Enhancing NHL Salary Evaluation through Dimensionality Reduction

Raphaël Fontaine
McGill University
Montreal, Canada
raphael.fontaine@mail.mcgill.ca

Partial Least Squares (PLS)

In [1]:
```
import common
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from sklearn.cross_decomposition import PLSRegression
```

Load Dataset

In [2]:
```
df = common.load_dataset()

# Split features and label
X_data, y_data = common.split_dataset(df)

X_columns = X_data.columns

# Split train and test data
X_train, y_train, X_test, y_test = common.split_train_test(X_data, y_data)

# Standardize the features
X_train, X_test = common.standard_scaler(X_train, X_test)
```

PLS Implementation

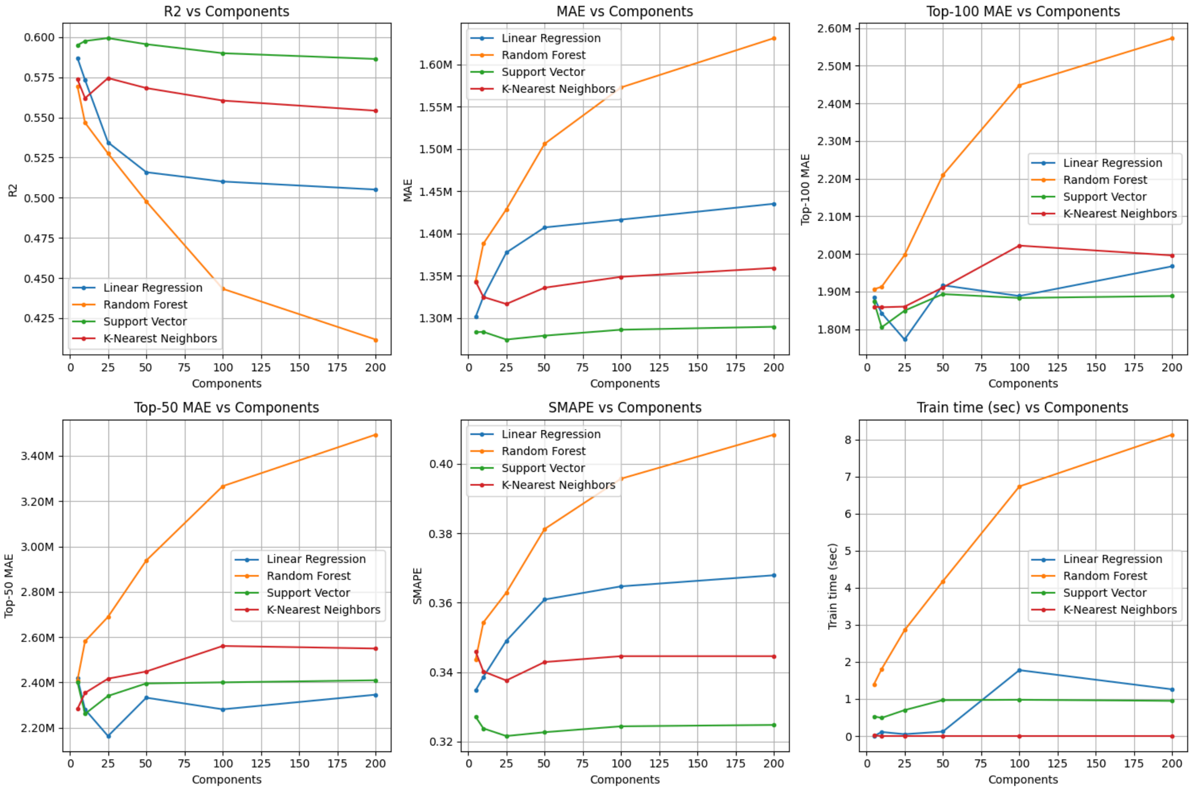
In [3]:
```
def PLS(n_components):
    X = X_train.astype(np.float64)
    n_samples, n_features = X.shape

    # Reshape y to column vector of floats
    y = y_train.reshape(-1, 1).astype(np.float64)

    # Initialize matrices full of zeros
    T = np.zeros((n_samples, n_components))     # Components
    W = np.zeros((n_features, n_components))    # Weights
    P = np.zeros((n_features, n_components))    # Loadings for X
    Q = np.zeros(n_components)                  # Loadings for y

    for i in range(n_components):
        # Compute weights w that maximize covariance between X and y
        w = X.T @ y
        w /= np.linalg.norm(w)  # Normalize to unit length

        # Project X onto w to find t
        t = X @ w

        tk = (t.T @ t)

        # Compute loadings p
        p = (X.T @ t) / tk

        # Compute loadings q
        q = (y.T @ t) / tk
        q = q.item()  # Convert to scalar

        # Deflate X and y
        X -= t @ p.T
        y -= q * t

        # Store results
        T[:, i] = t.ravel()
        P[:, i] = p.ravel()
        W[:, i] = w.ravel()
        Q[i] = q

    return T, W, P, Q
```

In [4]:
```
n_components = [5, 10, 25, 50, 100, 200]

results = []
PLS_results = []
for n in n_components:

    # Perform PLS and apply to the data
    T, W, P, Q = PLS(n)
    PLS_results.append((T, W, P, Q))

    X_train_pls = X_train @ W
    X_test_pls = X_test @ W

    # Train and evaluate the models
    results_df, predictions = common.train_and_evaluate(X_train_pls, y_train,
X_test_pls, y_test)

    results_df["Components"] = n

    results.append(results_df)

results_df = pd.concat(results)
results_df.groupby(by="Components")[results_df.columns].apply(lambda x: x).dro
p(columns=["Components"])
```

Out[4]:

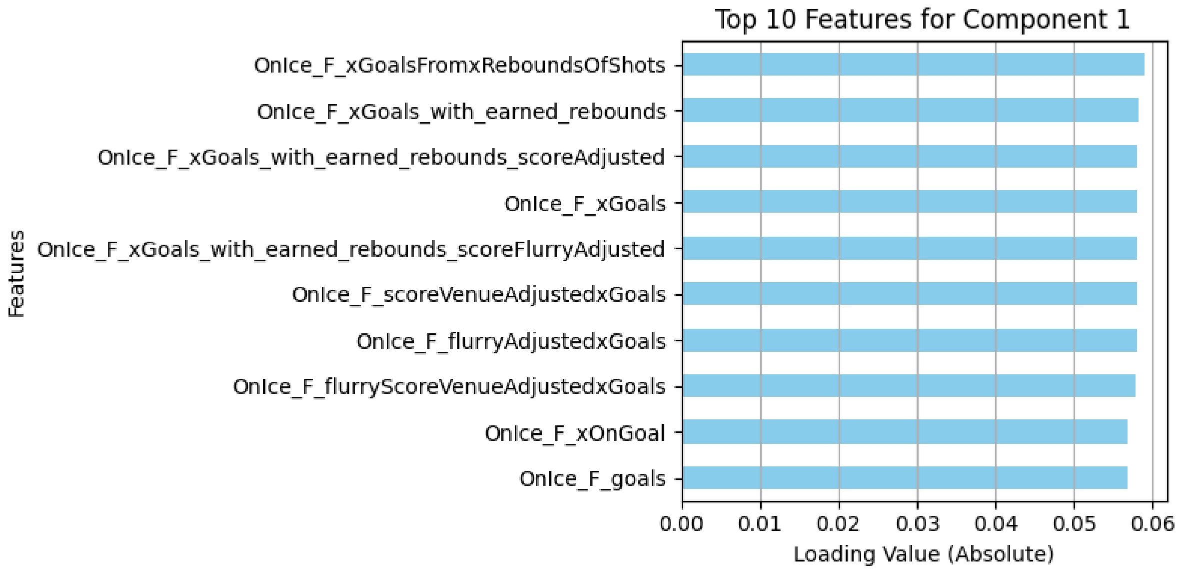
Components	Model	R2	MAE	Top-100 MAE	Top-50 MAE	SMAPE	Train time (sec)
5	Linear Regression	0.5871	1,301,688	1,884,016	2,420,104	0.3348	0.00
	Random Forest	0.5695	1,344,141	1,906,363	2,408,968	0.3437	1.40
	Support Vector	0.5950	1,283,262	1,873,306	2,402,444	0.3271	0.53
	K-Nearest Neighbors	0.5738	1,343,160	1,860,158	2,282,809	0.3459	0.02
10	Linear Regression	0.5733	1,325,284	1,842,915	2,280,512	0.3386	0.11
	Random Forest	0.5467	1,388,011	1,913,015	2,581,870	0.3543	1.81
	Support Vector	0.5976	1,283,728	1,805,279	2,261,418	0.3238	0.49
	K-Nearest Neighbors	0.5620	1,325,035	1,858,557	2,353,879	0.3402	0.00
25	Linear Regression	0.5345	1,377,533	1,772,794	2,163,478	0.3490	0.05
	Random Forest	0.5276	1,428,605	1,997,287	2,689,512	0.3629	2.86
	Support Vector	0.5994	1,274,635	1,849,491	2,340,399	0.3216	0.70
	K-Nearest Neighbors	0.5745	1,316,685	1,860,251	2,416,610	0.3376	0.00
50	Linear Regression	0.5159	1,407,256	1,916,936	2,332,747	0.3609	0.12
	Random Forest	0.4977	1,506,085	2,209,466	2,938,098	0.3812	4.18
	Support Vector	0.5956	1,279,232	1,893,485	2,395,726	0.3227	0.97
	K-Nearest Neighbors	0.5683	1,335,935	1,910,936	2,448,318	0.3429	0.00
100	Linear Regression	0.5101	1,416,501	1,888,406	2,281,440	0.3647	1.78
	Random Forest	0.4433	1,572,835	2,448,338	3,266,438	0.3957	6.74
	Support Vector	0.5900	1,286,268	1,883,272	2,400,313	0.3244	0.98
	K-Nearest Neighbors	0.5605	1,348,836	2,022,288	2,561,082	0.3446	0.00
200	Linear Regression	0.5051	1,435,270	1,967,415	2,346,021	0.3679	1.26
	Random Forest	0.4117	1,631,209	2,573,281	3,493,959	0.4084	8.14
	Support Vector	0.5864	1,289,766	1,888,387	2,409,532	0.3248	0.95
	K-Nearest Neighbors	0.5542	1,359,316	1,996,245	2,549,803	0.3446	0.00

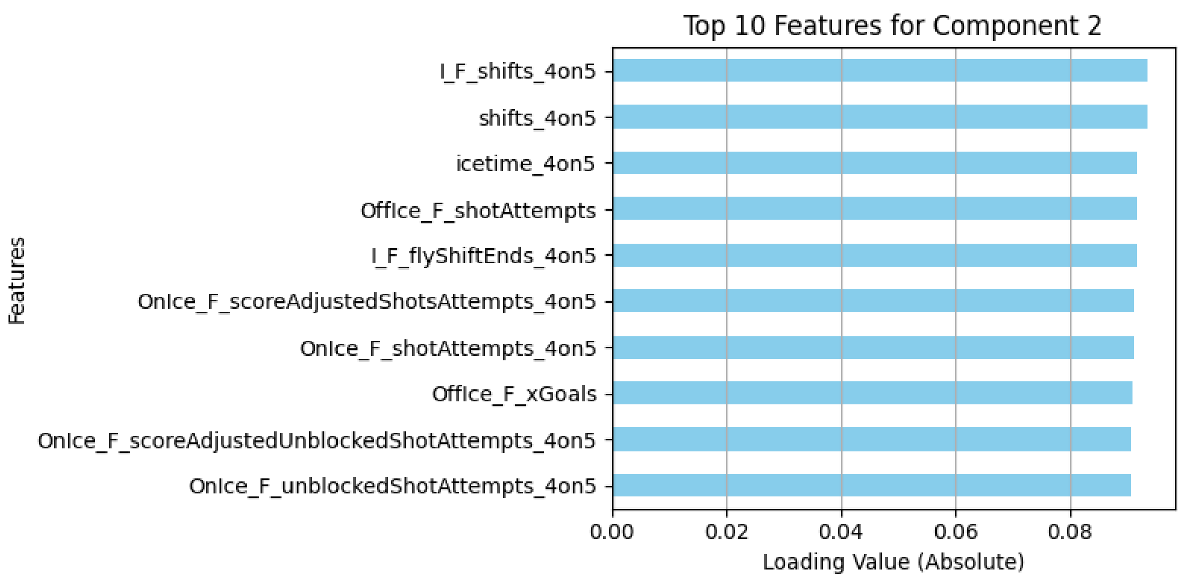
In [5]:
```
common.plot_metrics(results_df, "Components")
```

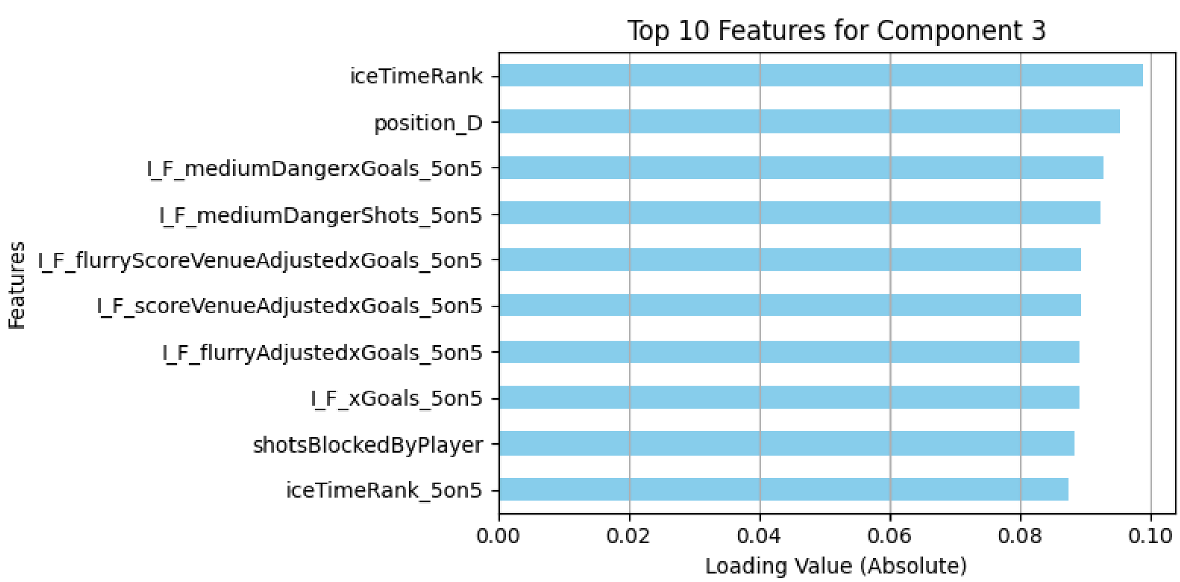


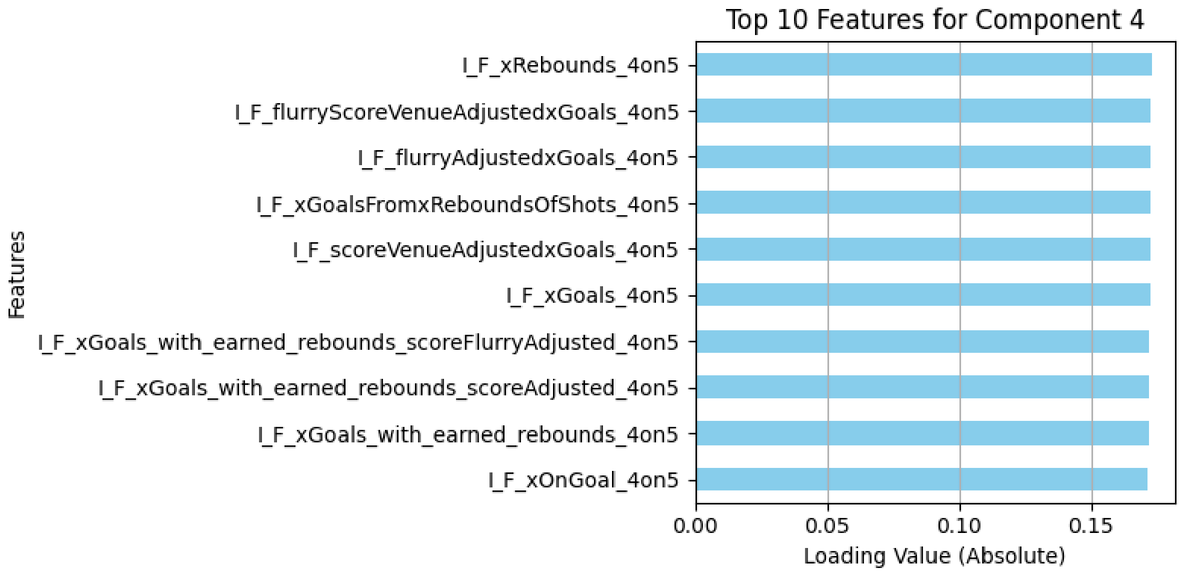
In [6]:

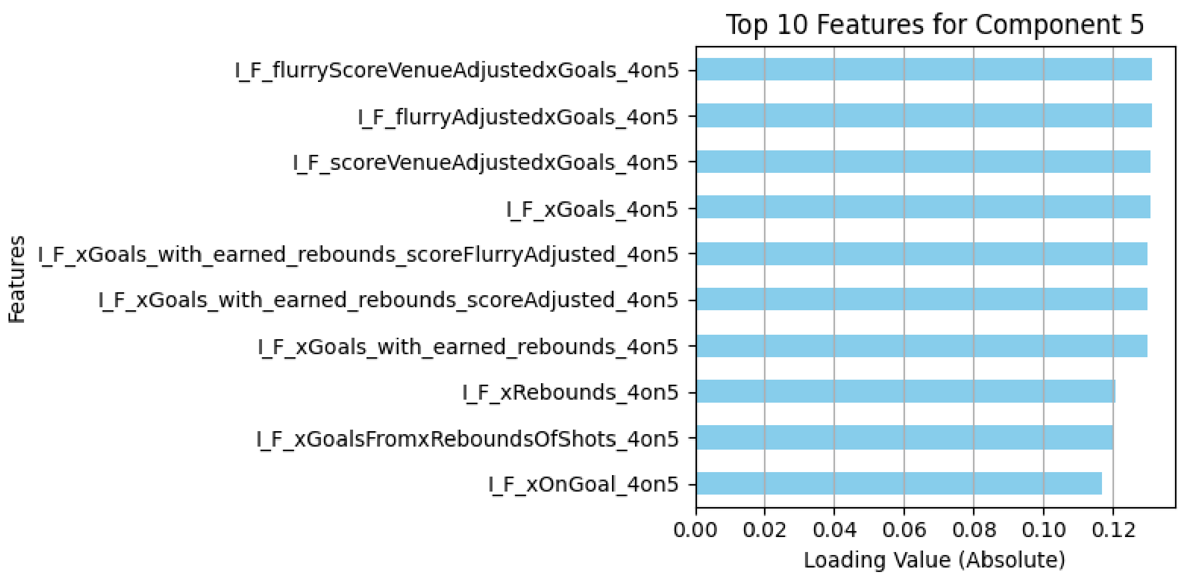
```
n_top_features = 10

loadings_df = pd.DataFrame(P, index=X_data.columns)

for comp in range(5):
    plt.figure(figsize=(8, 4))
    # Sort features by absolute loading values for the current component
    top_features = loadings_df.iloc[:, comp].abs().nlargest(n_top_features)

    # Plot
    top_features.sort_values().plot(kind='barh', color='skyblue')
    plt.title(f"Top {n_top_features} Features for Component {comp + 1}")
    plt.xlabel("Loading Value (Absolute)")
    plt.ylabel("Features")
    plt.grid(axis='x')
    plt.tight_layout()
    plt.show()
```


Top 10 Features for Component 1
OnIce_F_xGoalsFromxReboundsOfShots
OnIce_F_xGoals_with_earned_rebounds
OnIce_F_xGoals_with_earned_rebounds_scoreAdjusted
OnIce_F_xGoals
OnIce_F_xGoals_with_earned_rebounds_scoreFlurryAdjusted
OnIce_F_scoreVenueAdjustedxGoals
OnIce_F_flurryAdjustedxGoals
OnIce_F_flurryScoreVenueAdjustedxGoals
OnIce_F_xOnGoal
OnIce_F_goals
Features
0.00 0.01 0.02 0.03 0.04 0.05 0.06
Loading Value (Absolute)


Top 10 Features for Component 2
I_F_shifts_4on5
shifts_4on5
icetime_4on5
OffIce_F_shotAttempts
I_F_flyShiftEnds_4on5
OnIce_F_scoreAdjustedShotsAttempts_4on5
OnIce_F_shotAttempts_4on5
OffIce_F_xGoals
OnIce_F_scoreAdjustedUnblockedShotAttempts_4on5
OnIce_F_unblockedShotAttempts_4on5
Features
0.00 0.02 0.04 0.06 0.08
Loading Value (Absolute)


Top 10 Features for Component 3
iceTimeRank
position_D
I_F_mediumDangerxGoals_5on5
I_F_mediumDangerShots_5on5
I_F_flurryScoreVenueAdjustedxGoals_5on5
I_F_scoreVenueAdjustedxGoals_5on5
I_F_flurryAdjustedxGoals_5on5
I_F_xGoals_5on5
shotsBlockedByPlayer
iceTimeRank_5on5
Features
0.00 0.02 0.04 0.06 0.08 0.10
Loading Value (Absolute)


Top 10 Features for Component 4
Features
I_F_xRebounds_4on5
I_F_flurryScoreVenueAdjustedxGoals_4on5
I_F_flurryAdjustedxGoals_4on5
I_F_xGoalsFromxReboundsOfShots_4on5
I_F_scoreVenueAdjustedxGoals_4on5
I_F_xGoals_4on5
I_F_xGoals_with_earned_rebounds_scoreFlurryAdjusted_4on5
I_F_xGoals_with_earned_rebounds_scoreAdjusted_4on5
I_F_xGoals_with_earned_rebounds_4on5
I_F_xOnGoal_4on5
0.00
0.05
0.10
0.15
Loading Value (Absolute)


Top 10 Features for Component 5
Features
I_F_flurryScoreVenueAdjustedxGoals_4on5
I_F_flurryAdjustedxGoals_4on5
I_F_scoreVenueAdjustedxGoals_4on5
I_F_xGoals_4on5
I_F_xGoals_with_earned_rebounds_scoreFlurryAdjusted_4on5
I_F_xGoals_with_earned_rebounds_scoreAdjusted_4on5
I_F_xGoals_with_earned_rebounds_4on5
I_F_xRebounds_4on5
I_F_xGoalsFromxReboundsOfShots_4on5
I_F_xOnGoal_4on5
0.00
0.02
0.04
0.06
0.08
0.10
0.12
Loading Value (Absolute)

Sklearn PLS

In [7]:
```
results = []
for n in n_components:
    pls = PLSRegression(n_components=n)
    X_train_pls = pls.fit_transform(X_train, y_train)[0]
    X_test_pls = pls.transform(X_test)

    # Train and evaluate the models
    results_df, predictions = common.train_and_evaluate(X_train_pls, y_train,
X_test_pls, y_test)

    results_df["Components"] = n

    results.append(results_df)

results_df = pd.concat(results)
results_df.groupby(by="Components")[results_df.columns].apply(lambda x: x).dro
p(columns=["Components"])
```

Out[7]:

Components	Model	R2	MAE	Top-100 MAE	Top-50 MAE	SMAPE	Train time (sec)
5	Linear Regression	0.5871	1,301,688	1,884,016	2,420,104	0.3348	0.00
	Random Forest	0.5704	1,315,768	1,992,715	2,591,870	0.3373	1.04
	Support Vector	0.5913	1,291,255	1,909,250	2,449,940	0.3304	0.41
	K-Nearest Neighbors	0.5595	1,336,533	1,888,610	2,421,002	0.3438	0.00
10	Linear Regression	0.5733	1,325,284	1,842,915	2,280,512	0.3386	0.00
	Random Forest	0.5710	1,325,692	1,881,152	2,547,652	0.3376	1.43
	Support Vector	0.5764	1,318,906	1,856,640	2,304,904	0.3357	0.37
	K-Nearest Neighbors	0.5742	1,299,178	1,816,722	2,283,583	0.3311	0.00
25	Linear Regression	0.5345	1,377,533	1,772,794	2,163,478	0.3490	0.00
	Random Forest	0.5631	1,365,039	1,944,955	2,570,970	0.3468	2.40
	Support Vector	0.5471	1,363,322	1,785,004	2,191,026	0.3439	0.65
	K-Nearest Neighbors	0.5745	1,313,526	1,820,760	2,294,476	0.3346	0.00
50	Linear Regression	0.5159	1,407,256	1,916,936	2,332,747	0.3609	0.02
	Random Forest	0.5308	1,435,212	2,150,801	2,850,166	0.3654	3.23
	Support Vector	0.5435	1,371,151	1,854,169	2,272,515	0.3484	0.54
	K-Nearest Neighbors	0.5711	1,323,524	1,814,138	2,353,707	0.3379	0.00
100	Linear Regression	0.5101	1,416,501	1,888,406	2,281,440	0.3647	0.06
	Random Forest	0.4905	1,509,033	2,322,325	3,109,303	0.3836	4.64
	Support Vector	0.5378	1,385,282	1,850,568	2,273,441	0.3522	0.68
	K-Nearest Neighbors	0.5873	1,294,146	1,837,739	2,357,288	0.3292	0.00
200	Linear Regression	0.5051	1,435,270	1,967,415	2,346,021	0.3679	0.07
	Random Forest	0.4440	1,585,455	2,402,792	3,258,788	0.4003	6.79
	Support Vector	0.5346	1,390,313	1,911,683	2,342,793	0.3532	1.06
	K-Nearest Neighbors	0.5817	1,299,766	1,812,803	2,356,020	0.3308	0.00

In [8]:
```
common.plot_metrics(results_df, "Components")
```