Musical Acoustics

Homework 1: Helmholtz resonator and system impedance

Homework 1 Report

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Point a

Given a general Helmholtz resonator it is possible to draw a parallel from this object to a mass-spring system. In such a resonator it is possible to model the mass of the system as the mass of air which is contained in the neck of the resonator, while the the mass of air contained in the body can be associated to the stiffness of the resonator, thus making it similar to a spring in term of the effect it has on the system.

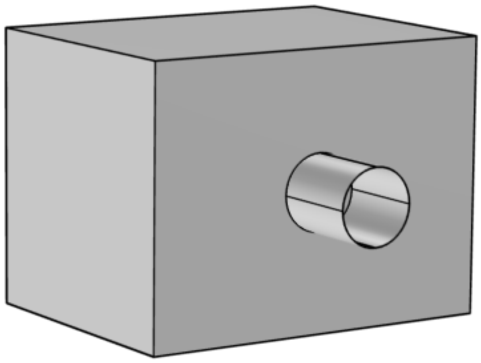

Figure 1: **Parallelepipedal Helmholtz resonator with hollow cylinder neck**

Since the geometries of the internal parallelepipedal are $H = 25cm$ for height, $W = 25cm$ for width and $L = 18cm$ for length and the dimensions of the neck are $R_n = 2.5cm$ for radius, $L_n = 6cm$ for length, the volume of the Parallelepipedal is:

$$V = HWL = 0.0112[m^3], \tag{1}$$

and the area of the neck is

$$S = \pi R_n^2 = 0.002[m^2]. \tag{2}$$

Thus the resonance frequency of the system can be calculated as

$$f_0^a = \frac{1}{2\pi}\sqrt{\frac{K}{M}} = \frac{c}{2\pi}\sqrt{\frac{S}{VL_n}} = 93.2688[Hz], \tag{3}$$

where $c = 343.6m/s$ is the speed of sound in air ($20°C$) and $L_n = 6cm$ is the length of its neck, K is the equivalent stiffness $\frac{\rho S^2 c^2}{V}$ and M is the mass of air contained in the neck $\rho S L_n$.

Point b

In real situation, the effective mass of air moving in and around the neck is a little larger than the geometric dimensions of the neck itself. Therefore, the effect of additional air mass out of the neck can be taken into account by adding a length correction. The equivalent model used here is for unflanged pipe for the outside edge and flanged for the inside edge (assuming that the side of the resonator is large enough to be approximated to an infinite plate), so the end length correction here is [1]

$$L_{c_{unflanged}} = 0.61a = 0.0152[m], \tag{4}$$

$$L_{c_{flanged}} = 0.85a = 0.0213[m]. \tag{5}$$

[1]The unflanged pipe model in Eq.(4) is from "Fletcher, Neville H., and Thomas D. Rossing. The physics of musical instruments, page200, equation(8.33)".

Then the total length of the neck is

$$L_{tot} = L_n + L_{c_{unflanged}} + L_{c_{flanged}} = 0.0965[m]. \tag{6}$$

By using the correction, the resonance frequency is

$$f_0^b = \frac{c}{2\pi}\sqrt{\frac{S}{VL_{tot}}} = 73.5442[Hz]. \tag{7}$$

Point c

Now a resitance R is introduced in order to model a mass - spring - resistance system which better represents a real system. This is due to the fact that the resistance of a system affects both the resonant frequencies and its behaviour in correspondence to those frequencies.
The resonance angular frequency of the system can be written as

$$\omega_0 = 2\pi f_0^b = 462.0917[rad/s]. \tag{8}$$

For the critically damped system,

$$\alpha = \omega_0 = \frac{R}{2m}. \tag{9}$$

The mass of air in the neck is

$$m = \rho S L_{tot} = 2.27372 \times 10^{-4}[kg]. \tag{10}$$

Then the resistance value is

$$R = 2m\omega_0 = 0.2101[kg/s]. \tag{11}$$

Point d

When the virtual elongation is kept into account, the impedance can be calculated using

$$Z = R + j(\omega m - \frac{K}{\omega}), \tag{12}$$

where the resistance is $R = 5 \times 10^{-4} kg/s$. Fig.2 shows the impedance of this system.

Point e

For the above system,

$$\alpha = \frac{R}{2m} = 1.0995[rad/s]. \tag{13}$$

Then the Q factor can be calculated as

$$Q = \frac{\omega_0}{2\alpha} = 210.1341, \tag{14}$$

and the time decay factor is

$$\tau = \frac{1}{\alpha} = 0.9094[s/rad]. \tag{15}$$

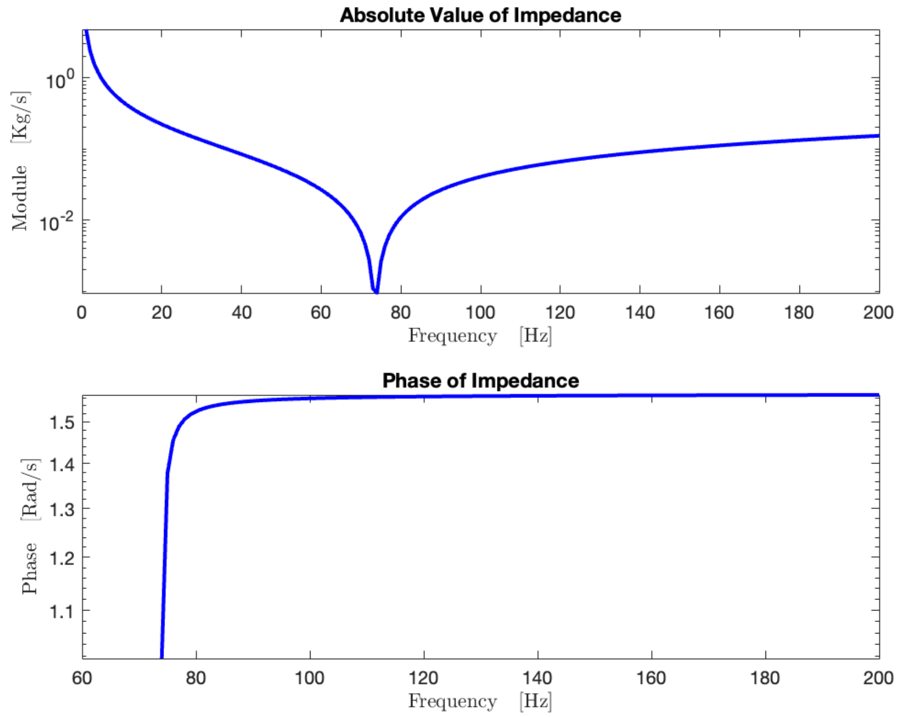

Figure 2: **Impedance graph of the system in d), the top represents its module, in logarithmic coordinates, and the bottom represents its phase.**

Point f

For a damped system, the Q factor and the resonance frequency can be derived from

$$Q = \frac{K}{R\omega_0}, \tag{16}$$

and

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}, \tag{17}$$

separately. The Q factor and the resonance frequency of the system are shown in Fig.3, with a range of R between $0kg/s$ and $0.5kg/s$.

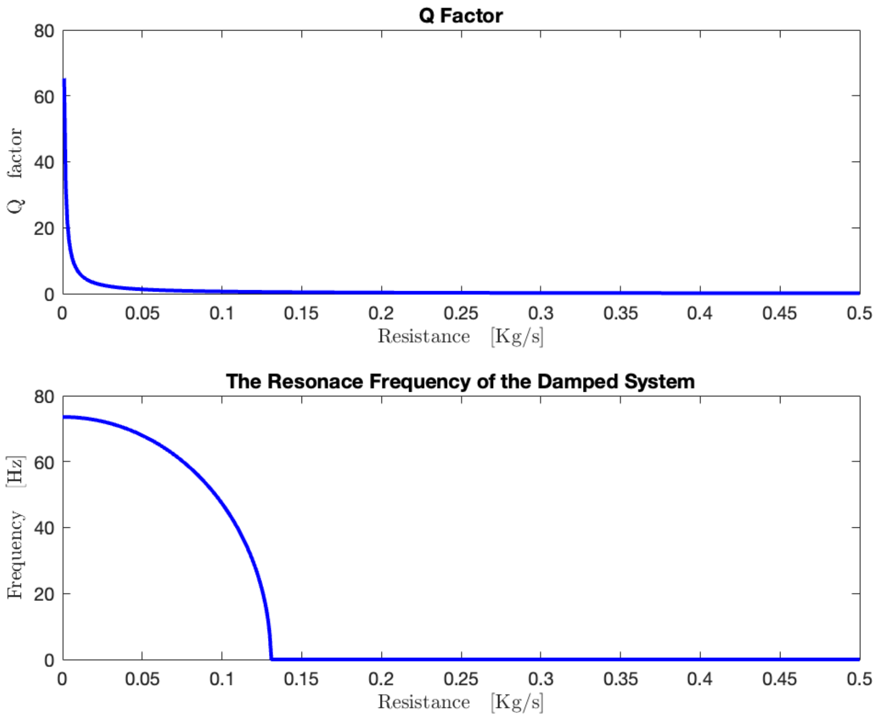

Figure 3: **The top graph represents the Q factor and the bottom represents the resonance frequency, both for the system in f).**