Enhancing NHL Salary Evaluation through Dimensionality Reduction

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Principal Component Analysis (PCA)

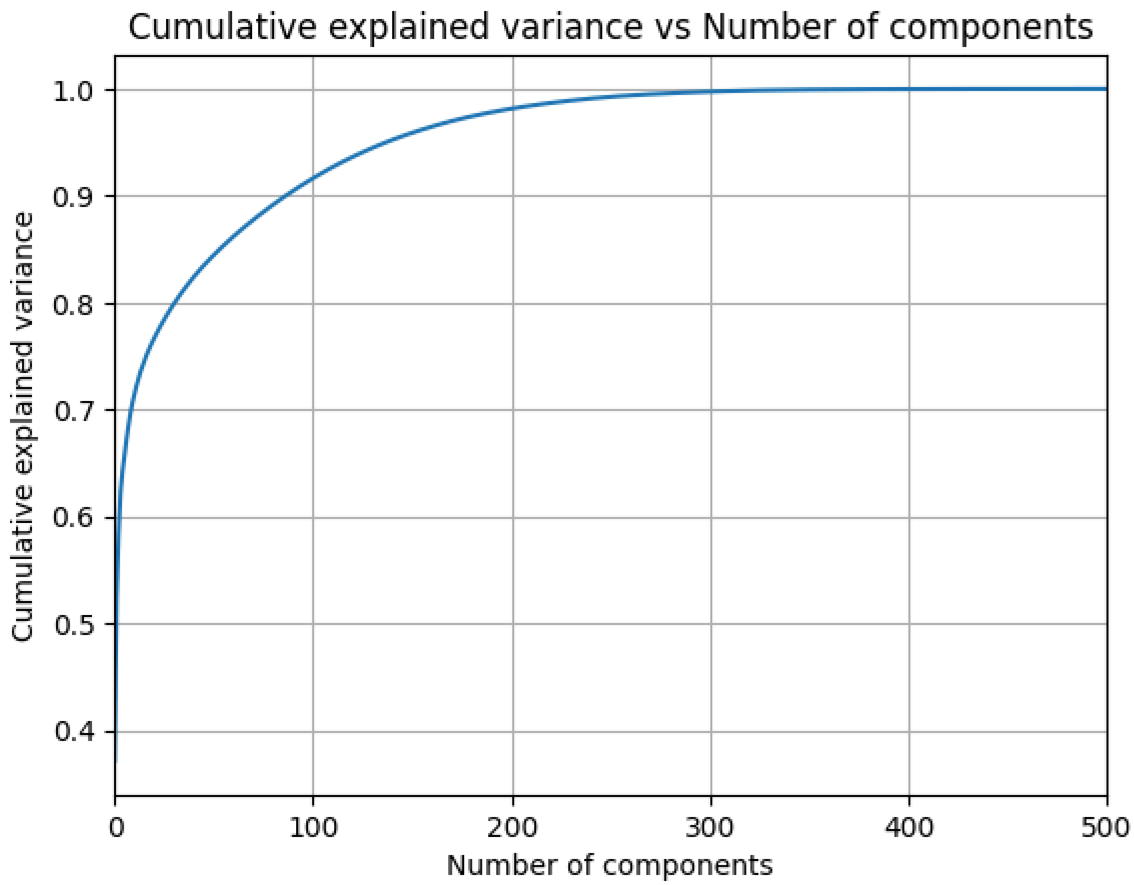
In [39]:
```
import common
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from sklearn.decomposition import PCA
from IPython.display import Markdown
```

In [40]:
```
df = common.load_dataset()

# Split features and label
X_data, y_data = common.split_dataset(df)

X_columns = X_data.columns

# Split train and test data
X_train, y_train, X_test, y_test = common.split_train_test(X_data, y_data)

# Standardize the features
X_train, X_test = common.standard_scaler(X_train, X_test)
```

Implementation 1: Singular Value Decomposition

In [41]:
```
X = X_train.copy()
n_samples, n_features = X.shape

# Center the data (subtract the mean of each feature)
X_centered = X - np.mean(X, axis=0)

# Singular Value Decomposition
U, S, Vt = np.linalg.svd(X_centered, full_matrices=False)

# Compute explained variance
explained_variance = (S**2) / (n_samples-1)
total_explained_variance = np.sum(explained_variance)
explained_variance_ratio = explained_variance / total_explained_variance

# Compute cumulative explained variance
cumsum = np.cumsum(explained_variance_ratio)
x_range = range(0, len(cumsum))
```

In [42]:
```
plt.clf()
plt.plot(range(0, len(cumsum)), cumsum)
plt.title("Cumulative explained variance vs Number of components")
plt.xlabel("Number of components")
plt.ylabel("Cumulative explained variance")
plt.xlim(0, 500)
plt.grid()
plt.show()
```



In [43]:
```python
str_output = ""

# Get the top 10 contributing features for the first 5 principal components
for i in range(5):
    # Get the component (row of Vt)
    component = Vt[i]

    # Get the indices of the top 10 contributing features
    top_features_idx = np.argsort(np.abs(component))[-10:][::-1]

    # Print the top 10 contributing features for the i-th principal component
    str_output += f"### Principal Component {i+1}:\n"
    str_output += f"Explained Variance: {explained_variance[i]:.4f}<br>"
    str_output += f"Explained Variance Ratio: {explained_variance_ratio[i]:.4f}<br>"
    str_output += f"Cumulative Explained Variance: {cumsum[i]:.4f}\n"

Markdown(str_output)
```

Out[43]:

Principal Component 1:

Explained Variance: 283.8259
Explained Variance Ratio: 0.3714
Cumulative Explained Variance: 0.3714

Principal Component 2:

Explained Variance: 115.4110
Explained Variance Ratio: 0.1510
Cumulative Explained Variance: 0.5224

Principal Component 3:

Explained Variance: 46.7740
Explained Variance Ratio: 0.0612
Cumulative Explained Variance: 0.5836

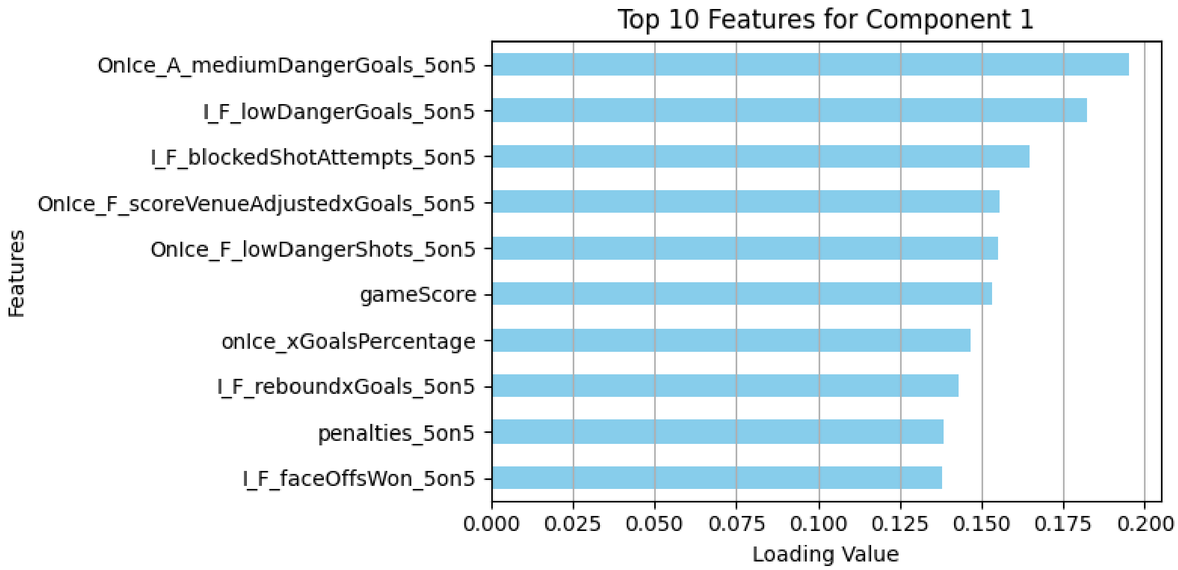
Principal Component 4:

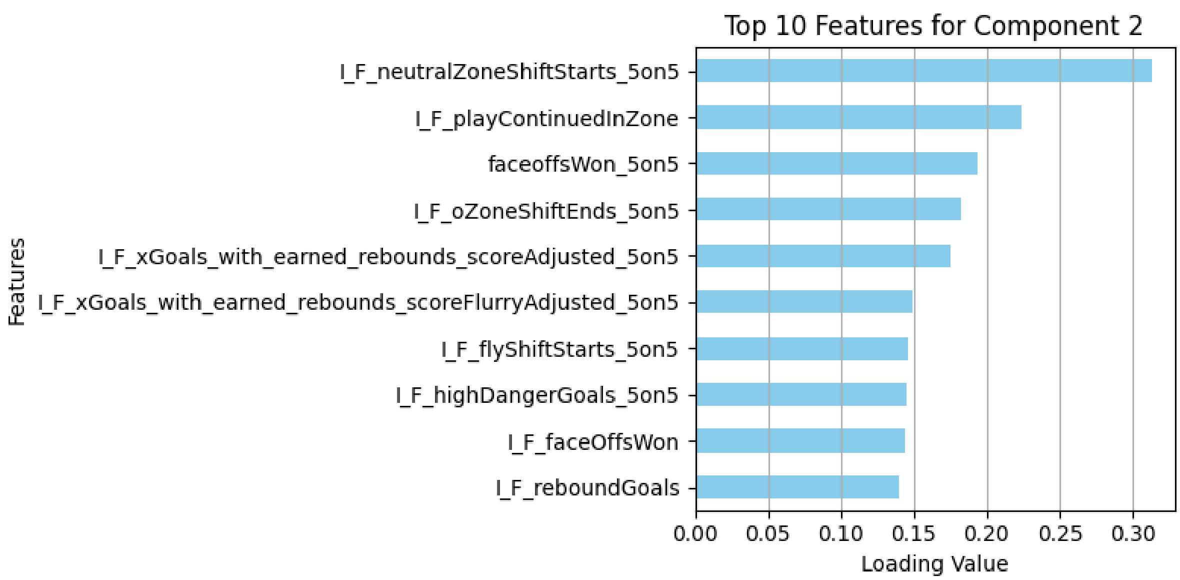
Explained Variance: 29.8602
Explained Variance Ratio: 0.0391
Cumulative Explained Variance: 0.6227

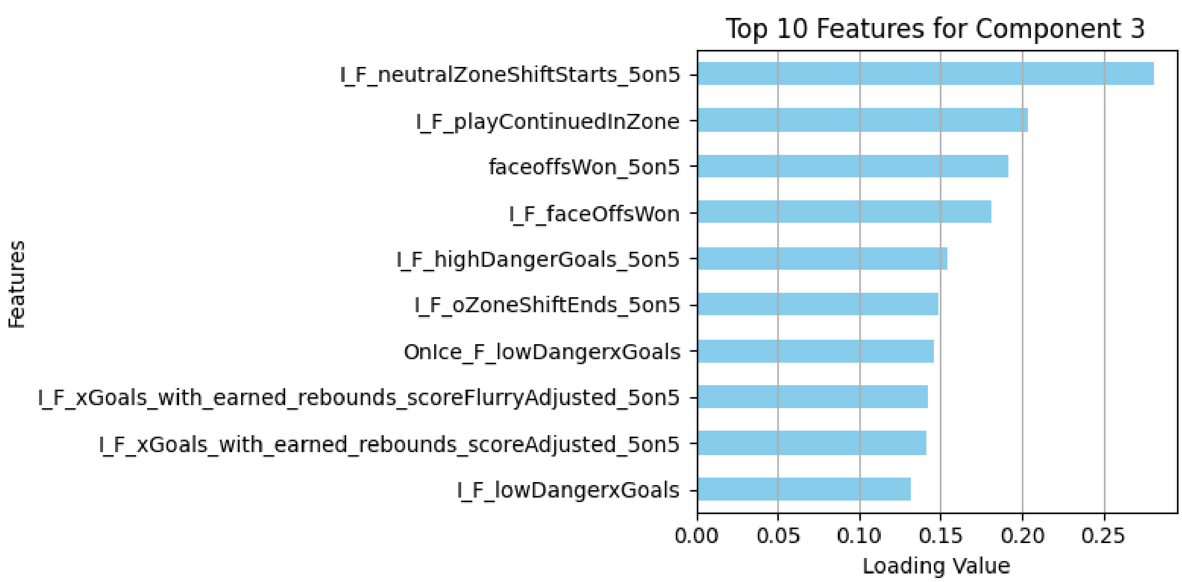
Principal Component 5:

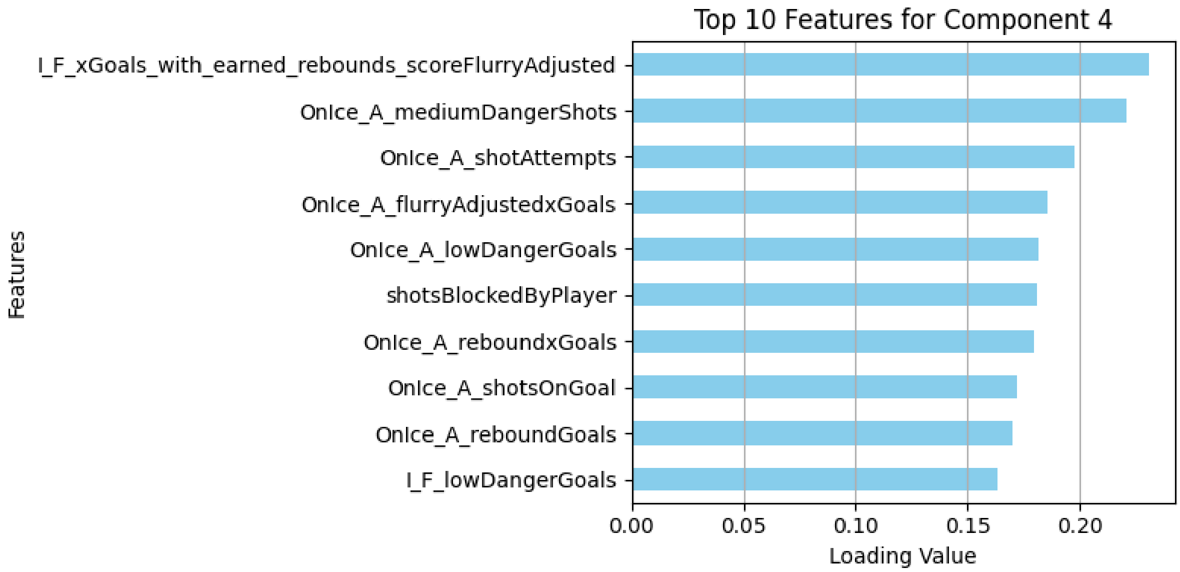
Explained Variance: 15.2144
Explained Variance Ratio: 0.0199
Cumulative Explained Variance: 0.6426

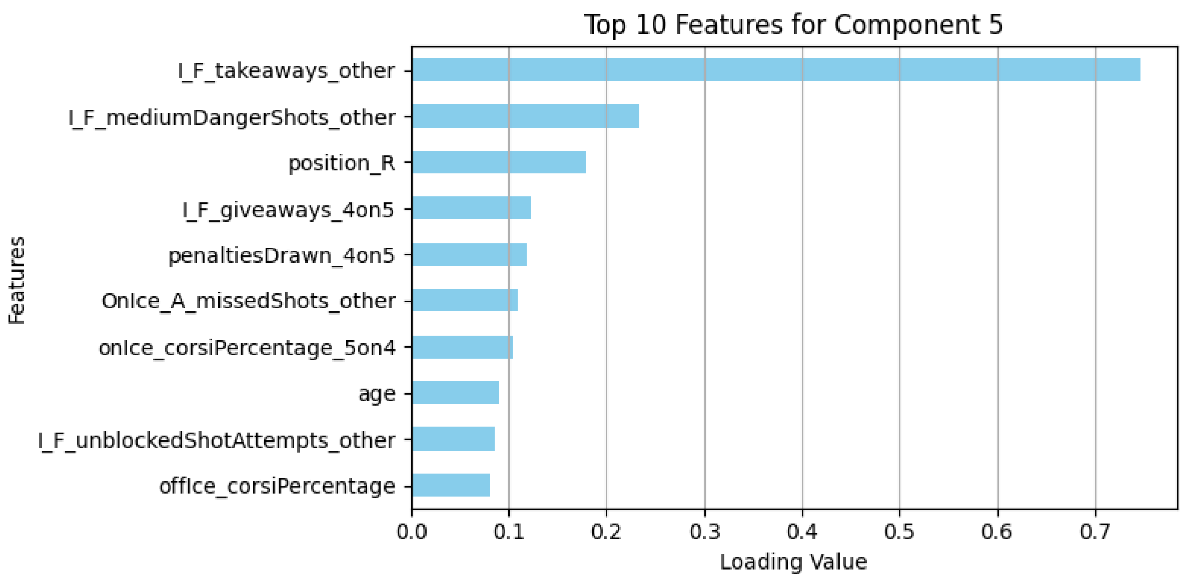
In [44]:
```python
n_top_features = 10

loadings_df = pd.DataFrame(Vt, index=X_data.columns)

for comp in range(5):
    plt.figure(figsize=(8, 4))
    # Sort features by absolute loading values for the current component
    top_features = loadings_df.iloc[:, comp].abs().nlargest(n_top_features)

    # Plot
    top_features.sort_values().plot(kind='barh', color='skyblue')
    plt.title(f"Top {n_top_features} Features for Component {comp + 1}")
    plt.xlabel("Loading Value")
    plt.ylabel("Features")
    plt.grid(axis='x')
    plt.tight_layout()
    plt.show()
```


Top 10 Features for Component 1
Features
OnIce_A_mediumDangerGoals_5on5
I_F_lowDangerGoals_5on5
I_F_blockedShotAttempts_5on5
OnIce_F_scoreVenueAdjustedxGoals_5on5
OnIce_F_lowDangerShots_5on5
gameScore
onIce_xGoalsPercentage
I_F_reboundxGoals_5on5
penalties_5on5
I_F_faceOffsWon_5on5
0.000 0.025 0.050 0.075 0.100 0.125 0.150 0.175 0.200
Loading Value


Top 10 Features for Component 2
Features
I_F_neutralZoneShiftStarts_5on5
I_F_playContinuedInZone
faceoffsWon_5on5
I_F_oZoneShiftEnds_5on5
I_F_xGoals_with_earned_rebounds_scoreAdjusted_5on5
I_F_xGoals_with_earned_rebounds_scoreFlurryAdjusted_5on5
I_F_flyShiftStarts_5on5
I_F_highDangerGoals_5on5
I_F_faceOffsWon
I_F_reboundGoals
0.00 0.05 0.10 0.15 0.20 0.25 0.30
Loading Value


Top 10 Features for Component 3
Features
I_F_neutralZoneShiftStarts_5on5
I_F_playContinuedInZone
faceoffsWon_5on5
I_F_faceOffsWon
I_F_highDangerGoals_5on5
I_F_oZoneShiftEnds_5on5
OnIce_F_lowDangerxGoals
I_F_xGoals_with_earned_rebounds_scoreFlurryAdjusted_5on5
I_F_xGoals_with_earned_rebounds_scoreAdjusted_5on5
I_F_lowDangerxGoals
0.00 0.05 0.10 0.15 0.20 0.25
Loading Value


Top 10 Features for Component 4
I_F_xGoals_with_earned_rebounds_scoreFlurryAdjusted
OnIce_A_mediumDangerShots
OnIce_A_shotAttempts
OnIce_A_flurryAdjustedxGoals
OnIce_A_lowDangerGoals
shotsBlockedByPlayer
OnIce_A_reboundxGoals
OnIce_A_shotsOnGoal
OnIce_A_reboundGoals
I_F_lowDangerGoals
Features
0.00
0.05
0.10
0.15
0.20
Loading Value

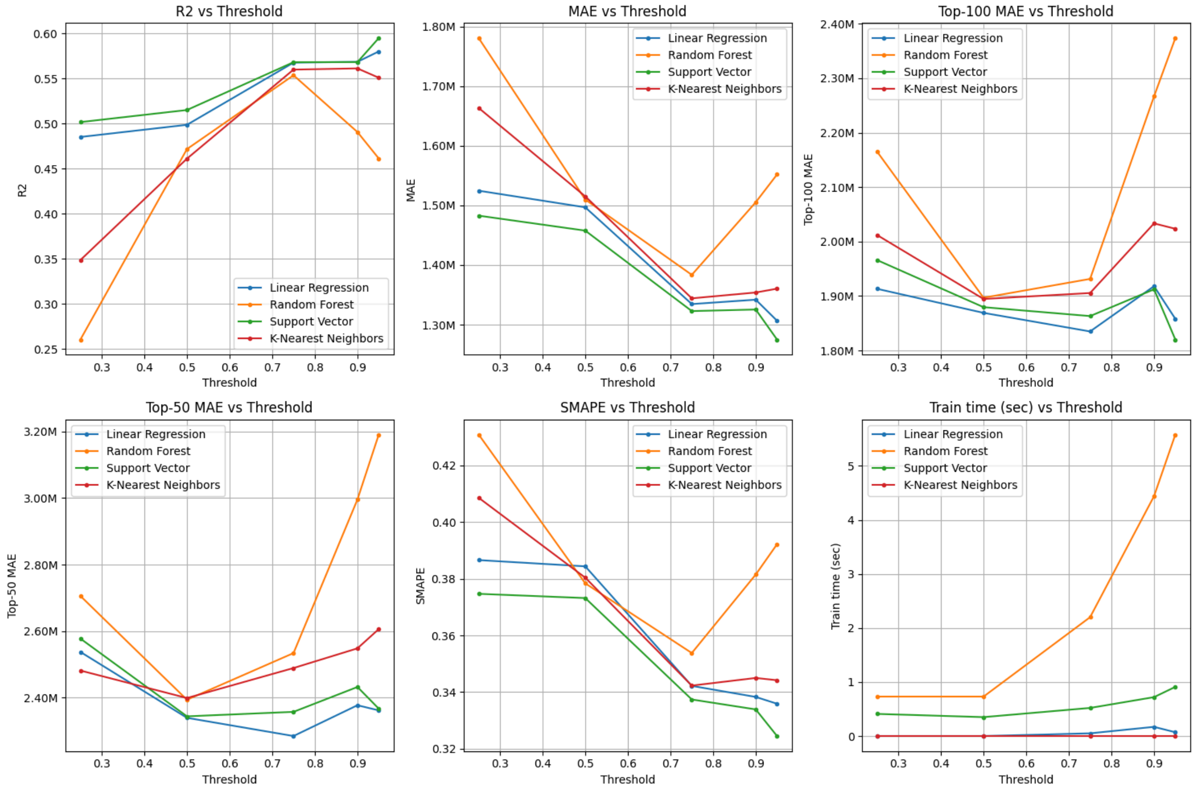

Top 10 Features for Component 5
I_F_takeaways_other
I_F_mediumDangerShots_other
position_R
I_F_giveaways_4on5
penaltiesDrawn_4on5
OnIce_A_missedShots_other
onIce_corsiPercentage_5on4
age
I_F_unblockedShotAttempts_other
offIce_corsiPercentage
Features
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Loading Value

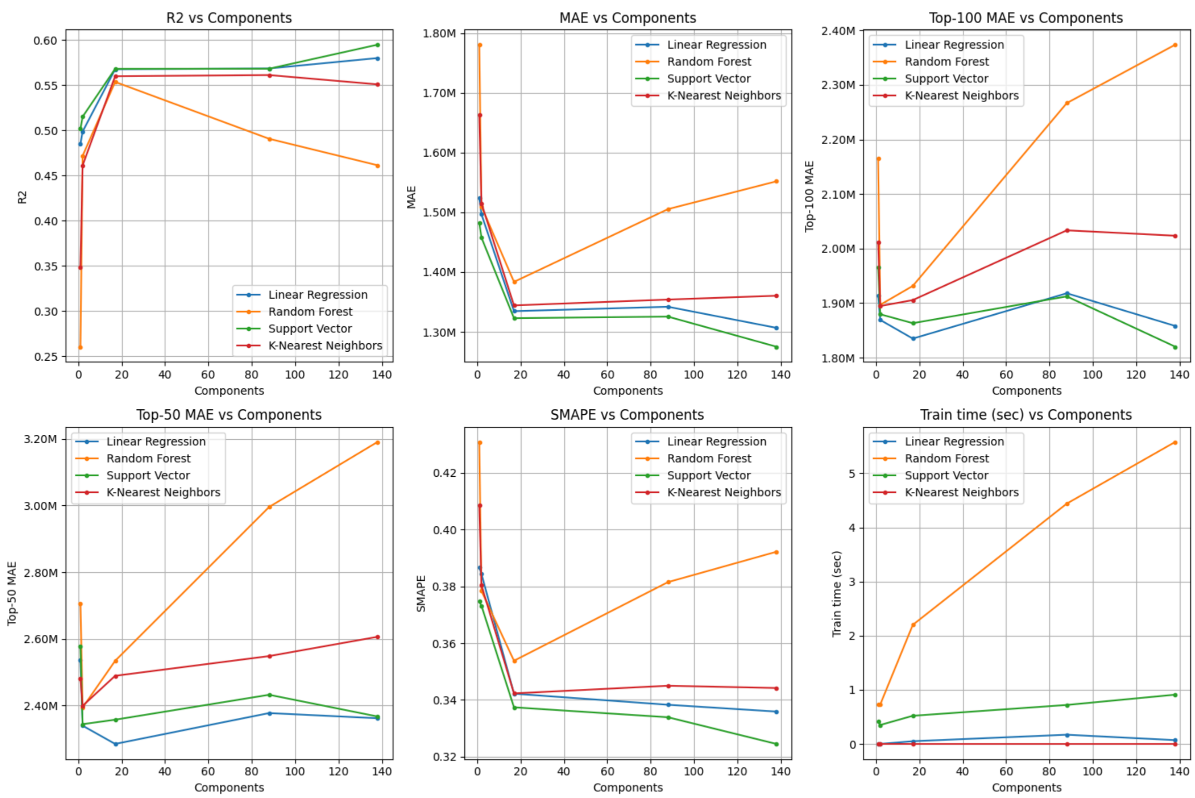
In [45]:
```
variance_thresholds = [0.25, 0.50, 0.75, 0.90, 0.95]

results = []
for variance_threshold in variance_thresholds:
    # Select the components
    selected_components = np.where(cumsum >= variance_threshold)[0][0] + 1

    # Transform data into principal component space
    X_train_pca = np.dot(X_centered, Vt.T)
    X_train_pca = X_train_pca[:, :selected_components]

    # Adjust the features for testing
    X_test_centered = X_test - np.mean(X, axis=0)
    X_test_pca = np.dot(X_test_centered, Vt.T)
    X_test_pca = X_test_pca[:, :selected_components]

    # Train and evaluate the model
    results_df, predictions = common.train_and_evaluate(X_train_pca, y_train,
X_test_pca, y_test)

    results_df["Components"] = selected_components
    results_df["Threshold"] = variance_threshold

    results.append(results_df)

results_df = pd.concat(results)
results_df.groupby(by="Threshold")[results_df.columns].apply(lambda x: x).drop
(columns=["Threshold"])
```

Out[45]:

Threshold	Model	R2	MAE	Top-100 MAE	Top-50 MAE	SMAPE	Train time (sec)	Components
0.25	**Linear Regression**	0.4851	1,524,628	1,913,240	2,537,464	0.3866	0.00	1
	Random Forest	0.2606	1,780,485	2,165,465	2,705,744	0.4309	0.73	1
	Support Vector	0.5016	1,482,780	1,966,063	2,577,555	0.3747	0.41	1
	K-Nearest Neighbors	0.3487	1,663,351	2,012,112	2,481,525	0.4086	0.00	1
0.50	**Linear Regression**	0.4986	1,496,771	1,869,011	2,339,426	0.3844	0.00	2
	Random Forest	0.4717	1,509,837	1,896,849	2,393,731	0.3784	0.73	2
	Support Vector	0.5150	1,457,700	1,879,472	2,343,895	0.3732	0.35	2
	K-Nearest Neighbors	0.4611	1,515,137	1,894,536	2,398,909	0.3804	0.00	2
0.75	**Linear Regression**	0.5675	1,334,535	1,834,847	2,284,677	0.3422	0.05	17
	Random Forest	0.5535	1,383,525	1,931,416	2,534,001	0.3538	2.20	17
	Support Vector	0.5680	1,322,788	1,863,102	2,357,359	0.3374	0.52	17
	K-Nearest Neighbors	0.5598	1,344,158	1,905,473	2,489,060	0.3423	0.00	17
0.90	**Linear Regression**	0.5684	1,341,923	1,917,990	2,377,233	0.3383	0.17	88
	Random Forest	0.4906	1,505,292	2,266,730	2,995,644	0.3815	4.44	88
	Support Vector	0.5681	1,325,428	1,912,240	2,432,140	0.3339	0.72	88
	K-Nearest Neighbors	0.5611	1,353,952	2,033,402	2,548,211	0.3450	0.00	88
0.95	**Linear Regression**	0.5799	1,306,634	1,858,033	2,361,751	0.3359	0.07	138
	Random Forest	0.4613	1,551,825	2,373,857	3,190,814	0.3922	5.58	138
	Support Vector	0.5947	1,275,063	1,819,979	2,366,959	0.3245	0.91	138
	K-Nearest Neighbors	0.5508	1,360,359	2,023,507	2,606,079	0.3442	0.00	138

In [46]:
```
common.plot_metrics(results_df, "Threshold")
common.plot_metrics(results_df, "Components")
```

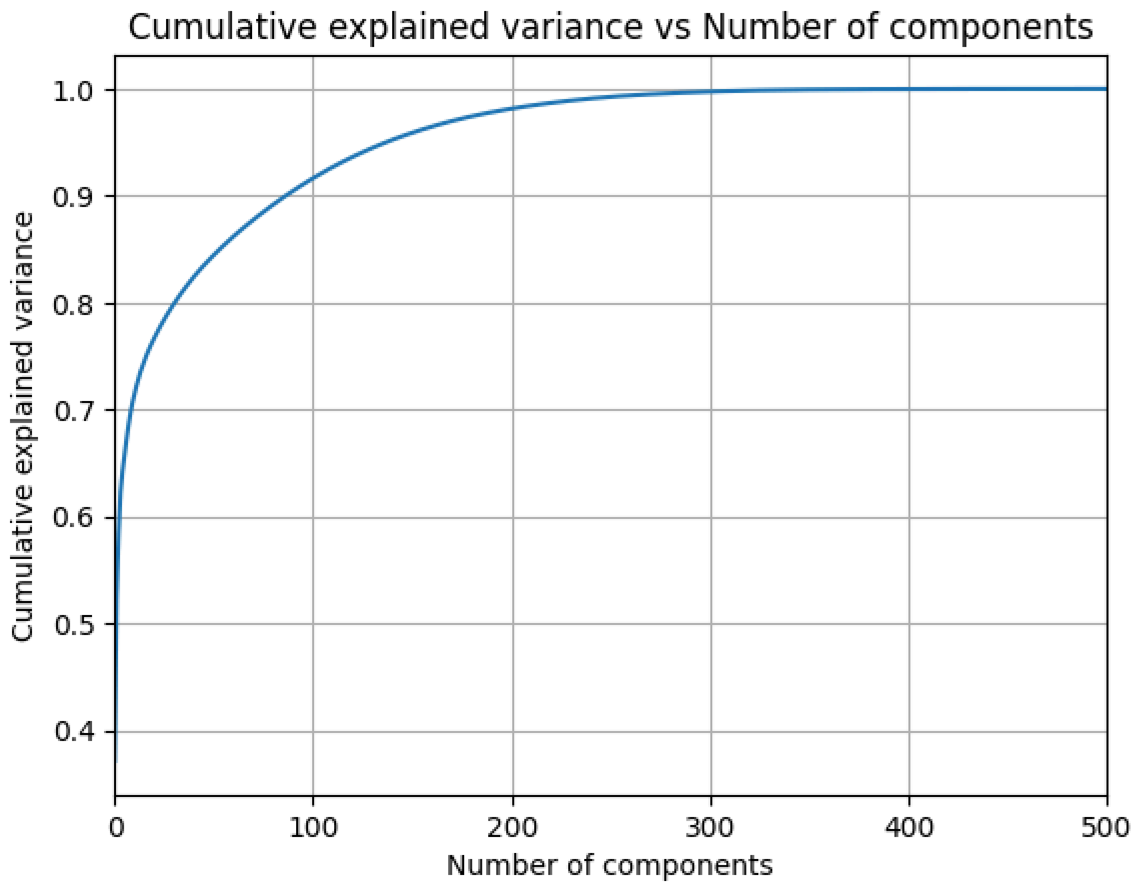




Implementation 2: Covariance Matrix and Eigenvalues

In [47]:
```
X = X_train.copy()
n_samples, n_features = X.shape

# Center the data (subtract the mean of each feature)
X_centered = X - np.mean(X, axis=0)

# Compute the covariance matrix
C = (X.T @ X)/(n_samples-1)

# Compute the eigenvalues and eigenvectors of the covariance matrix
eigenvalues, eigenvectors = np.linalg.eigh(C)

# Sort the eigenvalues and eigenvectors in descending order
eigenvalues = np.flip(eigenvalues, axis=0)
eigenvectors = np.flip(eigenvectors, axis=1)

# Clip to positive for numerical stability
eigenvalues[eigenvalues < 0.0] = 0.0

Vt = eigenvectors.T

# Compute explained variance
explained_variance = eigenvalues
total_explained_variance = np.sum(explained_variance)
explained_variance_ratio = explained_variance / total_explained_variance

# Compute cumulative explained variance
cumsum = np.cumsum(explained_variance_ratio)
x_range = range(0, len(cumsum))
```

In [48]:
```
plt.clf()
plt.plot(range(0, len(cumsum)), cumsum)
plt.title("Cumulative explained variance vs Number of components")
plt.xlabel("Number of components")
plt.ylabel("Cumulative explained variance")
plt.xlim(0, 500)
plt.grid()
plt.show()
```



In [49]:
```python
str_output = ""

# Get the top 10 contributing features for the first 5 principal components
for i in range(5):
    # Get the component (row of Vt)
    component = Vt[i]

    # Get the indices of the top 10 contributing features
    top_features_idx = np.argsort(np.abs(component))[-10:][::-1]

    # Print the top 10 contributing features for the i-th principal component
    str_output += f"### Principal Component {i+1}:\n"
    str_output += f"Explained Variance: {explained_variance[i]:.4f}<br>"
    str_output += f"Explained Variance Ratio: {explained_variance_ratio[i]:.4
f}<br>"
    str_output += f"Cumulative Explained Variance: {cumsum[i]:.4f}\n"

Markdown(str_output)
```

Out[49]:

Principal Component 1:

Explained Variance: 283.8259
Explained Variance Ratio: 0.3714
Cumulative Explained Variance: 0.3714

Principal Component 2:

Explained Variance: 115.4110
Explained Variance Ratio: 0.1510
Cumulative Explained Variance: 0.5224

Principal Component 3:

Explained Variance: 46.7740
Explained Variance Ratio: 0.0612
Cumulative Explained Variance: 0.5836

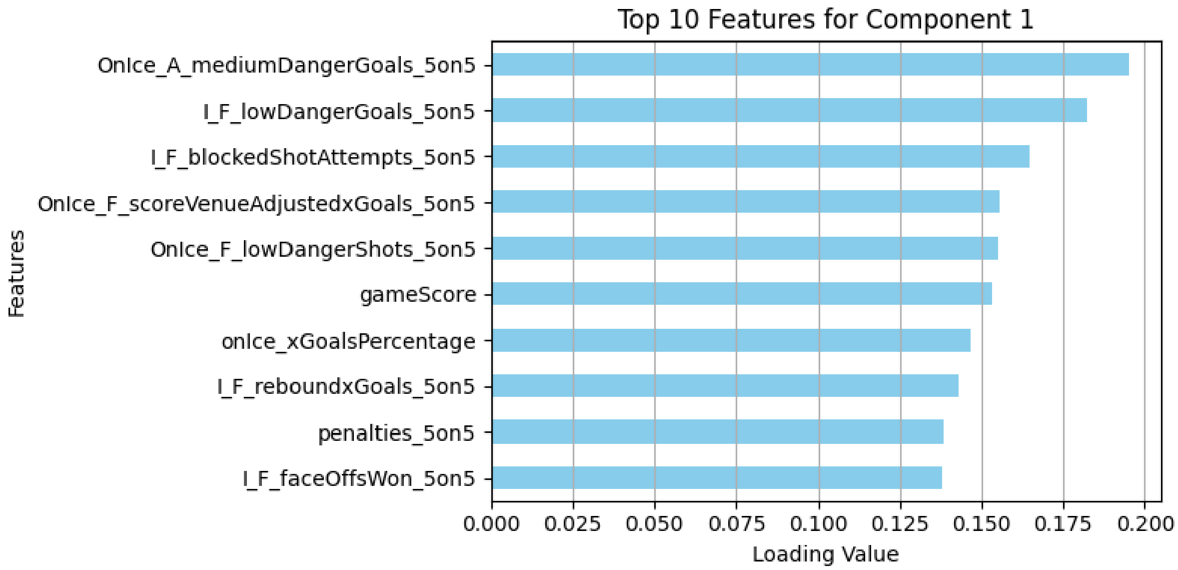
Principal Component 4:

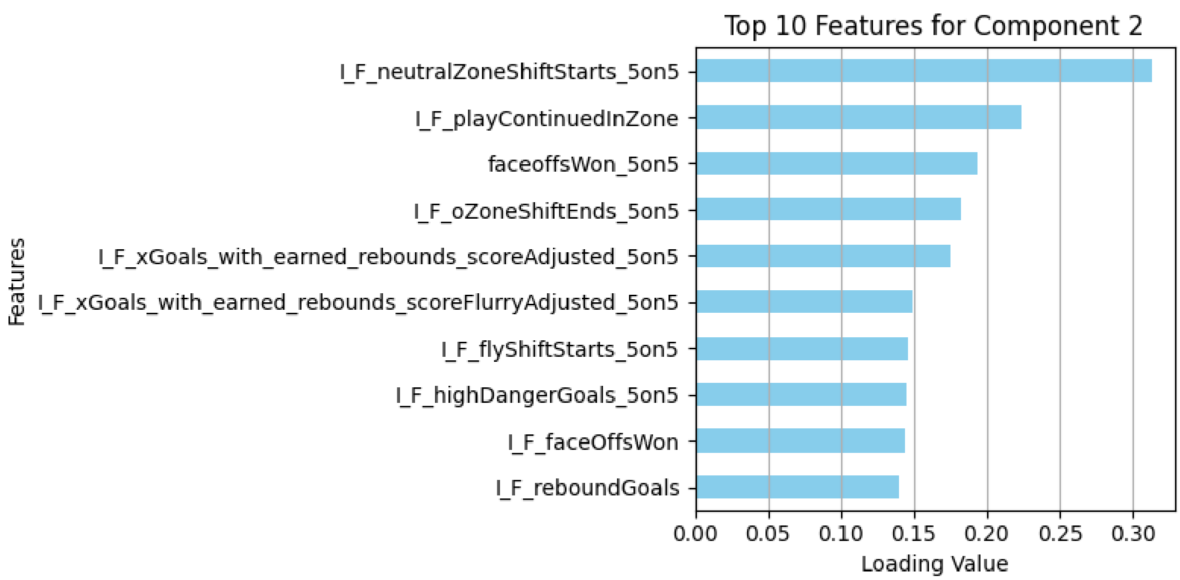
Explained Variance: 29.8602
Explained Variance Ratio: 0.0391
Cumulative Explained Variance: 0.6227

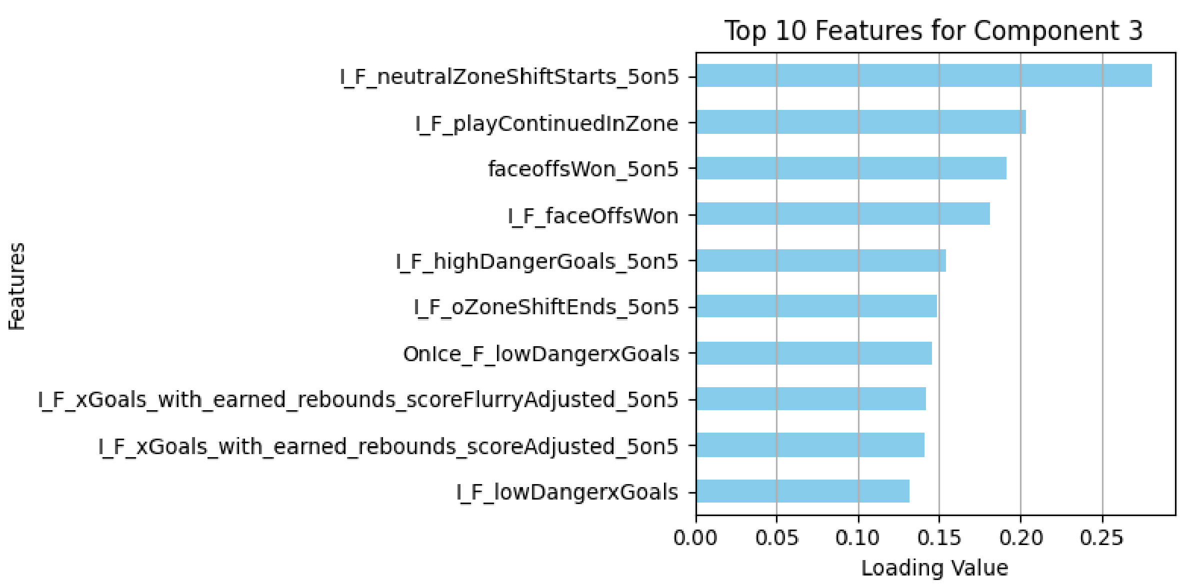
Principal Component 5:

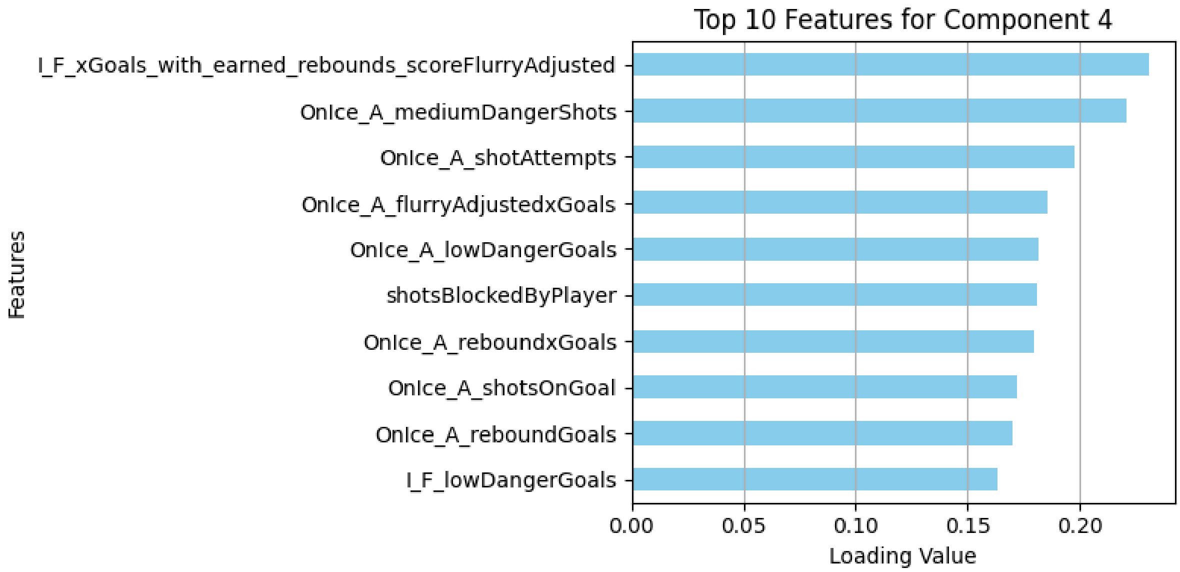
Explained Variance: 15.2144
Explained Variance Ratio: 0.0199
Cumulative Explained Variance: 0.6426

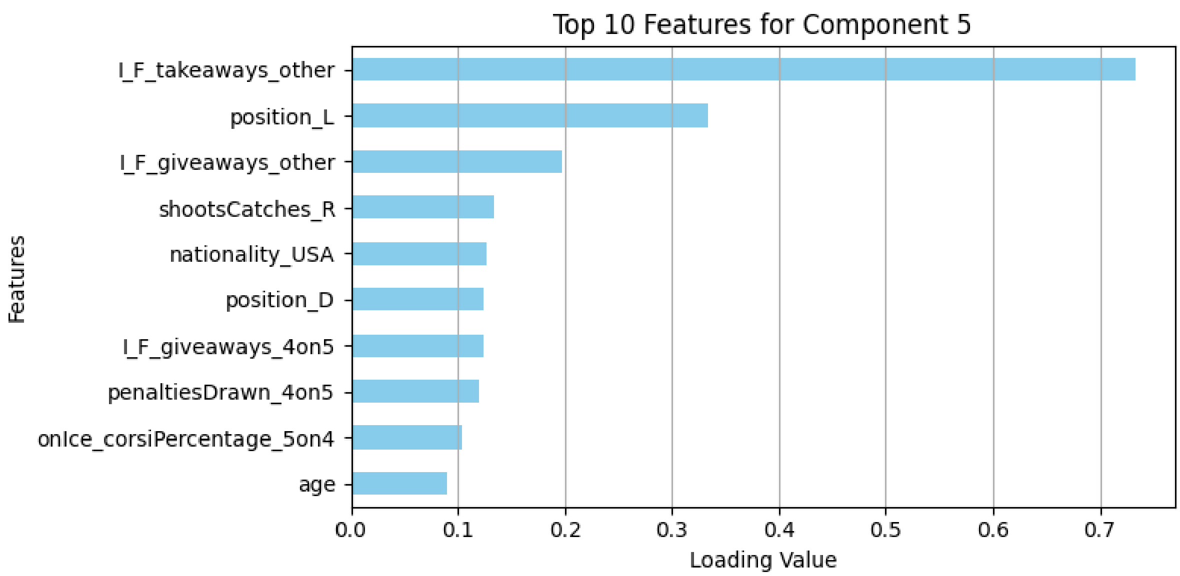
In [50]:
```
n_top_features = 10

loadings_df = pd.DataFrame(Vt, index=X_data.columns)

for comp in range(5):
    plt.figure(figsize=(8, 4))
    # Sort features by absolute loading values for the current component
    top_features = loadings_df.iloc[:, comp].abs().nlargest(n_top_features)

    # Plot
    top_features.sort_values().plot(kind='barh', color='skyblue')
    plt.title(f"Top {n_top_features} Features for Component {comp + 1}")
    plt.xlabel("Loading Value")
    plt.ylabel("Features")
    plt.grid(axis='x')
    plt.tight_layout()
    plt.show()
```


Top 10 Features for Component 1
OnIce_A_mediumDangerGoals_5on5
I_F_lowDangerGoals_5on5
I_F_blockedShotAttempts_5on5
OnIce_F_scoreVenueAdjustedxGoals_5on5
OnIce_F_lowDangerShots_5on5
gameScore
onIce_xGoalsPercentage
I_F_reboundxGoals_5on5
penalties_5on5
I_F_faceOffsWon_5on5
Features
0.000 0.025 0.050 0.075 0.100 0.125 0.150 0.175 0.200
Loading Value


Top 10 Features for Component 2
I_F_neutralZoneShiftStarts_5on5
I_F_playContinuedInZone
faceoffsWon_5on5
I_F_oZoneShiftEnds_5on5
I_F_xGoals_with_earned_rebounds_scoreAdjusted_5on5
I_F_xGoals_with_earned_rebounds_scoreFlurryAdjusted_5on5
I_F_flyShiftStarts_5on5
I_F_highDangerGoals_5on5
I_F_faceOffsWon
I_F_reboundGoals
Features
0.00 0.05 0.10 0.15 0.20 0.25 0.30
Loading Value


Top 10 Features for Component 3
I_F_neutralZoneShiftStarts_5on5
I_F_playContinuedInZone
faceoffsWon_5on5
I_F_faceOffsWon
I_F_highDangerGoals_5on5
I_F_oZoneShiftEnds_5on5
OnIce_F_lowDangerxGoals
I_F_xGoals_with_earned_rebounds_scoreFlurryAdjusted_5on5
I_F_xGoals_with_earned_rebounds_scoreAdjusted_5on5
I_F_lowDangerxGoals
Features
0.00 0.05 0.10 0.15 0.20 0.25
Loading Value


Top 10 Features for Component 4
I_F_xGoals_with_earned_rebounds_scoreFlurryAdjusted
OnIce_A_mediumDangerShots
OnIce_A_shotAttempts
OnIce_A_flurryAdjustedxGoals
OnIce_A_lowDangerGoals
shotsBlockedByPlayer
OnIce_A_reboundxGoals
OnIce_A_shotsOnGoal
OnIce_A_reboundGoals
I_F_lowDangerGoals
Features
0.00
0.05
0.10
0.15
0.20
Loading Value


Top 10 Features for Component 5
I_F_takeaways_other
position_L
I_F_giveaways_other
shootsCatches_R
nationality_USA
position_D
I_F_giveaways_4on5
penaltiesDrawn_4on5
onIce_corsiPercentage_5on4
age
Features
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Loading Value

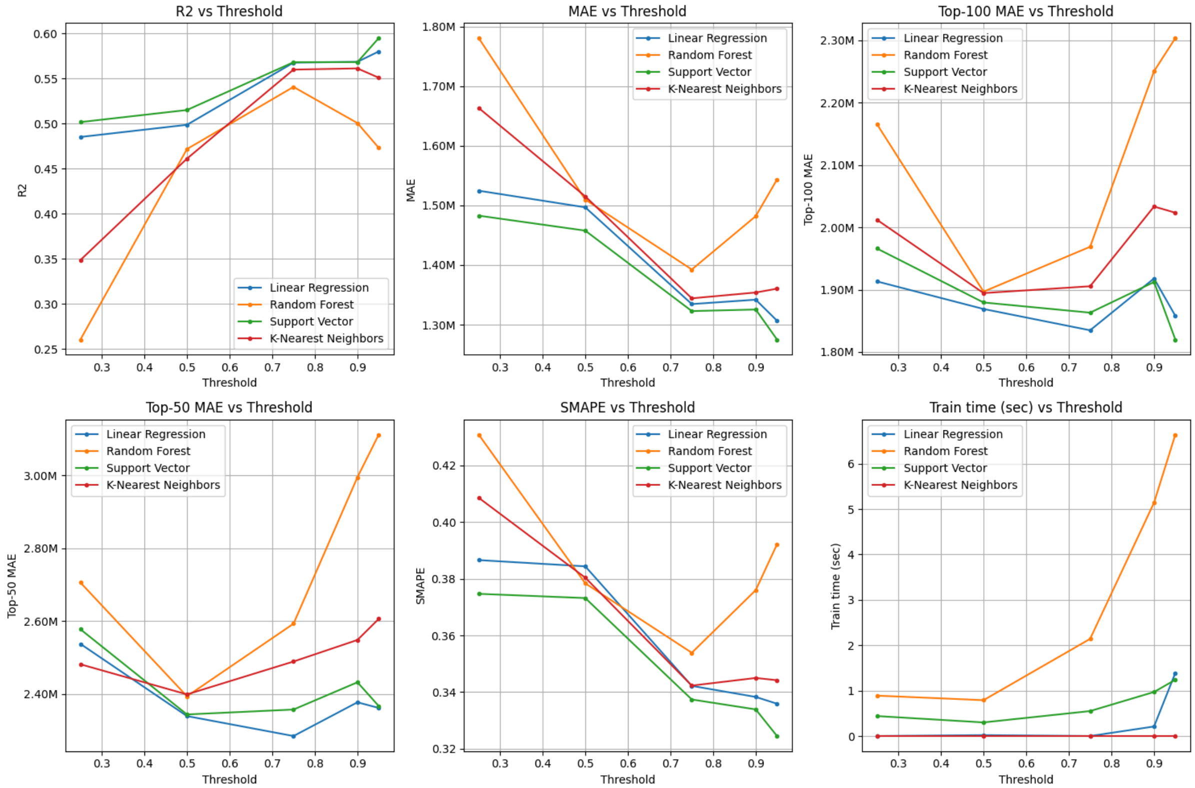
In [51]:

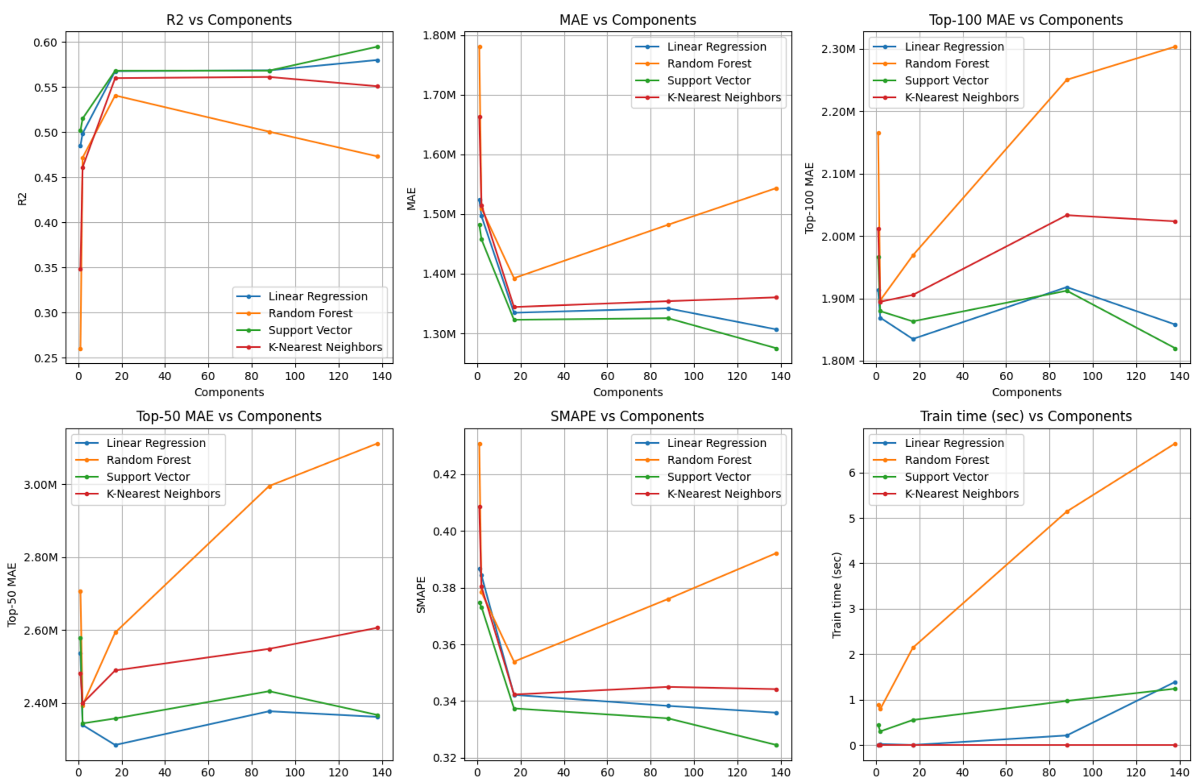
```
results = []
for variance_threshold in variance_thresholds:
    # Select the components
    selected_components = np.where(cumsum >= variance_threshold)[0][0] + 1

    # Transform data into principal component space
    X_train_pca = np.dot(X_centered, Vt.T)
    X_train_pca = X_train_pca[:, :selected_components]

    # Adjust the features for testing
    X_test_centered = X_test - np.mean(X, axis=0)
    X_test_pca = np.dot(X_test_centered, Vt.T)
    X_test_pca = X_test_pca[:, :selected_components]

    # Train and evaluate the model
    results_df, predictions = common.train_and_evaluate(X_train_pca, y_train,
X_test_pca, y_test)

    results_df["Components"] = selected_components
    results_df["Threshold"] = variance_threshold

    results.append(results_df)

results_df = pd.concat(results)
results_df.groupby(by="Threshold")[results_df.columns].apply(lambda x: x).drop
(columns=["Threshold"])
```

Out[51]:

Threshold	Model	R2	MAE	Top-100 MAE	Top-50 MAE	SMAPE	Train time (sec)	Components
0.25	Linear Regression	0.4851	1,524,628	1,913,240	2,537,464	0.3866	0.00	1
	Random Forest	0.2606	1,780,485	2,165,465	2,705,744	0.4309	0.89	1
	Support Vector	0.5016	1,482,780	1,966,063	2,577,555	0.3747	0.44	1
	K-Nearest Neighbors	0.3487	1,663,351	2,012,112	2,481,525	0.4086	0.00	1
0.50	Linear Regression	0.4986	1,496,771	1,869,011	2,339,426	0.3844	0.02	2
	Random Forest	0.4717	1,509,837	1,896,849	2,393,731	0.3784	0.79	2
	Support Vector	0.5150	1,457,700	1,879,472	2,343,895	0.3732	0.30	2
	K-Nearest Neighbors	0.4611	1,515,137	1,894,536	2,398,909	0.3804	0.00	2
0.75	Linear Regression	0.5675	1,334,535	1,834,847	2,284,677	0.3422	0.00	17
	Random Forest	0.5406	1,392,487	1,968,800	2,592,782	0.3539	2.14	17
	Support Vector	0.5680	1,322,788	1,863,102	2,357,359	0.3374	0.55	17
	K-Nearest Neighbors	0.5598	1,344,158	1,905,473	2,489,060	0.3423	0.00	17
0.90	Linear Regression	0.5684	1,341,923	1,917,990	2,377,233	0.3383	0.21	88
	Random Forest	0.5005	1,481,944	2,250,514	2,994,564	0.3760	5.14	88
	Support Vector	0.5681	1,325,428	1,912,240	2,432,140	0.3339	0.97	88
	K-Nearest Neighbors	0.5611	1,353,952	2,033,402	2,548,211	0.3450	0.00	88
0.95	Linear Regression	0.5799	1,306,634	1,858,033	2,361,751	0.3359	1.39	138
	Random Forest	0.4731	1,543,489	2,303,649	3,111,511	0.3922	6.64	138
	Support Vector	0.5947	1,275,063	1,819,979	2,366,959	0.3245	1.24	138
	K-Nearest Neighbors	0.5508	1,360,359	2,023,507	2,606,079	0.3442	0.00	138

In [52]:
```
common.plot_metrics(results_df, "Threshold")
common.plot_metrics(results_df, "Components")
```





Sklearn PCA

In [53]:

```
results = []
n_components = results_df["Components"].unique()

for n, variance_threshold in enumerate(variance_thresholds):
    # Apply sklearn PCA
    pca = PCA(n_components=n_components[n])
    X_train_pca = pca.fit_transform(X_train)
    X_test_pca = pca.transform(X_test)

    # Train and evaluate the model
    results_df, predictions = common.train_and_evaluate(X_train_pca, y_train,
X_test_pca, y_test)

    results_df["Components"] = n_components[n]
    results_df["Threshold"] = variance_threshold

    results.append(results_df)

results_df = pd.concat(results)
results_df.groupby(by="Threshold")[results_df.columns].apply(lambda x: x).drop
(columns=["Threshold"])
```

Out[53]:

Threshold	Model	R2	MAE	Top-100 MAE	Top-50 MAE	SMAPE	Train time (sec)	Components
0.25	Linear Regression	0.4851	1,524,628	1,913,240	2,537,464	0.3866	0.00	1
	Random Forest	0.2606	1,780,485	2,165,465	2,705,744	0.4309	0.84	1
	Support Vector	0.5016	1,482,780	1,966,063	2,577,555	0.3747	0.45	1
	K-Nearest Neighbors	0.3487	1,663,351	2,012,112	2,481,525	0.4086	0.00	1
0.50	Linear Regression	0.4986	1,496,771	1,869,011	2,339,426	0.3844	0.00	2
	Random Forest	0.4683	1,518,023	1,883,844	2,329,929	0.3805	0.77	2
	Support Vector	0.5150	1,457,700	1,879,472	2,343,895	0.3732	0.36	2
	K-Nearest Neighbors	0.4611	1,515,137	1,894,536	2,398,909	0.3804	0.00	2
0.75	Linear Regression	0.5674	1,334,550	1,834,987	2,284,685	0.3422	0.10	17
	Random Forest	0.5498	1,385,784	1,957,380	2,564,106	0.3541	2.48	17
	Support Vector	0.5680	1,322,608	1,863,055	2,357,856	0.3374	0.58	17
	K-Nearest Neighbors	0.5595	1,343,853	1,913,891	2,484,131	0.3422	0.00	17
0.90	Linear Regression	0.5697	1,331,858	1,801,437	2,269,818	0.3361	0.51	88
	Random Forest	0.4882	1,506,402	2,307,259	3,088,640	0.3807	5.17	88
	Support Vector	0.5684	1,321,029	1,852,178	2,369,520	0.3321	0.88	88
	K-Nearest Neighbors	0.5556	1,353,508	2,013,604	2,551,374	0.3445	0.00	88
0.95	Linear Regression	0.5763	1,312,559	1,875,333	2,376,525	0.3376	0.77	138
	Random Forest	0.4840	1,522,531	2,330,470	3,100,599	0.3863	7.24	138
	Support Vector	0.5910	1,277,788	1,869,213	2,424,537	0.3253	0.97	138
	K-Nearest Neighbors	0.5483	1,362,531	2,040,135	2,626,338	0.3447	0.00	138

In [54]:

```
common.plot_metrics(results_df, "Threshold")
common.plot_metrics(results_df, "Components")
```