Musical Acoustics

Homework 3: Horn design

Homework 3 Report

Students

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Part 1: Design of the exponential section

The geometry of the given exponential horn is as follows: The equivalent radius is $a = a_0 e^{mx}$, with $a_0 = 0.008$m and $m = 4.2\text{m}^{-1}$. The length of the horn is $L = 0.35$m (see figure 1).

Figure 1: Diagram of the exponential horn.

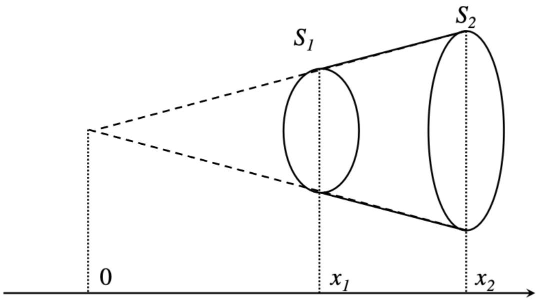

Figure 2: Diagram of a conical horn.

For horns that do not flare too rapidly, the wavefront could be assumed as a planar form. Two models are used here to calculate the input impedance of this exponential horn. The first is an approximate model, which takes the horn into multiple conical segments of the same length δ, see figure 2. Therefore, the input impedance $Z_1(\omega)$ can be calculated part by part from downstream to upstream, using the input of the previous step as the output of the next step. For each conical segment with a throat of area S_1, a mouth of area S_2, and length L_{tot}, the impedance is given by

$$Z_{in} = \frac{\rho c}{S_1}\left[\frac{jZ_L \frac{\sin(kL-\theta_2)}{\sin\theta_2} + (\rho c/S_2)\sin kL}{Z_L \frac{\sin(kL+\theta_1-\theta_2)}{\sin\theta_1 \sin\theta_2} - (j\rho c/S_2)\frac{\sin(kL+\theta_1)}{\sin\theta_1}}\right], \quad (1)$$

where ρ is the air density, c is the speed of sound in air, $k = \omega/c$ is the wave number, Z_L is the load impedance at output, $\theta_1 = \tan^{-1} kx_1$ and $\theta_2 = \tan^{-1} kx_2$.

For another method, we take the whole horn into consideration. The analytical expression of the impedance $Z_2(\omega)$ of an exponential horn (see figure 3) with length L is

$$Z_{in} = \frac{\rho c}{S_1}\left[\frac{Z_L \cos(bL+\theta) + j(\rho c/S_2)\sin bL}{jZ_L \sin bL + (\rho c/S_2)\cos(bL-\theta)}\right], \quad (2)$$

where $b^2 = k^2 - m^2$ and $\theta = \tan^{-1}(m/b)$.

At the first stage, the radiation of the horn is neglected.

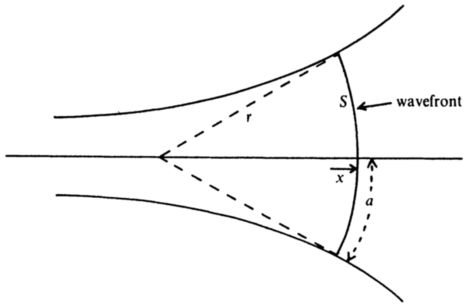

Figure 3: Diagram of an exponential horn.

In order to evaluate the similarity between the impedances $Z_1(\omega)$ and $Z_2(\omega)$, two metrics are used:

$$e_1 = \frac{1}{\omega_{max} - \omega_{min}} \int_{\omega_{min}}^{\omega_{max}} |Z_1(\omega) - Z_2(\omega)|^2 d\omega, \quad (3)$$

which evaluates the mean squared error between the two impedances, and

$$e_2 = \sum_1^4 min|argmax_\omega abs(Z_1(\omega)) - argmax_\omega abs(Z_2(\omega))|, \quad (4)$$

which evaluates the difference in frequency between the first four maximam of the two impedances. By analyzing the two metrices, we could find an appropriate value for segment length δ.

Point a

We use Matlab to calculate the two impedances $Z_1(\omega)$ and $Z_2(\omega)$ by equation 1 and 2, respectively. We set 1-1000 segments ($n_L = 1:1000$ in Matlab) when calculating $Z_1(\omega)$, which corresponds to $\delta = L_{tot}/n_L$. The result of e_1 is shown in figure 4. It is worth noting that e_1 relies a lot upon the frequency range and sampling. In this work, we use $f = 1 : 2000/1999 : 2000$ in Matlab.

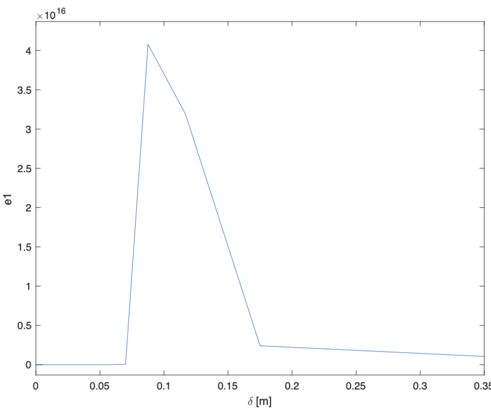

Figure 4: The error e_1.

Point b

The result of e_2 is shown in figure 5. The data corresponding to the arrow is $\delta^a = 0.0583[m]$, $e_2^a = 0$, which is the maximum length δ that guarantees the minimum value of the error function e_2. For the following work, we use δ^a as the length of the conical segment.

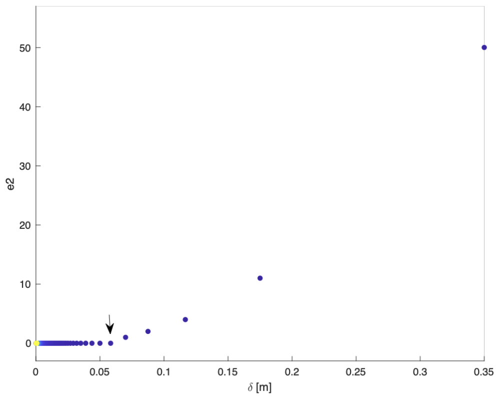

Figure 5: The error e_2.

Point c

Now we consider the radiation effect for the computation of the input impedance. For the approximated horn with multiple conical segments, the external air is modeled by a radiation impedance

$$Z_L(\omega) - Z_{L0}(\omega)\frac{S_p}{S_s}, \tag{5}$$

where Z_{L0} is the impedance of an unflanged cylindrical pipe of radius a_{cy}, given by

$$Z_{L0}(\omega) = 0.25\frac{\omega^2\rho}{\pi c} + 0.61j\frac{\rho\omega}{\pi a_{cy}}, \tag{6}$$

where S_p is the cross-sectional area of the cylinder, and S_s is the spherical wave front area at the open end of the cone. It can be approximated by

$$S_s = \frac{2S_p}{1+\cos\theta_f}, \tag{7}$$

where θ_f is the flaring angle of the last conical segment. Therefore, we can get the input impedance considering the radiation effect.
The results of $Z_1(\omega)$ for the two situations: with and without the radiation impedance are shown in figure 6. Intuitively, we can find when the radiation impedance is taken into account, the valleys and peaks of the input impedance curves become flat, and the corresponding frequencies become lower. Especially, the frequency shifts more when the frequency gets higher. The error function e_1 could also be used here to evaluate the mean square error between those two impedances, which is 1.5021×10^{16}.

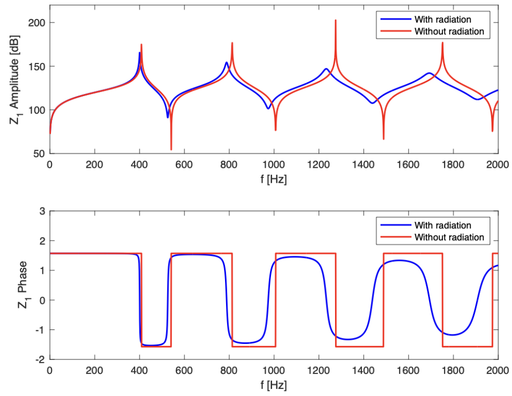

Figure 6: The input impedance of the approximated horn (blue curve: with radiation impedance, red curve: without radiation impedance).

Design of the compound horn

Consider now a compound horn, composed by a cylindrical pipe, whose length is $L_com = 0.85$ m, followed by the exponential horn obtained before, including the radiation load (see figure 7.

Figure 7: Diagram of the compound horn.

Point d

The input impedance of the approximated horn with radiation impedance obtained in point c can be directly used as the load impedance of the cylindrical part of the compound horn. The input impedance of a cylindrical pipe is

$$Z_{in} = \frac{\rho c}{S}\left[\frac{Z_L\cos kL + j\rho c/S\sin kL}{jZ_L\sin kL + \rho c/S\cos kL}\right], \tag{8}$$

where S is the area of the cross section. The input impedance of the compound horn is shown in figure 8 and the first ten maxima of the impedance are shown in table 1.

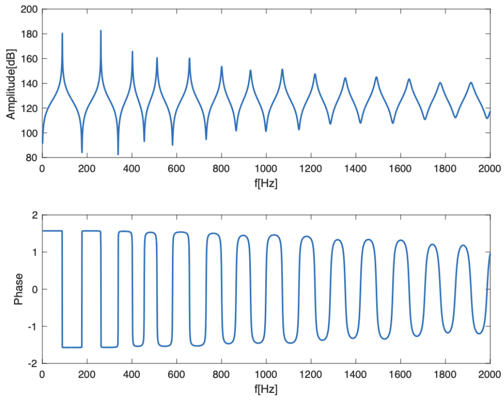

Figure 8: The input impedance of the approximated horn (blue curve: with radiation impedance, red curve: without radiation impedance).

No.	1	2	3	4	5
Freq.	89.044	261.13	402.201	512.256	657.328
No.	6	7	8	9	10
Freq.	800.4	929.464	1071.54	1217.61	1352.68

Table 1: The first ten maxima of the input impedance of the compound horn.

Point e

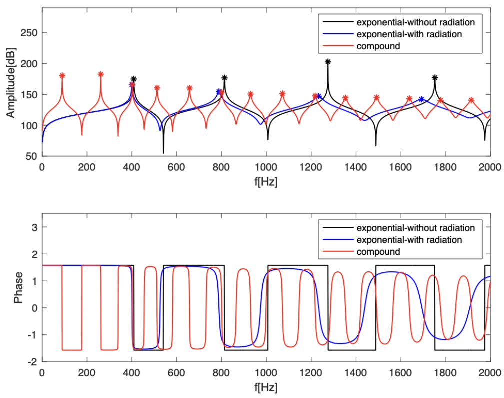

Figure 9: The input impedance of the approximated horn (blue curve: with radiation impedance, red curve: without radiation impedance).

Figure 9 shows the impedance curves for the different setups: exponential horn, exponential horn with impedance radiation and compound horn with impedance radiation. In order to evaluate the inharmonicity of the succession of impedance maxima, the frequency value for the peaks are selected. We increase the frequency range to 20kHz for a sufficient number of peaks to show the trend, then all the peak frequencies are divided by the fundamental frequency to show the harmonicity. Figure 10 shows the results. Thee two lines corresponding to the harmonicity (1:2:3:4...) and odd-harmonicity (1:3:5:7...) are used for reference. For the exponential horn, the impedance curve becomes a bit lower when the radiation impedance is taken into consideration, which means that it is closer to the harmonicity situation. Intuitively, this horn is more like a harmonic horn. When the cylindrical pipe is combined with the exponential horn, as a compound horn, it becomes more like odd-harmonicity situation.

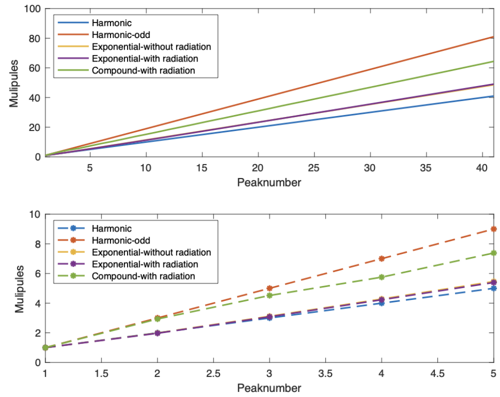

Figure 10: The harmonicity of the exponential horn with and without radiation impedance, and compound horn with radiation impedance. The x-axis is for the peak number from the impedance curve, and the y-axis is for the multiples of the fundamental frequency.

In order to get some more general ideas of how the harmonicity changes with the geometry of the horn, we change two of the parameters: the length of the cylindrical pipe and the length of the exponential horn. The results are shown in figure 11 and 12.

When the length of the cylinder goes from 0.001 m to 2.101 m, the horn behaves from more harmonic to more odd-harmonic. We know that for a cylindrical pipe, the frequency of the maxima of the impedance curve is odd-harmonic while for the exponential horn, especially for the shape used in this work, as we mentioned before it is more harmonic. Therefore, when the length of the cylinder is very short, the horn is basically an exponential horn, which is more like harmonic and vice versa. The principle is similar when changing the length of the exponential horn.

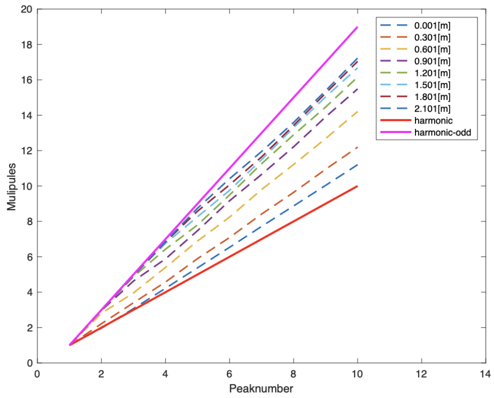

Figure 11: The harmonicity of the compound horn with different length of the cylindrical part (shown in legend). The x-axis is for the peak number from the impedance curve, and the y-axis is for the multiples of the fundamental frequency.

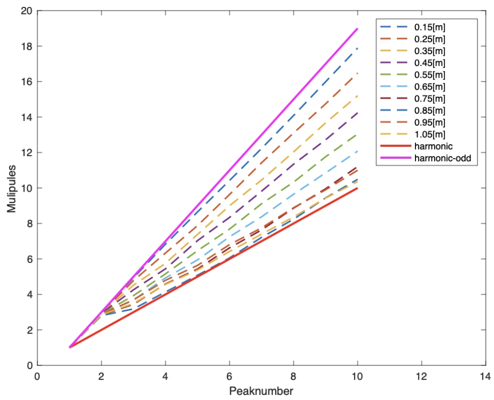

Figure 12: The harmonicity of the compound horn with different length of the exponential horn part (shown in legend). The x-axis is for the peak number from the impedance curve, and the y-axis is for the multiples of the fundamental frequency.