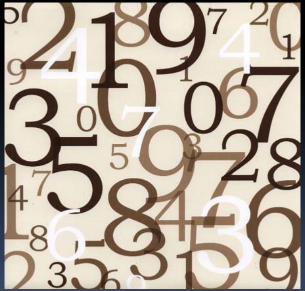
UC Berkeley
Teaching Professor
Dan Garcia

# CS61C

Great Ideas
in
Computer Architecture
(a.k.a. Machine Structures)

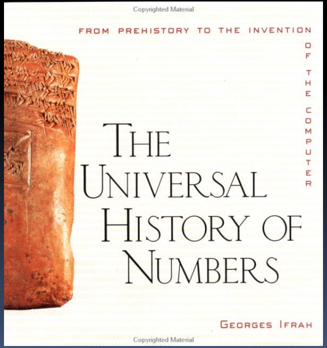
UC Berkeley
Teaching Professor
Lisa Yan

## Number Representation



Great book ⇒
The Universal History
of Numbers

by Georges Ifrah


From Prehistory to the Invention of the Computer
The Universal History of Numbers
Georges Ifrah

cs61c.org

# Remote Scan of Student's Room Before Test Violated His Privacy, Judge Rules

A federal judge said Cleveland State University violated the Fourth Amendment when it used software to scan a student's bedroom, a practice that has grown during the Covid-19 pandemic.

By AMANDA HOLPUCH and APRIL RUBIN

The right to privacy of the student, Aaron M. Ogletree, outweighed the interests of Cleveland State University, ruled Judge J. Philip Calabrese of the U.S. District Court for the Northern District of Ohio. The judge ordered lawyers for Mr. Ogletree and the university to discuss potential remedies for the case.

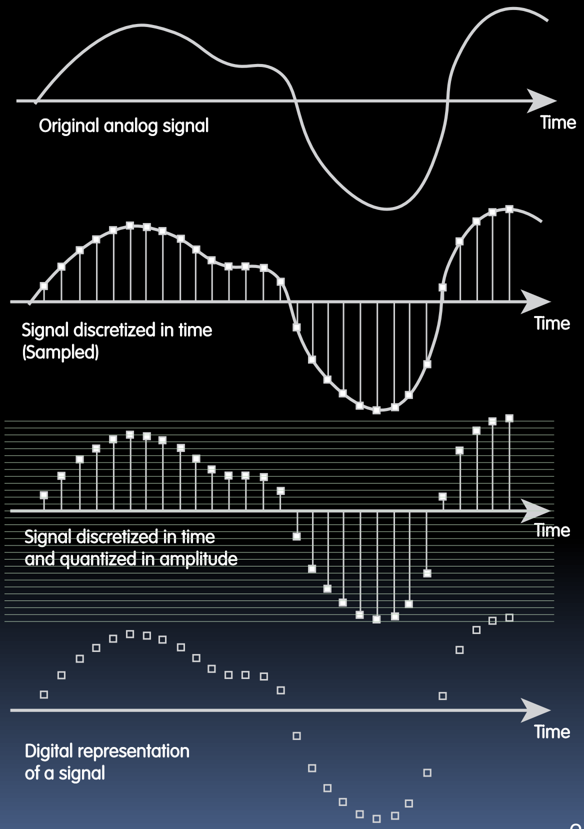
The use of virtual software to remotely monitor test takers exploded during the first years of the coronavirus pandemic, when millions of students were suddenly required to take classes online to minimize the spread of the disease. Students and privacy experts have raised concerns about these programs, which can detect keystrokes and collect feeds from a computer's camera and microphone.

New York Times 2022-08-25

`https://www.nytimes.com/2022/08/25/us/remote-testing-student-home-scan-privacy.html`

# Data input: Analog → Digital

- Real world is analog!
- To import analog information, we must do two things
  - Sample
    - E.g., for a CD, every 44,100ths of a second, we ask a music signal how loud it is.
  - Quantize
    - For every one of these samples, we figure out where, on a 16-bit (65,536 tic-mark) "yardstick", it lies.


Original analog signal
Time
Signal discretized in time (Sampled)
Time
Signal discretized in time and quantized in amplitude
Time
Digital representation of a signal
Time

# Digital data not necessarily born Analog…

hof.povray.org

# BIG IDEA: Bits can represent anything!!

- **Characters?**
  - 26 letters ⇒ 5 bits ($2^5 = 32$)
  - upper/lower case + punctuation ⇒ 7 bits (in 8) ("ASCII")
  - standard code to cover all the world's languages ⇒ 8,16,32 bits ("Unicode") `www.unicode.com`
- **Logical values?**
  - 0 → False, 1 → True
- **colors ? Ex:** ***Red (00)*** ***Green (01)*** ***Blue (11)***
- **locations / addresses? commands?**
- **MEMORIZE: N bits ⇔ at most $2^N$ things**

☑

## L02a Number Representation: How many bits to represent π (Pi)?

| | |
|---|---|
| 1 | A |
| 9 (π = 3.14, so that's 011 "." 001 100) | B |
| 64 (Since Macs are 64-bit machines) | C |
| Every bit the machine has! | D |
| ∞ | E |

Respond at **pollev.com/ddg**

Text **DDG** to **22333** once to join, then **A, B, C, D, or E**

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# Binary
# Decimal
# Hex

www.mathsisfun.com/numbers/numbers-numerals-digits.html

## Number vs Numeral

### Numeral

A symbol or name that stands for a number

*e.g., 4 , four , quatro , IV , IIII , …*

…and **Digits** are symbols that make numerals

Above the abstraction line

Abstraction Line

Below the abstraction line

### Number

The "idea" in our minds…there is only ONE of these

*e.g., the concept of "4"*

## Base 10 (Ten) #s, Decimals

Digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Example:

$3271 = 3271_{10} =$

$(3 \times 10^3) + (2 \times 10^2) + (7 \times 10^1) + (1 \times 10^0)$

# Base 2 (Two) #s, Binary (to Decimal)

0b

binary

Digits: 0, 1 (binary digits → bits)

Example: "1101" in binary? ("0b1101")

$1101_2 = \underline{(1 \times 2^3)} + \underline{(1 \times 2^2)} + \underline{(0 \times 2^1)} + \underline{(1 \times 2^0)}$

$= 8 + 4 + 0 + 1$

$= 13$

## Base 16 (Sixteen) #s, Hexadecimal (to Decimal)

**Digits:** 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F
10,11,12,13,14,15

0x

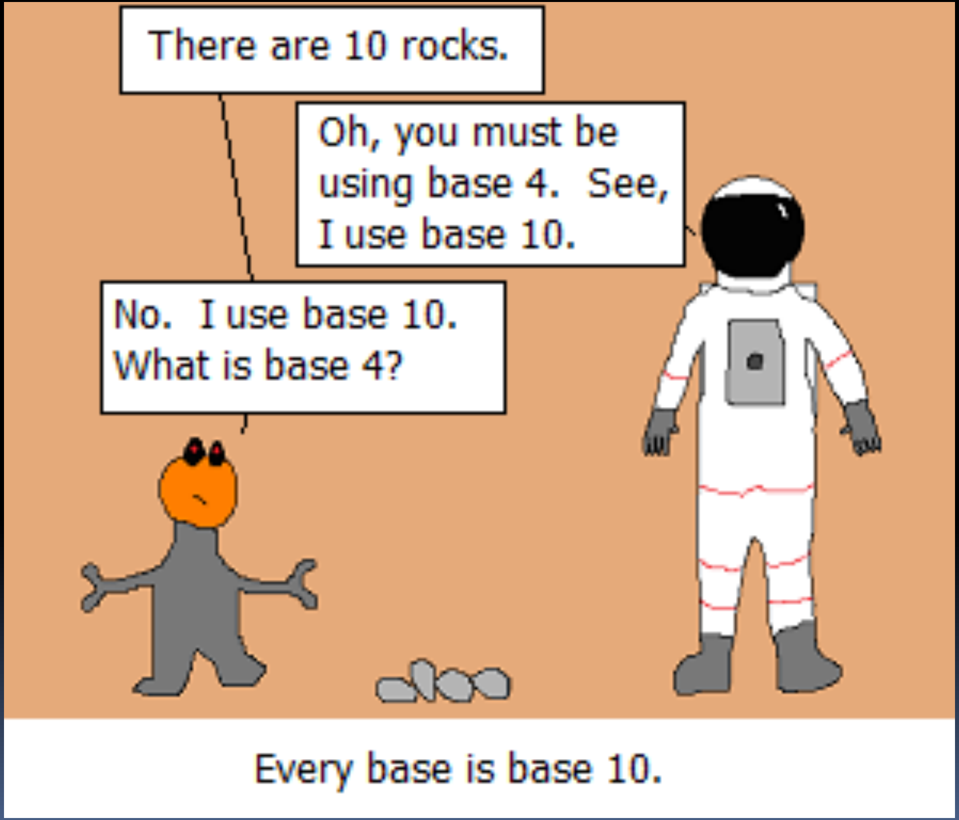
**Example: "A5" in Hexadecimal?**

$0xA5 = A5_{16} = (\underline{10}x16^{1}) + (\underline{5}x16^{0})$

$= 160 + 5$

$= 165$

## Every Base is Base 10…


There are 10 rocks.
Oh, you must be using base 4. See, I use base 10.
No. I use base 10. What is base 4?
Every base is base 10.

# Convert from Decimal to Binary

0 → 0b

another method:
除二取余

- E.g., 13 to binary?
- Start with the columns

~~13~~
~~5~~
~~1~~
0

| $2^3$=8 | $2^2$=4 | $2^1$=2 | $2^0$=1 |
|---|---|---|---|
| 1 | 1 | 0 | 1 |

- Left to right, is (column) ≤ number **n**?
  - If yes, put how many of that column fit in **n**, subtract col * that many from **n**, keep going.
  - If not, put 0 and keep going. (and Stop at 0)

# Convert from Decimal to Hexadecimal

- E.g., 165 to hexadecimal?
- Start with the columns

~~165~~
~~5~~
0

| $16^3$=4096 | $16^2$=256 | $16^1$=16 | $16^0$=1 |
|---|---|---|---|
| 0 | 0 | (10) A | 5 |

- Left to right, is (column) ≤ number n?
  - If yes, put how many of that column fit in n, subtract col * that many from n, keep going.
  - If not, put 0 and keep going. (and Stop at 0)

# Convert Binary ←→ Hexadecimal

- Binary → Hex? Easy!
  - Always left-pad with 0s to make full 4-bit values, then look up!
  - E.g., `0b11110` to Hex?
    - `0b11110` → `0b00011110`
    - Then look up: `0x1E`
- Hex → Binary? Easy!
  - Just look up, drop leading 0s
    - `0x1E`→`0b00011110`→`0b11110`

| D | H | B |
|---|---|---|
| 00 | 0 | 0000 |
| 01 | 1 | 0001 |
| 02 | 2 | 0010 |
| 03 | 3 | 0011 |
| 04 | 4 | 0100 |
| 05 | 5 | 0101 |
| 06 | 6 | 0110 |
| 07 | 7 | 0111 |
| 08 | 8 | 1000 |
| 09 | 9 | 1001 |
| 10 | A | 1010 |
| 11 | B | 1011 |
| 12 | C | 1100 |
| 13 | D | 1101 |
| 14 | E | 1110 |
| 15 | F | 1111 |

## Decimal vs Hexadecimal vs Binary

- 4 Bits
  - 1 "Nibble"
  - 1 Hex Digit = 16 things
- 8 Bits
  - 1 "Byte"
  - 2 Hex Digits = 256 things
  - Color is usually
    0-255 Red,
    0-255 Green,
    0-255 Blue.
    #D0367F= 🟥

| D | H | B |
|---|---|---|
| 00 | 0 | 0000 |
| 01 | 1 | 0001 |
| 02 | 2 | 0010 |
| 03 | 3 | 0011 |
| 04 | 4 | 0100 |
| 05 | 5 | 0101 |
| 06 | 6 | 0110 |
| 07 | 7 | 0111 |
| 08 | 8 | 1000 |
| 09 | 9 | 1001 |
| 10 | A | 1010 |
| 11 | B | 1011 |
| 12 | C | 1100 |
| 13 | D | 1101 |
| 14 | E | 1110 |
| 15 | F | 1111 |

## Which base do we use?

- **Decimal:** great for humans, especially when doing arithmetic
- **Hex:** if human looking at long strings of binary numbers, its much easier to convert to hex and see 4 bits/symbol
  - Terrible for arithmetic on paper
- **Binary:** what computers use;
  you will learn how computers do +, -, *, /
  - To a computer, numbers always binary
  - Regardless of how number is written:
  - $32_{ten}$ == $32_{10}$ == 0x20 == $100000_2$ == 0b100000
  - Use subscripts “ten”, “hex”, “two” in book, slides when might be confusing

# The computer knows it, too…

```c
#include <stdio.h>
int main() {
    const int N = 1234;

    printf("Decimal: %d\n",N);
    printf("Hex:     %x\n",N);
    printf("Octal:   %o\n",N);

    printf("Literals (not supported by all compilers):\n");
    printf("0x4d2         = %d (hex)\n", 0x4d2);
    printf("0b10011010010 = %d (binary)\n", 0b10011010010);
    printf("02322         = %d (octal, prefix 0 - zero)\n", 0x4d2);
    return 0;
}
```

***Output***

```
Decimal: 1234
Hex:     4d2
Octal:   2322
Literals (not supported by all compilers):
0x4d2         = 1234 (hex)
0b10011010010 = 1234 (binary)
02322         = 1234 (octal, prefix 0 - zero)
```

☑

## Number Representations

# What to do with representations of numbers?

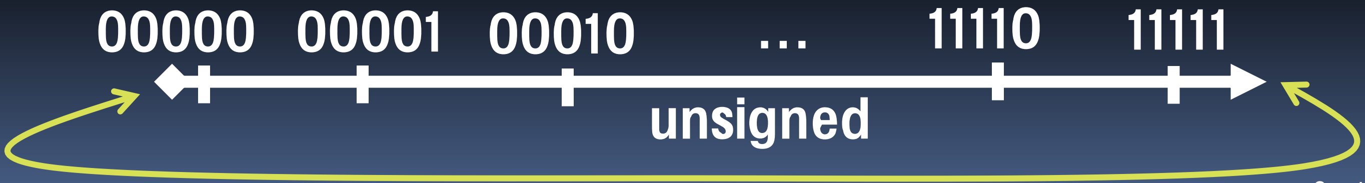
- **What to do with number representations?**
  - Add them
  - Subtract them
  - Multiply them
  - Divide them
  - Compare them

```
    1   0   1   0
+   0   1   1   1
--------------------
```

- **Example: 10 + 7 = 17**
  - …so simple to add in binary that we can build circuits to do it!
  - Subtraction just as you would in decimal
  - Comparison: How do you tell if X > Y ?

# What if too big?

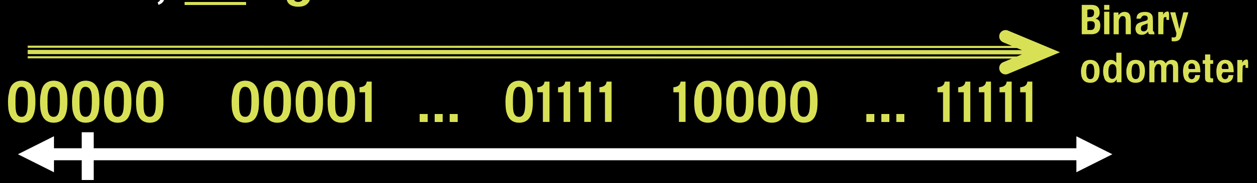
- Binary bit patterns are simply representatives of numbers. Abstraction!
  - Strictly speaking they are called "numerals".
- Numerals really have an ∞ number of digits
  - with almost all being same (00…0 or 11…1) except for a few of the rightmost digits
  - Just don't normally show leading digits
- If result of add (or -, *, / ) cannot be represented by these rightmost HW bits, we say overflow occurred



## How to Represent Negative Numbers?

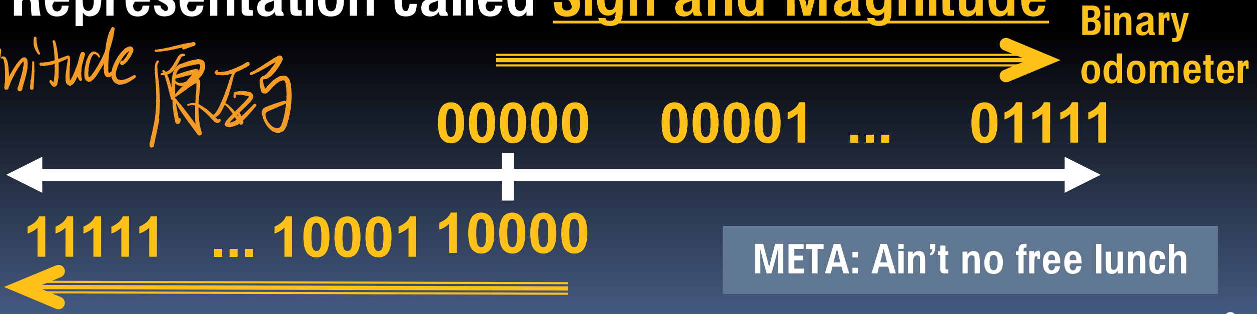
(C's `unsigned int`, C18's `uintN_t`)

- So far, unsigned numbers



- Obvious solution: define leftmost bit to be sign!
  - 0 ➔ + 1 ➔ – …and rest of bits are numerical value
- Representation called Sign and Magnitude

sign and magnitude 原码



META: Ain't no free lunch

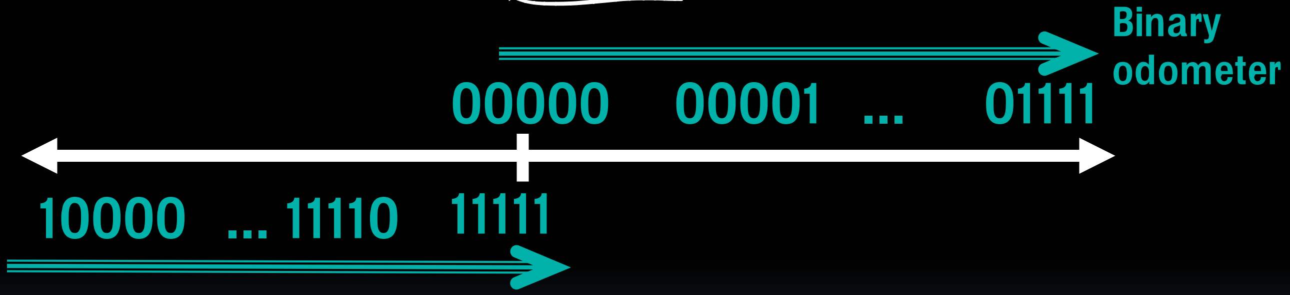
Garcia, Yan

# Shortcomings of Sign and Magnitude?

- Arithmetic circuit complicated
  - Special steps depending on if signs are the same or not
- Also, <u>two</u> zeros
  - 0x00000000 = $+0_{ten}$ positive zero
  - 0x80000000 = $-0_{ten}$ negative zero
  - What would two 0s mean for programming? 二进制里程表
- ↗ Also, incrementing "binary odometer", sometimes increases values, and sometimes decreases! ↙
- Therefore sign and magnitude used only in signal processors 信号处理器

# Another try: complement the bits

- Example: $7_{10} = 00111_2$ $-7_{10} = 11000_2$
- Called One's Complement 一的补码
- Note: positive numbers have leading 0s, negative numbers have leadings 1s.



- What is -00000 ? Answer: 11111
- How many positive numbers in N bits?
- How many negative numbers?

## Shortcomings of One’s Complement?

- Arithmetic still somewhat complicated
- Still two zeros
  - 0x00000000 = $+0_{ten}$
  - 0xFFFFFFFF = $-0_{ten}$
- Although used for a while on some computer products, one’s complement was eventually abandoned because another solution was better.

☑

二进制

补码

## Two's Complement & Bias Encoding

# Standard Negative # Representation

- **Problem is the negative mappings "overlap" with the positive ones (the two 0s). Want to shift the negative mappings left by one.**
  - Solution! For negative numbers, complement, then add 1 to the result
- **As with sign and magnitude, & one's compl. leading 0s → positive, leading 1s → negative**
  - 000000...xxx is ≥ 0, 111111...xxx is < 0
  - except 1…1111 is -1, not -0 (as in sign & mag.)
- **This representation is Two's Complement**
  - This makes the hardware simple!

**(C's `int`, C18's `intN_t`, aka a "signed integer")**

# Two's Complement Formula

- Can represent positive **and negative** numbers in terms of the bit value times a power of 2:

$$d_{31} \times \boxed{-(2^{31})} + d_{30} \times 2^{30} + \ldots + d_2 \times 2^2 + d_1 \times 2^1 + d_0 \times 2^0$$

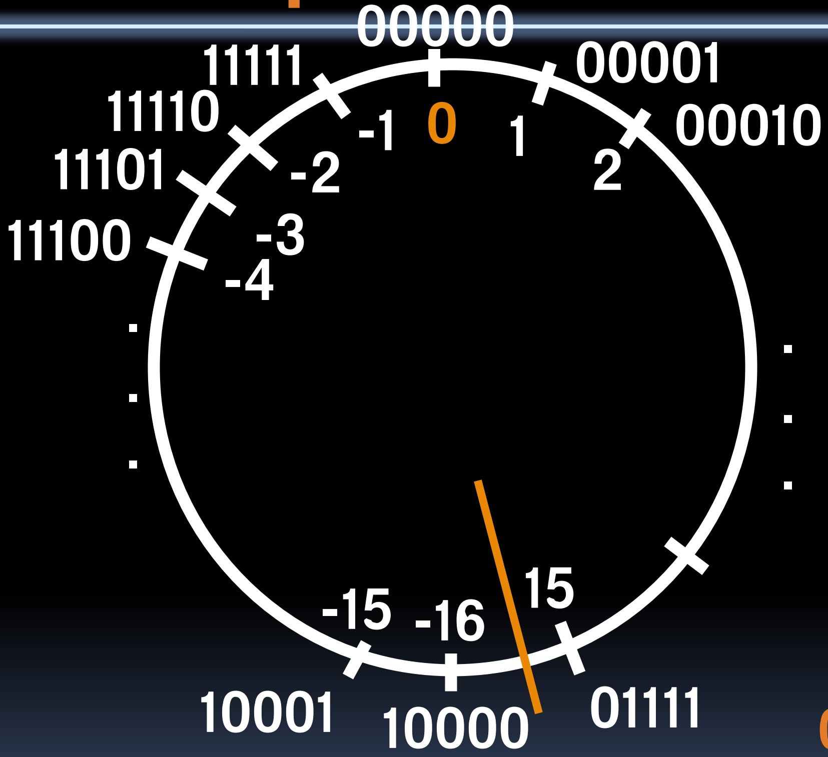
- Example: $1101_{two}$ in a nibble?

$$= 1\text{x}-(2^3) + 1\text{x}2^2 + 0\text{x}2^1 + 1\text{x}2^0$$
$$= -2^3 + 2^2 + 0 + 2^0$$
$$= -8 + 4 + 0 + 1$$
$$= -8 + 5$$
$$= -3_{ten}$$

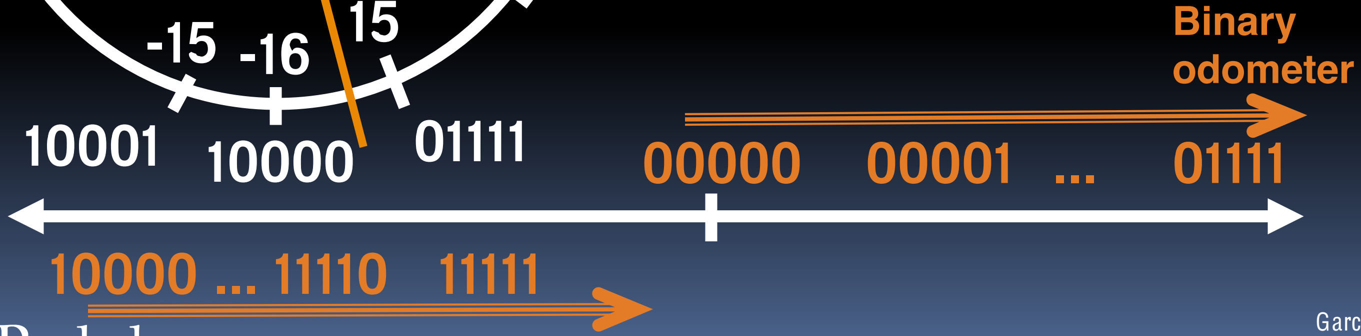
**Example: -3 to +3 to -3 (again, in a nibble):**

```
x   : 1101two -3
x'  : 0010two
+1  : 0011two  3
()' : 1100two
+1  : 1101two -3
```

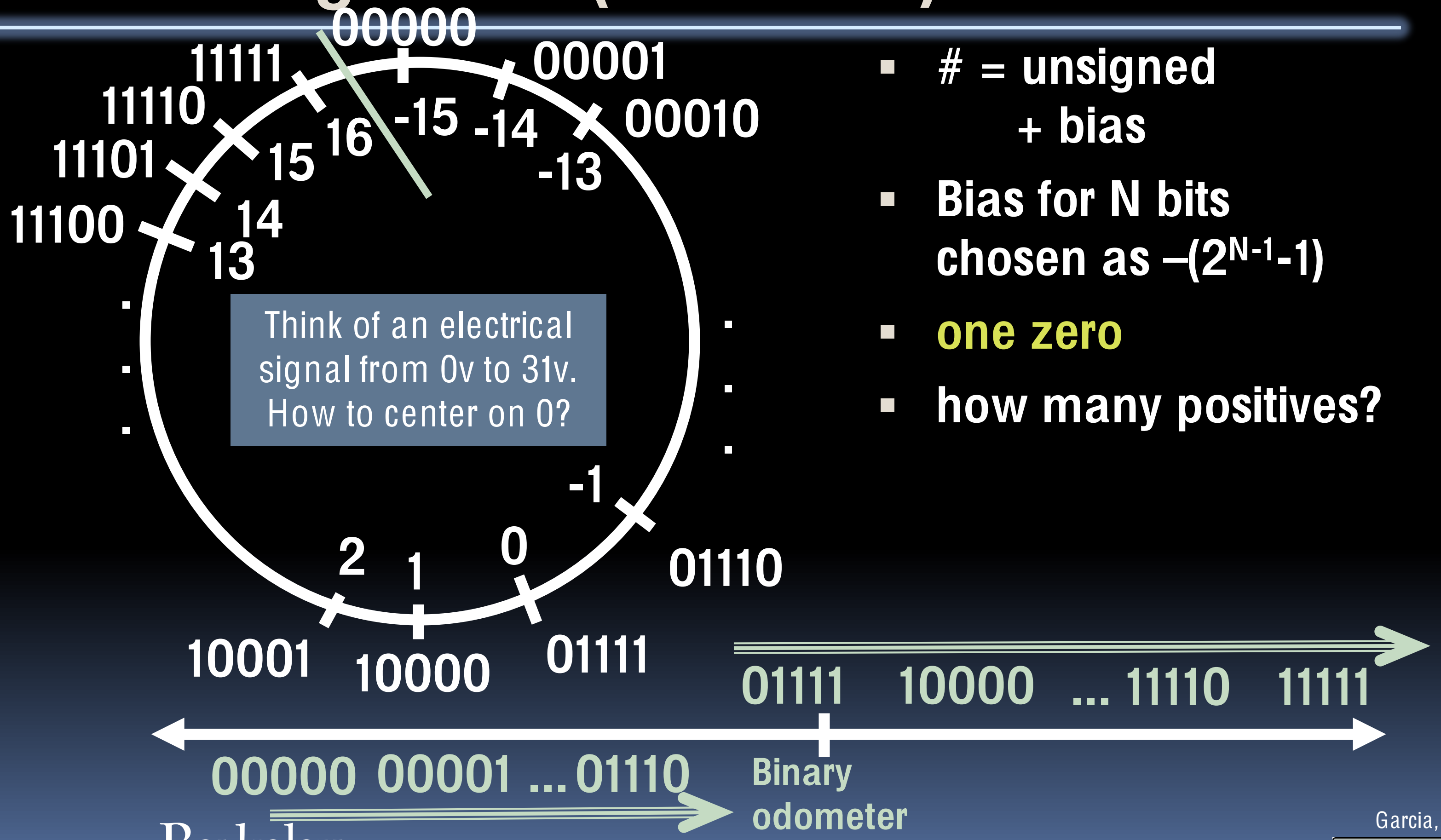
## Two's Complement Number "line": N = 5



- $2^{N-1}$ non-negatives
- $2^{N-1}$ negatives
- one zero
- how many positives?



## Bias Encoding: N = 5 (bias = -15)



Think of an electrical signal from 0v to 31v. How to center on 0?

- # = unsigned + bias
- Bias for N bits chosen as $-(2^{N-1}-1)$
- one zero
- how many positives?

## L02b How best to represent -12.75? (explain shifting binary point)

- 2s Complement (but shift binary point)
- Bias (but shift binary point)
- Combination of 2 encodings
- Combination of 3 encodings
- We can't

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2s Complement (but shift binary point)

Bias (but shift binary point)

Combination of 2 encodings

Combination of 3 encodings

We can't

Poll locked. Responses not accepted.

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- Combination of 2 encodings
- Combination of 3 encodings
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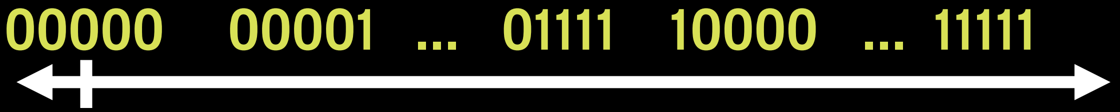
Powered by Poll Everywhere

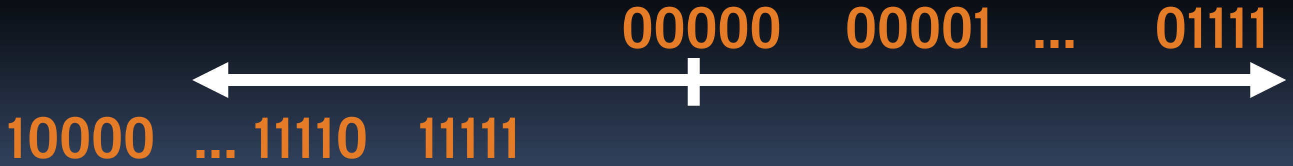
# And in summary...

META: We often make design decisions to make HW simple

- We represent “things” in computers as particular bit patterns: N bits ⇒ $2^N$ things
- These 5 integer encodings have different benefits; 1s complement and sign/mag have most problems.
- unsigned (C18’s `uintN_t`) :



- Two’s complement (C18’s `intN_t`) universal, learn!



- Overflow: numbers ∞; computers finite, errors!

META: Ain’t no free lunch