

既然带我走吧

CS 61B
Spring 2021

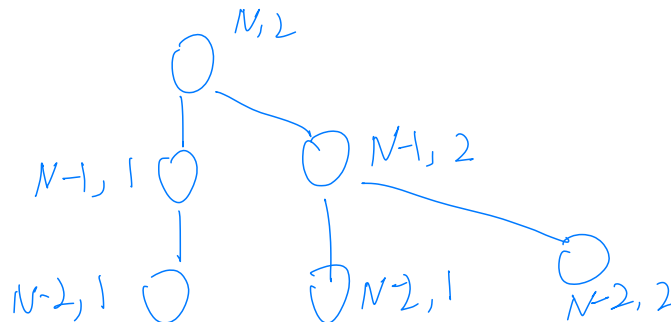
More Asymptotics and ADTs
Exam Prep Discussion 7: March 1, 2021

1 Asymptotics is Fun!

- (a) Using the function g defined below, what is the runtime of the following function calls? Write each answer in terms of N .

```
1 void g(int N, int x) {  
2     if (N == 0) {  
3         return;  
4     }  
5     for (int i = 1; i <= x; i++) {  
6         g(N - 1, i);  
7     }  
8 }
```

$g(N, 1): \Theta(N)$
 $g(N, 2): \Theta(N^2)$



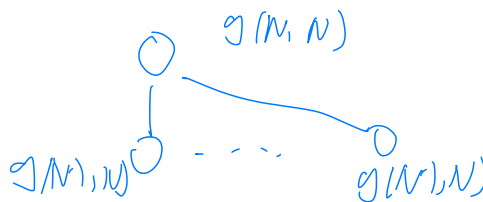
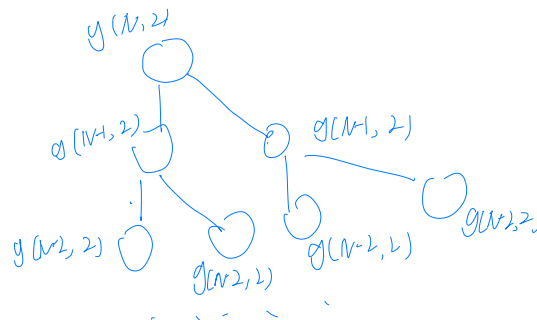
$$1+2+3+\dots+N = \frac{(1+N)N}{2} = N^2$$

- (b) Suppose we change line 6 to $g(N - 1, x)$ and change the stopping condition in the for loop to $i \leq f(x)$ where $f(x)$ returns a random number between 1 and x , inclusive. For the following function calls, find the tightest Ω and big O bounds.

```
1 void g(int N, int x) {  
2     if (N == 0) {  
3         return;  
4     }  
5     for (int i = 1; i <= f(x); i++) {  
6         g(N - 1, x);  
7     }  
8 }
```

$g(N, 2): \Omega(N), O(2^N)$
 $g(N, N): \Omega(N), O(N^N)$

the best run time case is that $N = 0$.



$$\frac{1+N^N}{1-N} = N^N$$

N^2

2 Flip Flop

Suppose we have the flip function as defined below. Assume the method `unknown` returns a random integer between 1 and N , exclusive, and runs in constant time. For each definition of the flop method below, give the best and worst case runtime of flop in $\Theta(\cdot)$ notation as a function of N .

```

1 public static void flip(int N) {
2     if (N <= 100) {
3         return;
4     }
5     int stop = unknown(N);
6     for (int i = 1; i < N; i++) {
7         if (i == stop) {
8             flop(i, N);
9             return;
10        }
11    }
12 }
```

(a) `public static void flop(int i, int N) {`
 `flip(N - i);`
`}`

Best Case: $\Theta(\sqrt{N})$, Worst Case: $\Theta(\sqrt{N})$

(b) `public static void flop(int i, int N) {`
 `int minimum = Math.min(i, N - i);`
 `flip(minimum);`
 `flip(minimum);`
`}`

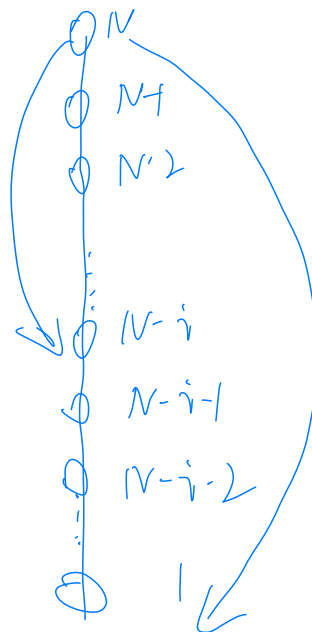
Best Case: $\Theta(\sqrt{N})$, Worst Case: $\Theta(\sqrt{N})$

(c) `public static void flop(int i, int N) {`
 `flip(i);`
 `flip(N - i);`
`}`

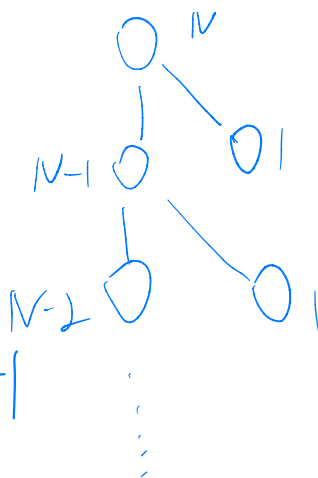
Best Case: $\Theta(\sqrt{N})$, Worst Case: $\Theta(N^2)$

stop = 1

stop = N-1



math.min($\frac{N}{2}, \frac{N}{2}$)
 stop always equals to $\frac{N}{2}$



3 Prime Factors

Determine the best and worst case runtime of `prime_factors` in $\Theta(\cdot)$ notation as a function of N .

```

1  int prime_factors(int N) {
2      int factor = 2;
3      int count = 0;
4      while (factor * factor <= N) {
5          while (N % factor == 0) {
6              System.out.println(factor);
7              count += 1;
8              N = N / factor;
9          }
10         factor += 1;
11     }
12     return count;
13 }
```

Best Case: $\Theta(\log N)$, Worst Case: $\Theta(\sqrt{N})$

4 ADT Selection

Suppose we have a TA Shreyas who teaches multiple discussion sections! A student may frequent more than one discussion section. For each situation below, choose the best ADT(s) out of the following — Map, Set, List — and explain how you can use the ADT(s) to solve the problem. Each subpart is independent of the previous. One answer may involve multiple ADTs. There may be multiple efficient answers for each problem.

multiple ADTs

1. Storing all the Students in Shreyas's first section in alphabetical order.

Put the Students in a List in alphabetical order.

2. Storing all the Students by their section, where Students within a section are sorted alphabetically.

Use a Map, where each Section maps to an alphabetically ordered List of Students in that section.

3. Storing the Students in *all* of Shreyas's sections. There shouldn't be duplicates.

Use a Set. Add all the Students to the Set.

4. Determining all the Students that attend more than one of Shreyas's sections.

Use two Sets. The first Set will store all the students seen so far, and the second Set will be our answer. Iterate through the Students in each Section and, if the first set contains the Student,

5. Quickly getting a Student by sid. add it to the second set. Otherwise, add the Student to the first set.

Use a Map, where each sid maps to one Student.

6. Quickly getting a Student by name or sid. Names aren't necessarily unique.

Use two Maps, where the first map works as described in the previous part and the second map maps names to a Set of Students of the given name.

7. Cycling through the Students in one discussion section.

Put the Students in a List, a LinkedList recommended. Repeat removing from the front and reinserting at the back.