Musical Acoustics

Homework Laboratory 4: Brass instrument simulation

Homework Laboratory 4 Report

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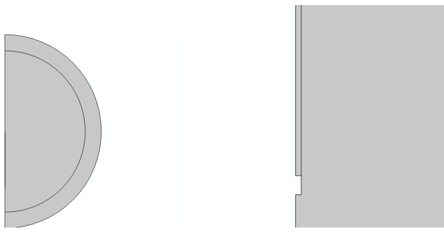
In this report, we implement the model of a trumpet in Comsol Multiphysics in order to simulate its acoustic response. The FEM simulation is used to compute the input impedance of the trumpet in the frequency domain. In order to reduce the complexity and computation time, the 2D axisymmetric model is used. Since we just care about the pressure field without the mechanical properties, only involves the physics of acoustic pressure. Each of the component of the trumpet is added step by step.

Exercise 1: Impendance of a tube

- Geometry:

First we create a tube by setting a rectangle with length 1.37 m and width 0.6 cm. A half circle with radius 2 m is added to model the open air sphere centered at the end of the tube. In order to create a space for the input pressure at the upstream, we create a rectangular with length 20 mm and width 0.6 cm, which keeps the same as the radius of the tube. Since the performance of the PML is related to the meshing wavelength condition, we could choose a random layer thickness. In that case, we set the PML by adding a circle with radius $1.2\times$ 2 m. Then we take the difference between the circles and input space to make it blank. The profiles of the geometry are shown in Figure 1.



(a) (b)

Figure 1: The geometry of the tube.

- PML:

We set PML in *Definition* with spherical type and center it at the end of the tube. The PML scaling factor is 0.5 and PML scaling curvature parameter is 5.

- Materials:

All domains are defined as air.

- Pressure Acoustics, Frequency Domain:

All domains are applied for *Pressure Acoustics* and *Initial Values* as 0 Pa. The *Axial Symmetry* is set to ensure the axisymmetric model. Only the outer of the PML is set as *Sound Hard Boundary*. The input pressure at the upstream is set by *Port* with circular type, mode 0 and amplitude 1.1 Pa (see Figure 2). At last, select the outer of the tube as *Interior Sound Hard Boundary*.

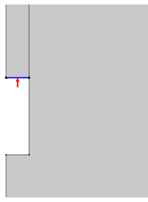


Figure 2: The pressure set at the input.

- Mesh:

The maximum element size of the mesh is set to satisfy the 5 points per wavelength condition. *Free Triangular* is applied to all the domain without the PML part. Then *Mapped* is set to PML part, also with the size to satisfy the 5 points per wavelength condition. Then *Distribution* is used contains 5 elements. The mesh is shown in Figure 3.

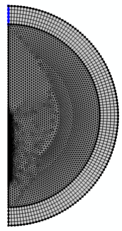


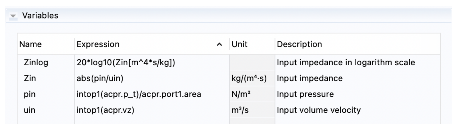
Figure 3: The mesh of the tube component.

- Variables:

The input acoustic impedance is given by

$$Z(\omega) = 20log[P(\omega)/U(\omega)], \tag{1}$$

where $P(\omega)$ is the pressure and $U(\omega)$ is the volume velocity of the input end. The impedance is in logarithmic scale. First an *Integration* is set as *intop1* to get the operator of the input surface. Then we defined a set of variables shown in Figure 5.



Variables

Name	Expression	Unit	Description
Zinlog	20*log10(Zin[m^4*s/kg])		Input impedance in logarithm scale
Zin	abs(pin/uin)	kg/(m⁴·s)	Input impedance
pin	intop1(acpr.p_t)/acpr.port1.area	N/m²	Input pressure
uin	intop1(acpr.vz)	m³/s	Input volume velocity

Figure 4: The defination of the variables.

- Study:

Frequency Domain study is used to evaluate the acoustics field. The frequency range is from 50Hz to 1200 Hz, with 255 samples.

- Results:

In order to plot the frequency dependant input impedance, *1D Plot Group* is set, containing *Global* plot with expression Zinlog.

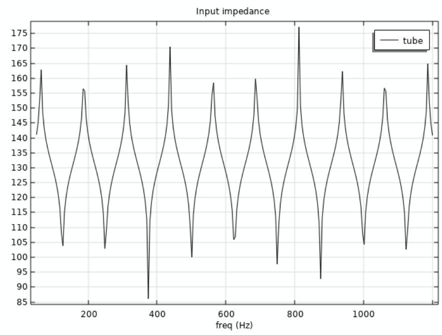


Figure 5: The simulation results for input impedance of the tube.

- Comments:

To validate the simulation, the input impedance of the tube can be calculated by

$$Z = Z_0 \left[\frac{Z_L \cos(kL) + jZ_0 \sin(kL)}{jZ_L \sin(kL) + Z_0 \cos(kL)} \right], \tag{2}$$

where $Z_0 = \frac{\rho c}{S}$, with ρ as density of the air, c as the sound speed, and L is the length of the tube. Z_L is the radiation impedance at the end of the tube and k is the wave number. Since we haven't considered the thickness of the tube model in comsol, here the unflanged radiation impedance is chosen as [1]

$$Z_l = 0.25 \frac{\omega^2 \rho}{\pi c} + 0.61j \frac{\rho \omega}{\pi a}, \tag{3}$$

where a is the radius of the tube. The results of the impedance through these two methods (keeps the same sampling frequency) are shown in figure 6. Intuitively, we could say the model of tube set up in comsol is reliable from the comparison results.

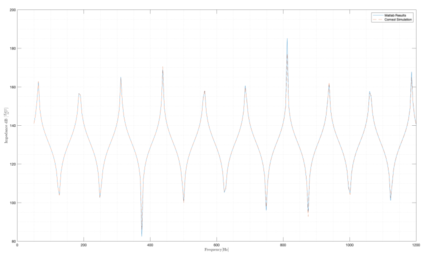


Figure 6: The comparison of the input impedance curve of the tube.

The trumpet works at the maxima of the input impedance curve. From this stage, we take a look at the maxima performance when there is only a tube. For an open pipe, the frequency of the impedance maxima follows an odd fashion, like 1:3:5:7... Therefore, to check this, the frequencies of the maxima value from the modeling are selected to compare with the sequence of odd multiples of the resonant frequency in figure 7. The frequency is very close to odd multiples of growth, which indicates that the tube is in harmony. In fact, in the trumpet, what we want is to get all harmonics instead of only odd harmonics. Adding the bell and mouthpiece should help to achieve this, which will be elaborated in the following.

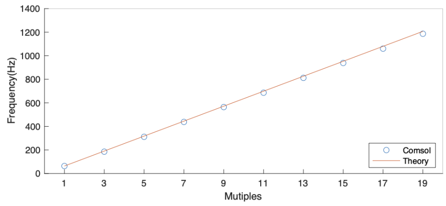


Figure 7: The comparison of the impedance maxima between comsol model and odd sequence.

The general idea of PML, materials, pressure acoustics, mesh and variables will be the same in the following modeling.

Exercise 2: Tube with bell

- Geometry:

An exponential horn of length 0.2 m is attached to the end of the tube that has been set up before. The tube now becomes shorter to keep the total length as before. The profile of the bell part is shown in figure 8.

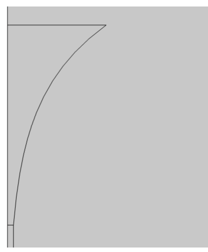


Figure 8: The geometry of the bell.

- Results:

The simulation results for the tube with bell is shown in figure 9.

- Comments:

The bell has a function to be the impedance adapter to the air, which makes the brass instruments very efficient acoustically. The frequency of the maxima of the input impedance curve for tube with bell are shown in table 1. The peaks 2, 5, and 8 are quite harmonically related with 1:3:5. Also, the first resonance frequency corresponding to the first peak is not used in trumpet. The ratio of the frequency for each peak (without the

[1] The equation 3 is given by musical acoustics homework 3.

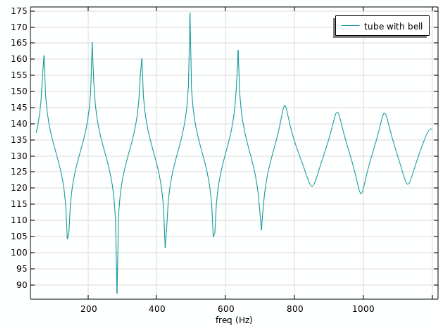


Figure 9: The simulation results for input impedance of the tube with bell.

No.	1	2	3	4
Freq.	72.55	212.35	356.67	496.47
5	6	7	8	9
636.27	771.57	924.90	1060.20	1195.49

Table 1: The frequency of the maxima of the input impedance curve for tube with bell.

first peak) is 2.0 : 3.6 : 4.7 : 6.0 : 7.3 : 8.7 : 10.0 : 11.3.
Compare the results of the model with and without the bell in figure 10. It is easy to find that the resonance peaks shift towards higher frequency, which modifies the harmonic relation. The bell raises the lower resonances toward a harmonic sequence. The pedal tone in trumpet is created by the effect of the bell, which is an octave below the note corresponding to the second peak. Another obvious effect is that the amplitude of the impedance becomes smaller dramatically from 700 Hz onwards.

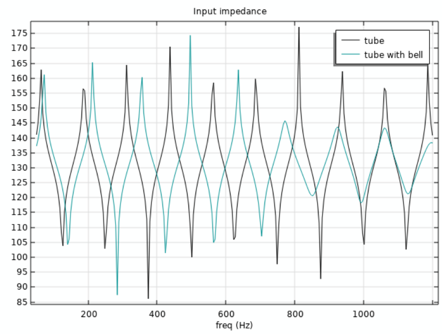


Figure 10: The simulation results for input impedance of the tube and tube with bell.

Exercise 3: Mouthpiece

- Geometry:

Now model the mouthpiece of the trumpet. A half circle of radius 0.008m and a polygon with short side equal to one eighth of the tube radius are attached together to shape the mouthpiece. Since the length of the mouthpiece is about 0.08 times the length of the tube, a smaller free field of radius 0.08 $\times$ 2m and PML with radius 0.08 $\times$ 1.2 $\times$ 2m are used. The geometry is shown in figure 11.

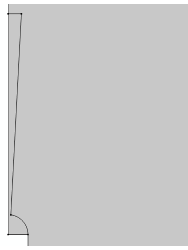


Figure 11: The geometry of the mouthpiece.

- Results:

The simulation results for the mouthpiece is shown in figure 12.

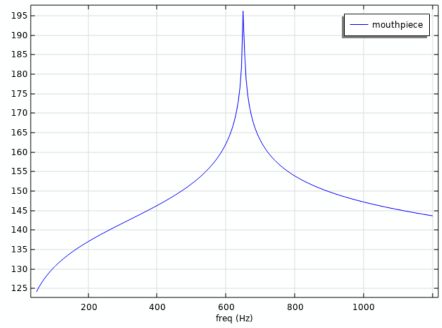


Figure 12: The simulation results for input impedance of the mouthpiece.

- Comments:

The mouthpiece includes a cup cavity as an acoustics compliance and a constricted backbore as a series inertance, acts as a cavity resonator, which leads to a strong resonant at 649.80Hz. From this, we could imagine that the existence of the mouthpiece could influence the input impedance, especially at the frequency band near to the resonance. Compare the impedance curve for the tube with bell and mouthpiece in figure 13. The resonance of the mouthpiece is close to the fifth peak of the tube with bell, which is in the harmonic relation. This could enhance the playability of the trumpet.

Exercise 4: Complete model

- Geometry:

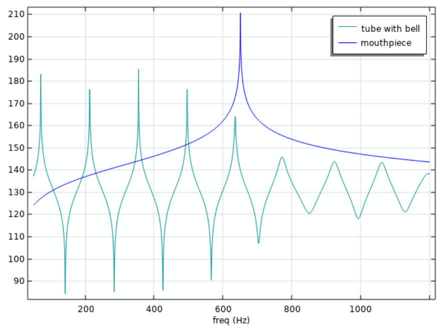


Figure 13: The simulation results for input impedance of the tube with bell and mouthpiece.

At last, we combine the tube with bell together with the mouthpiece. The length of the tube becomes shorter again to compensate the existence of the mouthpiece. The size of the free field and PML return to the original one. The geometry is shown in figure 14.

Figure 14: The geometry of the trumpet.

- Results:

The simulation results of the whole trumpet is shown in figure 15.

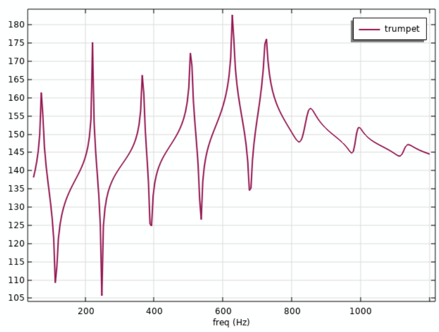


Figure 15: The simulation results for input impedance of the trumpet.

- Comments:

The input impedance for the four models are shown in figure 16. By comparing the impedance curve of the trumpet with the one without mouthpiece, it proves the assumption that the mouthpiece resonance emphasizes impedance peaks in the major playing range. It is clear that the frequency becomes a bit higher for the first four peaks while lower for the upward peaks. This time the ratio of the frequency for each peak (without the first peak) is 2.0 : 3.3 : 4.6 : 5.7 : 6.6 : 7.7 : 9.0 : 10.2, which is more harmonic than the tube with bell. (The expectation of the ratio for a good quality trumpet should be like 2:3:4:5...) In that case, the mouthpiece tunes the instrument to be more harmonic.
Now we take a look at the amplitude of the input impedance. However, the sampling frequency we use in the model is not enough if we want to get the accurate amplitude of the input impedance. Therefore we increase the sampling frequency as 1 Hz and redo the simulation. The results are shown in figure 17. Comparing the amplitude of the tube with bell and the trumpet, it can be concluded that the mouthpiece balances the resonance peak heights over the playing range, which helps the player to move smoothly between different tones.

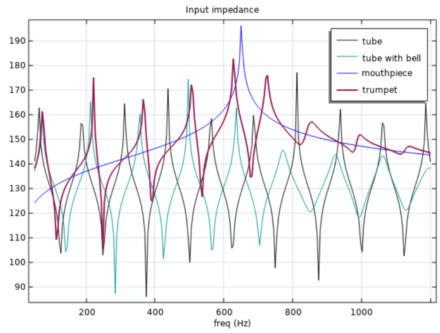


Figure 16: The simulation results for input impedance of the four models.

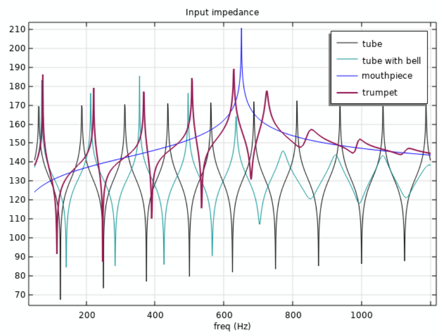


Figure 17: The simulation results for input impedance of the four models with higher sampling frequency.