Enhancing NHL Salary Evaluation through Dimensionality Reduction

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Feature Selection

In [1]:
```
import common
import pandas as pd
import numpy as np
from sklearn.preprocessing import MinMaxScaler
```

In [2]:
```
df = common.load_dataset()

# Split features and label
X_data, y_data = common.split_dataset(df)

X_columns = X_data.columns

# Split train and test data
X_train, y_train, X_test, y_test = common.split_train_test(X_data, y_data)
```

Feature Selection

Remove features with low variance

In [3]:
```
scaler = MinMaxScaler()
X_train_scaled = scaler.fit_transform(X_train)
X_test_scaled = scaler.transform(X_test)

# Calculate the variance
variance = np.var(X_train_scaled, axis=0)
variance_df = pd.DataFrame(variance, index=X_columns, columns=['variance'])

# Display the top 10 features with the highest variance
print("Features with the highest variance")
display(variance_df.sort_values(by='variance', ascending=False).head(20))

# Display the top 10 features with the lowest variance
```

```python
print("Features with the lowest variance")
display(variance_df.sort_values(by='variance', ascending=True).head(20))
```

Features with the highest variance

	variance
nationality_CAN	0.248886
shootsCatches_R	0.232368
position_D	0.228136
nationality_USA	0.186624
position_L	0.159966
position_R	0.133075
season	0.104679
nationality_SWE	0.093311
I_F_shifts_5on4	0.068052
shifts_5on4	0.068052
I_F_oZoneShiftStarts_5on4	0.067822
I_F_flyShiftEnds_5on4	0.063645
icetime_5on4	0.054445
onIce_xGoalsPercentage_5on4	0.054253
I_F_shifts_4on5	0.054040
shifts_4on5	0.054040
I_F_neutralZoneShiftEnds_4on5	0.051402
faceoffsLost	0.050664
faceoffsLost_5on5	0.049979
I_F_dZoneShiftStarts_4on5	0.049933

Features with the lowest variance

	variance
nationality_NLD	0.000000
nationality_SVN	0.000558
nationality_BLR	0.001394
offIce_corsiPercentage_4on5	0.002534
nationality_LVA	0.002785
nationality_NOR	0.002785
offIce_fenwickPercentage_4on5	0.003308
I_F_reboundGoals_4on5	0.003333
offIce_corsiPercentage_5on4	0.004083
nationality_FRA	0.004171
penalityMinutesDrawn_4on5	0.004307
offIce_fenwickPercentage_5on4	0.004616
nationality_DEU	0.004725
I_F_playStopped_4on5	0.005943
team_SJS	0.006106
I_F_dZoneGiveaways_5on4	0.006157
offIce_xGoalsPercentage_4on5	0.006251
team_SEA	0.006382
offIce_xGoalsPercentage_5on4	0.006469
I_F_mediumDangerGoals_5on4	0.006663

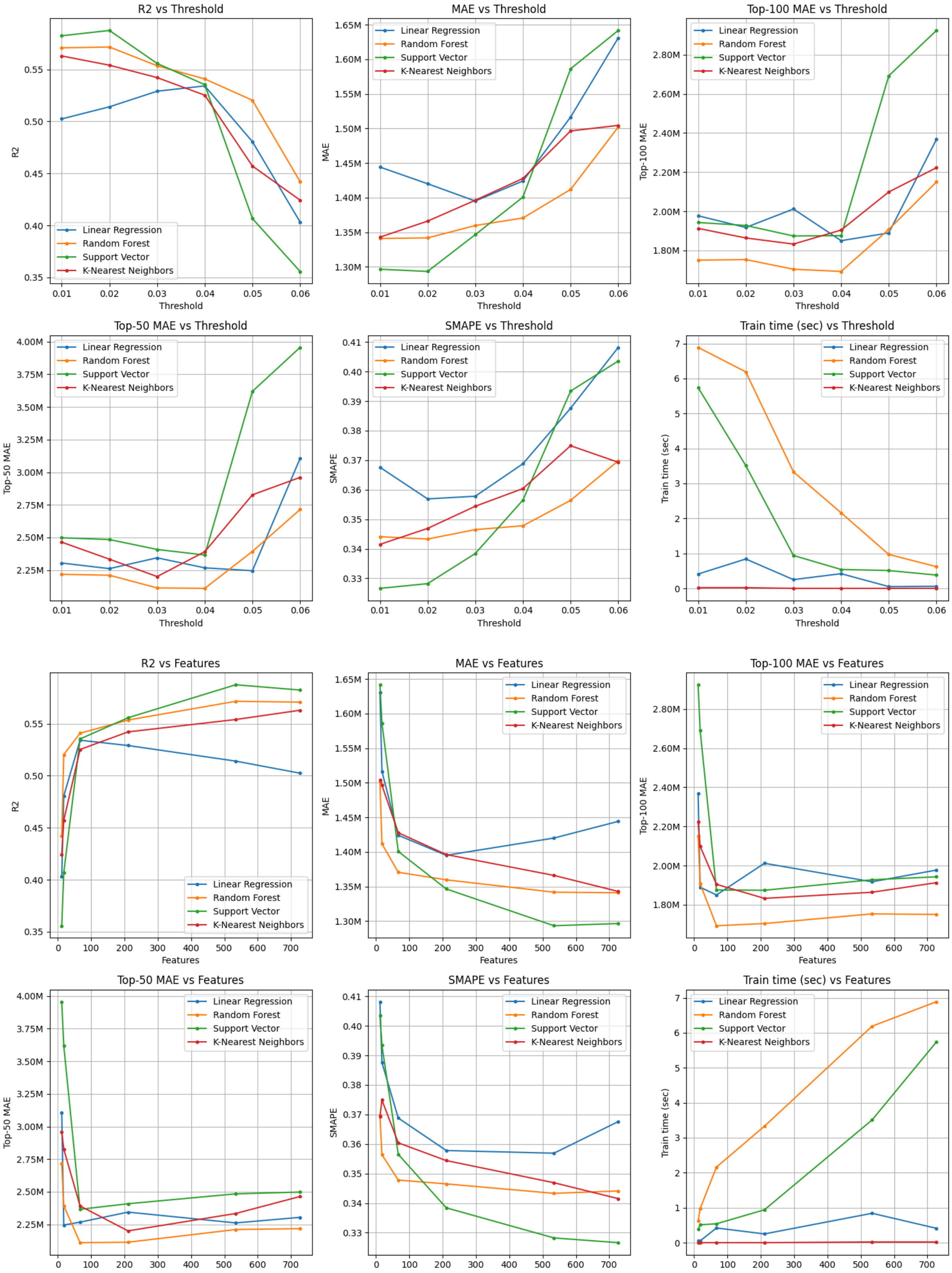
In [4]:
```
variance_thresholds = [0.01, 0.02, 0.03, 0.04, 0.05, 0.06]
results = []

for threshold in variance_thresholds:
    selected_features = np.where(variance > threshold)[0]
    X_train_reduced = X_train[:, selected_features]
    X_test_reduced = X_test[:, selected_features]

    # Standardize the data
    X_train_reduced, X_test_reduced = common.standard_scaler(X_train_reduced,
X_test_reduced)

    # Train and evaluate the model
    results_df, predictions = common.train_and_evaluate(X_train_reduced, y_tra
in, X_test_reduced, y_test)

    # Save the results in another df with the specific threshold
    results_df["Features"] = X_train_reduced.shape[1]
    results_df["Threshold"] = threshold

    results.append(results_df)

results_df = pd.concat(results)
results_df.groupby(by="Threshold")[results_df.columns].apply(lambda x: x).drop
(columns=["Threshold"])
```

Out[4]:

Threshold	Model	R2	MAE	Top-100 MAE	Top-50 MAE	SMAPE	Train time (sec)	Features
0.01	Linear Regression	0.5024	1,444,133	1,977,317	2,304,512	0.3676	0.41	727
	Random Forest	0.5707	1,340,970	1,750,731	2,218,032	0.3441	6.89	727
	Support Vector	0.5823	1,296,489	1,942,936	2,497,923	0.3266	5.74	727
	K-Nearest Neighbors	0.5629	1,342,834	1,912,944	2,464,192	0.3415	0.02	727
0.02	Linear Regression	0.5140	1,420,031	1,917,548	2,261,717	0.3569	0.84	534
	Random Forest	0.5715	1,341,772	1,753,626	2,211,082	0.3433	6.19	534
	Support Vector	0.5874	1,293,308	1,927,914	2,484,424	0.3282	3.51	534
	K-Nearest Neighbors	0.5540	1,366,021	1,864,098	2,334,045	0.3469	0.02	534
0.03	Linear Regression	0.5290	1,395,072	2,011,941	2,344,456	0.3578	0.25	211
	Random Forest	0.5535	1,359,619	1,704,305	2,113,901	0.3465	3.33	211
	Support Vector	0.5557	1,346,444	1,874,234	2,408,629	0.3384	0.94	211
	K-Nearest Neighbors	0.5421	1,396,171	1,832,835	2,200,864	0.3544	0.00	211
0.04	Linear Regression	0.5341	1,424,041	1,850,242	2,267,687	0.3688	0.42	67
	Random Forest	0.5408	1,370,534	1,693,126	2,110,504	0.3478	2.16	67
	Support Vector	0.5353	1,400,692	1,875,733	2,365,925	0.3565	0.54	67
	K-Nearest Neighbors	0.5252	1,427,714	1,904,417	2,391,179	0.3604	0.00	67
0.05	Linear Regression	0.4805	1,516,414	1,889,712	2,245,816	0.3876	0.05	18
	Random Forest	0.5203	1,411,781	1,907,684	2,393,466	0.3564	0.97	18
	Support Vector	0.4066	1,586,163	2,692,241	3,619,580	0.3934	0.51	18
	K-Nearest Neighbors	0.4571	1,496,368	2,099,004	2,827,101	0.3749	0.00	18

Threshold	Model	R2	MAE	Top-100 MAE	Top-50 MAE	SMAPE	Train time (sec)	Features
0.06	**Linear Regression**	0.4033	1,630,867	2,367,750	3,106,159	0.4082	0.06	12
	Random Forest	0.4421	1,502,002	2,149,980	2,714,460	0.3698	0.62	12
	Support Vector	0.3558	1,641,583	2,923,764	3,956,213	0.4036	0.38	12
	K-Nearest Neighbors	0.4243	1,504,398	2,222,808	2,960,498	0.3693	0.00	12

In [5]:
```
common.plot_metrics(results_df, "Threshold")
common.plot_metrics(results_df, "Features")
```



Univariate feature selection

In [6]:
```
def pearson_correlation(X, y):
    # Center the data
    X_centered = X - X.mean()
    y_centered = y - y.mean()

    # Calculate the covariance
    covariance = X_centered.T @ y_centered / (X.shape[0] - 1)

    # Calculate the standard deviations
    std_X = X.std()
    std_y = y.std()

    # Calculate Pearson's r
    r = covariance / (std_X * std_y)

    return r
```

In [7]:
```
scaler = MinMaxScaler()
X_train_scaled = scaler.fit_transform(X_train)
X_test_scaled = scaler.transform(X_test)

# Calculate the Pearson correlation
pearson_corr = pearson_correlation(X_train_scaled, y_train)
correlation_df = pd.DataFrame(pearson_corr, index=X_columns, columns=['correla
tion'])

# Display the top 10 features with the highest correlation
display(correlation_df.sort_values(by='correlation', ascending=False).head(2
0))

# Display the top 10 features with the lowest correlation
display(correlation_df.sort_values(by='correlation', ascending=True).head(20))
```

	correlation
I_F_oZoneShiftStarts_5on4	0.657052
icetime_5on4	0.624534
OnIce_F_shotAttempts_5on4	0.594658
OnIce_F_scoreAdjustedShotsAttempts_5on4	0.594658
OnIce_F_unblockedShotAttempts_5on4	0.582603
OnIce_F_scoreAdjustedUnblockedShotAttempts_5on4	0.582603
I_F_shifts_5on4	0.581040
shifts_5on4	0.581040
OnIce_F_blockedShotAttempts_5on4	0.577918
OnIce_F_xOnGoal_5on4	0.576197
OnIce_F_shotsOnGoal_5on4	0.575519
OnIce_F_missedShots_5on4	0.574642
OnIce_F_unblockedShotAttempts_other	0.565104
OnIce_F_scoreAdjustedUnblockedShotAttempts_other	0.565104
OnIce_F_lowDangerxGoals_5on4	0.563693
OnIce_F_lowDangerShots_5on4	0.562545
OnIce_A_scoreAdjustedUnblockedShotAttempts_5on4	0.559855
OnIce_A_unblockedShotAttempts_5on4	0.559855
OnIce_F_xGoalsFromxReboundsOfShots_5on4	0.556338
OnIce_F_shotsOnGoal_other	0.553692

	correlation
iceTimeRank	-0.347112
iceTimeRank_5on5	-0.268340
offIce_corsiPercentage	-0.173838
offIce_fenwickPercentage	-0.170056
offIce_xGoalsPercentage	-0.169862
offIce_corsiPercentage_other	-0.140132
offIce_fenwickPercentage_other	-0.130943
OffIce_F_xGoals	-0.113007
I_F_hits_5on5	-0.112420
I_F_hits	-0.104416
nationality_CAN	-0.098920
OffIce_F_shotAttempts	-0.082560
I_F_dZoneShiftEnds_4on5	-0.081138
position_D	-0.071298
I_F_flyShiftStarts_4on5	-0.069511
OnIce_A_blockedShotAttempts_4on5	-0.069488
OnIce_A_goals_4on5	-0.068182
icetime_4on5	-0.065750
OnIce_A_shotAttempts_4on5	-0.063001
OnIce_A_scoreAdjustedShotsAttempts_4on5	-0.063001

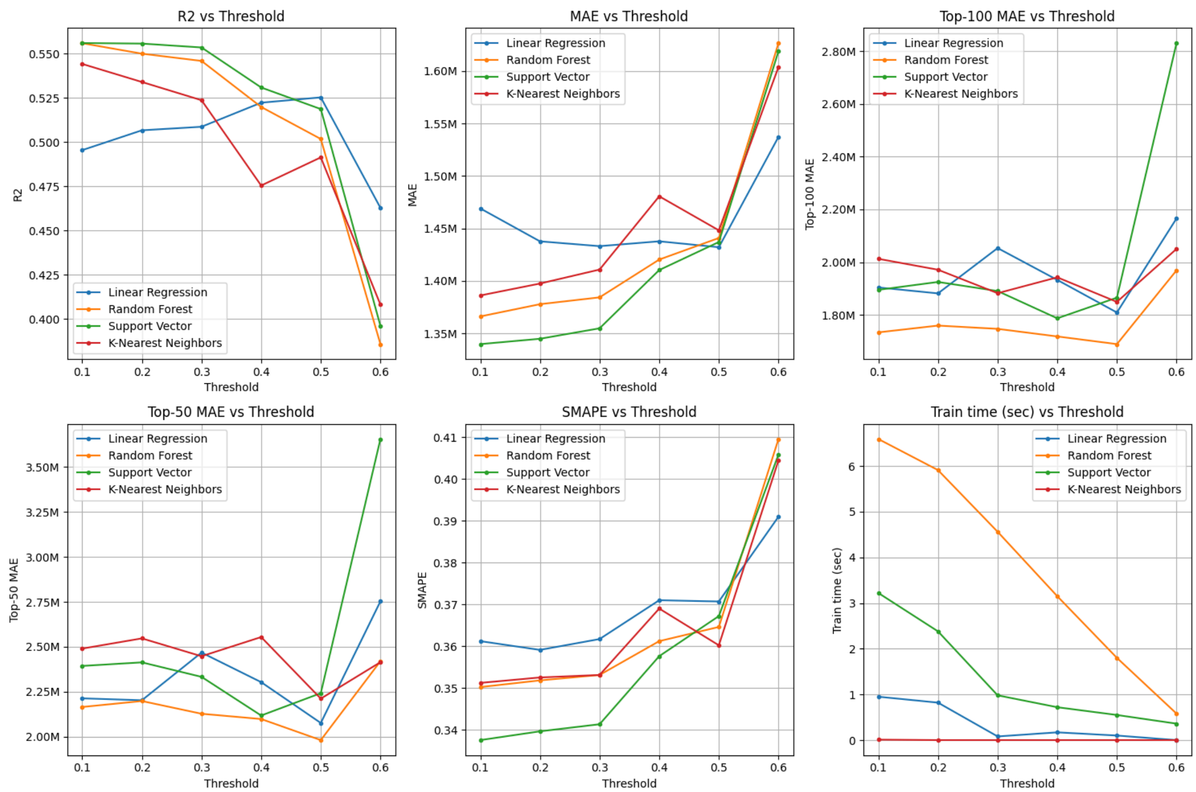
In [8]:
```
thresholds = [0.1, 0.2, 0.3, 0.4, 0.5, 0.6]

results = []
for threshold in thresholds:
    selected_features = np.where(np.absolute(pearson_corr) > threshold)[0]
    X_train_reduced = X_train_scaled[:, selected_features]
    X_test_reduced = X_test_scaled[:, selected_features]

    # Standardize the data (scaler)
    X_train_reduced, X_test_reduced = common.standard_scaler(X_train_reduced, 
X_test_reduced)

    # Train and evaluate the model
    results_df, predictions = common.train_and_evaluate(X_train_reduced, y_tra
in, X_test_reduced, y_test)

    # Save the results in another df with the specific threshold
    results_df["Features"] = X_train_reduced.shape[1]
    results_df["Threshold"] = threshold

    results.append(results_df)

results_df = pd.concat(results)
results_df.groupby(by="Threshold")[results_df.columns].apply(lambda x: x).drop
(columns=["Threshold"])
```

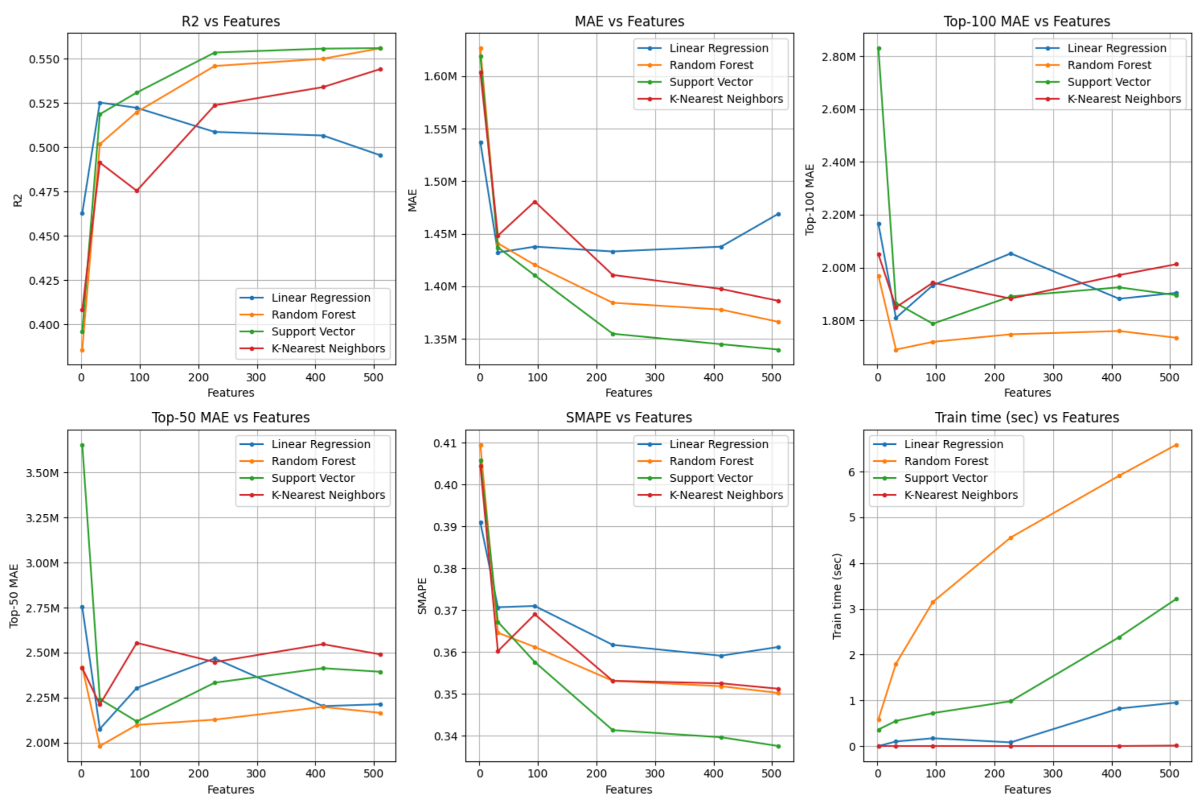
Out[8]:

Threshold	Model	R2	MAE	Top-100 MAE	Top-50 MAE	SMAPE	Train time (sec)	Features
0.1	Linear Regression	0.4955	1,469,050	1,903,977	2,213,647	0.3612	0.95	511
	Random Forest	0.5559	1,366,071	1,733,658	2,165,628	0.3502	6.59	511
	Support Vector	0.5560	1,339,682	1,894,952	2,393,776	0.3375	3.22	511
	K-Nearest Neighbors	0.5442	1,386,066	2,012,540	2,490,112	0.3512	0.01	511
0.2	Linear Regression	0.5067	1,437,679	1,881,445	2,203,010	0.3591	0.82	413
	Random Forest	0.5500	1,377,752	1,759,376	2,198,645	0.3518	5.91	413
	Support Vector	0.5557	1,344,777	1,924,828	2,413,609	0.3396	2.38	413
	K-Nearest Neighbors	0.5340	1,397,426	1,971,142	2,546,844	0.3525	0.00	413
0.3	Linear Regression	0.5087	1,433,091	2,053,276	2,468,015	0.3617	0.08	228
	Random Forest	0.5459	1,384,257	1,746,963	2,128,031	0.3531	4.56	228
	Support Vector	0.5535	1,354,750	1,890,353	2,333,057	0.3413	0.98	228
	K-Nearest Neighbors	0.5237	1,410,763	1,882,182	2,447,578	0.3531	0.00	228
0.4	Linear Regression	0.5223	1,437,706	1,932,520	2,303,616	0.3710	0.17	95
	Random Forest	0.5199	1,420,296	1,718,210	2,098,734	0.3612	3.15	95
	Support Vector	0.5309	1,410,298	1,786,934	2,118,174	0.3576	0.72	95
	K-Nearest Neighbors	0.4755	1,480,634	1,942,326	2,554,244	0.3690	0.00	95
0.5	Linear Regression	0.5253	1,432,065	1,809,026	2,076,659	0.3707	0.10	32
	Random Forest	0.5018	1,440,867	1,689,103	1,981,560	0.3646	1.80	32
	Support Vector	0.5187	1,437,032	1,865,292	2,242,419	0.3672	0.55	32
	K-Nearest Neighbors	0.4914	1,448,172	1,849,235	2,212,024	0.3602	0.00	32

Threshold	Model	R2	MAE	Top-100 MAE	Top-50 MAE	SMAPE	Train time (sec)	Features
0.6	**Linear Regression**	0.4629	1,537,285	2,165,741	2,752,814	0.3910	0.00	2
	Random Forest	0.3859	1,626,744	1,968,846	2,420,005	0.4096	0.58	2
	Support Vector	0.3962	1,618,978	2,830,753	3,654,712	0.4059	0.36	2
	K-Nearest Neighbors	0.4082	1,603,915	2,050,231	2,414,581	0.4045	0.00	2

In [9]:
```
common.plot_metrics(results_df, "Threshold")
common.plot_metrics(results_df, "Features")
```





Random Selection

In [10]:
```
# Standardize the data
X_train, X_test = common.standard_scaler(X_train, X_test)
```

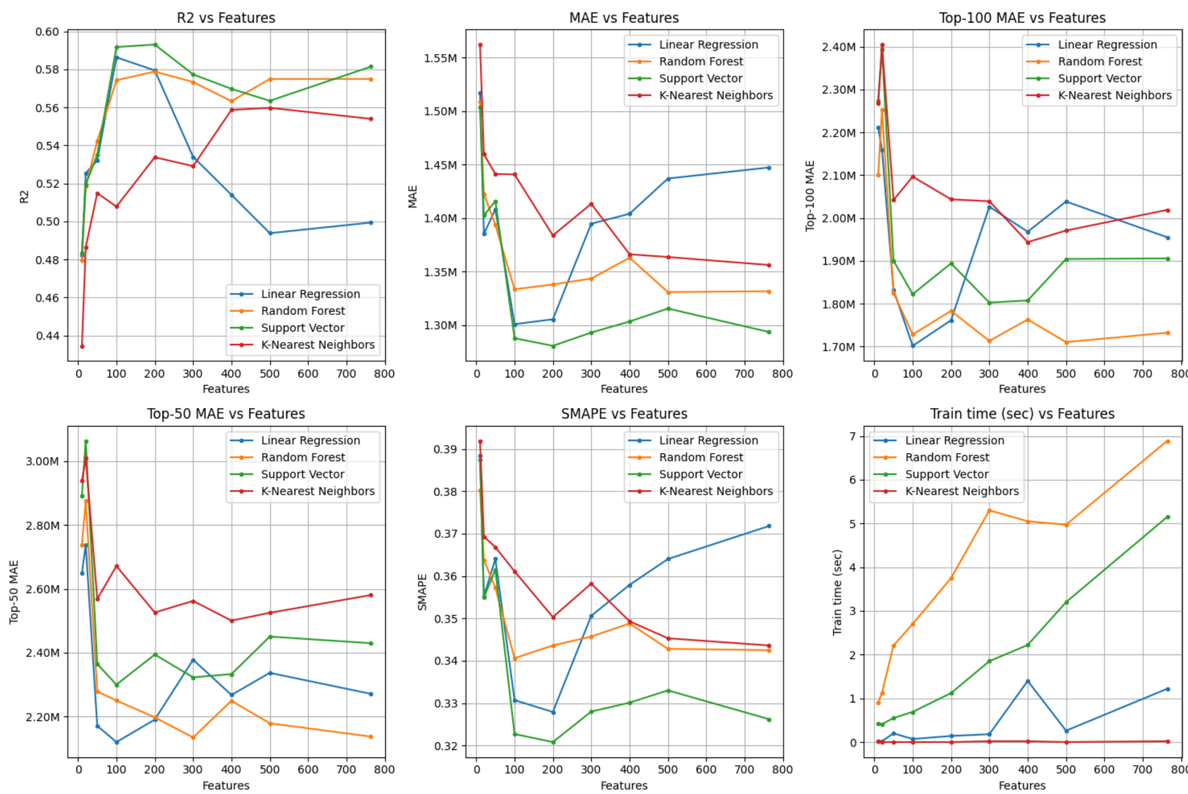
In [11]:
```
n_features = [10, 20, 50, 100, 200, 300, 400, 500, X_data.shape[1]-1]

results = []

for n in n_features:
    # Randomly select n features
    rng = np.random.default_rng(seed=12345)
    selected_features = rng.choice(X_train.shape[1], size=n, replace=False)
    X_train_reduced = X_train[:, selected_features]
    X_test_reduced = X_test[:, selected_features]

    # Train and evaluate the model
    results_df, predictions = common.train_and_evaluate(X_train_reduced, y_tra
in, X_test_reduced, y_test)

    results_df["Features"] = n

    results.append(results_df)

results_df = pd.concat(results)
results_df.groupby(by="Features")[results_df.columns].apply(lambda x: x).drop
(columns=["Features"])
```

Out[11]:

Features	Model	R2	MAE	Top-100 MAE	Top-50 MAE	SMAPE	Train time (sec)
10	**Linear Regression**	0.4822	1,516,986	2,211,265	2,649,530	0.3884	0.02
	Random Forest	0.4796	1,509,157	2,100,143	2,737,145	0.3803	0.90
	Support Vector	0.4836	1,503,715	2,273,302	2,892,071	0.3875	0.42
	K-Nearest Neighbors	0.4345	1,562,287	2,267,887	2,937,656	0.3919	0.02
20	**Linear Regression**	0.5254	1,385,284	2,159,127	2,736,993	0.3552	0.02
	Random Forest	0.5186	1,422,653	2,253,787	2,874,604	0.3638	1.13
	Support Vector	0.5194	1,402,562	2,394,130	3,063,242	0.3549	0.41
	K-Nearest Neighbors	0.4865	1,459,765	2,404,914	3,008,550	0.3693	0.00
50	**Linear Regression**	0.5321	1,408,249	1,832,230	2,171,290	0.3640	0.20
	Random Forest	0.5423	1,393,628	1,825,231	2,278,476	0.3572	2.21
	Support Vector	0.5352	1,416,004	1,900,335	2,364,856	0.3615	0.55
	K-Nearest Neighbors	0.5149	1,441,322	2,041,872	2,568,063	0.3668	0.00
100	**Linear Regression**	0.5864	1,300,907	1,701,469	2,120,465	0.3307	0.07
	Random Forest	0.5743	1,333,489	1,727,869	2,250,647	0.3406	2.70
	Support Vector	0.5918	1,287,790	1,822,080	2,300,212	0.3227	0.69
	K-Nearest Neighbors	0.5078	1,440,904	2,096,645	2,671,528	0.3611	0.00
200	**Linear Regression**	0.5795	1,305,477	1,761,106	2,190,760	0.3279	0.14
	Random Forest	0.5789	1,338,019	1,783,135	2,197,795	0.3436	3.75
	Support Vector	0.5931	1,280,671	1,894,051	2,394,359	0.3208	1.12
	K-Nearest Neighbors	0.5338	1,383,844	2,043,542	2,525,929	0.3503	0.00
300	**Linear Regression**	0.5340	1,394,833	2,026,289	2,378,097	0.3506	0.18
	Random Forest	0.5734	1,343,508	1,712,939	2,134,928	0.3457	5.30
	Support Vector	0.5774	1,293,067	1,802,472	2,323,035	0.3280	1.85
	K-Nearest Neighbors	0.5291	1,413,438	2,039,094	2,562,262	0.3582	0.02
400	**Linear Regression**	0.5140	1,404,114	1,967,911	2,267,881	0.3579	1.40
	Random Forest	0.5633	1,363,045	1,763,038	2,249,719	0.3488	5.05
	Support Vector	0.5698	1,303,399	1,807,374	2,333,151	0.3301	2.22
	K-Nearest Neighbors	0.5587	1,366,267	1,943,712	2,500,617	0.3493	0.02

Features	Model	R2	MAE	Top-100 MAE	Top-50 MAE	SMAPE	Train time (sec)
500	**Linear Regression**	0.4938	1,437,090	2,038,397	2,337,069	0.3640	0.26
	Random Forest	0.5750	1,330,938	1,710,139	2,179,325	0.3428	4.97
	Support Vector	0.5635	1,315,526	1,904,263	2,450,611	0.3330	3.20
	K-Nearest Neighbors	0.5598	1,363,691	1,970,665	2,525,004	0.3453	0.00
764	**Linear Regression**	0.4994	1,447,414	1,954,974	2,271,386	0.3718	1.22
	Random Forest	0.5750	1,331,716	1,732,098	2,137,533	0.3425	6.89
	Support Vector	0.5815	1,293,600	1,905,611	2,429,886	0.3262	5.15
	K-Nearest Neighbors	0.5540	1,356,219	2,018,904	2,580,622	0.3436	0.02

In [12]:
```
common.plot_metrics(results_df, "Features")
```



Curated Features

In [13]:
```
ten_features = [
    "age",
    "games_played",
    "icetime",
    "gameScore",
    "position_D",
    "I_F_xGoals",
    "I_F_shotAttempts",
    "I_F_points",
    "I_F_goals",
    "OnIce_A_xGoals"
]

twenty_features = ten_features + [
    "OnIce_A_xGoals",
    "I_F_primaryAssists",
    "I_F_hits",
    "OnIce_F_shotAttempts",
    "I_F_shotsOnGoal",
    "I_F_xOnGoal",
    "OnIce_F_highDangerShots",
    "I_F_xGoals_5on5",
    "I_F_points_5on5",
    "I_F_goals_5on5"
]

thirty_features = twenty_features + [
    "shifts",
    "OnIce_F_xGoals_5on5",
    "onIce_xGoalsPercentage",
    "OnIce_F_xGoals",
    "OnIce_A_xGoals",
    "onIce_corsiPercentage",
    "onIce_fenwickPercentage",
    "I_F_xRebounds",
    "I_F_xPlayContinuedInZone",
    "I_F_rebounds"
]

forthy_features = thirty_features + [
    "OnIce_A_highDangerShots",
    "OnIce_A_mediumDangerShots",
    "penalties_5on5",
    "I_F_dZoneGiveaways_5on5",
    "I_F_xGoals_5on4",
    "OnIce_F_goals_5on4",
    "faceoffsWon",
    "OnIce_F_rebounds_5on5",
    "OnIce_F_highDangerShots_5on5",
    "OnIce_F_mediumDangerShots_5on5"
]
```

In [14]:

```
results = []

for features in [ten_features, twenty_features, thirty_features, forthy_featur
es]:
    selected_features = [X_columns.get_loc(feature) for feature in features]
    X_train_reduced = X_train[:, selected_features]
    X_test_reduced = X_test[:, selected_features]

    # Train and evaluate the models
    results_df, predictions = common.train_and_evaluate(X_train_reduced, y_tra
in, X_test_reduced, y_test)

    results_df["Features"] = len(features)

    results.append(results_df)

results_df = pd.concat(results)
results_df.groupby(by="Features")[results_df.columns].apply(lambda x: x).drop
(columns=["Features"])
```

Out[14]:

Features	Model	R2	MAE	Top-100 MAE	Top-50 MAE	SMAPE	Train time (sec)
10	Linear Regression	0.5972	1,304,619	1,787,843	2,170,968	0.3332	0.02
	Random Forest	0.5480	1,396,831	1,835,874	2,268,770	0.3555	1.00
	Support Vector	0.5421	1,431,640	2,061,207	2,600,320	0.3649	0.35
	K-Nearest Neighbors	0.5208	1,420,575	2,038,903	2,636,208	0.3571	0.02
20	Linear Regression	0.5923	1,306,572	1,790,465	2,139,256	0.3311	0.00
	Random Forest	0.5517	1,402,172	1,758,680	2,235,437	0.3589	1.51
	Support Vector	0.5654	1,382,579	1,808,086	2,252,780	0.3510	0.47
	K-Nearest Neighbors	0.5101	1,455,013	2,010,293	2,527,197	0.3611	0.00
30	Linear Regression	0.5867	1,317,756	1,885,756	2,305,836	0.3354	0.05
	Random Forest	0.5524	1,409,961	1,789,828	2,244,629	0.3603	1.64
	Support Vector	0.5673	1,363,986	1,802,121	2,261,938	0.3434	0.61
	K-Nearest Neighbors	0.5244	1,416,097	1,895,139	2,441,433	0.3515	0.00
40	Linear Regression	0.5846	1,326,821	1,856,544	2,262,506	0.3380	0.06
	Random Forest	0.5581	1,374,543	1,800,319	2,252,110	0.3527	1.83
	Support Vector	0.5750	1,334,249	1,855,225	2,299,091	0.3359	0.55
	K-Nearest Neighbors	0.5262	1,410,998	2,030,788	2,573,531	0.3523	0.00

In [15]:

```
common.plot_metrics(results_df, "Features")
```