Computer Systems

Week 01 - Number systems

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- Number systems
- Decimal to Binary Conversion.
- Operations on binary number.
- Signed numbers.
- Arithmetic Operations with Signed Numbers

- ***Lab 01:*** *Conversion between number systems.*

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Number system

Objectives

- Review the decimal number system
- Count in the binary number system
- Convert from decimal to binary and from binary to decimal
- Apply arithmetic operations to binary numbers
- Determine the 1's and 2's complements of a binary number
- Express signed binary numbers in sign-magnitude, 1's complement, 2's complement, and floating-point format
- Carry out arithmetic operations with signed binary numbers

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Contents

- Convert between the binary and hexadecimal number systems
- Add numbers in hexadecimal form
- Convert between the binary and octal number systems

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Decimal Numbers

The position of each digit in a weighted number system is assigned a weight based on the **base** or **radix** of the system. The radix of decimal numbers is ten, because only ten symbols (0 through 9) are used to represent any number.

The column weights of decimal numbers are powers of ten that increase from right to left beginning with $10^0 = 1$:

…10^5 10^4 10^3 10^2 10^1 10^0.

For fractional decimal numbers, the column weights are negative powers of ten that decrease from left to right:

10^2 10^1 10^0. 10^{-1} 10^{-2} 10^{-3} 10^{-4} …

Decimal Numbers

Decimal numbers can be expressed as the sum of the products of each digit times the column value for that digit. Thus, the number 9240 can be expressed as

$$(9 \times 10^3) + (2 \times 10^2) + (4 \times 10^1) + (0 \times 10^0)$$

or

$$9 \times 1{,}000 + 2 \times 100 + 4 \times 10 + 0 \times 1$$

Example

Express the number 480.52 as the sum of values of each digit.

Solution

$$480.52 = (4 \times 10^2) + (8 \times 10^1) + (0 \times 10^0) + (5 \times 10^{-1}) + (2 \times 10^{-2})$$

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Binary Numbers

For digital systems, the binary number system is used. Binary has a radix of two and uses the digits 0 and 1 to represent quantities.

The column weights of binary numbers are powers of two that increase from right to left beginning with $2^0 = 1$:

$\ldots 2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0.$

For fractional binary numbers, the column weights are negative powers of two that decrease from left to right:

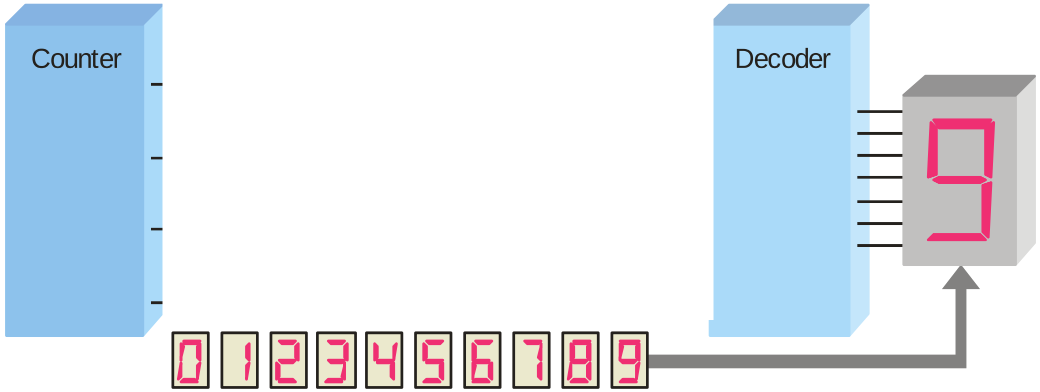
$2^2\ 2^1\ 2^0.\ 2^{-1}\ 2^{-2}\ 2^{-3}\ 2^{-4} \ldots$

Binary Numbers

A binary counting sequence for numbers from zero to fifteen is shown.

Notice the pattern of zeros and ones in each column.

Digital counters frequently have this same pattern of digits:



Decimal Number	Binary Number
0	0 0 0 0
1	0 0 0 1
2	0 0 1 0
3	0 0 1 1
4	0 1 0 0
5	0 1 0 1
6	0 1 1 0
7	0 1 1 1
8	1 0 0 0
9	1 0 0 1
10	1 0 1 0
11	1 0 1 1
12	1 1 0 0
13	1 1 0 1
14	1 1 1 0
15	1 1 1 1

Binary Conversions

The decimal equivalent of a binary number can be determined by adding the column values of all of the bits that are 1 and discarding all of the bits that are 0.

Example Convert the binary number 100101.01 to decimal.

Solution Start by writing the column weights; then add the weights that correspond to each 1 in the number.

2^5	2^4	2^3	2^2	2^1	$2^0.$	2^{-1}	2^{-2}
32	16	8	4	2	1 .	½	¼
1	0	0	1	0	1.	0	1
32			+4		+1		+¼ = 37¼

Binary Conversions

You can convert a decimal whole number to binary by reversing the procedure. Write the decimal weight of each column and place 1's in the columns that sum to the decimal number.

Example Convert the decimal number 49 to binary.

Solution The column weights double in each position to the right. Write down column weights until the last number is larger than the one you want to convert.

2^6 2^5 2^4 2^3 2^2 2^1 2^0.

64 32 16 8 4 2 1.

0 1 1 0 0 0 1.

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Binary Conversions

You can convert a decimal fraction to binary by repeatedly multiplying the fractional results of successive multiplications by 2. The carries form the binary number.

Example Convert the decimal fraction 0.188 to binary by repeatedly multiplying the fractional results by 2.

Solution

0.188 x 2 = 0.376	carry = 0	MSB
0.376 x 2 = 0.752	carry = 0	
0.752 x 2 = 1.504	carry = 1	
0.504 x 2 = 1.008	carry = 1	
0.008 x 2 = 0.016	carry = 0	

Answer = .00110 (for five significant digits)

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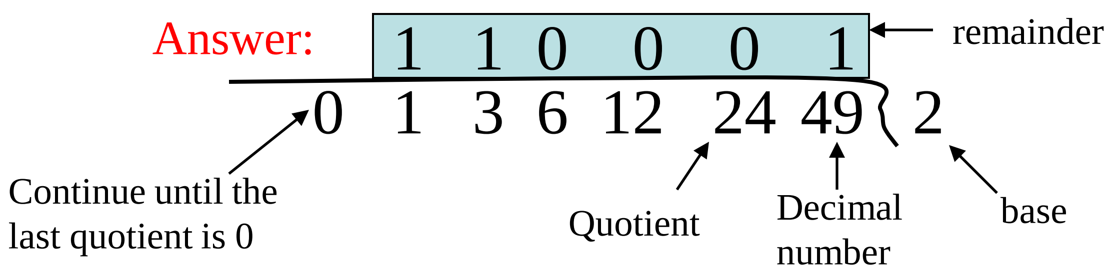
Binary Conversions

You can convert decimal to any other base by repeatedly dividing by the base. For binary, repeatedly divide by 2:

Example Convert the decimal number 49 to binary by repeatedly dividing by 2.

Solution You can do this by "reverse division" and the answer will read from left to right. Put quotients to the left and remainders on top.



Binary Addition

The rules for binary addition are

0 + 0 = 0	Sum = 0, carry = 0
0 + 1 = 1	Sum = 1, carry = 0
1 + 0 = 1	Sum = 1, carry = 0
1 + 1 = 10	Sum = 0, carry = 1

When an input carry = 1 due to a previous result, the rules are

1 + 0 + 0 = 01	Sum = 1, carry = 0
1 + 0 + 1 = 10	Sum = 0, carry = 1
1 + 1 + 0 = 10	Sum = 0, carry = 1
1 + 1 + 1 = 11	Sum = 1, carry = 1

Binary Addition

Example Add the binary numbers 00111 and 10101 and show the equivalent decimal addition.

Solution

$$\begin{array}{r@{\quad}r} {\color{red}{0\,1\,1\,1}} & \\ 00111 & 7 \\ 10101 & 21 \\ \hline 11100 & {\color{red}{=}}\ 28 \end{array}$$

Binary Subtraction

The rules for binary subtraction are

$$0 - 0 = 0$$
$$1 - 1 = 0$$
$$1 - 0 = 1$$
$$10 - 1 = 1 \quad \text{with a borrow of 1}$$

Example Subtract the binary number 00111 from 10101 and show the equivalent decimal subtraction.

Solution

$$\begin{array}{rcr} {\scriptstyle 1\,1\,1\,} & & \\ 10101 & & 21 \\ \underline{00111} & & \underline{7} \\ 01110 & = & 14 \end{array}$$

Binary Multiplication

The four basic rules for multiplying bits are as follows:

$$0 \times 0 = 0$$
$$0 \times 1 = 0$$
$$1 \times 0 = 0$$
$$1 \times 1 = 1$$

Example Multiplication the binary number 101 x 111

Solution

```
    111            7
  x 101          x 5
  -----          ---
    111           35
   000
 + 111
 ------
 100011
```

Binary Division

Division in binary follows the same procedure as division in decimal.

Examples Perform the following binary divisions:

(a) $110 \div 11$ **(b)** $110 \div 10$

Solution

(a)
$$\begin{array}{r} \mathbf{10} \\ 11\overline{)110} \\ \underline{11} \\ 000 \end{array} \qquad \begin{array}{r} 2 \\ 3\overline{)6} \\ \underline{6} \\ 0 \end{array}$$

(b)
$$\begin{array}{r} \mathbf{11} \\ 10\overline{)110} \\ \underline{10} \\ 10 \\ \underline{10} \\ 00 \end{array} \qquad \begin{array}{r} 3 \\ 2\overline{)6} \\ \underline{6} \\ 0 \end{array}$$

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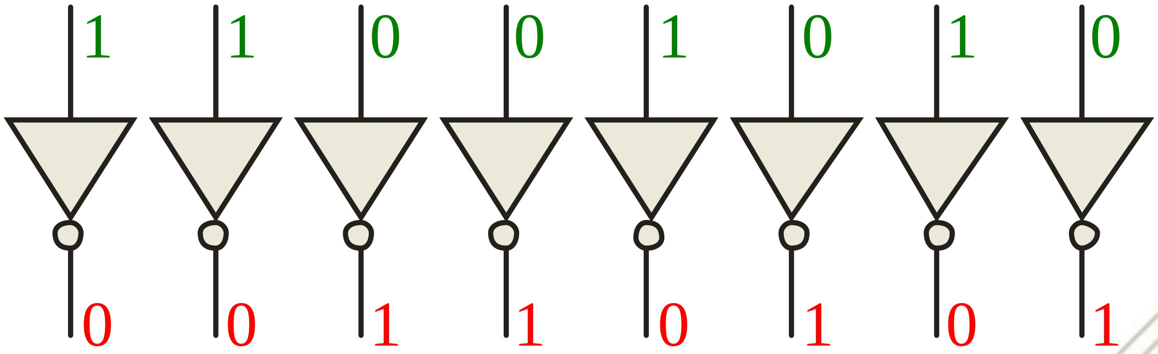
1's Complement

The 1's complement of a binary number is just the inverse of the digits. To form the 1's complement, change all 0's to 1's and all 1's to 0's.

For example, the 1's complement of 11001010 is
00110101

In digital circuits, the 1's complement is formed by using inverters:



2's Complement

The 2's complement of a binary number is found by adding 1 to the LSB of the 1's complement.

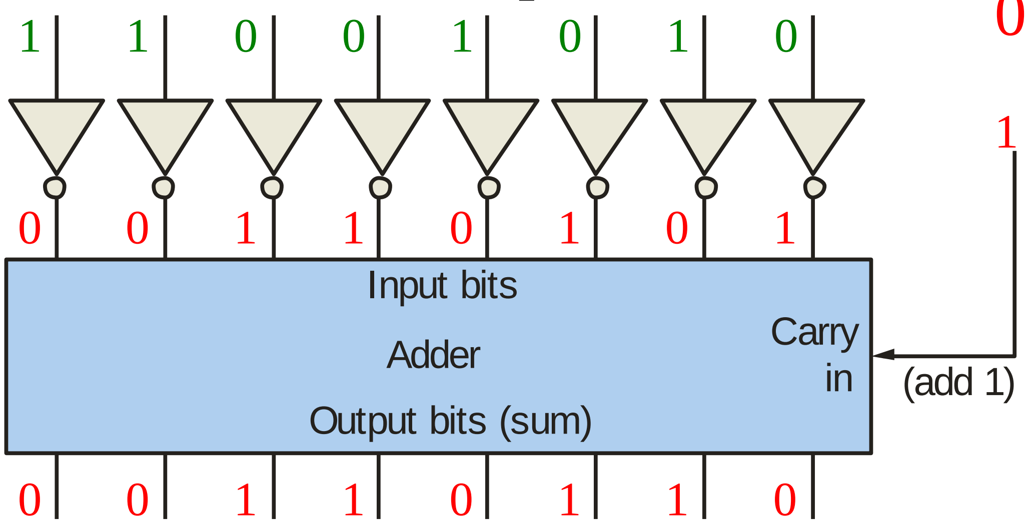
Recall that the 1's complement of 11001010 is

00110101 (1's complement)

To form the 2's complement, add 1:

+1

00110110 (2's complement)



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Signed Binary Numbers

There are several ways to represent signed binary numbers. In all cases, the MSB in a signed number is the sign bit, that tells you if the number is positive or negative.

Computers use a modified 2's complement for signed numbers. Positive numbers are stored in *true* form (with a 0 for the sign bit) and negative numbers are stored in *complement* form (with a 1 for the sign bit).

For example, the positive number 58 is written using 8-bits as 00111010 (true form).

Sign bit

Magnitude bits

Signed Binary Numbers

Negative numbers are written as the 2's complement of the corresponding positive number.

The negative number –58 is written as:

–58 = 11000110 (complement form)

Sign bit Magnitude bits

An easy way to read a signed number that uses this notation is to assign the sign bit a column weight of –128 (for an 8-bit number). Then add the column weights for the 1's.

Example Assuming that the sign bit = –128, show that 11000110 = –58 as a 2's complement signed number:

Solution Column weights: –128 64 32 16 8 4 2 1.

–128	64	32	16	8	4	2	1	
1	1	0	0	0	1	1	0	
–128	+64				+4	+2		= –58

Arithmetic Operations with Signed Numbers

Using the signed number notation with negative numbers in 2's complement form simplifies addition and subtraction of signed numbers.

Rules for **addition**: Add the two signed numbers. Discard any final carries. The result is in signed form.

Examples:

00011110 = +30	00001110 = +14	11111111 = –1
00001111 = +15	11101111 = –17	11111000 = –8
00101101 = +45	11111101 = –3	~~1~~ 11110111 = –9

Discard carry

Arithmetic Operations with Signed Numbers

Note that if the number of bits required for the answer is exceeded, overflow will occur. This occurs only if both numbers have the same sign. The overflow will be indicated by an incorrect sign bit.

Two examples are:

01000000 = +128
01000001 = +129
10000001 = ~~–126~~

10000001 = –127
10000001 = –127
Discard carry → ~~1~~00000010 = ~~+2~~

Wrong! The answer is incorrect and the sign bit has changed.

Arithmetic Operations with Signed Numbers

Rules for **subtraction**: 2's complement the subtrahend and add the numbers. Discard any final carries. The result is in signed form.

Repeat the examples done previously, but subtract:

$$\begin{array}{rl} 00011110 & (+30) \\ -\ \underline{00001111} & -(+15) \end{array} \qquad \begin{array}{rl} 00001110 & (+14) \\ -\ \underline{11101111} & -(-17) \end{array} \qquad \begin{array}{rl} 11111111 & (-1) \\ -\ \underline{11111000} & -(-8) \end{array}$$

2's complement subtrahend and add:

$$\begin{array}{rl} 00011110 & = +30 \\ \underline{11110001} & = -15 \\ \cancel{1}00001111 & = +15 \end{array} \qquad \begin{array}{rl} 00001110 & = +14 \\ \underline{00010001} & = +17 \\ 00011111 & = +31 \end{array} \qquad \begin{array}{rl} 11111111 & = -1 \\ \underline{00001000} & = +8 \\ \cancel{1}00000111 & = +7 \end{array}$$

Discard carry

Discard carry

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Hexadecimal Numbers

Hexadecimal uses sixteen characters to represent numbers: the numbers 0 through 9 and the alphabetic characters A through F.

Large binary number can easily be converted to hexadecimal by grouping bits 4 at a time and writing the equivalent hexadecimal character.

Example Express 1001 0110 0000 1110_2 in hexadecimal:

Solution Group the binary number by 4-bits starting from the right. Thus, 960E

Decimal	Hexadecimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
10	A	1010
11	B	1011
12	C	1100
13	D	1101
14	E	1110
15	F	1111

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Hexadecimal Numbers

Hexadecimal is a weighted number system. The column weights are powers of 16, which increase from right to left.

$$\text{Column weights} \begin{cases} 16^3 & 16^2 & 16^1 & 16^0. \\ 4096 & 256 & 16 & 1\ . \end{cases}$$

Decimal	Hexadecimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
10	A	1010
11	B	1011
12	C	1100
13	D	1101
14	E	1110
15	F	1111

Example Express $1A2F_{16}$ in decimal.

Solution Start by writing the column weights:

4096 256 16 1

1 A 2 F_{16}

$1(4096) + 10(256) + 2(16) + 15(1) = 6703_{10}$

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Octal Numbers

Octal uses eight characters the numbers 0 through 7 to represent numbers. There is no 8 or 9 character in octal.

Binary number can easily be converted to octal by grouping bits 3 at a time and writing the equivalent octal character for each group.

Decimal	Octal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	10	1000
9	11	1001
10	12	1010
11	13	1011
12	14	1100
13	15	1101
14	16	1110
15	17	1111

Example

Express 1 001 011 000 001 110_2 in octal:

Solution

Group the binary number by 3-bits starting from the right. Thus, 113016_8

Octal Numbers

Octal is also a weighted number system. The column weights are powers of 8, which increase from right to left.

Column weights	8^3	8^2	8^1	8^0	.
	512	64	8	1	.

Decimal	Octal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	10	1000
9	11	1001
10	12	1010
11	13	1011
12	14	1100
13	15	1101
14	16	1110
15	17	1111

Example Express 3702_8 in decimal.

Solution Start by writing the column weights:

512 64 8 1

3 7 0 2_8

$3(512) + 7(64) + 0(8) + 2(1) = 1986_{10}$

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Quiz

1. For the binary number 1000, the weight of the column with the 1 is

   a. 4

   b. 6

   c. 8

   d. 10

Quiz

2. The 2's complement of 1000 is

   a. 0111

   b. 1000

   c. 1001

   d. 1010

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Quiz

3. The fractional binary number 0.11 has a decimal value of

a. ¼

b. ½

c. ¾

d. none of the above

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Quiz

4. The hexadecimal number 2C has a decimal equivalent value of

   a. 14

   b. 44

   c. 64

   d. none of the above

Quiz

5. The octal number 72 has a decimal equivalent value of

  a. 58

  b. 64

  c. 78

  d. 92

Quiz

6. Convert hexadecimal 0xFE5C to Binary number

   a. 1111 1110 0101 1100

   b. 1111 1001 0101 1101

   c. 1110 0101 1001 1100

   d. 1111 1110 0001 0010

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Quiz

7. Sum of 0100 + 0101

   a. 1101

   b. 1010

   c. 1110

   d. 1001

Quiz

8. Subtraction binary 1110 – 0011

   a. 1010

   b. 1001

   c. 1100

   d. 1011

Quiz

9. Multiplication binary 0100 x 0111

   a. 11101

   b. 11100

   c. 11010

   d. 10011

Quiz

10. Presentation of -25 in binary with 8 bit

    a. 1100 0101

    b. 1111 0001

    c. 1110 0111

    d. 1110 1100

Quiz

Answers:

1. c	6. a
2. b	7. d
3. c	8. d
4. b	9. b
5. a	10. c

Thank you!

Any questions?