results in a higher expected payoff. When we look at the certificate's Vega, we can see that the maximum value for the sensitivity to volatility is recorded at inception, which helps to explain the discrepancy in the initial price offer.

2.4.5 Rho

The rho of an option measure the sensitivity with respect to the level of interest rates r:

$$\rho = \frac{\partial C}{\partial r} = K \Delta t e^{-r\Delta t} N(d_2)$$

From a financial point of view, an increase of r has two effects: the discount factor becomes lower, but the risk neutral distribution, which is a log-Normal with mean equal to r, is shifted to the right, thus increasing the expected value of the payoff. **Figure 8** shows the movements of ρ during the life of the Certificate. It is difficult to assess the net effect on a purely intuitive basis.

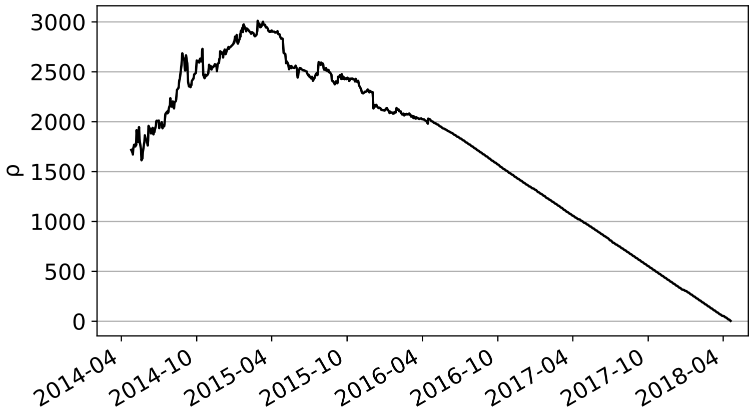

Figure 8: Rho of the Certificate

2.5 Validity of the Black-Scholes model

To price this Certificate, we employed the Black-Scholes model, which is the most commonly used pricing model due to its simplicity; nevertheless, it is also acknowledged that some assumptions are surrealistic and not empirically supported:

- The first assumption is that the underlying stock price S_t follows a Geometric

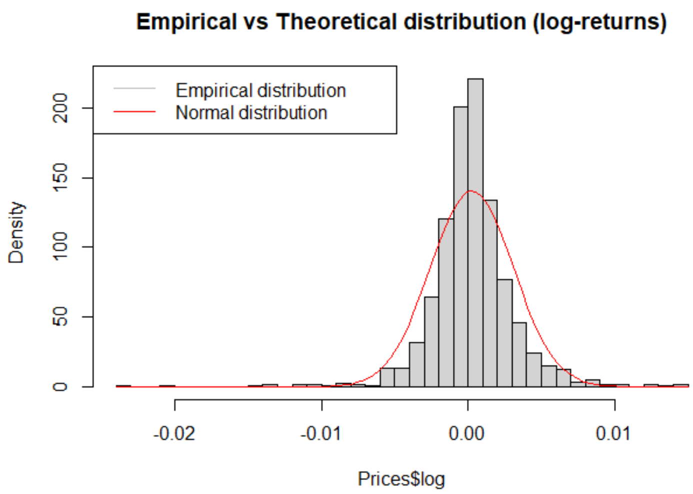

Figure 9: Empirical distribution of log-returns

Brownian Motion with constant μ and σ, which implies a Normal distribution of the log-returns. The true data distribution exhibits "fat tails", implying a larger probability of extreme log-returns. Furthermore, financial time series typically show a leptokurtic distribution, which has a larger kurtosis than the Normal distribution. To demonstrate this, we plotted the distribution of the log-returns of the data presented in **Figure 9** against a Normal distribution with the same moments. The observations pointed out are proven empirically.

- According to the underlying assumptions of the Black-Scholes model, stocks follow a random walk, which indicates that the price of the underlying stock might go up or down with the same probability at any given point in time. This assumption, however, does not hold since stock prices are affected by a multitude of factors, none of which have the same probability of influencing stock price movements.

- Independence of log-returns: it is empirically proven that if we have a look at the

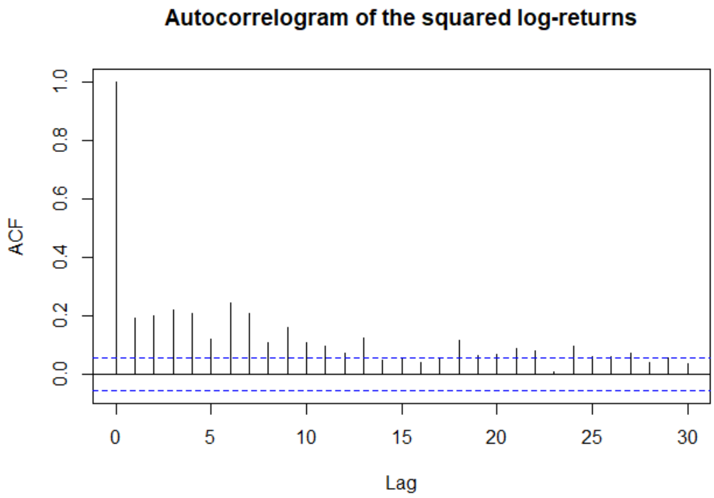

Figure 10: Autocorrelation of the squared log-returns

autocorrelation graph of the log-returns, the non serial correlation holds. However, if we focus on the autocorrelation graph of squared log-returns in **Figure 10**, which we created in R, we can detect a significant autocorrelation on all of the lags. This indicates that while the log-returns can be uncorrelated over time, they are not independent of one another, and hence the hypothesis fails.

- Fixed volatility σ: if the Black-Scholes model is correct, the implicit volatility must

Figure 11: Volatility surface

be equal to the Geometric Brownian motion volatility, and it is independent of the strike price K and the time to maturity Δt. In practice, volatility is not constant, but we have a volatility surface that is heavily influenced by the strike K (volatility smile) and the time to maturity Δt. These concerns can be seen in action in **Figure 11**, which displays the volatility surface of the FTSE MIB index from Bloomberg Terminal. At the same time, because volatility is one of the model inputs that has

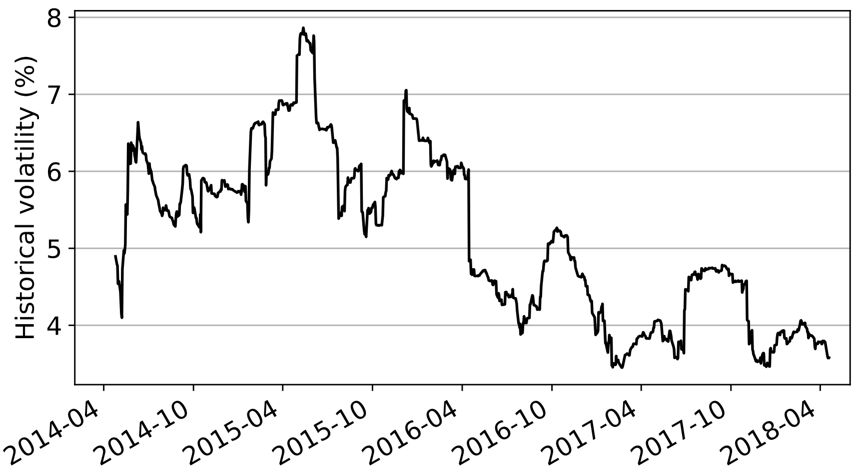

Figure 12: Historical volatility

the biggest impact on the resulting option price, breaking this assumption might have significant consequences. The volatility over time can be seen in **Figure 12**.

- Another assumption in the model is that the risk-free interest rate is known and will remain constant during the option's life. Although this assumption looks to be more realistic in practice than the known constant volatility, it is not as simple. In reality, the risk-free rate curve has inverted and reached negative values during the index's existence. Although there is no need for interest rates to be positive (negative interest rates can be included in the model), the Black-Scholes assumption is violated since rates have not been constant (**Figure 2**). Furthermore, the EURIBOR is no longer an adequate proxy for the risk-free rate. The EURIBOR is the fundamental rate of interest used in inter-bank lending between banks in the European Union. Since it is now recognised that big banks can become insolvent following the financial crisis, they implicitly supply credit risk and hence are not risk-free.

- No dividends. This problem is not crucial since it is possible to generalize the model to the case in which the underlying pays a continuous dividend yield q. The intuition is that in order to have S_T at maturity it is too much to start today with S, since in this case we would have S_T at maturity plus the dividend yield continuously compounded. On the other hand, it is enough to start today with $Se^{-q\Delta t}$ and reinvest continuously the dividends in this stock itself. In this way you can find the Black-Scholes adjusted formula simply substituting $Se^{-q\Delta t}$ instead of S in the original equation. This is just an approximation but since the Certificate underlying doesn't pay dividends (as reported in the Key Investor Information Document), this is not a problem.

- No taxes, no transaction costs, no transfer fees, no option exercise or award fees. In this way is possible to delta hedge without transaction costs and without liquidity constraints, which is of course not true in the real world. In fact, the fund charges 1.43% commission per year.

3 Qualitative Analysis

In this part, we will conduct a qualitative study from the perspectives of both the investor and the issuer, concentrating on their respective motivations for purchasing or issuing this Certificate in the market. To begin with, we clarify the Certificate's nature: it is a European Call option on a non-dividend-paying underlying that offers the buyer the right but not the obligation to purchase a transformation of the underlying index at a specific moment in time. The option's underlying is an index that tracks the performance of a fund that invests primarily in fixed income instruments, and because the value of these products is inversely proportional to the risk-free rate, we can say that this option provides the buyer a short position on the risk-free rate. In other words, any delta-positive derivative based on the fund results in a short interest rate position.

3.1 Investor's viewpoint

We want to find out why a potential investor would want to buy the Certificate in this section. It is commonly known that there are three primary motivations to participate in the market in general: speculation, arbitrage opportunity, and risk hedging. Looking at the pricing, we can see that EUR 90 is significantly greater than the fair value of EUR 33.05 founded using the Black-Scholes model. This suggests that no arbitrage opportunities exist in this case. However, a prospective investor may utilise this Certificate for speculative purposes. Also, because we know the payoff at maturity was beneficial, it stands to reason that someone would want to invest in it. However, due to the unconventional structure of the contract, it is unlikely that a retail investor would purchase this Certificate. It is also not impossible because some investors simply seek assistance from their bank's financial advisor, who can provide a more detailed explanation of the Certificate. In practice, such products are uncommon among ordinary investors. However, it could be used as a speculative instrument among institutional investors. Next to this, we believe that this Certificate can be utilised to hedge. This concept is reinforced by the presence of a volatility-limiting mechanism, as well as the fact that the certificate is ultimately a call option, with a loss which is limited to the initial premium. Given the possible use of this Certificate to hedge another position, and because this call option provides a short position on risk-free rates, it is likely that the buyer has an initial long position on certain financial assets that are long on risk-free rates. Given this result, we feel that commercial banks are the most likely buyers of this Certificate, as their business is based mostly on the interest paid by their money-borrowing clients. Also other financial institutions, such as mutual funds or other fixed-income funds, may be interested in obtaining these certificates if they need to hedge against fluctuations in interest rates. When utilised as a hedging strategy, the Certificate's risks are mitigated by the purchasing party's principal business. Higher interest rates would have a negative impact on the performance of the fixed income fund on which the Certificate is based. Profits from a commercial bank's principal business as a money lender would grow. Furthermore, as previously noted, the loss from the certificates would be limited to the premium paid for the Certificates themselves. Finally, we want to mention the prospective charges and commissions that will be utilised to cover the Fund's running expenditures, including marketing and share distribution. There are two potential one-off charges taken before or after the investment: 5.00% for entry and 3.00% for exit, which are maximum rates. In certain cases, the charges paid may be lower. Also there is an ongoing charges taken from the Fund over a year, which is 1.43%.

3.2 Issuer's viewpoint

The final section of the project discusses the rationale for issuing this Certificate from the standpoint of Commerzbank. The first and most obvious explanation is the gap between the Certificate's initial purchase price (EUR 90) and its fair value at inception (EUR 33.05). This results in an immediate enormous profit. This may be explained in part by the cost of having a bullish position on interest rates, which was nearly a guaranteed bet to be lucrative at the height of the financial crisis in 2014. Another factor for the issuance is the necessity to safeguard against risk linked with the bank's own books. Even though we know that after the financial crisis, the major authorities strengthened the capital requirements for banks to trade risky assets, a proprietary trading component remains and must be hedged. A bank can create and sell an option in this manner to minimize some personal risks. Another intriguing feature is the modest number of Certificates issued ($30,000$), with a total value of EUR 2.7 million. This little sum may indicate that it was a structured security built on demand for a specific customer, which would explain the non-standard value computation and payout. Another rationale might be client relationship management from the issuer's standpoint. Commerzbank's main responsibility over the past 20 years has been to develop the infrastructure necessary to produce the structured financial instruments that their clients need for capital protection, leverage, or other objectives. Taking the passage of time into consideration, the interest rate environment was extremely stressful in 2014, and investors were primarily looking for yield and fixed-income equivalent products.

3.3 Conclusion

To summarise, in a time of significantly high uncertainty, with investors searching for hedging and capital protection solutions, the certificates would have been a simple way to expand the market and restore client trust for Commerzbank. Moreover, if we consider the potential investor perspective and the results of our ex-post analysis, it can be stated that investing in the Certificate would have been a profitable option. This is clear due to the difference between the original price (EUR 90) and the Settlement Amount (EUR 135.73), even if the Fair Price at issue should have been closer to EUR 33.05.

4 Listing

```r
## INIZIALIZATION


# Clean the environment
rm(list = ls())
library(readxl)

# Import dataset
Prices <- read_excel("C:/Users/marco/OneDrive - Universita
Commerciale Luigi Bocconi/University/Bocconi/MAFINRISK/Courses/
Derivatives/Assigments/Assignment Project/Prices.xlsx")


## Empirical distribution

# Empirical histogram
hist(Prices$log, probability = T, breaks = 50, main = paste("
Empirical vs Theoretical distribution (log-returns)"))

# Normal distribution with same moments
curve(dnorm(x,mean=mean(Prices$log),sd=sd(Prices$log)), add=
TRUE, col="red")
legend("topleft", legend = c("Empirical distribution","Normal
distribution"), col = c("grey","red"), lty = c(1,1))


## Autocorrelogram of the squared log-returns

acf((Prices$log)^2, main = paste("Autocorrelogram of the
squared log-returns"))
```

Listing 1: Wrong distributional hypotesis of Black-Scholes model