Enhancing NHL Salary Evaluation through Dimensionality Reduction

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Random Projection

In [5]:
```
import common
import pandas as pd
import numpy as np
from sklearn.random_projection import GaussianRandomProjection, SparseRandomPr
ojection
```

In [6]:
```
df = common.load_dataset()

# Split features and label
X_data, y_data = common.split_dataset(df)

X_columns = X_data.columns

# Split train and test data
X_train, y_train, X_test, y_test = common.split_train_test(X_data, y_data)

# Standardize the features
X_train, X_test = common.standard_scaler(X_train, X_test)
```

Johnson-Lindenstrauss Lemma

In [7]:
```
X = X_train.copy()
n_samples, n_features = X.shape

# Determine the minimum of components k for different values of epsilon, using
the Johnson-Lindenstrauss Lemma
epsilons = [0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90]

n_components = []

for eps in epsilons:
    denominator = (eps**2 / 2) - (eps**3 / 3)
    k = (4 * np.log(n_samples) / denominator).astype(np.int64)
    n_components.append(k)

# Print results as a table
jl_df = pd.DataFrame(data={'Epsilon': epsilons, 'Number of components': n_comp
onents})
display(jl_df)

# Discard the number of components that are bigger than the dataset dimension
jl_df = jl_df[jl_df['Number of components'] < n_features]
```

	Epsilon	Number of components
0	0.1	7014
1	0.2	1888
2	0.3	909
3	0.4	557
4	0.5	392
5	0.6	303
6	0.7	250
7	0.8	219
8	0.9	202

Gaussian Random Matrix Projection

In [8]:

```
results = []
for eps, n_component in jl_df.to_numpy():
    n_component = int(n_component)

    # Generate a dense Gaussian random matrix
    rng = np.random.default_rng(12345)
    scale = 1.0 / np.sqrt(n_component)
    random_matrix = rng.normal(size=(n_features, n_component), scale=scale).as
type(X.dtype, copy=False)

    # Project data onto random matrix
    X_train_projected = np.dot(X, random_matrix)
    X_test_projected = np.dot(X_test, random_matrix)

    # Train and evaluate the models
    results_df, predictions = common.train_and_evaluate(X_train_projected, y_t
rain, X_test_projected, y_test)

    results_df["Components"] = n_component
    results_df["Epsilon"] = eps

    results.append(results_df)

results_df = pd.concat(results)
results_df.groupby(by="Epsilon")[results_df.columns].apply(lambda x: x).drop(c
olumns=["Epsilon"])
```

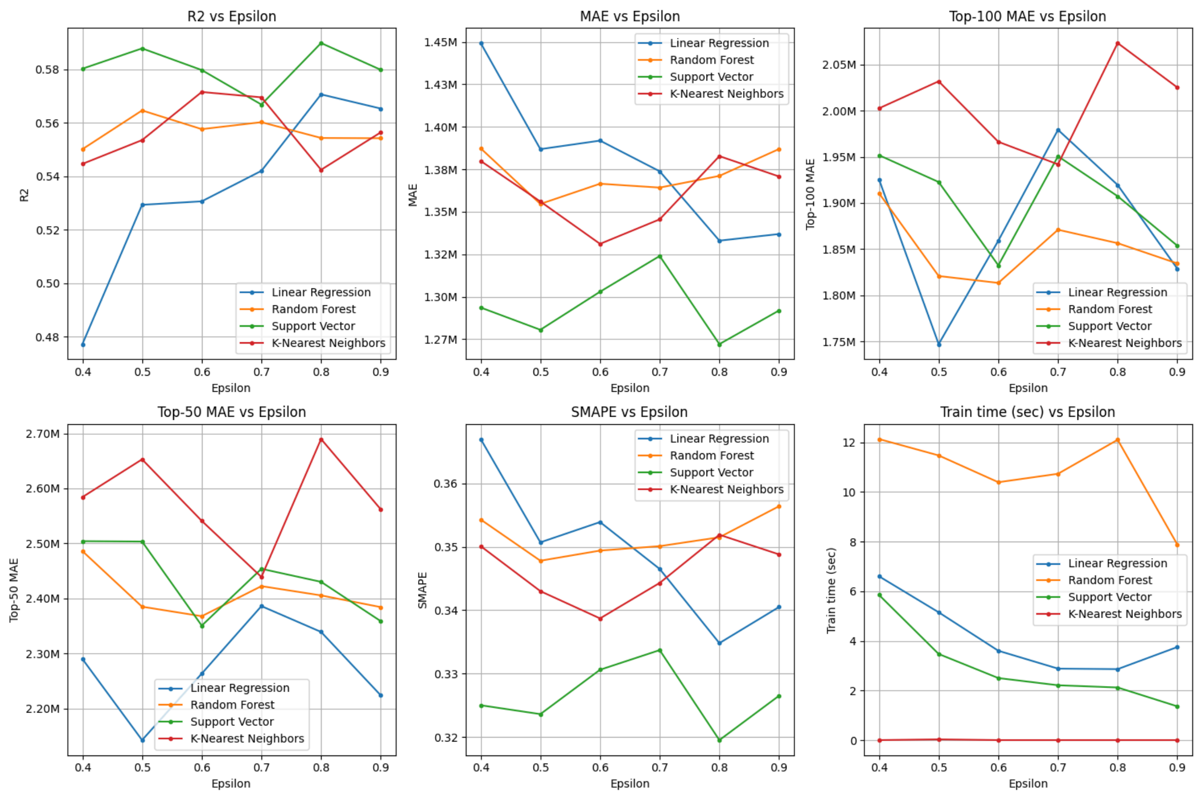
Out[8]:

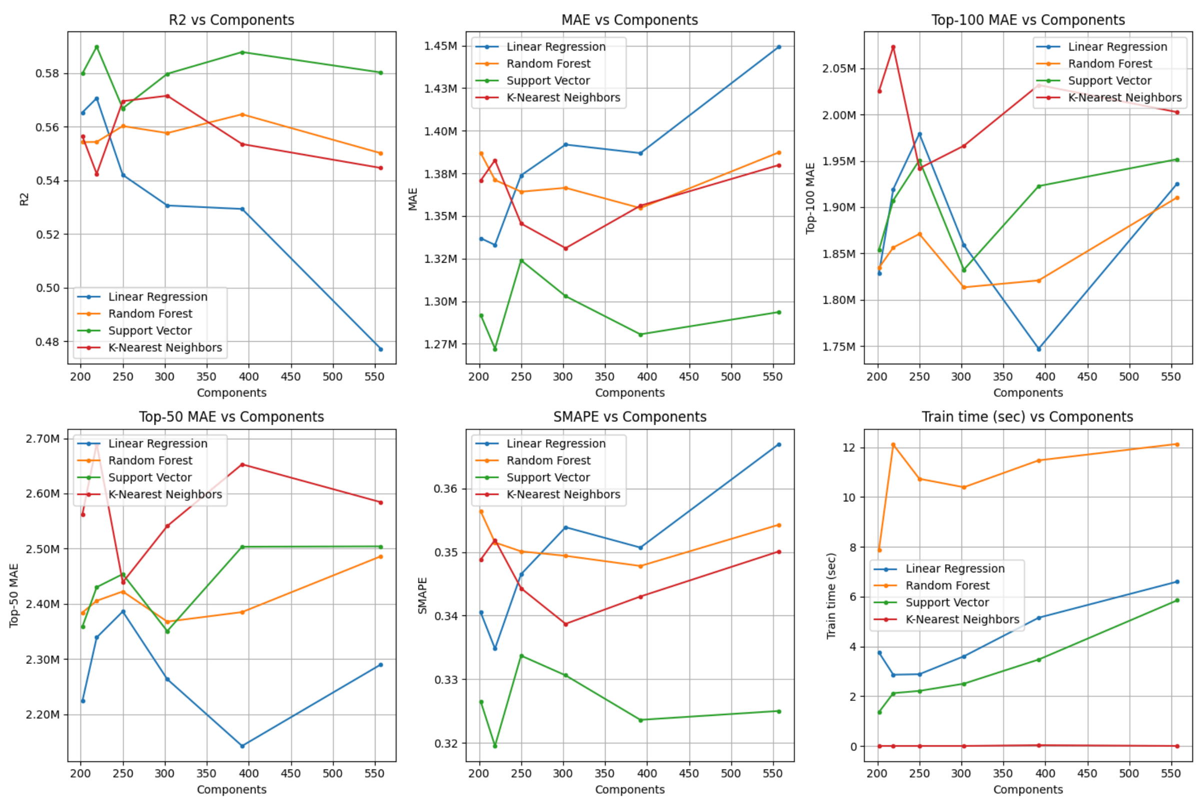
Epsilon	Model	R2	MAE	Top-100 MAE	Top-50 MAE	SMAPE	Train time (sec)	Components
0.4	Linear Regression	0.4772	1,449,343	1,925,584	2,289,866	0.3670	6.60	557
	Random Forest	0.5501	1,387,369	1,910,571	2,486,037	0.3543	12.13	557
	Support Vector	0.5802	1,293,677	1,951,716	2,504,092	0.3250	5.85	557
	K-Nearest Neighbors	0.5446	1,379,937	2,002,477	2,584,236	0.3501	0.00	557
0.5	Linear Regression	0.5293	1,386,897	1,747,167	2,142,316	0.3507	5.15	392
	Random Forest	0.5646	1,354,649	1,820,897	2,384,940	0.3478	11.47	392
	Support Vector	0.5878	1,280,588	1,922,684	2,503,436	0.3236	3.47	392
	K-Nearest Neighbors	0.5535	1,356,066	2,031,758	2,652,892	0.3430	0.03	392
0.6	Linear Regression	0.5306	1,391,917	1,859,150	2,263,253	0.3539	3.60	303
	Random Forest	0.5576	1,366,545	1,813,436	2,367,626	0.3494	10.39	303
	Support Vector	0.5797	1,302,989	1,832,474	2,350,518	0.3306	2.50	303
	K-Nearest Neighbors	0.5715	1,331,155	1,966,218	2,541,004	0.3387	0.00	303
0.7	Linear Regression	0.5420	1,373,855	1,979,209	2,385,997	0.3465	2.88	250
	Random Forest	0.5602	1,364,271	1,871,017	2,422,313	0.3501	10.73	250
	Support Vector	0.5668	1,324,016	1,950,564	2,453,819	0.3337	2.21	250
	K-Nearest Neighbors	0.5695	1,345,593	1,941,830	2,438,948	0.3443	0.00	250
0.8	Linear Regression	0.5706	1,333,041	1,919,475	2,339,018	0.3348	2.86	219
	Random Forest	0.5543	1,371,156	1,856,456	2,405,458	0.3515	12.10	219
	Support Vector	0.5898	1,272,156	1,907,310	2,430,132	0.3195	2.12	219
	K-Nearest Neighbors	0.5424	1,382,793	2,073,160	2,689,819	0.3519	0.00	219

Epsilon	Model	R2	MAE	Top-100 MAE	Top-50 MAE	SMAPE	Train time (sec)	Components
0.9	Linear Regression	0.5653	1,336,946	1,828,793	2,223,758	0.3405	3.75	202
	Random Forest	0.5542	1,386,972	1,834,528	2,384,162	0.3564	7.88	202
	Support Vector	0.5798	1,291,950	1,853,697	2,358,625	0.3265	1.36	202
	K-Nearest Neighbors	0.5564	1,370,809	2,025,079	2,562,036	0.3488	0.00	202

In [9]:

```
common.plot_metrics(results_df, "Epsilon")
common.plot_metrics(results_df, "Components")
```





Sparse Random Matrix Projection

In [10]:
```
# Density of sparse matrices
p = 1 / np.sqrt(n_features)

results = []
for eps, n_component in jl_df.to_numpy():
    n_component = int(n_component)

    # Generate random sparse matrix
    rng = np.random.default_rng(12345)
    k = n_component

    random_matrix = rng.choice(
        [-np.sqrt(1/(k*p)), 0, np.sqrt(1/(k*p))],
        size=(n_features, n_component),
        p=[p/2, 1-p, p/2]
    ).astype(X.dtype, copy=False)

    # Project data onto random sparse matrix
    X_train_projected = np.dot(X, random_matrix)
    X_test_projected = np.dot(X_test, random_matrix)

    # Train and evaluate the models
    results_df, predictions = common.train_and_evaluate(X_train_projected, y_t
rain, X_test_projected, y_test)

    results_df["Components"] = n_component
    results_df["Epsilon"] = eps

    results.append(results_df)

results_df = pd.concat(results)
results_df.groupby(by="Epsilon")[results_df.columns].apply(lambda x: x).drop(c
olumns=["Epsilon"])
```

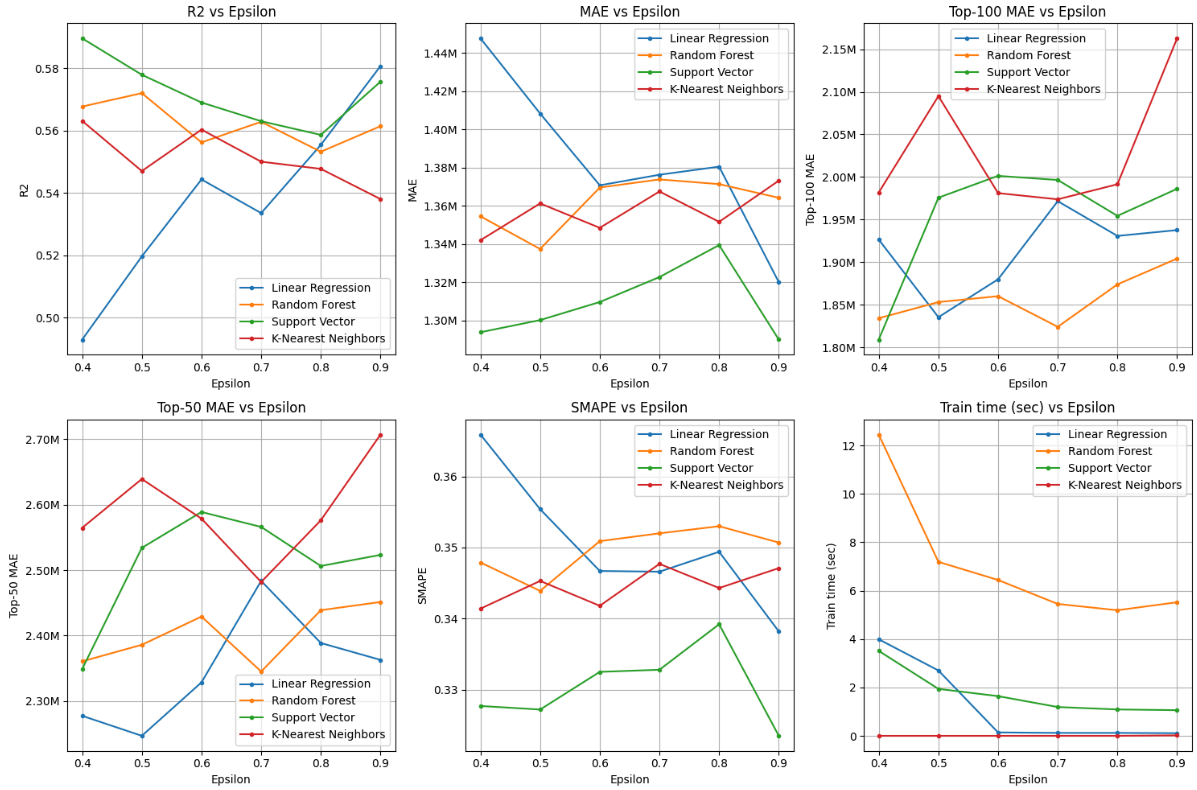
Out[10]:

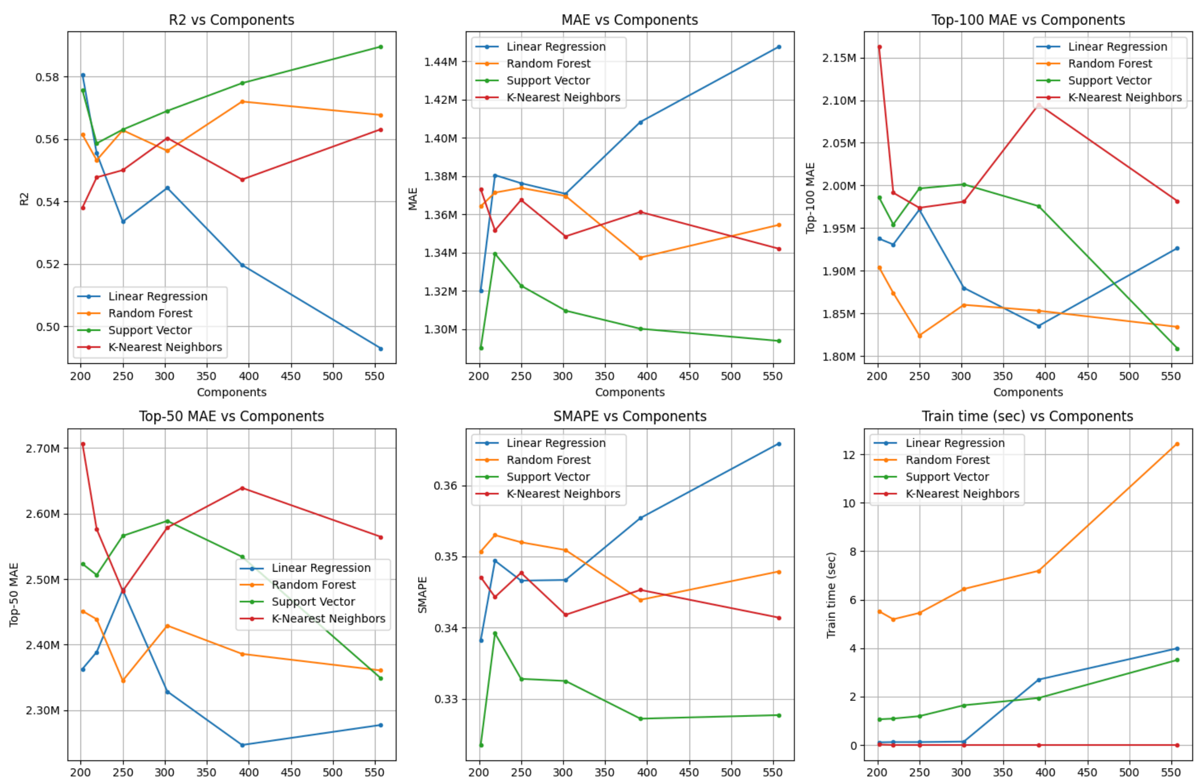
Epsilon	Model	R2	MAE	Top-100 MAE	Top-50 MAE	SMAPE	Train time (sec)	Components
0.4	Linear Regression	0.4929	1,447,730	1,926,613	2,277,496	0.3659	3.99	557
	Random Forest	0.5677	1,354,581	1,834,340	2,360,674	0.3479	12.45	557
	Support Vector	0.5896	1,293,912	1,809,285	2,348,889	0.3277	3.51	557
	K-Nearest Neighbors	0.5631	1,342,076	1,981,836	2,564,548	0.3414	0.00	557
0.5	Linear Regression	0.5196	1,408,368	1,835,555	2,246,959	0.3554	2.70	392
	Random Forest	0.5720	1,337,485	1,853,278	2,385,929	0.3439	7.19	392
	Support Vector	0.5779	1,300,279	1,975,796	2,534,010	0.3272	1.94	392
	K-Nearest Neighbors	0.5470	1,361,326	2,094,948	2,639,123	0.3453	0.00	392
0.6	Linear Regression	0.5443	1,370,806	1,880,034	2,328,615	0.3467	0.14	303
	Random Forest	0.5562	1,369,647	1,860,205	2,429,080	0.3509	6.44	303
	Support Vector	0.5690	1,309,732	2,001,370	2,588,757	0.3325	1.64	303
	K-Nearest Neighbors	0.5603	1,348,553	1,981,113	2,578,589	0.3418	0.00	303
0.7	Linear Regression	0.5335	1,376,338	1,971,659	2,483,549	0.3466	0.12	250
	Random Forest	0.5628	1,373,886	1,824,280	2,345,156	0.3520	5.45	250
	Support Vector	0.5630	1,322,790	1,996,544	2,565,925	0.3328	1.19	250
	K-Nearest Neighbors	0.5500	1,367,573	1,973,876	2,481,862	0.3477	0.00	250
0.8	Linear Regression	0.5554	1,380,573	1,930,908	2,388,718	0.3494	0.12	219
	Random Forest	0.5532	1,371,430	1,874,032	2,438,786	0.3530	5.19	219
	Support Vector	0.5586	1,339,521	1,954,430	2,506,329	0.3392	1.09	219
	K-Nearest Neighbors	0.5477	1,351,692	1,991,498	2,576,106	0.3443	0.00	219

Epsilon	Model	R2	MAE	Top-100 MAE	Top-50 MAE	SMAPE	Train time (sec)	Components
0.9	**Linear Regression**	0.5807	1,320,126	1,937,740	2,362,910	0.3382	0.11	202
	Random Forest	0.5614	1,364,272	1,904,327	2,451,333	0.3507	5.52	202
	Support Vector	0.5757	1,290,071	1,986,179	2,523,237	0.3235	1.06	202
	K-Nearest Neighbors	0.5380	1,373,240	2,162,742	2,707,002	0.3471	0.02	202

In [11]:

```
common.plot_metrics(results_df, "Epsilon")
common.plot_metrics(results_df, "Components")
```





Sklearn Random Projection

Gaussian Random Projection

In [12]:
```
results = []
for eps, n_component in jl_df.to_numpy():
    n_component = int(n_component)

    # Train and test using sklearn
    grp = GaussianRandomProjection(n_components=n_component, eps=eps, random_s
tate=12345)
    X_train_grp = grp.fit_transform(X_train)
    X_test_grp = grp.transform(X_test)

    # Train and evaluate the model
    results_df, predictions = common.train_and_evaluate(X_train_grp, y_train,
X_test_grp, y_test)

    results_df["Components"] = n_component
    results_df["Epsilon"] = eps

    results.append(results_df)

results_df = pd.concat(results)
results_df.groupby(by="Epsilon")[results_df.columns].apply(lambda x: x).drop(c
olumns=["Epsilon"])
```

Out[12]:

Epsilon	Model	R2	MAE	Top-100 MAE	Top-50 MAE	SMAPE	Train time (sec)	Components
0.4	Linear Regression	0.4192	1,469,078	1,908,124	2,260,402	0.3688	0.24	557
	Random Forest	0.5620	1,359,857	1,823,444	2,357,254	0.3503	8.96	557
	Support Vector	0.5801	1,310,526	1,851,573	2,370,715	0.3308	3.13	557
	K-Nearest Neighbors	0.5510	1,365,011	1,965,775	2,518,604	0.3475	0.00	557
0.5	Linear Regression	0.5385	1,378,102	1,756,509	2,200,566	0.3490	0.16	392
	Random Forest	0.5579	1,371,821	1,868,950	2,435,015	0.3526	7.61	392
	Support Vector	0.5863	1,302,174	1,811,184	2,292,809	0.3271	1.86	392
	K-Nearest Neighbors	0.5583	1,342,546	1,972,078	2,569,981	0.3415	0.00	392
0.6	Linear Regression	0.5328	1,410,192	1,754,458	2,214,650	0.3519	0.13	303
	Random Forest	0.5545	1,376,722	1,800,616	2,329,936	0.3547	5.78	303
	Support Vector	0.5834	1,310,403	1,848,103	2,344,505	0.3302	1.53	303
	K-Nearest Neighbors	0.5570	1,338,400	1,984,778	2,527,290	0.3401	0.00	303
0.7	Linear Regression	0.5434	1,401,641	1,790,909	2,206,982	0.3521	0.12	250
	Random Forest	0.5626	1,369,404	1,797,362	2,337,838	0.3534	5.24	250
	Support Vector	0.5817	1,315,331	1,876,547	2,356,015	0.3322	1.12	250
	K-Nearest Neighbors	0.5619	1,337,662	1,973,702	2,511,175	0.3398	0.00	250
0.8	Linear Regression	0.5631	1,357,184	1,871,351	2,253,201	0.3454	0.13	219
	Random Forest	0.5557	1,374,977	1,842,586	2,387,345	0.3531	5.60	219
	Support Vector	0.5874	1,300,504	1,890,460	2,357,412	0.3292	1.26	219
	K-Nearest Neighbors	0.5630	1,326,890	1,969,850	2,517,957	0.3374	0.00	219

Epsilon	Model	R2	MAE	Top-100 MAE	Top-50 MAE	SMAPE	Train time (sec)	Components
0.9	**Linear Regression**	0.5639	1,349,141	1,911,611	2,290,718	0.3430	0.10	202
	Random Forest	0.5633	1,358,921	1,811,341	2,349,612	0.3501	4.86	202
	Support Vector	0.5916	1,290,090	1,873,204	2,330,786	0.3259	0.90	202
	K-Nearest Neighbors	0.5659	1,324,999	1,946,208	2,472,329	0.3385	0.00	202

Sparse Matrix Random Projection

In [13]:
```
results = []
for eps, n_component in jl_df.to_numpy():
    n_component = int(n_component)

    # Train and test using sklearn
    grp = SparseRandomProjection(n_components=n_component, eps=eps, random_sta
te=12345)
    X_train_grp = grp.fit_transform(X_train)
    X_test_grp = grp.transform(X_test)

    # Train and evaluate the models
    results_df, predictions = common.train_and_evaluate(X_train_grp, y_train,
X_test_grp, y_test)

    results_df["Components"] = n_component
    results_df["Epsilon"] = eps

    results.append(results_df)

results_df = pd.concat(results)
results_df.groupby(by="Epsilon")[results_df.columns].apply(lambda x: x).drop(c
olumns=["Epsilon"])
```

Out[13]:

Epsilon	Model	R2	MAE	Top-100 MAE	Top-50 MAE	SMAPE	Train time (sec)	Components
0.4	Linear Regression	0.4871	1,441,535	1,938,102	2,303,167	0.3631	0.22	557
	Random Forest	0.5674	1,351,966	1,803,927	2,346,078	0.3476	8.46	557
	Support Vector	0.5833	1,301,843	1,899,314	2,430,388	0.3284	3.28	557
	K-Nearest Neighbors	0.5778	1,337,588	1,943,292	2,448,738	0.3414	0.01	557
0.5	Linear Regression	0.5162	1,404,633	1,941,664	2,372,015	0.3564	0.13	392
	Random Forest	0.5524	1,391,693	1,881,728	2,434,763	0.3576	6.56	392
	Support Vector	0.5772	1,297,153	1,942,015	2,533,461	0.3274	1.88	392
	K-Nearest Neighbors	0.5454	1,392,000	1,936,565	2,469,002	0.3545	0.00	392
0.6	Linear Regression	0.5276	1,389,957	1,843,873	2,226,917	0.3523	0.12	303
	Random Forest	0.5486	1,383,252	1,817,536	2,341,322	0.3536	6.89	303
	Support Vector	0.5746	1,316,245	1,822,899	2,312,214	0.3318	1.48	303
	K-Nearest Neighbors	0.5660	1,337,420	1,933,053	2,459,587	0.3393	0.02	303
0.7	Linear Regression	0.5585	1,343,418	1,642,964	2,032,249	0.3357	0.13	250
	Random Forest	0.5676	1,364,784	1,855,125	2,355,645	0.3514	5.18	250
	Support Vector	0.5776	1,290,019	1,855,029	2,430,698	0.3269	1.20	250
	K-Nearest Neighbors	0.5521	1,370,646	1,997,861	2,510,903	0.3493	0.00	250
0.8	Linear Regression	0.5793	1,317,984	1,810,111	2,211,133	0.3343	0.08	219
	Random Forest	0.5530	1,374,642	1,863,367	2,404,346	0.3509	4.88	219
	Support Vector	0.5862	1,288,950	1,849,318	2,322,432	0.3264	1.00	219
	K-Nearest Neighbors	0.5789	1,324,909	1,877,110	2,391,925	0.3394	0.00	219

Epsilon	Model	R2	MAE	Top-100 MAE	Top-50 MAE	SMAPE	Train time (sec)	Components
0.9	**Linear Regression**	0.5643	1,354,683	1,886,447	2,299,880	0.3399	0.10	202
	Random Forest	0.5565	1,377,184	1,858,645	2,393,417	0.3521	5.05	202
	Support Vector	0.5625	1,334,153	1,923,577	2,423,074	0.3363	0.99	202
	K-Nearest Neighbors	0.5547	1,351,077	1,997,335	2,513,720	0.3448	0.00	202

In [14]:
```
common.plot_metrics(results_df, "Epsilon")
common.plot_metrics(results_df, "Components")
```