

## ICPC Template Manual



作者: 贺梦杰

July 26, 2019

## Contents

1	<del>基</del> 础 1.1	IO 优化	. 4
2	搜索		5
3	动态	规划	6
4	字符	串	7
	4.1	KMP	. 8
5	数据		10
	5.1	并查集	. 11
	5.2	线段树	. 12
		5.2.1 基础操作	. 12
		5.2.2 单点更新	. 12
		5.2.3 区间更新	. 13
		5.2.4 区间查询	. 13
	5.3	树状数组	. 15
		5.3.1 单点修改,区间查询	. 15
		5.3.2 区间修改, 单点查询	. 15
		5.3.3 区间修改,区间查询	. 15
	5.4	二维树状数组	. 16
		5.4.1 单点修改,区间查询	
		5.4.2 区间修改,区间查询	
6	图论		19
U	到化 6.1	最小生成树	
	0.1	6.1.1 Prim	
		6.1.2 Kruskal	
		·	
	6.0	6.1.3 次小生成树	
	6.2	最近公共祖先	
	c o	6.2.1 倍增	
	6.3	强连通分量	
		6.3.1 Tarjan	
		6.3.2 缩点 DAG	
	6.4	最短路	
		6.4.1 Dijkstra	
	6.5	网络流	
		6.5.1 最大流	
		6.5.1.1 Edmonds Karp	
		6.5.1.2 Dinic	20

CONTENTS

			6.5.1. 6.5.1.																	31 31
	7.2	快速幂 7.1.1 7.1.2 矩阵快 组合数	递归; 循环; 速幂	形式 形式 · ·	 			  		•	 		•		 	 			•	 33 33 33
8	计算	几何																		36
9	其他																			37

# Chapter 1

基础

1.1. IO 优化 CHAPTER 1. 基础

## 1.1 IO 优化

```
1 ios::sync_with_stdio(false);
2 cin.tie(0);
```

# Chapter 2

# 搜索

# Chapter 3 动态规划

# Chapter 4 字符串

### 4.1 KMP

```
1 int *Get_next(string str)
2
   {
3
       int *ptr = new int[str.length()];
       //申请next数组
4
5
       ptr[0] = 0;
                               //首位next值为0
6
       int i = 1;
                               //初始化
7
       int j = 0;
                               //初始化
8
       int len = str.length(); //模式串长度
       while (i < len)</pre>
9
10
11
           if (str[i] == str[j])
12
           {
13
               ptr[i] = j + 1;
14
               j++;
15
               i++; //确定前缀后缀相同的长度
           }
16
           else
17
18
           {
19
               //不同时
20
               if (j != 0)
                   j = ptr[j - 1]; //j回到前一个字符的next值位置
21
22
               else
23
               {
                   ptr[i] = 0; //回到模式串的第一个字符
24
25
                   i++;
               }
26
27
           }
28
29
       return ptr;
30
  }
31
  int KMP(string s, string p)
32 {
33
       int *next = Get_next(p);
       //获得next数组
34
35
       int i = 0;
36
       int j = 0;
37
       int len = s.length();
       while (i < len)</pre>
38
39
40
           if (s[i] == p[j])
41
42
               i++;
43
               j++; //匹配
44
               if (j >= p.length())
                   return i - j;
45
46
           }
47
           else
48
           {
49
               //字符不相同回到前一个字符的next值位置
               if (j != 0)
50
51
                   j = next[j - 1];
```

```
52 else

53 i++;

54 }

55 }

56 return -1;

57 }
```

来源: https://www.bilibili.com/video/av47471886? from=search & seid=4651914725266859344

# Chapter 5

# 数据结构

## 5.1 并查集

```
1 #define MAX 1010
2 struct node
3 {
4
       int par;
5
       //int rank;
6
       //路径压缩后 rank=1或2 rank失去了意义
7
       int data;
8 };
9 node ns[MAX];
10 void Init()
11 {
12
       for (int i = 1; i < MAX; i++)</pre>
13
14
           ns[i].par = i;
       }
15
16
   }
   int Find(int i)
17
18 {
19
       if (ns[i].par == i)
20
21
           //返回根结点
22
           return i;
23
24
       ns[i].par = Find(ns[i].par);
25
       //路径压缩
26
       return ns[i].par;
27
   }
28 void Union(int i, int j)
29 {
       int pi = Find(i);
30
       int pj = Find(j);
31
       if (pi != pj)
32
33
34
           ns[pi].par = pj;
35
       }
36 }
```

## 5.2 线段树

### 5.2.1 基础操作

```
1 const int N = 1e5 + 10;
 2 #define ls(a) (a << 1)
3 #define rs(a) (a << 1 | 1)</pre>
4 struct node
5 {
6
       int val;
7
       int lazy;
8
   };
9 node tree[N << 2];</pre>
10 int a[N];
11 void PushUp(int rt)
12 {
13
       tree[rt].val = tree[ls(rt)].val + tree[rs(rt)];
14 }
   void PushDown(int ls, int rs, int rt)
15
16 {
17
       tree[ls(rt)].val += ls * tree[rt].lazy;
18
       tree[rs(rt)].val += rs * tree[rt].lazy;
19
       tree[ls(rt)].lazy += tree[rt].lazy;
20
       tree[rs(rt)].lazy += tree[rt].lazy;
21
       tree[rt].lazy = 0;
22
   }
23 void Build(int left, int right, int rt)
24
25
       if (left == right)
26
27
            tree[rt].val = a[left];
28
            return;
29
        int mid = (left + right) >> 1;
30
       Build(left, mid, ls(rt));
31
32
       Build(mid + 1, right, rs(rt));
33
       PushUp(rt);
34
       //向上更新
35
   }
```

### 5.2.2 单点更新

```
void Update(int left, int right, int rt, int pos, int val)
1
^{2}
   {
3
       if (left == right && left == pos)
4
        {
            tree[rt].val += val;
5
6
            return;
7
8
       int mid = (left + right) >> 1;
9
       if (tree[rt].lazy)
10
```

```
PushDown(mid - left + 1, right - mid, rt);
11
12
       if (mid >= pos)
13
14
           Update(left, mid, ls(rt), pos, val);
       else if (pos > mid)
15
16
           Update(mid + 1, right, rs(rt), pos, val);
17
       PushUp(rt);
18 }
   例题: https://www.luogu.org/problemnew/show/P3372
   5.2.3
           区间更新
   void Update(int left, int right, int rt, int s, int t, int val)
2
3
       if (left >= s && right <= t)</pre>
4
           tree[rt].val += (right - left + 1) * val;
5
6
           tree[rt].lazy += val;
7
            return;
8
        }
       int mid = (left + right) >> 1;
9
10
       if (tree[rt].lazy)
11
       {
12
           PushDown(mid - left + 1, right - mid, rt);
13
        }
14
        if (mid < s)
15
           Update(mid + 1, right, rs(rt), s, t, val);
16
       else if (mid >= t)
17
           Update(left, mid, ls(rt), s, t, val);
18
       else
19
20
           Update(left, mid, ls(rt), s, t, val);
21
           Update(mid + 1, right, rs(rt), s, t, val);
22
23
       PushUp(rt);
24 }
   5.2.4
           区间查询
   void Query(int left, int right, int s, int t, int rt)
1
2
   {
3
       if (left >= s && right <= t)</pre>
4
       {
5
            return tree[rt].val;
6
7
       int mid = (left + right) >> 1;
8
       if (tree[rt].lazy)
           PushDown(mid - left + 1, right - mid, rt);
9
10
       long long sum = 0;
11
        if (mid < s)
12
            sum += Query(mid + 1, right, rs(rt), s, t, val);
13
       else if (mid >= t)
```

```
14
           sum += Query(left, mid, ls(rt), s, t, val);
       else
15
16
       {
           sum += Query(left, mid, ls(rt), s, t, val);
17
18
           sum += Query(mid + 1, right, rs(rt), s, t, val);
19
       }
20
       return sum;
21 }
   例题: https://www.luogu.org/problemnew/show/P3373
```

## 5.3 树状数组

推荐阅读: https://www.cnblogs.com/RabbitHu/p/BIT.html

### 5.3.1 单点修改,区间查询

```
1 #define N 1000100
2 long long c[N];
3 int n,q;
   int lowbit(int x)
4
5
   {
6
        return x&(-x);
7
   }
   void change(int x,int v)
8
9
   {
        while(x<=n)</pre>
10
11
        {
12
            c[x]+=v;
13
            x+=lowbit(x);
14
        }
15
   }
16
   long long getsum(int x)
17
18
        long long ans=0;
19
        while(x>=1)
20
21
            ans+=c[x];
22
            x-=lowbit(x);
23
24
        return ans;
25
   }
   例题: https://loj.ac/problem/130
```

### 5.3.2 区间修改,单点查询

引入差分数组来解决树状数组的区间更新

```
    //初始化
    change(i,cur-pre);
    //区间修改
    change(l,x);
    change(r+1,-x);
    //单点查询
    getsum(x)
    例题: https://loj.ac/problem/131
```

### 5.3.3 区间修改,区间查询

```
    1 //初始化
    2 change(c1,i,cur-pre);
    3 change(c2,i,i*(cur-pre));
    4 //为什么这么写? 你需要写一下前缀和的表达式
```

```
5 //区间修改
6 change(c1,1,x);
7 change(c2,1,1*x);
8 change(c1,r+1,-x);
9 change(c2,r+1,-(r+1)*x);
10 //区间查询
11 temp1=1*getsum(c1,1-1)-getsum(c2,1-1);
12 temp2=(r+1)*getsum(c1,r)-getsum(c2,r);
13 ans=temp2-temp1
例题: https://loj.ac/problem/132
```

## 5.4 二维树状数组

### 5.4.1 单点修改,区间查询

```
1 #define N 5050
2 long long tree[N][N];
3 long long n,m;
4 long long lowbit(long long x)
5
        return x&(-x);
6
7
   }
   void change(long long x,long long y,long long val)
8
9
   {
10
       long long init_y=y;
11
        //这里注意n,m的限制
12
       while(x<=n)</pre>
13
       {
            y=init_y;
14
15
            while(y<=m)</pre>
16
            {
17
                tree[x][y]+=val;
18
                y+=lowbit(y);
19
20
            x+=lowbit(x);
21
        }
22
23
   long long getsum(long long x,long long y)
24
   {
25
       long long ans=0;
26
        long long init_y=y;
27
       while(x>=1)
28
        {
29
            y=init_y;
30
            while(y>=1)
31
32
                ans+=tree[x][y];
                y-=lowbit(y);
33
34
35
            x-=lowbit(x);
36
37
       //这里画图理解
```

```
38
       return ans;
39 }
40 //初始化
41 change(x,y,k);
42 //二维前缀和
43 ans = getsum(c,d)+getsum(a-1,b-1)-getsum(a-1,d)-getsum(c,b-1);
   例题: https://loj.ac/problem/133
           区间修改, 区间查询
   5.4.2
1 #define N 2050
 2 long long t1[N][N];
 3 long long t2[N][N];
4 long long t3[N][N];
5 long long t4[N][N];
6 long long n,m;
 7 long long lowbit(long long x)
8
   {
9
       return x&(-x);
10
   }
   long long getsum(long long x,long long y)
11
12
   {
13
        long long ans=0;
14
        long long init_y=y;
15
        long long init_x=x;
16
       while(x>=1)
17
       {
18
           y=init_y;
19
           while(y>=1)
20
            {
                ans+=(init_x+1)*(init_y+1)*t1[x][y];
21
22
                ans-=(init_y+1)*t2[x][y];
23
                ans-=(init_x+1)*t3[x][y];
24
                ans+=t4[x][y];
25
                y-=lowbit(y);
26
27
            x-=lowbit(x);
28
        }
29
       return ans;
30
   }
31
   void change(long long x, long long y, long long val)
32 {
33
        long long init_x=x;
34
       long long init_y=y;
35
       while(x<=n)</pre>
36
37
           y=init_y;
38
           while(y<=m)</pre>
39
40
                t1[x][y]+=val;
41
                t2[x][y]+=init_x*val;
42
                t3[x][y]+=init_y*val;
43
                t4[x][y]+=init_x*init_y*val;
```

```
44
               y+=lowbit(y);
           }
45
46
           x+=lowbit(x);
47
       }
48
   }
49
   //区间修改
50 change(c+1,d+1,x);
51 change(a,b,x);
52 change(a,d+1,-x);
53 change(c+1,b,-x);
54 //区间查询
55 ans=getsum(c,d)+getsum(a-1,b-1)-getsum(c,b-1)-getsum(a-1,d);
   例题: https://loj.ac/problem/135
```

# Chapter 6

# 图论

6.1. 最小生成树 CHAPTER 6. 图论

## 6.1 最小生成树

#### 6.1.1 Prim

```
1 #define inf 0x3f3f3f3f
2 \text{ const int } N = 2e5 + 20;
3 struct node
4 {
5
        long long u, v, w;
6
        node(int uu, int vv, int ww) : u(uu), v(vv), w(ww)
7
        {
8
9
        bool operator<(node n) const</pre>
10
        {
11
            return w > n.w;
12
        }
13 };
14 long long n, m;
15 priority_queue<node> q;
16 vector<pair<long long, long long>> G[N];
17 bool vis[N];
18 long long Prim()
19
   {
20
        long long ans = 0;
21
        for (auto ele : G[1])
22
        {
23
            q.push(node(1, ele.first, ele.second));
24
        }
25
        memset(vis, 0, sizeof(vis));
26
        vis[1] = 1;
27
        int t = n - 1;
        while (t--)
28
29
30
            node top = q.top();
31
            q.pop();
32
            while (vis[top.v])
33
34
                top = q.top();
35
                q.pop();
36
37
            ans += top.w;
            cout<<ans<<endl;</pre>
38
39
            vis[top.v] = 1;
            for (auto ele : G[top.v])
40
41
            {
42
                q.push(node(top.v, ele.first, ele.second));
43
            }
44
        }
45
        return ans;
46 }
```

6.1. 最小生成树 CHAPTER 6. 图论

#### 6.1.2 Kruskal

```
基于并查集
1 const long long MAXN = 2e5 + 20;
2 \quad {\tt struct} \  \, {\tt Edge}
3 {
4
        long long u, v, w;
5
        bool operator<(Edge e) const</pre>
6
7
            return w > e.w;
8
9
        Edge(long long uu, long long vv, long long ww) : u(uu), v(vv), w(ww)
10
11
        }
12 };
   long long fa[MAXN];
14 long long Find(long long x)
15 {
16
        if (fa[x] == -1)
17
            return x;
18
        else
            return fa[x] = Find(fa[x]);
19
20 }
21 priority_queue<Edge> q;
22 long long n; //点
23 long long m; //边
  long long Kruskal()
24
25
   {
26
        memset(fa, -1, sizeof(fa));
27
        long long cnt = 0;
28
        long long ans = 0;
29
        long long fu, fv;
        while (!q.empty())
30
31
32
            Edge now = q.top();
            q.pop();
33
            fu = Find(now.u);
34
35
            fv = Find(now.v);
36
            if (fu != fv)
37
                 //cout<<"add:"<<now.u<<" "<<now.v<<" "<<now.w<<endl;</pre>
38
39
                 ans += now.w;
                fa[fu] = fv;
40
                 cnt++;
41
42
            if (cnt == n - 1)
43
44
                break;
45
        if (cnt < n - 1)</pre>
46
47
            return -1;
48
        return ans;
49
   }
```

例题: https://loj.ac/problem/123

## 6.1.3 次小生成树

倍增 LCA 维护最小生成树

代码过长.....

例题: https://loj.ac/problem/10133

6.2. 最近公共祖先

## 6.2 最近公共祖先

### 6.2.1 倍增

```
1 #define N 500050
2 int depth[N];
3 int fa[N][20];
4 vector<int> v[N];
5 int lg[N]; //log2(n) floor
6 int n, m, s;
7
   void init()
8
   {
9
       lg[0] = -1;
10
       //floor
11
       for (int i = 1; i <= n; i++)
12
13
            lg[i] = lg[i >> 1] + 1;
14
15
   }
   void DFS(int cur, int pre)
16
17
   {
18
       depth[cur] = depth[pre] + 1;
19
       fa[cur][0] = pre;
20
       for (int i = 1; (1 << i) <= depth[cur]; i++)</pre>
21
22
            fa[cur][i] = fa[fa[cur][i - 1]][i - 1];
23
24
       for (auto ele : v[cur])
25
26
            if (ele != pre)
27
            {
28
                DFS(ele, cur);
29
30
        }
31
32
   int LCA(int a, int b)
33
   {
34
       // assume depth[a]>=depth[b]
35
        if (depth[a] < depth[b])</pre>
36
            swap(a, b);
37
       // reset to the same depth
38
       while (depth[a] > depth[b])
39
40
            a = fa[a][lg[depth[a] - depth[b]]];
41
            //up
42
        }
        if (a == b)
43
44
            return a;
       for (int k = \lg[depth[a]] + 1; k >= 0; k--)
45
46
47
            if (fa[a][k] != fa[b][k])
48
            {
49
                a = fa[a][k];
```

```
50 b = fa[b][k];
51 //up
52 }
53 }
54 return fa[a][0];
55 }
例题: https://www.luogu.org/problemnew/show/P3379
```

## 6.3 强连通分量

### 6.3.1 Tarjan

```
1 const int N = 10500;
2 vector<int> G[N];
3 bool vis[N];
4 int dfn[N];
5 int low[N];
6 int cnt;
7 stack<int> s;
8 int n, m;
   void init()
9
10
11
       memset(vis, 0, sizeof(vis));
12
       memset(dfn, 0, sizeof(dfn));
13
       cnt = 0;
14
   }
15
   void DFS(int cur)
16
   {
17
       dfn[cur] = low[cur] = ++cnt;
18
        s.push(cur);
19
       vis[cur] = 1;
20
       for (auto ele : G[cur])
21
            if (!dfn[ele])
22
23
            {
24
                DFS(ele);
25
                low[cur] = min(low[cur], low[ele]);
26
            else if (vis[ele])
27
28
            {
                low[cur] = min(low[cur], dfn[ele]);
29
30
            }
31
32
       if (dfn[cur] == low[cur])
33
            while (1)
34
35
            {
36
                int t = s.top();
37
                s.pop();
                vis[t] = 0;
38
39
                if (t == cur)
40
                    break;
41
            }
42
        }
43
   }
```

例题: https://www.luogu.org/problemnew/show/P2863

### 6.3.2 缩点 DAG

利用强连通分量缩点为有向无环图 Tarjan 加入染色 6.3. 强连通分量 CHAPTER 6. 图论

```
if (dfn[cur] == low[cur])
1
2
3
       sum++;
       while (1)
4
5
6
            int t = s.top();
7
            s.pop();
            color[t] = sum;
8
            pnum[sum]++;
9
            vis[t] = 0;
10
            if (t == cur)
11
12
                break;
13
        }
14 }
```

例题: https://www.luogu.org/problemnew/show/P2341

6.4. 最短路 CHAPTER 6. 图论

## 6.4 最短路

### 6.4.1 Dijkstra

```
非负权图,单源最短路径
   O((N+M)\log M)
1 #define N 100100
2 #define 11 long long
3 #define inf 2147483647
4 vector<pair<int, ll>> G[N];
5 struct edge
6 {
7
       int to;
8
       ll weight;
       edge(int i, ll w) : to(i), weight(w){}
9
10
       bool operator<(const edge &e) const{return weight > e.weight;}
   };
11
12 11 dis[N];
13 bool vis[N];
14 int n, m, s;
   void Dijkstra(int start)
15
   {
16
17
       for (int i = 1; i <= n; i++){dis[i] = inf;}</pre>
18
       priority_queue<edge> q;
19
       q.push(edge(start, 0));
20
       dis[start] = 0;
21
       while (!q.empty())
22
23
           edge now = q.top();
24
           q.pop();
25
           if (!vis[now.to])
26
27
                vis[now.to] = 1;
               for (auto ele : G[now.to])
28
29
                {
                    if (!vis[ele.first] && now.weight + ele.second < dis[ele.first])</pre>
30
31
                    {
32
                        dis[ele.first] = now.weight + ele.second;
33
                        q.push(edge(ele.first, dis[ele.first]));
34
                    }
                }
35
36
           }
37
       }
38
   }
   例题: https://www.luogu.org/problemnew/show/P3371
   例题: https://www.luogu.org/problemnew/show/P4779
```

## 6.5 网络流

### 6.5.1 最大流

#### 6.5.1.1 Edmonds Karp

```
1 #define ll long long
2 #define inf 0x3f3f3f3f
3 const int M = 205;
4 ll c[M][M];
5 int pre[M];
6 11 flow[M];
7 int n, m;
8
  11 BFS(int s, int t)
9
   {
10
        queue<int> q;
11
        memset(pre, -1, sizeof(pre));
12
        flow[s] = inf;
13
        pre[s] = 0;
14
        q.push(s);
15
        while (!q.empty())
16
17
            int u = q.front();
18
            q.pop();
19
            if (u == t)
20
            {
21
                break;
22
            for (int v = 1; v <= m; v++)
23
24
25
                if (c[u][v] > 0 \&\& pre[v] == -1\&\&v!=s)
26
27
                     pre[v] = u;
28
                     flow[v] = min(c[u][v], flow[u]);
29
                     q.push(v);
30
                }
31
            }
32
33
        if (pre[t] == -1)
34
            return -1;
35
        return flow[t];
36
37
   11 Edmonds_Karp(int s, int t)
38
   {
39
        ll inc;
40
        11 \text{ ans} = 0;
41
        int k, last;
        while ((inc = BFS(s, t)) != -1)
42
43
44
            k = t;
            while (k != s)
45
46
            {
47
                last = pre[k];
48
                c[last][k] -= inc;
```

```
49
               c[k][last] += inc;
50
               k = last;
           }
51
52
           ans += inc;
53
           //cout<<"cur:"<<inc<<endl;</pre>
54
       }
55
       return ans;
56
   }
   例题: https://www.luogu.org/problemnew/show/P2740
   6.5.1.2 Dinic
   多路增广, 当前弧优化
1 #define inf 0x3f3f3f3f3f
2 #define N 1200
3 #define M 245000
4 #define ll long long
5 //反向边的存在 实际边数的两倍
6 struct Edge
7
   {
8
       11 to;
       11 w;
9
10
       ll next;
11
  };
12 Edge edge[M];
13 ll dep[N];
14 ll head[N];
15 ll cur[N]; //当前弧优化
16 \ 11 \ cnt = 0;
17 11 n, m;
18 void Init()
19
  {
20
       memset(head, -1, sizeof(head));
21
   }
22
   void add(ll u, ll v, ll w)
23
24
       edge[cnt].to = v;
25
       edge[cnt].w = w;
26
       edge[cnt].next = head[u];
27
       head[u] = cnt++;
28
29
  void Add(ll u, ll v, ll w)
30
  {
31
       add(u, v, w);
32
       add(v, u, 0); //反向边
33
  11 s, t;
34
   bool BFS()
35
36
   {
37
       queue<ll> q;
       memset(dep, -1, sizeof(dep));
38
39
       dep[s] = 0;
40
       q.push(s);
```

```
while (!q.empty())
41
42
            11 now = q.front();
43
44
            q.pop();
45
            for (ll i = head[now]; i != -1; i = edge[i].next)
46
47
                if (edge[i].w > 0 && dep[edge[i].to] == -1)
48
                {
49
                     dep[edge[i].to] = dep[now] + 1;
50
                     q.push(edge[i].to);
51
                }
52
            }
53
54
        if (dep[t] == -1)
            return false; //不存在分层图
55
56
        return true;
57
   11 DFS(11 now, 11 flow)
58
59
   {
        if (now == t)
60
61
            return flow;
62
        11 \text{ used} = 0;
63
        for (ll &i = cur[now]; i != -1; i = edge[i].next)
64
            //&: 当前弧优化
65
66
            if (dep[edge[i].to] == dep[now] + 1 && edge[i].w)
67
                11 inc = DFS(edge[i].to, min(flow - used, edge[i].w));
68
69
                if (inc > 0)
70
71
                     edge[i].w -= inc;
72
                     edge[i ^ 1].w += inc;
73
                     used += inc;
74
                     if (flow == used)
75
                         break;
76
                }
            }
77
78
        }
        if (!used)
79
80
            dep[now] = -1;
81
        return used;
82
83
   11 Dinic()
84
   {
85
        11 \text{ ans} = 0;
86
        while (BFS())
87
        {
88
            for (ll i = 1; i <= n; i++)
89
            {
90
                cur[i] = head[i];
91
92
            ans += DFS(s, inf);
93
        }
```

```
94 return ans;
95 }
例题: https://www.luogu.org/problemnew/show/P3376
6.5.1.3 ISAP
```

最高标号预留推进

# Chapter 7

# 数学

7.1. 快速幂 CHAPTER 7. 数学

## 7.1 快速幂

### 7.1.1 递归形式

```
1 #define mod 1000000007
2
   long long quick_pow(long long a, long long b)
3
   {
4
       if (b == 0)
5
           return 1;
6
       long long temp = quick_pow(a, b >> 1);
 7
       if (b & 1)
           return a % mod * temp % mod * temp % mod;
8
9
       else
10
           return temp % mod * temp % mod; //快速幂
11
  }
```

### 7.1.2 循环形式

#### 如果递归形式栈溢出可使用循环形式

```
1 #define mod 1000000007
   long long quick_pow(long long x,long long n)
2
3
4
       long long ret=1;
5
       long long temp=x%mod;
6
       while(n)
7
        {
            if(n&1)
8
9
            {
10
                ret=(ret*temp)%mod;
11
12
            temp=(temp*temp)%mod; //偶次
13
           n>>=1;
14
        }
15
       return ret;//结果
16 }
```

## 7.2 矩阵快速幂

```
1 #define N 105
2 int m;//矩阵阶
   struct matrix
3
4
5
       long long a[N][N];
6
       matrix(){memset(a, 0, sizeof(a));}
7
   };
8
   matrix matrix mul(matrix m1, matrix m2)
9
   {
10
       matrix ans;
       for (int k = 0; k < m; k++)
11
12
13
            for (int i = 0; i < m; i++)</pre>
```

7.3. 组合数取模 CHAPTER 7. 数学

```
14
            {
15
                if (m1.a[i][k]) //剪枝
16
                {
                     for (int j = 0; j < m; j++)
17
18
                         if (m2.a[k][j]) //剪枝
19
20
                             ans.a[i][j] = (ans.a[i][j] + (m1.a[i][k] * m2.a[k][j]) %
21
        mod) % mod;
22
                         }
23
                     }
24
                }
25
            }
26
        }
27
        return ans;
28
   }
29
   matrix quick_pow_matrix(matrix m1, long long k)
30
   {
31
        matrix ans;//递归写法 可能堆栈溢出
        for (int i = 0; i < m; i++)</pre>
32
            ans.a[i][i] = 1;
33
34
        while (k)
35
        {
36
            if (k & 1)
37
                ans = matrix_mul(ans, m1);
            m1 = matrix_mul(m1, m1);
38
39
            k >>= 1;
40
        }
41
        return ans;
42
   }
```

## 7.3 组合数取模

```
1 #define N 100010
2 #define mod 1000000007
3 #define 11 long long
4 ll fac[N];//阶乘
5 ll inv[N];//阶乘逆元
6
  void init()
7
   {
8
       fac[0]=1;
9
       for(int i=1;i<N;i++)</pre>
10
11
           fac[i]=(fac[i-1]*i)%mod;
12
       }
       inv[N-1]=quick_pow(fac[N-1],mod-2);//费马小定理 求逆元a^(p-1)%p=1 a的逆元为a^(
13
14
       //如果 ax%p=1 , 那么x的最小正整数解就是 a 模p的逆元
15
       for(int i=N-2;i>=0;i--)
16
17
           inv[i]=inv[i+1]*(i+1)%mod;
18
       }
```

7.3. 组合数取模 *CHAPTER 7.* 数学

```
19  }
20  11 C(11 a,11 b)
21  {
22    if(b>a) return 0;
23    if(b==0) return 1;
24    return fac[a]*inv[b]%mod*inv[a-b]%mod;
25  }
```

# Chapter 8 计算几何

# Chapter 9

# 其他