CAGR (Compound Annual Growth Rate)

The general model used is ubiquitous is many other financial instruments. The interest is computed from the balance/net value at a particular point in time and thus varies throughout the investment's lifespan. Besides, it is assumed that the investment's value does not fluctuate erratically or experience any unpredictable change. Instead, it is founded upon a so-called interest rate which is the key to determining the systematic ascension or decline in the value. The interest rate is tantamount to the rate of change, as it indirectly influences the gradient of the investment's value. (Note: The derivation below assumes that the interest rates are in their original multiplicative forms, not in percentages.)

Let
$P = Principal$
$n = Duration\ in\ years$
$a = Annual\ interest\ rate$
$I_1 = Initial\ yearly\ interest = Pi$
$T = Net\ value\ or\ total$

The net value in the first year, T_1, is as follows:

$$\begin{aligned} T_1 &= P + I_1 \\ &= P + Pi \\ &= P(1+i) \end{aligned}$$

The second year's interest, I_2, is as follows:

$$I_2 = T_1 i$$

Thus, the second year's balance, T_2, would be as below:

$$\begin{aligned} T_2 &= T_1 + I_2 \\ &= T_1 + T_1 i \\ &= T_1(1+i) \\ &= P(1+i)(1+i) \\ &= P(1+i)^2 \end{aligned}$$

If we were to continue to derive the balance of the subsequent years, we would conclude that:

$$T_n = P(1+i)^n$$

With some elementary algebraic manipulations, the interest rate, a, and the duration, n can be computed:

$$i = \sqrt[n]{\frac{T_n}{P}} - 1$$

$$n = \frac{\log \frac{T_n}{P}}{\log(1+i)}$$