Project Proposal

Project Title: Production Planning in Manufacturing

Group Members: Fong Kirst (9075075516) Kelly He (9078291425)

1. Brief description of the problem. If you'll be using real data, where will you find it and how much will you need?

Production planning is essential for the management of manufacturing to produce the right number of products to satisfy customer demand over a specific time horizon and maximize profit. Our final project is aiming to match production and sourcing decisions to meet market demand subject to production capacity, workforce availability, and overtime restrictions. The objective of the problem is to maximize the profit or minimize the total cost.

We will solve this problem using two types of mathematical models:

a. Deterministic Production Planning Model

   In this approach, we will use the best guess of demand d_i for a period of time, i.e. assume the demand is given in a time t, to model and solve the production planning problem.

b. Stochastic Production Planning Model

   In this approach, the demand d_i in a period of time is uncertain. However, we are given a set of probabilities associated with demand to find a solution that is feasible for all or almost all the possible data and optimizes the expected performance of the model.

2. Type of model (LP, QP, MIP, etc.) and an approximate count of the number of variables and constraints in the model.

I. Deterministic Production Planning (LP & MIP)

a. Decision variables

   i. $x(t)$ is the regular production produced in a period time t

ii. $y(t)$ is the inventory level at the end of each period of time t

iii. $z\,(t) \in \{0, 1\}$

- 1 if production occurs in time t
- 0 otherwise

b. Parameters

i. $f(t)$ is the workforce cost of producing in time t

ii. $c(t)$ is the cost of a unit of production in time t

iii. $h(t)$ is the cost of the storage in time t

iv. C is the capacity of manufacturing

v. $d(t)$ is the demand of the product in time t

c. Constraints

i. Capacity constraint

$$x(t) \leq Cz(t) \qquad \forall t \in \{1, 2, \dots, n\}$$

ii. Nonnegativity and integer constraints

$$x(t),\ y(t) \geq 0, \text{ and } z(t) \in \{0, 1\} \qquad \forall t \in \{1, 2, \dots, n\}$$

iii. Storage balance constraint

$$y(t-1) + x(t) = d(t) + y(t) \qquad \forall t \in \{1, 2, \dots, n\}$$

d. Objective

Minimize

$$\sum_{t=1}^{n} c(t)x(t) + \sum_{1}^{n} f(t)z(t) + \sum_{1}^{n} h(t)y(t)$$

II. Stochastic Production Planning Model

a. Decision variables

i. $r_1, r_2, \dots, r_m$ are the amounts of raw materials that required to produce *n* different products, where $i = 1, 2, \dots, m$ and $r_i \geq 0$

ii. $q, q_2, \dots, q_n$ are the quantities of *n* different products, where $q_j \geq 0$.

iii. $d_1, d_2, \dots, d_n$ are the demand for product j, where $d_i \geq 0$.

b. Parameters

i. A_{ij}, A is nonnegative.

ii. $c \in \mathcal{R}_+^m$, is the cost for raw material r. The total cost is $c^T r$.

iii. $p \in \mathcal{R}_+^n$, is the vector of product prices. The total profit is $p^T s - c^T r$.

iv. $\pi_1, \pi_2, \dots, \pi_K$ are the probabilities of a set of *K* possible demand vectors $d^{(1)}, \dots, d^{(K)}$, where $1^T \pi = 1, \pi \succcurlyeq 0$.

v. $s_j = min\{q_j,\ d_j\}$, is the number of units of product j sold, where $s_j \geq 0$.

- If $q_j > d_j$, $q_j - d_j$ is the amount of product j produced but not sold.
- If $q_j < d_j$, $d_j - q_j$ is the amount of unmet demand.

vi. *C* is the manufacturing capacity.

c. Constraints

i. $r \succcurlyeq Aq$, because manufacturing one unit of product j requires at least A_{ij} units of raw material i.

ii. $q \succcurlyeq 0,\ r \succcurlyeq 0$.

d. Objective:

Case I: Choose r and q before knowing

We incorporate the probabilities of demand into the model and maximize the expected profit.

Maximize

$$-c^T r + \sum_{k=1}^{K} \pi_k p^T \min\{q, d^k\}$$

Subject to

$$r, q \geqslant 0, r \geqslant Aq, \qquad k = 1, \dots, K$$

Case II: Choose r ahead of time, and then q after d is known

In this case we have variables $r \in \mathcal{R}_+^m$ and $q^k \in \mathcal{R}_+^n, k = 1, \dots, K$, where q^k is the product we produce if d^k turns out to be the actual product demand. Then, the objective is to maximize the expected profit

Maximize

$$-c^T r + \sum_{k=1}^{K} \pi_k \, p^T q^k$$

Subject to

$$r, d^k, q^k \geqslant 0, \; r \geqslant Aq^k, \qquad \text{k} = 1, \dots \text{K}$$