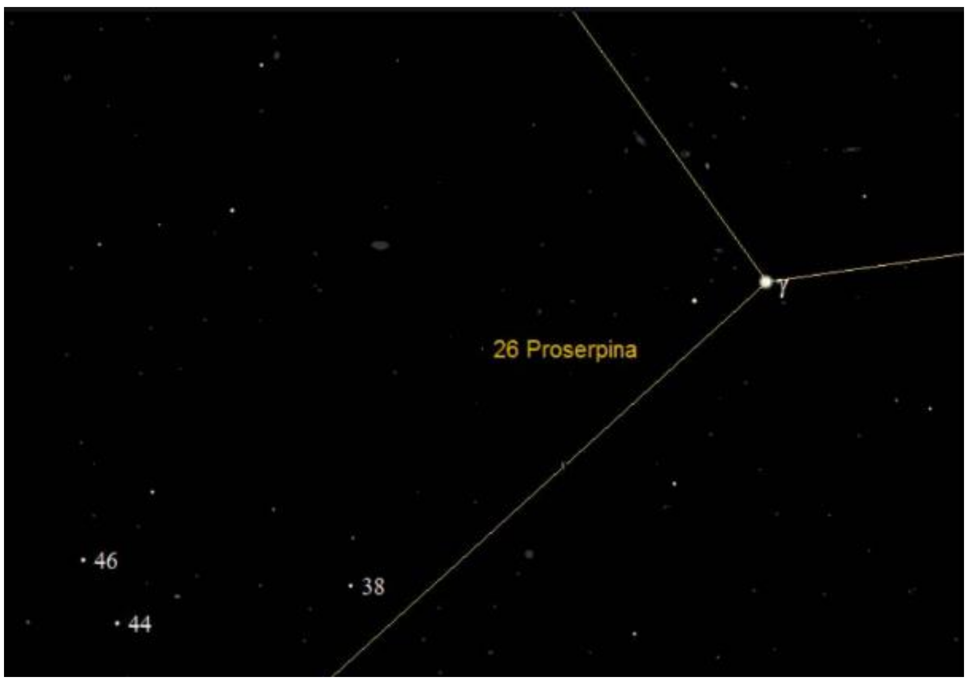
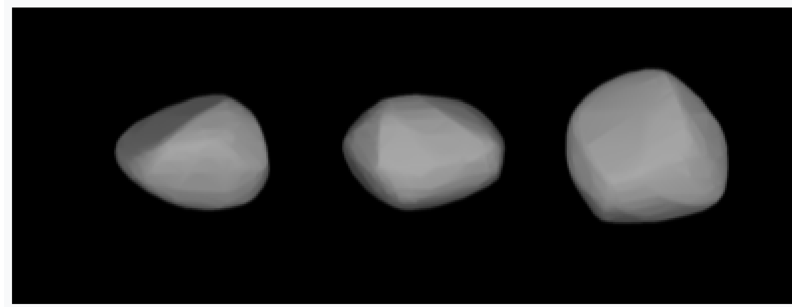
Lab 3: Astrometry

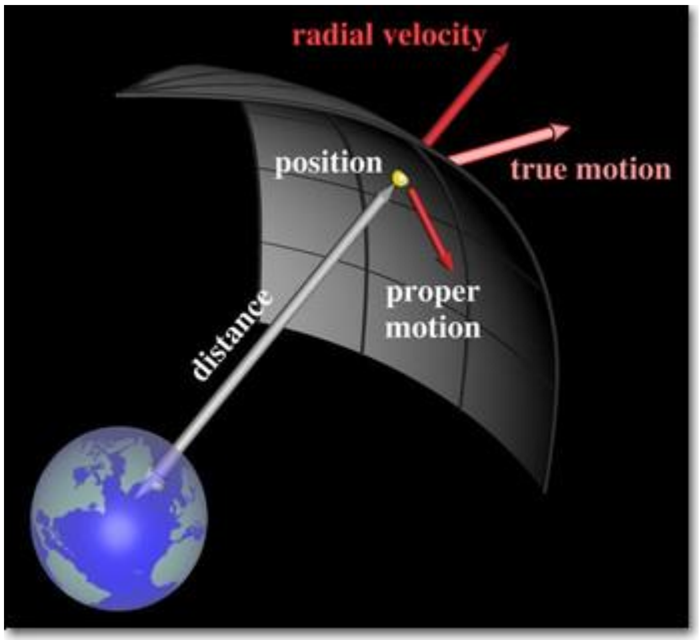
Let's calculate **the proper motion** (= motion of a celestial object on sky) of the asteroid "**26 Proserpina**" (discovered in 1853)

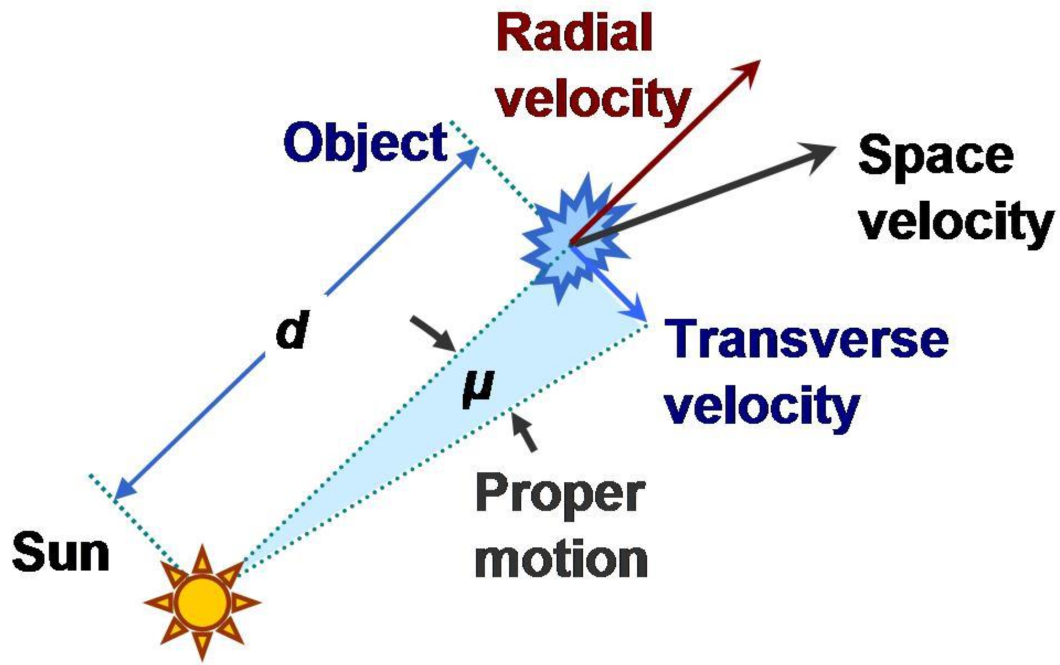




Three-dimensional model of 26 Proserpina created based on light-curve inversions.

Lab 3: Astrometry





Proper motion is measured in units of angular distance (e.g., degrees) or angular velocity (e.g., degrees/year).

Lab 3: Astrometry

In order to measure a proper motion, we need:

(1) imaging observations from multiple epochs;

(2) proper reduction of the observed data;

(3) to measure precise positions of the asteroid in celestial coordinates on images ← **This is called "Astrometry."**

In Lab 3, we will do (2) and (3) using data already taken for (1).

Lab 3: Astrometry

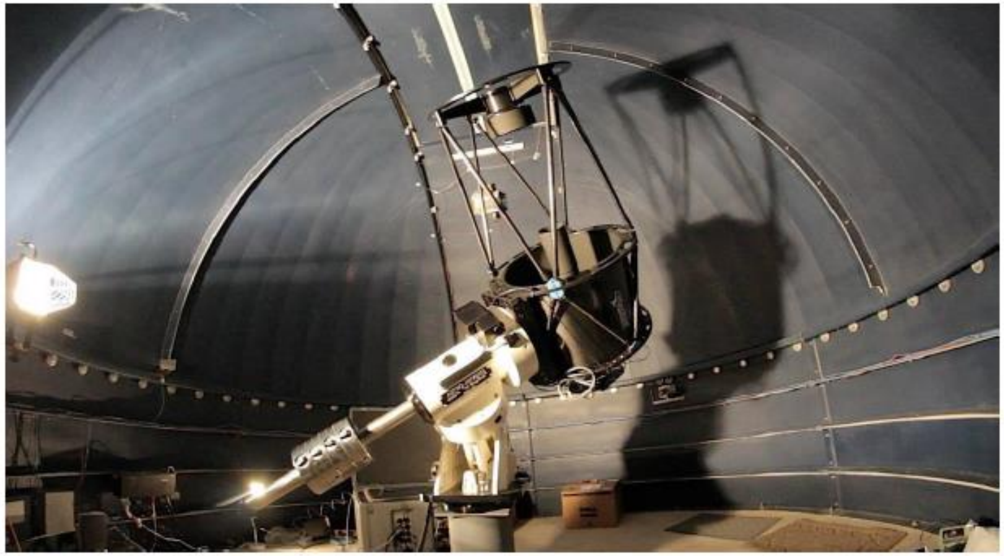


Figure 1: *A picture from inside the dome of the 50-cm DIT telescope on Mt. Joy.*

Table 1: Nominal observatory and instrument properties

Site	Mt. Joy, NM
Geographic location	32°54'10"N 105°31'46"W
Primary mirror	50-cm (diameter)
F/ratio-focal length	F/6.8 – 3454 mm
Camera	Apogee Alta U16M (www.ccd.com)
Detector	TE-cooled Kodak KAF-16803 CCD
CCD format	4096 × 4096, 9.0 μm pixels (unbinned)
Nominal pixel scale	0.53 arc seconds/pixel
Field-of-view	36.2 arc minutes (square)
Optical Filters	g, r, i, z, L, B, clear
Limiting Magnitude	$r \approx 17.5/18.5$ mag. (10/60 seconds)

CCD FITS files form DIT are available from the following web page

https://drive.google.com/drive/folders/1qaKMqBI1H4NdcqhATHQdzqTF-o6M3kV_?usp=sharing

Lab 3: Astrometry

How do we measure precise positions of the asteroid (note that this is not 26 Proserpina) in celestial coordinates on images?

Lab 3: Astrometry

- The positions of (bright) stars are already known to a good precision, and almost of all stars are stationary.

- We can use the positions of stars to obtain **2-dim astrometric mapping solutions** between observed images and their known coordinates. (Note that this is very similar to the wavelength solution using the known wavelengths of reference lines and their positions on observed images.)

- By applying the astrometric solutions to the positions of the asteroid, we can estimate its celestial coordinates. The change of the celestial coordinates is proper motion.

Lab 3: Astrometry

What do we need to know/understand?

- how to conduct **basic reduction of CCD images**, such as flat-fielding, dark subtraction, etc;
- the **celestial coordinate system** and the definition of the term "**magnitude**" for astronomical brightness;
- how to use astronomical catalogs (specifically **USNO B-1.0**) that have positions and brightness of stars and image viewer service (specifically **ALADIN**);
- how to obtain **2-dim mapping solutions** based on Affine matrix transformation and linear least squares fit;

Lab 3: Astrometry

Lab #3 "Astrometry from CCD Images"

Lab report is due at 4pm on Dec 6 (Monday). Submission on Quercus

Asteroids do not concern me, Admiral. – Darth Vader

1. Overview and Goals

In this lab we will use CCD images to measure the positions of stars relative to the celestial coordinate system and determine proper motions (= the apparent motion of a celestial object) of asteroids. We will use data from the Dunlap Institute telescope (DIT), which was a 50-cm robotic telescope located on Mt. Joy in the Sacramento Mountains of New Mexico, U.S.A. The purpose of this activity is to learn astronomical skills that we will apply to track the position of asteroids (or minor planets). For this lab we will use data that is already posted on the webpage so that we can get off to a quick start and determine the properties of the imaging camera and CCD of DIT. You will learn how to query professional astronomical databases and conduct affine transformation.

Schedule: This is a five-week lab between November 1 and December 6. There will be no class on November 8 during the reading week. Group-led discussions will happen on November 15, 22, and 29. (Check the schedule of Group-led discussion.)

The following sections of this document provide a step-by-step description of: raw astronomical CCD images; CCD calibration and correction; measurement of star positions and comparison to standard star catalogs; and finally, measurement of the position of an asteroid.

Lab 3: Astrometry

2. Key steps

As listed below, there are 10 key steps for this assignment of determining proper motions (= motions of a celestial object on sky) of the asteroid **26-Proserpina**. Before working on the data for the asteroid, however, you need to use data for **NGC 7331** galaxy image in order to learn steps 1–8. And then you can reduce the asteroid data (**26-Proserpina**) with the same steps and extend the analysis with steps 9 and10 to find the associated proper motion of the asteroid.

The key steps are:

1) Download the DIT CCD FITS files. Read/display them using the Python PyFITS library (§3.1);
2) Reduce the data by applying systematic corrections including dark and flat field (§3.3);
3) Compare your image with the digital sky survey (using ALADIN) to confirm field is correct (§4);
4) Measure the positions of the stars in your CCD image (§4.1);
5) Cross-correlate the stars in your image with those in the USNO (US Naval Observatory) B-1.0 star catalog (§4.2);
6) Compute "standard coordinates" for the USNO B-1.0 stars and match the two lists (§4.3);
7) Perform a linear least squares fit to find the "plate constants" (§5);
8) Examine the residuals between the measurements and the fit;
9) Use your Python code to compute the position (and associated errors) of the asteroid;
10) Determine your observed asteroid's proper motion (i.e., arcsec per hour) and its measured error, and generate figure(s) that illustrates its observed motion on the plane of the sky.

Lab 3: Astrometry

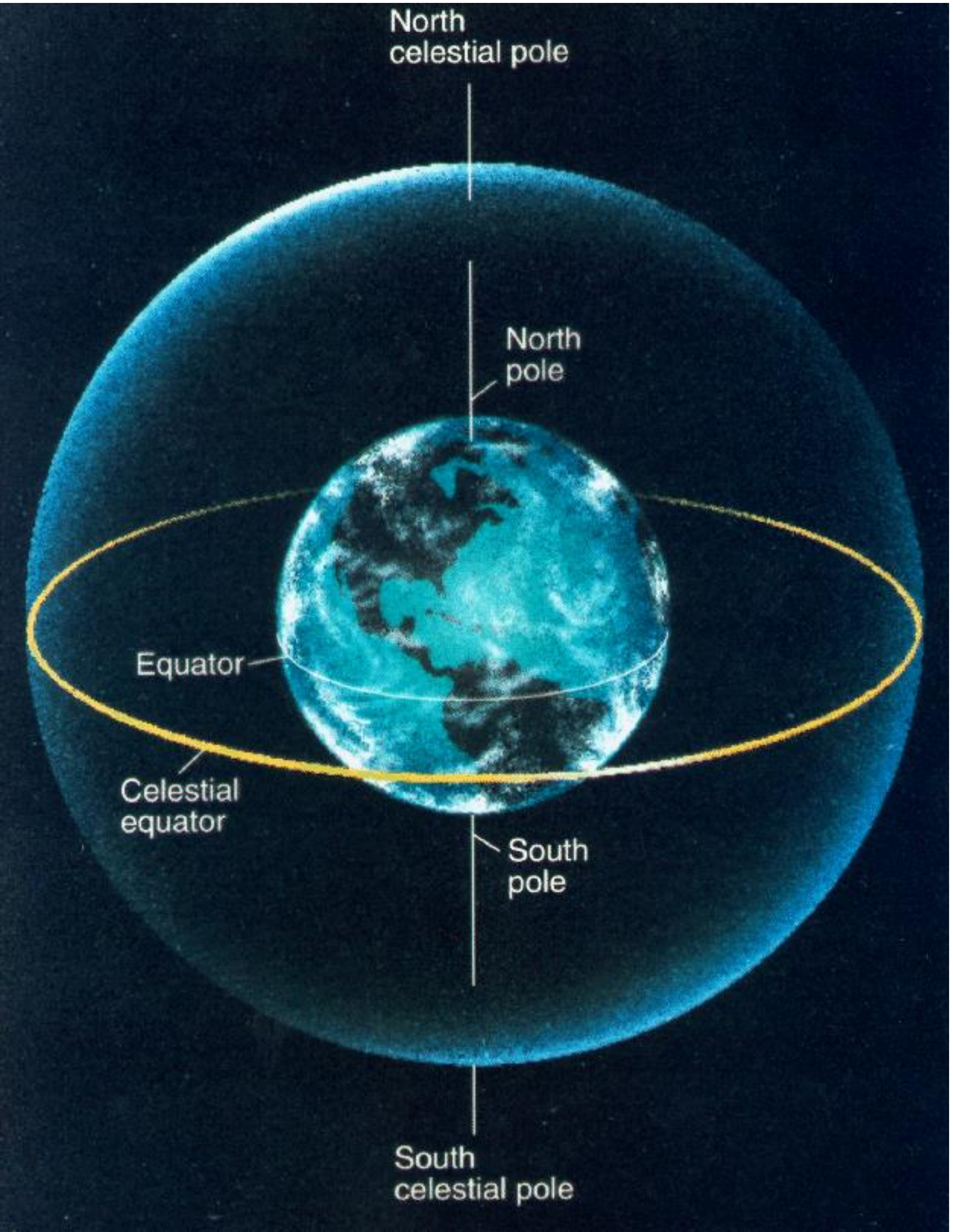
Basic CCD characteristics and operations are in previous lectures.

The Lab #3 document has detailed instructions of how to conduct basic CCD data reduction. So follow the instructions to reduce the data.

We already know what FITS files are and how to use DS9 to examine the FITS file images.

Celestial Coordinate Systems

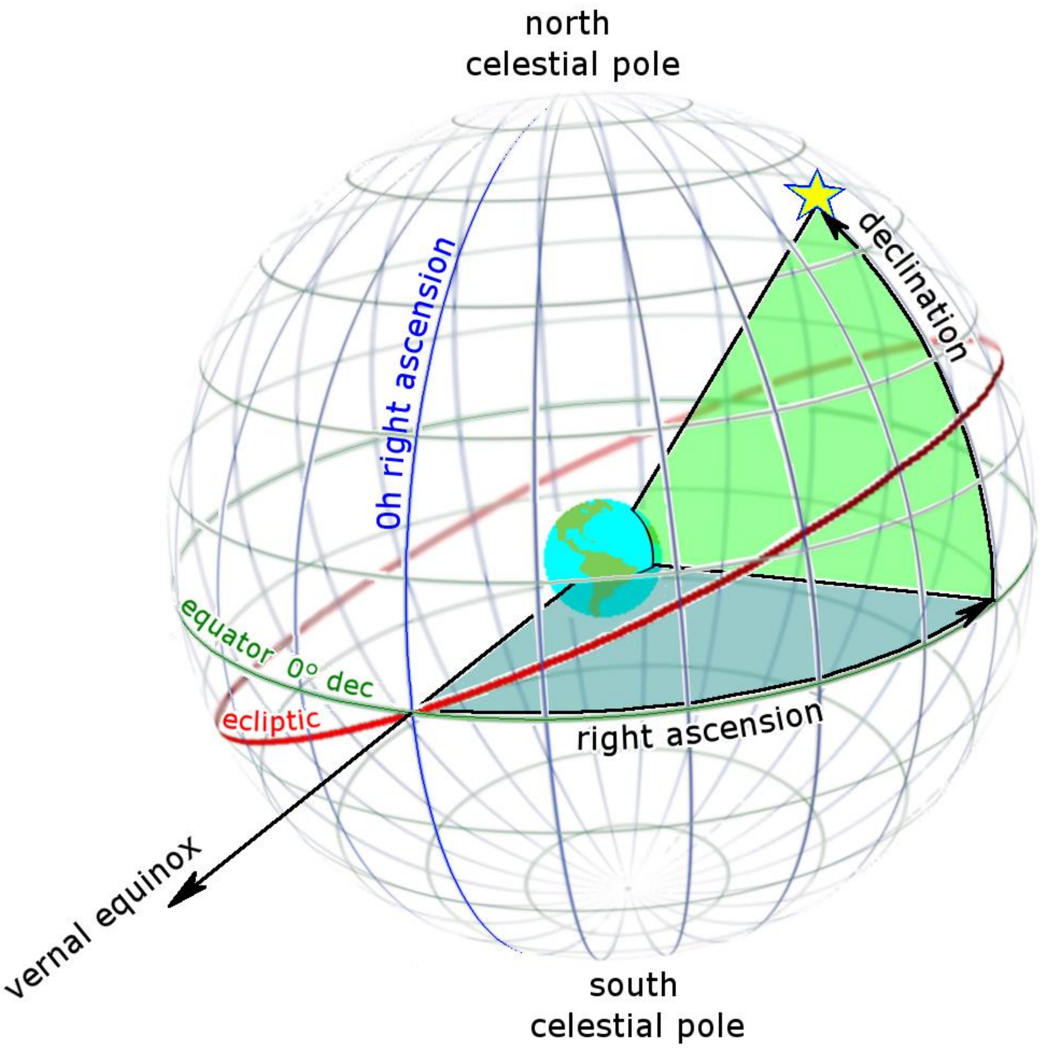
The Celestial Sphere: North and South Poles



The celestial equator is an extension of Earth's equator to the surface of the celestial sphere.

The north/south celestial pole is an extension of Earth's north/south pole.

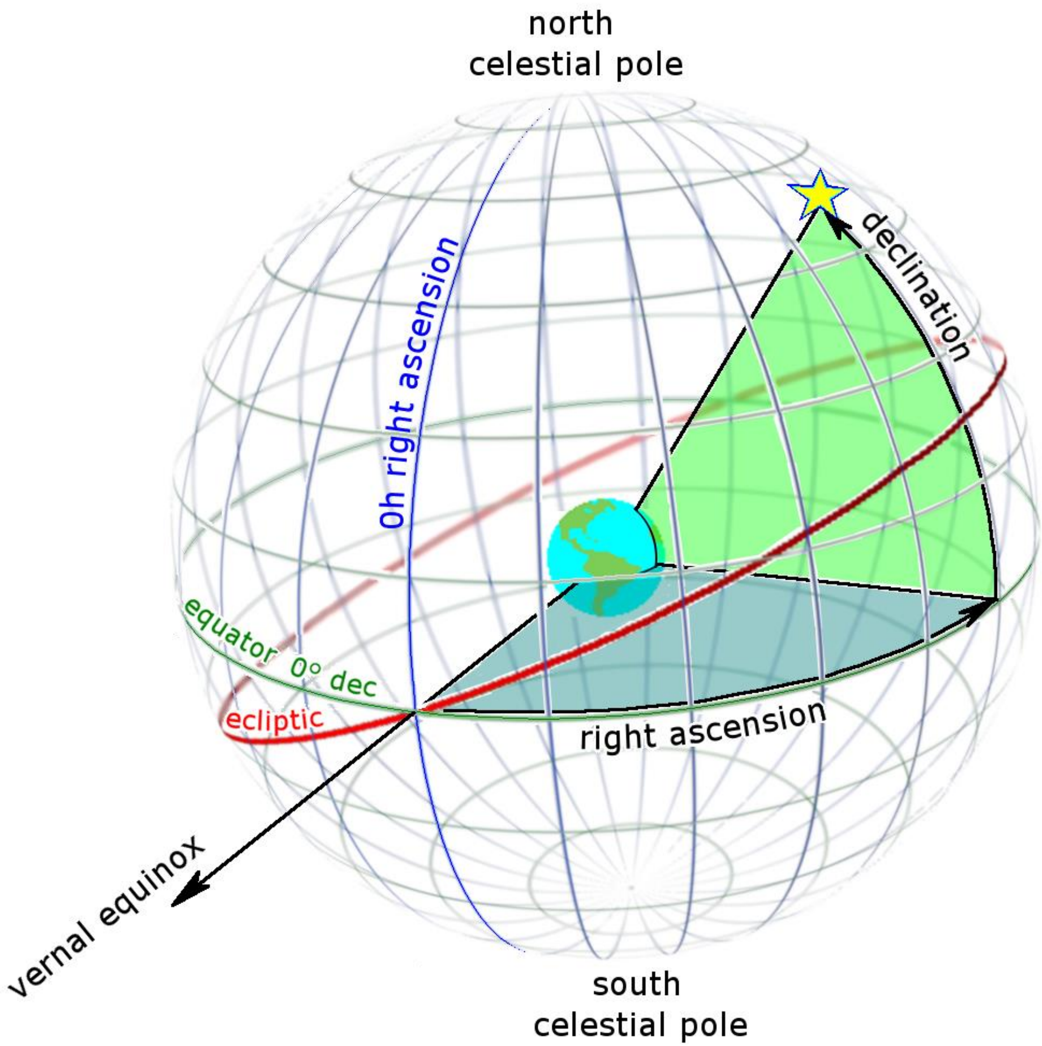
(R.A., DEC) and Vernal Equinox



Ecliptic Plane represents the motion of the Sun and solar system objects.

Vernal equinox is the ascending node of the Sun on the ecliptic plane when it meets with the celestial equator.

(R.A., DEC) and Vernal Equinox



Right Ascension (R.A.): extension of the longitude to the celestial sphere. It varies in the range of 0–360 degrees, or 0–24 hours. (So 1 hour is 15 degrees.)

Declination (DEC): extension of the latitude to the celestial sphere (vernal equinox). It varies between –90 degrees to +90 degrees.

(R.A., DEC) and Vernal Equinox

Angular Size:

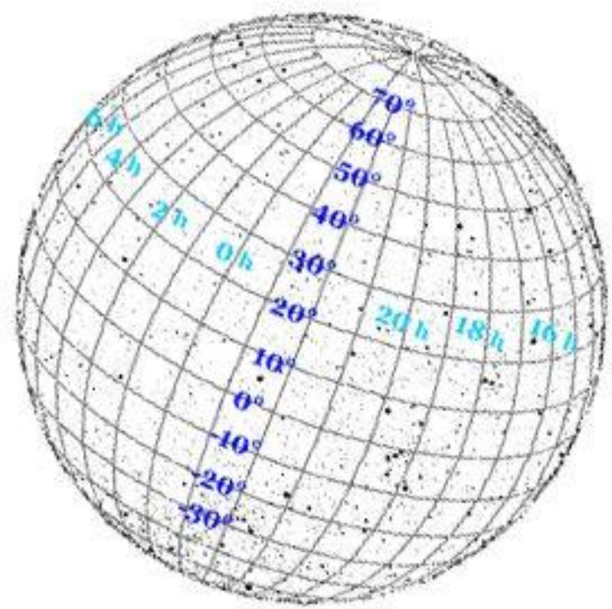
1 degree(°)= 60 arcminutes(′) = 3600 arcseconds(″)

Right Ascension:

Use both hour/minute/second and °/′/″ units.

1 hour = 15 degrees

1 second = 15 arcseconds



Example:

NGC 7331 Coordinate

(R.A., DEC) =

(22h37m04s, +34°24′55″)

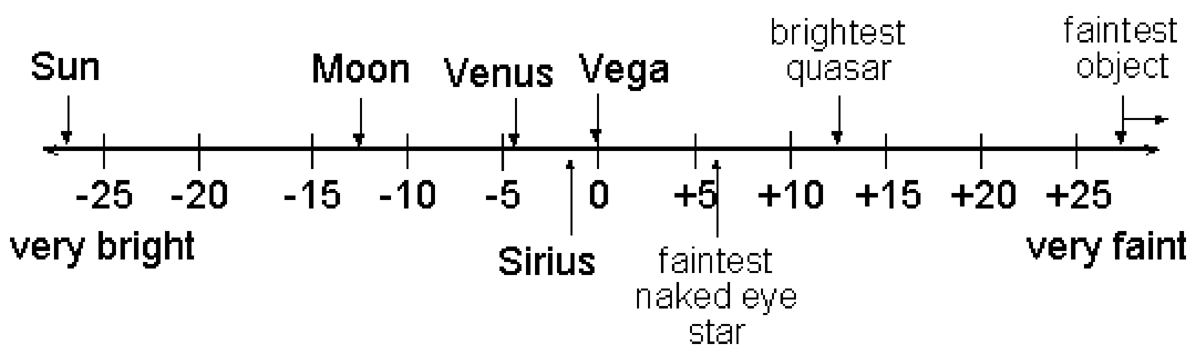
Astronomical Magnitude System

<u>Magnitude = –2.512 log (Object Flux) + Constant</u>

"Magnitude" represents object flux (= brightness) in logarithmic scale with a negative sign.

➔

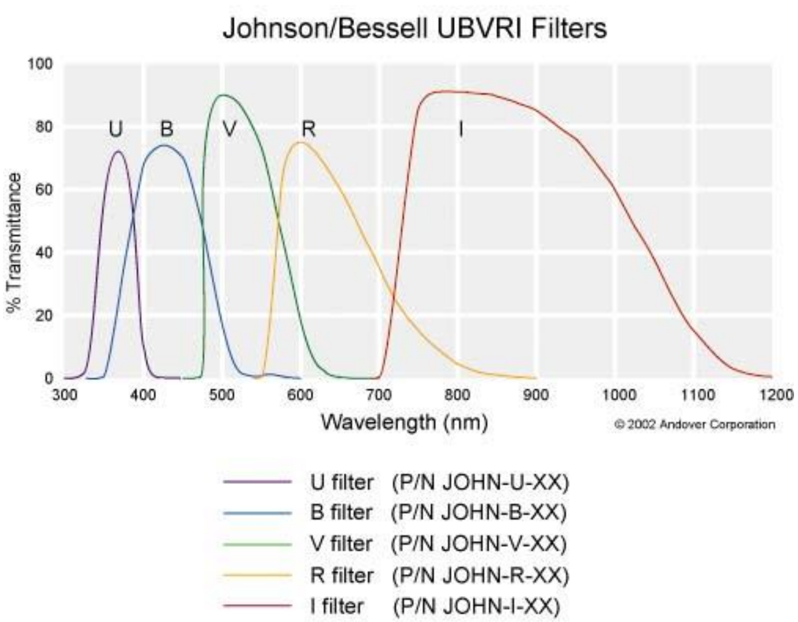
- A bright object has a smaller magnitude
- 1 magnitude difference is about 2.5 times difference in brightness (= flux)
- 5 magnitude difference is about 100 times difference in brightness since $2.512^5 = 100$.

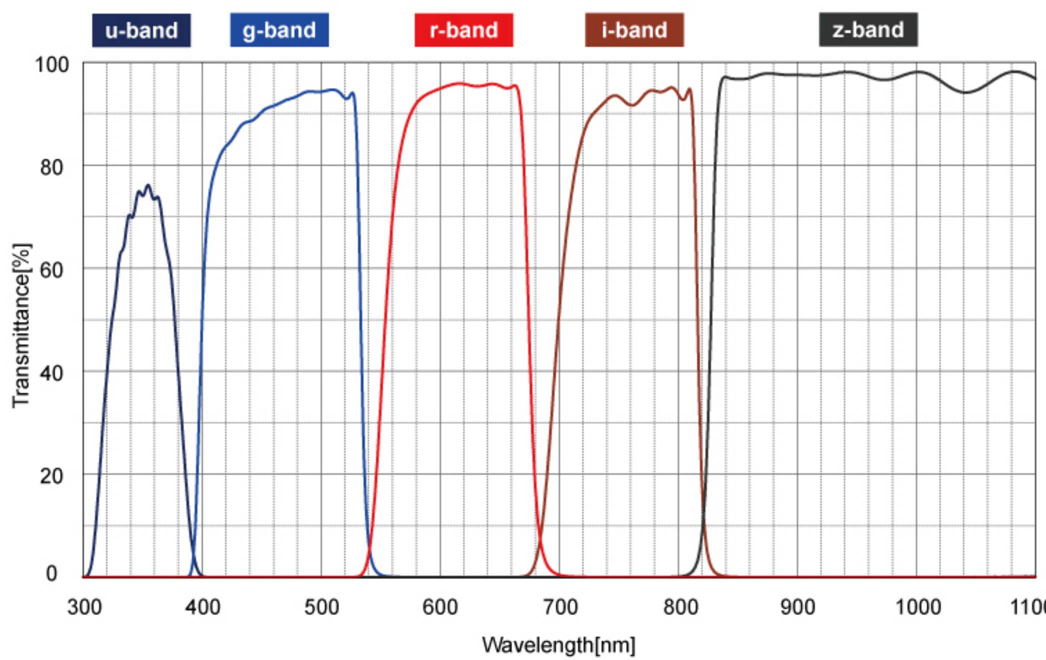


Apparent brightnesses of some objects in the magnitude system.

Astronomical Filter (= Band) Systems for CCDs

- CCD observations are made with filters located in front of the CCD.
- Filters transmit photons of certain wavelengths, determined by the filter transmission curve → we have a good idea about average wavelengths of photons detected (without dispersing the light).
- Examples below for the UBVRI & ugriz filter/band systems.
- Example: **r = 15 mag** usually means that the magnitude of an object is 15 when measured with the r-band filter.





Online Resources

6. Online Resources

The key online resource for this assignment is the **USNO B-1.0** catalog from which you can retrieve the coordinates and brightness (= magnitudes) of stars around your targets: NGC 7331 and 26 Proserpina. The catalog is available at

https://vizier.cds.unistra.fr/viz-bin/VizieR-3?-source=I/284/out&-out.max=50&-out.form=HTML%20Table&-out.add=_r&-out.add=_RAJ,_DEJ&-sort=_r&-oc.form=sexa

Note that you can access the catalog in your Python code as explained above and §9. More information about the USNO B-1.0 catalog can be found at

http://tdc-www.harvard.edu/software/catalogs/ub1.html

ALADIN is a simple and excellent tool to see images of any part of sky available at

https://aladin.u-strasbg.fr/aladin.gml

You can download ALADIN Desktop on your computer and check images around your targets. In addition, you can overlap the USNO B-1.0 catalog on the image. For example, type "NGC 7331" for Command, choose SDSS, J2000 (for Frame), and File-Catalog(VizieR)-I-Astrometric Data-USNO-B1.0 Catalog-Load.

The JPL/NASA maintains a few web pages relevant to asteroid, including

https://ssd.jpl.nasa.gov/tools/sbdb_lookup.html

for Small-Body Database Lookup where you can retrieve information about 26 Perserpina and

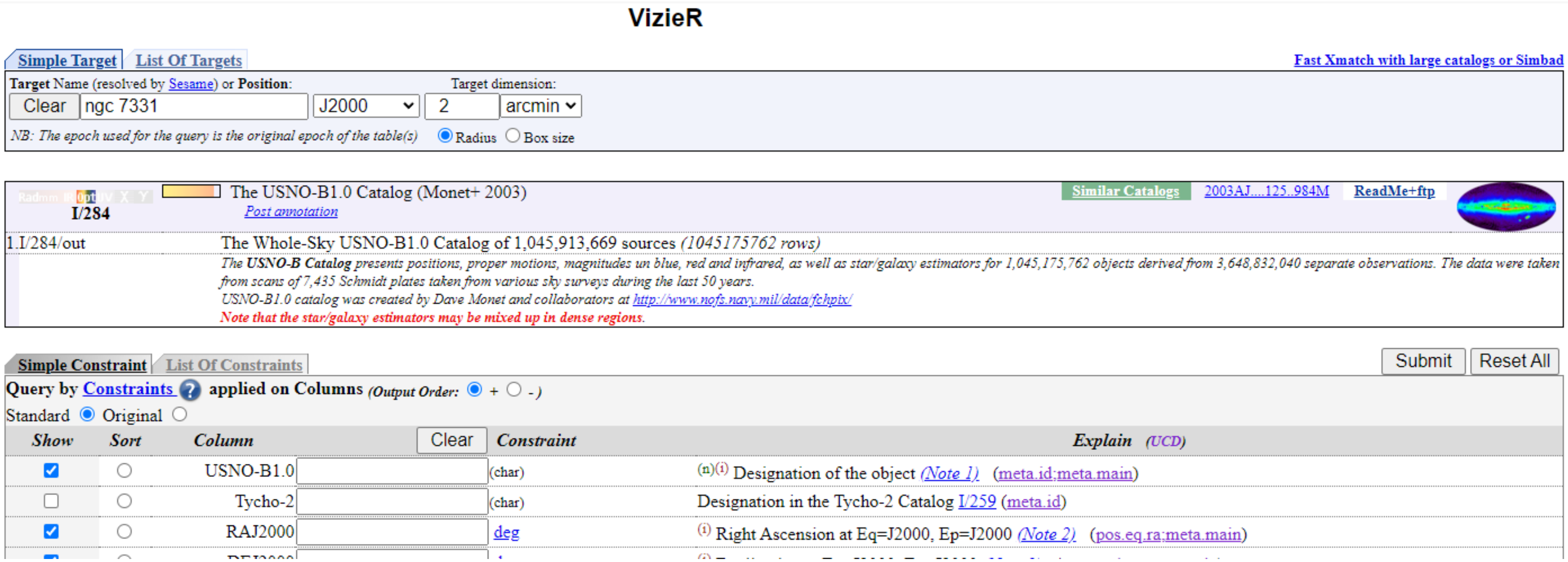
https://ssd.jpl.nasa.gov/tools/sbwobs.html

the observability of small bodies. The following link has a collection of these services:

https://ssd.jpl.nasa.gov/tools

Online Resources

USNO B-1.0 Catalog from VizieR web page.



Stars within 2 arcminute from NGC 7331 with R1mag > 15 (= R-band magnitude larger than 15) are queried.
You can do this within Python scripts (see the lab document).

I/284/out The USNO-B1.0 Catalog (Monet+ 2003) 2003AJ....125..984

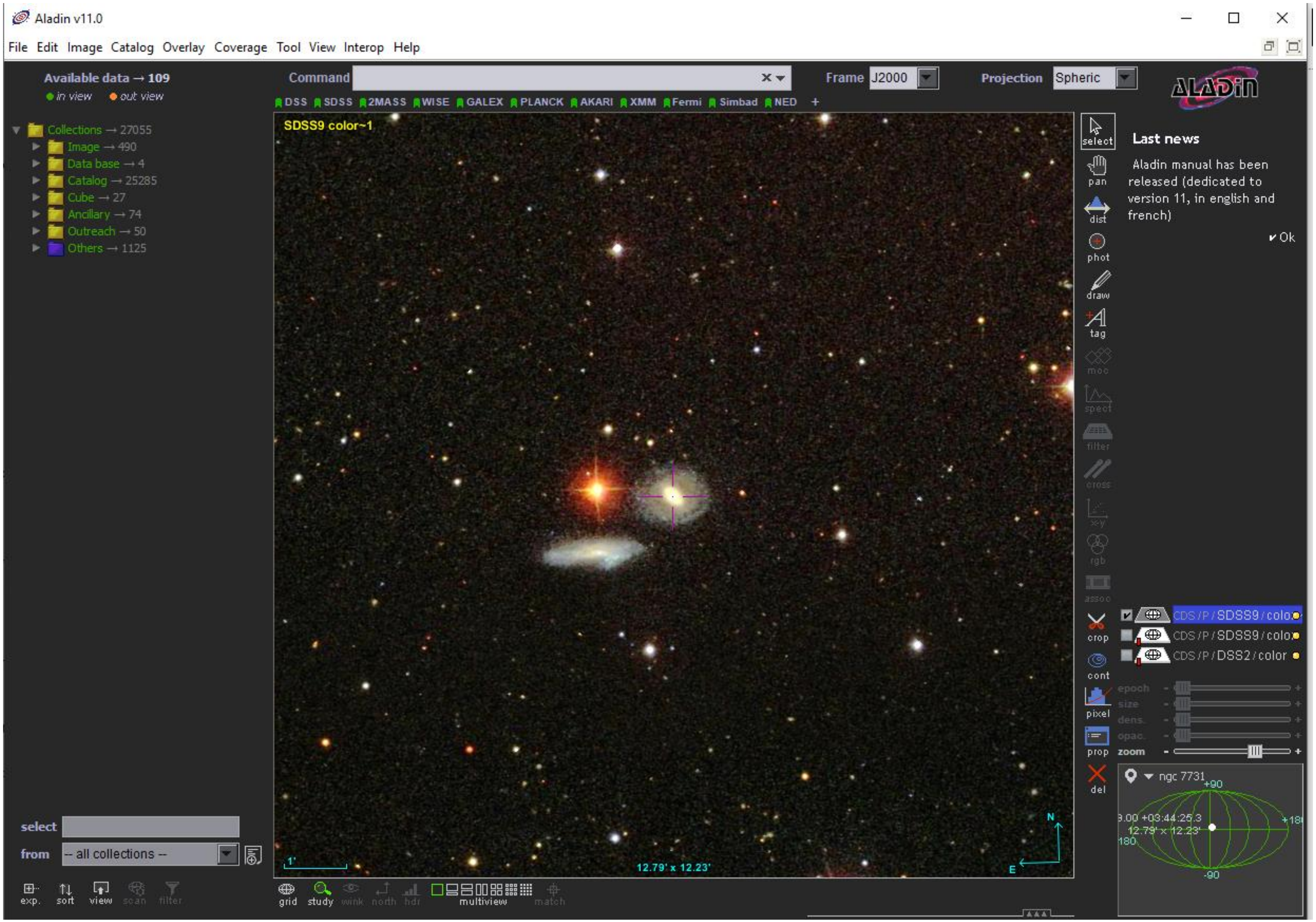
Post annotation The Whole-Sky USNO-B1.0 Catalog of 1,045,913,669 sources *(1045175762 rows)*

start AladinLite plot the output query using TAP/SQL

Full	_r arcmin	_RAJ2000 "h:m:s"	_DEJ2000 "d:m:s"	USNO-B1.0	RAJ2000 deg	DEJ2000 deg	e_RAJ2000 mas	e_DEJ2000 mas	Epoch yr	pmRA mas/yr	pmDE mas/yr	Ndet	B1mag mag	R1mag mag	B2mag mag	R2mag mag	Imag mag
1	0.9599	22 37 00.60336	+34 24 19.3320	1244-0510878	339.252514	+34.405370	166	160	1975.6	0	0	5	16.29	15.31	12.35	12.20	11.14
2	0.9979	22 36 59.29344	+34 25 03.9612	1244-0510867	339.247056	+34.417767	85	41	1976.0	0	0	5	18.29	16.84	18.55	18.14	17.07
3	1.0620	22 36 59.18736	+34 24 38.3004	1244-0510865	339.246614	+34.410639	187	78	1976.0	0	0	5	19.77	16.84	18.62	17.20	16.50
4	1.1795	22 37 09.31272	+34 25 26.4900	1244-0510926	339.288803	+34.424025	111	999	1971.6	0	0	2		17.22	17.35		
5	1.2206	22 37 09.29016	+34 25 32.5524	1244-0510925	339.288709	+34.425709	999	745	1971.6	0	0	2		17.93	17.26		
6	1.3690	22 36 59.40744	+34 23 59.2404	1243-0510547	339.247531	+34.399789	79	66	1976.0	0	0	5	20.09	17.68	20.17	18.10	17.57
7	1.4098	22 37 08.96472	+34 25 56.7624	1244-0510921	339.287353	+34.432434	224	72	1975.6	0	0	5	18.92	17.69	13.96	16.97	17.14
8	1.4458	22 37 06.64608	+34 26 18.1428	1244-0510915	339.277692	+34.438373	342	780	1953.6	0	0	2	15.79	15.35			
9	1.4937	22 37 11.18472	+34 24 38.6028	1244-0510935	339.296603	+34.410723	89	49	1976.0	0	0	5	17.93	16.69	17.58	16.75	16.25

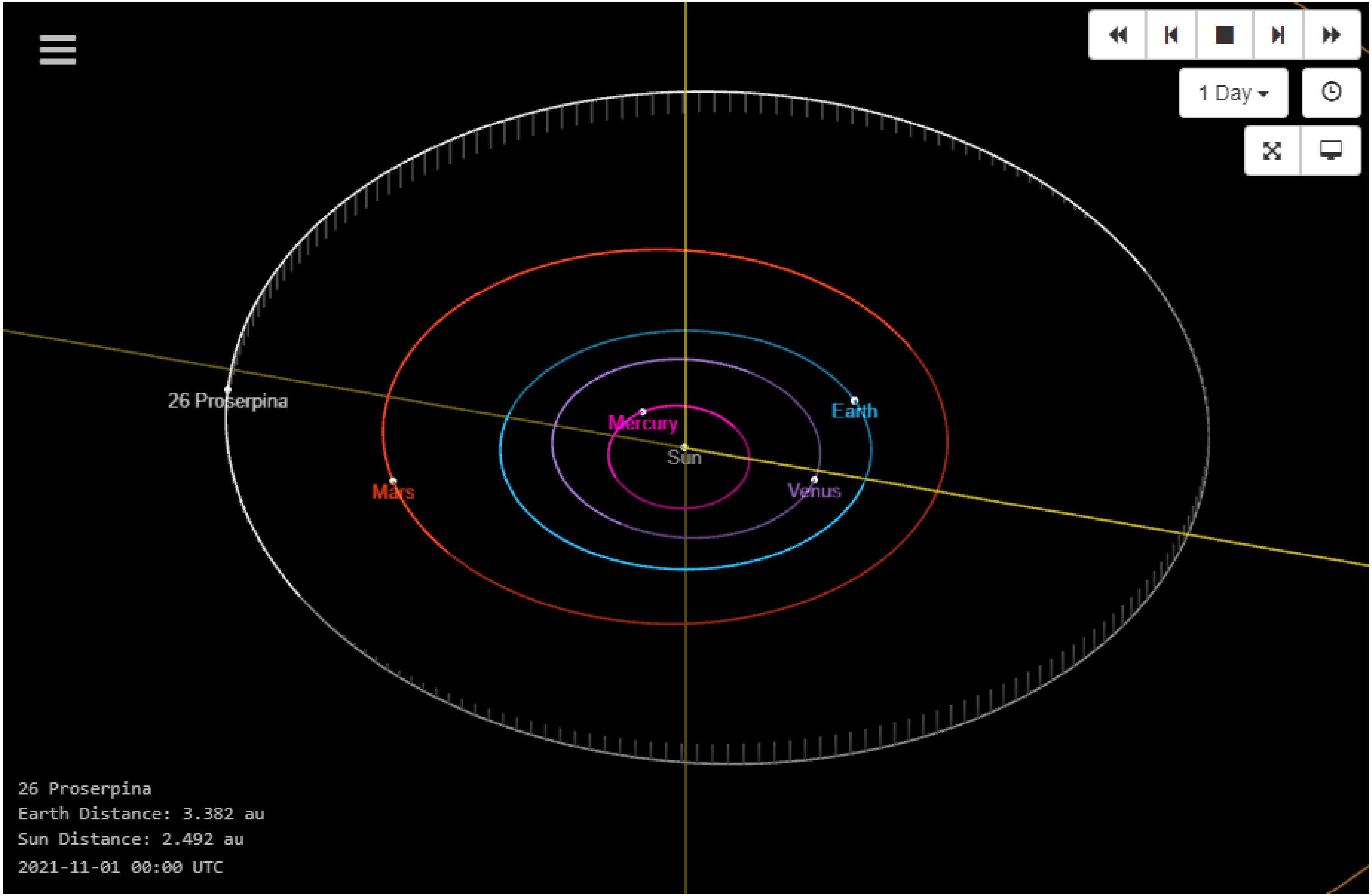
Online Resources

ALADIN Desktop (download and run)



Online Resources

JPL/NASA Small-Body Database Lookup



2D Astrometric Mapping Solution

It's between the coordinates of stars (X, Y) and their pixel positions (x, y) on CCD images.

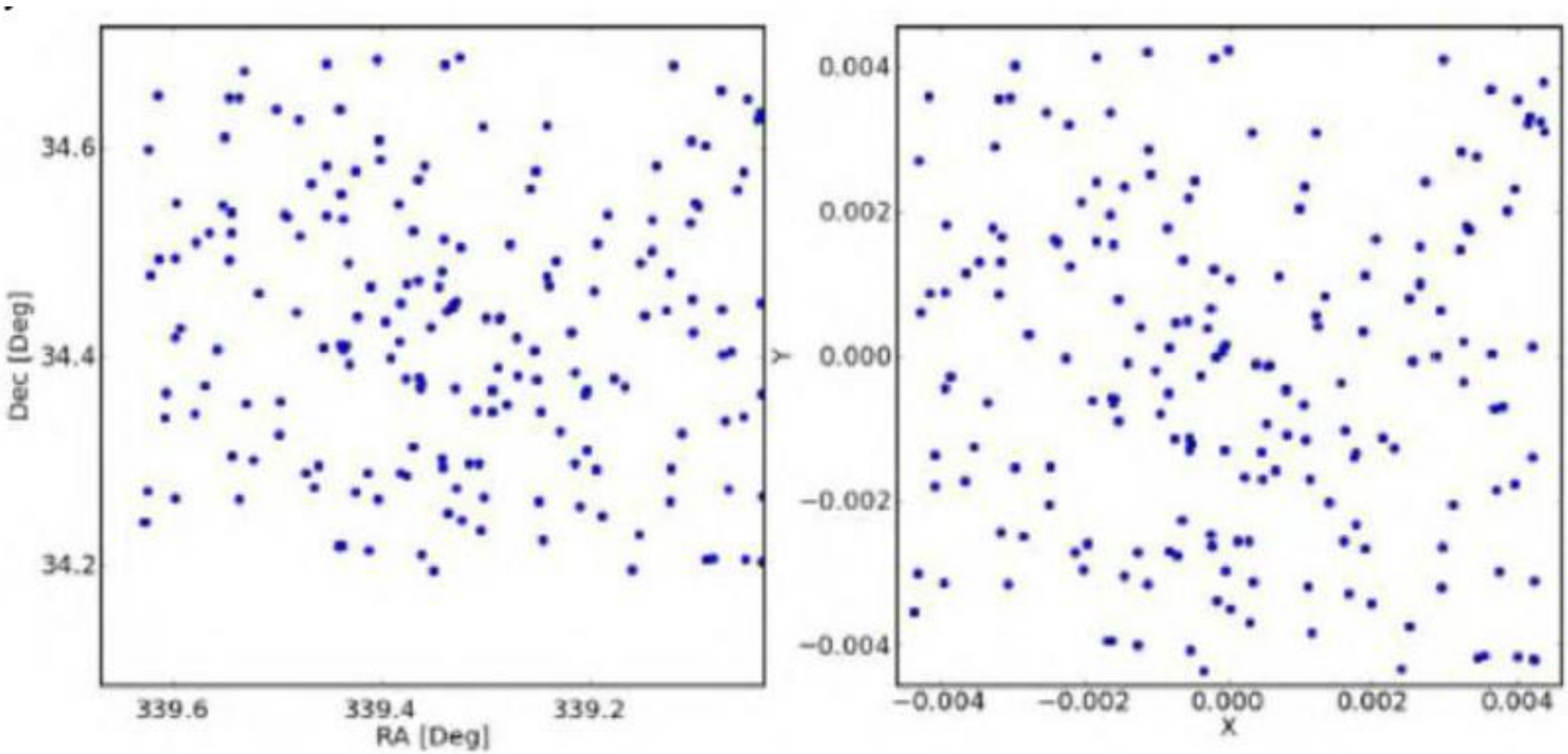
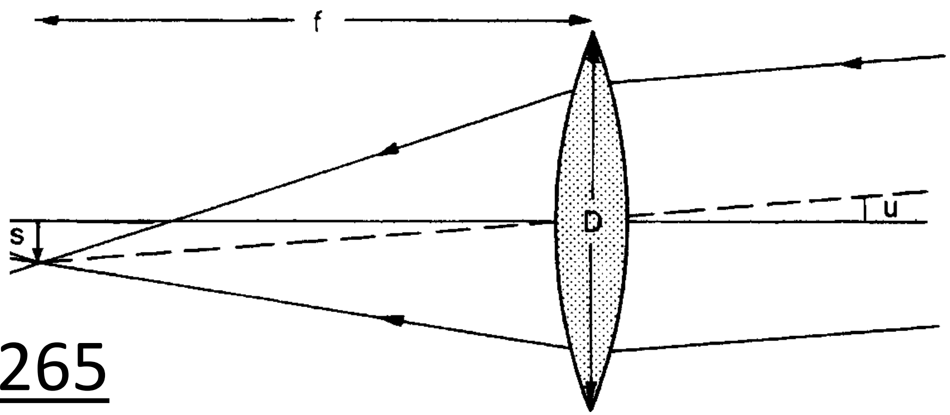


Figure 9: (Left) *The catalog positions of 199 USNO B-1.0 stars within a 30 × 30 arcminute square field centered at the position of NGC 7331 (RA = 339°.325; DEC= +34°.444 2000.0) with r magnitude brighter than 14.5 mag.* (Right) *The same data plotted in standard coordinates (see Eq. 2)—the projection from celestial coordinates on the unit sphere tangent plane.*

Conversion between (X, Y) and (x, y)

Telescope Plate Scale: the relation between an angular distance on the sky and a physical distance in the telescopic focal plane (= CCD)

- D: diameter of the primary
- f: **focal length of telescope**
- f/# (focal ratio) = f/D
- u: angular distance on the sky in arcsec(″)
- s: linear distance at the focal plane



$$\underline{s = f \times u = (f/\#) \times D \times u('')/206265}$$

→ If the focal length is 100 m,
it gives a plate scale of ~ 2″/mm (or 0.5 mm/″)
Note: 1 radian = 206265″, tan u ≈ u if u << 1 (u in radian)

<u>(Example)</u> The Palomar 5 meter telescope has the focal ratio of f/16. Using the above equation, we can obtain the plate scale of 2.56″/mm (or 0.388 mm/″). If its CCD has 2048 pixels and each pixel is 18 micron size, the field of view is 1.8 arcminute.

Conversion between (X, Y) and (x, y)

- The CCD has plate scale (pixel per arcseconds) that is defined by the focal length (f) of the camera and the size of the CCD pixels (p)
- For an "ideal" camera and detector the x, y position on the detector can be found by,

$$x = f(X/p) + x_0 \qquad \Leftarrow \qquad \Delta x \times p = f \times X$$

$$y = f(Y/p) + y_0 \qquad\qquad \Delta y \times p = f \times Y$$

- Of course a number of factors make this non-ideal with optical distortion, anamorphic magnification and the CCD rotation on sky

Conversion between (X, Y) and (x, y)

In reality, there is a rotation between (X, Y) and (x, y) planes (see §5 of the Lab document):

$$x = \frac{f}{p}(X\cos\theta - Y\sin\theta) + x_0$$
$$y = \frac{f}{p}(X\sin\theta + Y\cos\theta) + y_0$$
,

The constants a_{ij} refer to the scale, shear, and orientation of the image, and x_0 and y_0 are pointing offsets in pixels.

In matrix form, we have

$$\mathbf{x} = \mathbf{TX}$$

where

$$\mathbf{T} = \begin{pmatrix} (f/p)a_{11} & (f/p)a_{12} & x_0 \\ (f/p)a_{21} & (f/p)a_{22} & y_0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\mathbf{X} = (X, Y, 1)$ and $\mathbf{x} = (x, y, 1)$.

Conversion between (X, Y) and (x, y)

$$\mathbf{T} = \begin{pmatrix} (f/p)a_{11} & (f/p)a_{12} & x_0 \\ (f/p)a_{21} & (f/p)a_{22} & y_0 \\ 0 & 0 & 1 \end{pmatrix}$$

The plate constants $\boldsymbol{a}_{\mathbf{ij}}$ refer to the **scale**, **shear**, and **orientation** of the image, and x_0 and y_0 are pointing offsets in pixels.

Obtaining a 2D astrometric mapping solution is the determination of the plate constants.

Conversion between (X, Y) and (x, y)

Affine Transformation **(from MathWorld)**: An affine transformation is any transformation that preserves collinearity (i.e., all points lying on a line initially still lie on a line after transformation) and ratios of distances (e.g., the midpoint of a line segment remains the midpoint after transformation). In this sense, affine indicates a special class of projective transformations that do not move any objects from the affine space to the plane at infinity or conversely. Geometric contraction, expansion, dilation, reflection, rotation, shear, similarity transformations, spiral similarities, and translation are all affine transformations, as are their combinations. In general, an affine transformation is a composition of rotations, translations, dilations, and shears.

Examples of Affine Transformation in matrix form:

Scaling (factors $a, b \neq 0$): $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax \\ by \end{pmatrix}$

Reflection (in line $y = x$): $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ x \end{pmatrix}$

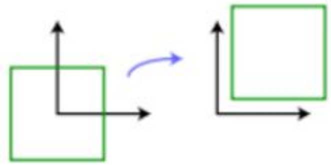
Shearing (x-dir., factor s): $\begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + sy \\ y \end{pmatrix}$

Rotation (countercl., $45°$): $\frac{\sqrt{2}}{2} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \frac{\sqrt{2}}{2} \begin{pmatrix} x - y \\ x + y \end{pmatrix}$

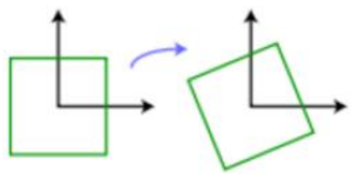
Conversion between (X, Y) and (x, y)

Examples of Affine Transformation in matrix form:

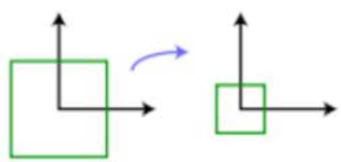
- **Translation** by vector $\vec{t}$: $\bar{p}_1 = \bar{p}_0 + \vec{t}$.



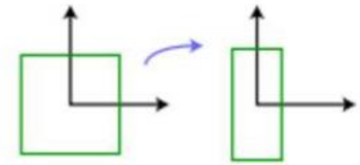
- **Rotation** counterclockwise by θ: $\bar{p}_1 = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \bar{p}_0$.



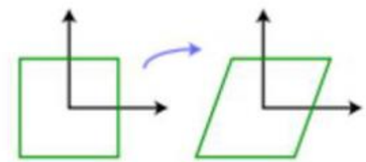
- **Uniform scaling** by scalar a: $\bar{p}_1 = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} \bar{p}_0$.



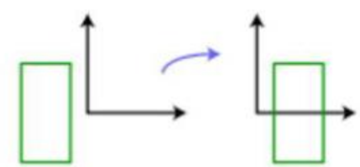
- **Nonuniform scaling** by a and b: $\bar{p}_1 = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \bar{p}_0$.



- **Shear** by scalar h: $\bar{p}_1 = \begin{bmatrix} 1 & h \\ 0 & 1 \end{bmatrix} \bar{p}_0$.



- **Reflection** about the y-axis: $\bar{p}_1 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \bar{p}_0$.



Schedule

Group-led Discussion Schedule (under development)

Date	Groups	Minimum Progress Milestones
Oct 18	A, B, C	Wavelength solution of 1-d Neon spectrum
Oct 25	D, E, F, G	Blackbody temperature; Wavelength solution of 2-d OH skyline spectrum
Nov 15	H, I, J	To Step 6 with NGC 7331 data
Nov 22	K, L, M	To Step 8 with NGC 7331 data
Nov 29	N, O, P, Q	To Step 9 with 26 Proserpina data

It always takes longer than expected to complete practical assignment and a paper style report.

So start early!!!