Base case: when $n=0$, $(1+x)^n = 1$, $1+nx = 1$, therefore $(1+x)^n \geq 1+nx$

Inductive Hypothesis: Assume that $(1+x)^k \geq 1+kx$, for some value of $n=k$, where $k \in N$.

Inductive Step: For $n=k+1$,

$$(1+x)^{k+1} = (1+x)^k(1+x) \geq (1+kx)(1+x) = 1+kx+x+kx^2 \geq 1+kx+x = 1+(k+1)x$$

By induction, we could prove its correctness.

Base case: For $n=1$, $a_1 = 1$, $3^{(2^n)} = 9$, thus $a_n \leq 3^{(2^n)}$

Inductive Hypothesis: Assume that $a_n \leq 3^{(2^n)}$ for $n=k$, where $k \geq 1$

Inductive Step: for $n=k+1$ $a_{k+1} = 3a_n^2 \leq 3[3^{(2^n)}]^2 = 3 \cdot 3^{2^{n+1}} = 3^{2^{n+1}+1}$

While $3^{2^{n+1}+1} > 3^{2^{n+1}}$, that means our hypothesis doesn't work.

Base case: For $n=1$ $a_1 = 1$, $3^{(2^n-1)} = 3$, thus $a_n \leq 3^{(2^n-1)}$

Inductive Hypothesis: Assume that $a_n \leq 3^{(2^n-1)}$ for $n=k$ where $k \geq 1$

Inductive Step: For $n=k+1$, $a_{k+1} = 3a_k^2 \leq 3 \cdot 3^{2^{n+1}-2} = 3^{2^{n+1}-1}$

For every $n \geq 1$, we have $2^n-1 \leq 2^n$ and therefore $3^{2^n-1} \leq 3^{2^n}$. This means that our modified hypothesis which we proved in part (b) does indeed imply what we wanted to prove in part (a).

Base case: When $n=1$, $1 = 1\times 2^0$

Inductive Hypothesis: Assume that the statement is true for all $1 \le m \le n$, where n is arbitrary.

Now, we need to consider $n+1$, if $n+1$ is divisible by 2, then we can apply our inductive hypothesis to $(n+1)/2$ and use its representation to express $n+1$ in the desired form.

$$(n+1)/2 = c_k 2^k + c_{k-1}\cdot 2^{k-1} + \dots + c_1\cdot 2^1 + c_0 2^0$$

$$n+1 = c_k 2^{k+1} + c_{k-1}\cdot 2^k + \dots + c_1\cdot 2^2 + c_0\cdot 2^1 + 0\cdot 2^0$$

If $n+1$ is odd, then n is even, $n = c_k\cdot 2^k + c_{k-1}\cdot 2^{k-1} + \dots + c_1\cdot 2^1 + 0\cdot 2^0$

$$n+1 = c_k\cdot 2^k + c_{k-1}\cdot 2^{k-1} + \dots + c_1\cdot 2^1 + 1\times 2^0$$

Therefore, the statement is true.

It's easy to prove that all the numbers are integers.

Base Case: when $n=3$, $F_3 = F_1 + F_2 = 2$, the claim is correct.

Inductive Hypothesis: Assume that, F_{3k} is even

for $n = 3k+3$

$$F_{3k+3} = F_{3k+2} + F_{3k+1}$$
$$= F_{3k} + F_{3k+1} + F_{3k+1}$$
$$= F_{3k} + 2F_{3k+1}$$

Since, F_{3k} is even and $2F_{3k+1}$ is even, F_{3k+3} is even.

Therefore, the statement is true.