Musical Acoustics

Homelab 2: Electric Analogs

Homelab 2 Report

Students

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Ex 1 – Model the response of a single Helmholtz resonator

Given the following parameters it is asked to model a single Helmholtz resonator through an electric equivalent: $V_0 = 0.1[m^3]$, $l = 0.1[m]$, $S = 100[m^2]$, $c = 343[m/s]$, $\rho = 1.2[Kg/m^3]$.

Point a

As in HW1 we apply the virtual neck elongation. For this case we choose to add a flanged elongation for the end of the pipe that goes inside the resonator and one unflanged for the end that goes outside. The length of the neck which is considered is then:

$$l_{virtual} = l + (0.85_{flanged} + 0.61_{unflanged})\sqrt{S/\pi} \quad (1)$$

which results in a total length of $8.337[m]$. From this it is possible to derive the electric equivalent of the resonator which is an RLC circuit as in Fig.1 with the following parameters:

$$R = \rho c/S = 4.116[\Omega] \quad (2)$$

$$L = \rho l/S = 0.100[H] \quad (3)$$

$$C = V/(\rho c^2) = 7.083 \times 10^{-07}[F] \quad (4)$$

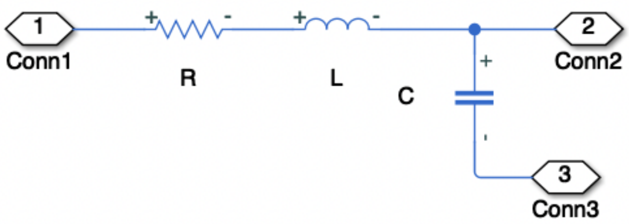


Figure 1: Electrical equivalent of the resonator as modeled inside Simulink

From this it is possible to start a simulation inside Simulink. By giving an impulsive voltage input and measuring the current flow in the capacitor we are able to compute the frequency response of the circuit. By dividing the fft of the response by the fft of the input impulse, which results to be a constant, we obtain said frequency response

$$H(\omega) = \frac{\mathcal{F}(I_c(t))}{\mathcal{F}(V_{in}(t))} \quad (5)$$

We take the absolute part and plot in a db scale plot. The results are shown in Fig.2

Point b

The accuracy of the model should now be checked against the analytical results. The natural frequency of the resonator is computed as follows:

$$f_0 = \frac{c}{2\pi}\sqrt{\frac{S}{Vl}} = 597.8675[Hz] \quad (6)$$

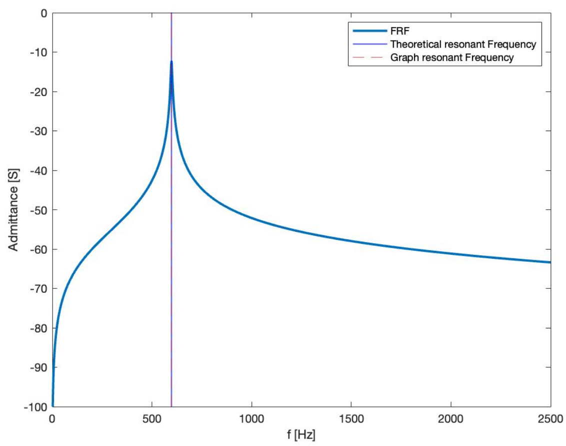


Figure 2: Frequency Response of the Electric-Equivalent of an Helmholtz Resonator

From the graph we get that the frequency of the peak is at $597.99[Hz]$. From these results we get a percentage error given by

$$error_{\%} = \left| \frac{f_0 - f_{0graph}}{f_0} \right| = 2.11 \times 10^{-6}\% \quad (7)$$

where $f_{0_{graph}}$ is the resonant frequency obtained from the graph. This error is probably due to the sampling error. As we can also see in Fig.2 the results are practically the same.

Ex 2 - Combine more resonators in a tree and analyze the response

Point a

Using the circuit in Fig.1 as the basic building block for the full equivalent circuit to the resonator tree, we can build a NxK Resonator tree where N is the height of the tree and K is the amount of resonators that are attached to the right of the previous one. We build a 2x2 tree as shown in Fig.3 using said building blocks. Taking as input the same impulsive voltage and as output the current on the capacitor of the top-right leaf, via Eq.5 we can compute its frequency response. The results are shown in Fig.4

Point b

Now with the same method we try to make a 2x3 and a 3x2 tree to see what changes when the parameters are modified from the 2x2 tree. The geometries are shown in Fig.5a and Fig.5b, while the resulting frequency responses are shown in Fig.6, all measured at the top-right leaf

Fig.6 shows something interesting. The length N of the tree directly affects the number of resonances in the frequency response so that they are equal to N+1. We

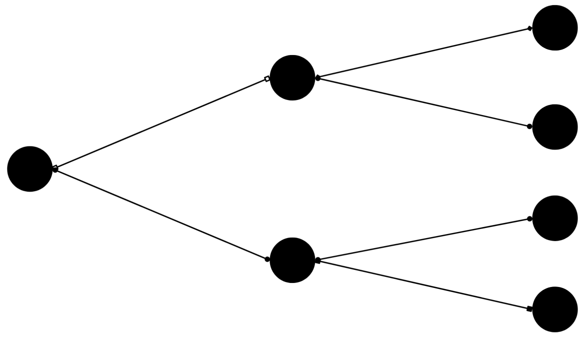


Figure 3: 2x2 Helmholtz Resonator Tree

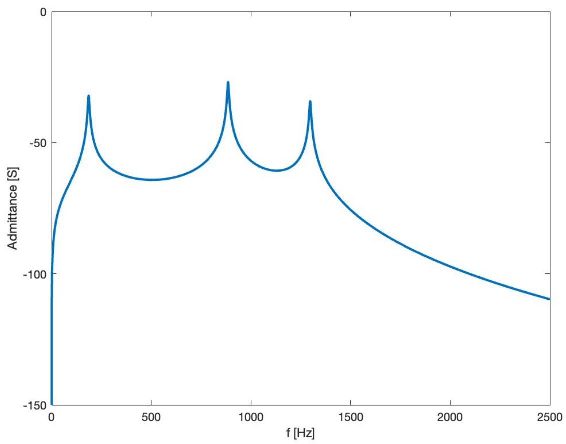
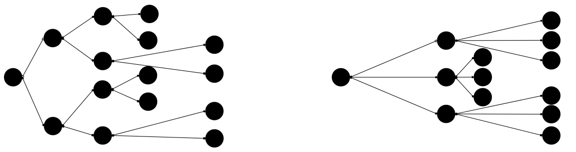


Figure 4: 2x2 Helmholtz Resonator Tree Impulse Response

can clearly see that the 2x3 tree shows 3 resonances while the 3x2 shows 4. The K parameter, on the other hand, shifts the resonant frequencies away from the resonant frequency of the single resonator. For higher values of K the lower resonances get lower and the higher ones get higher. This is further proved with the comparison in Fig.9 where it's clear that by having 5 parallel resonators the behaviour is the one stated above. These statements have meaning only until we consider complete trees. When incomplete trees are taken into account the situation changes dramatically, the following figures will have numbered leaves to try to avoid confusion. As we can see from Fig.8, which takes as output leaf 5 as shown in Fig.7. This shows 6 resonances and 1 anti-resonance with mutiplicity 2. Further analysis will be presented below.



(a) (b)

Figure 5: 3x2 and 2x3 Resonator Tree geometry

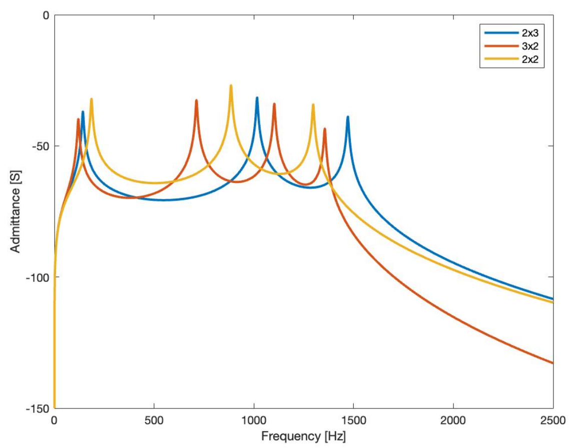


Figure 6: 2x3 and 3x2 Helmholtz Resonator Tree Impulse Responses against 2x2

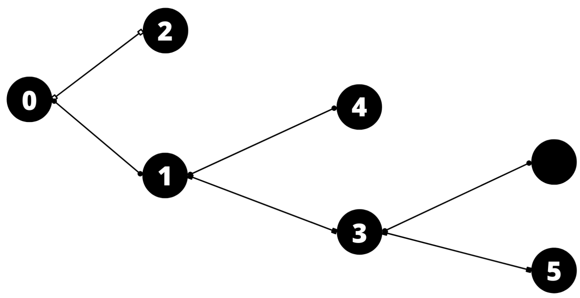


Figure 7: Incomplete 3x2 Helmholtz Resonator Tree Geometry

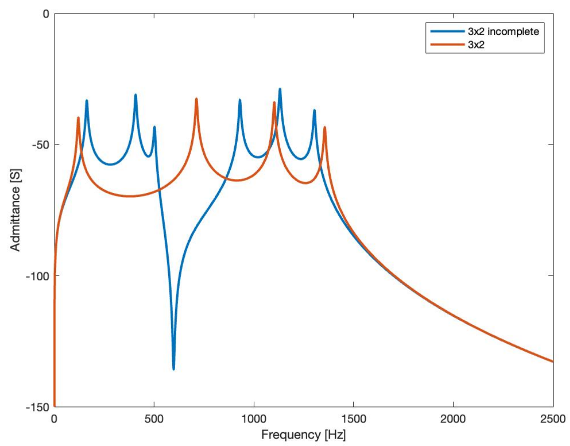


Figure 8: Incomplete 3x2 Helmholtz Resonator Tree Impulse Response

Point c

Now we investigate the difference that the choice of output leaf makes on the frequency response. Taking for example the 3x2 tree in Fig.5a we can now try to compute the frequency response on each level of the

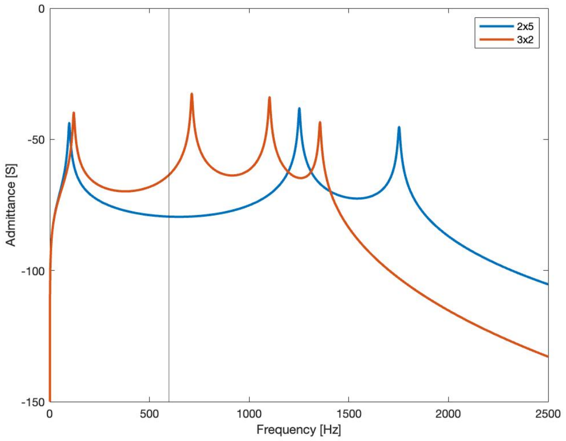


Figure 9: Comparison between a 2x5 and a 3x2 tree

tree since computing it on different leaves on the same level gives the same results. The plot in Fig.10 shows the results

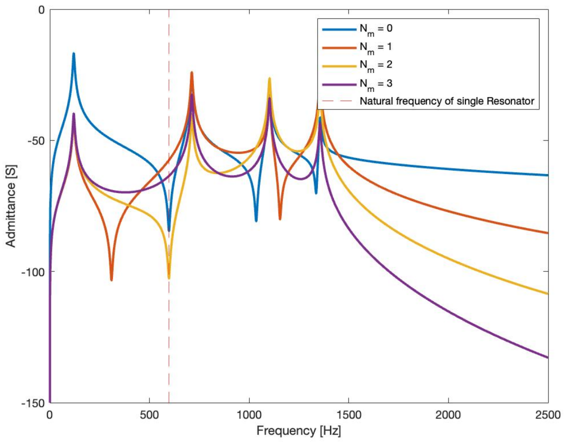


Figure 10: 3x2 Helmholtz Resonator Tree Impulse Response for each position

One thing is immediately apparent: the closer we do our measurement to the roots of the tree the more anti-resonances we find. In particular we find that the number of anti-resonances found is equal to $N - N_m$ where N_m is the height at which we take the measurement. Especially interesting is that, if measured at the root or at the second level, the frequency response shows an anti-resonance at the resonant frequency of the single Helmholtz resonator.
Now we take again into consideration cases of incomplete trees, in particular the ones shown in Fig.7 and Fig.11. Results for the first one are in Fig.12, for the second in Fig.13
Although it's hard to come to conclusions in these cases which are so far from the case of a complete tree we will try to give a qualitative approach to characterize the number of resonances and anti-resonances that happen in one of these cases:

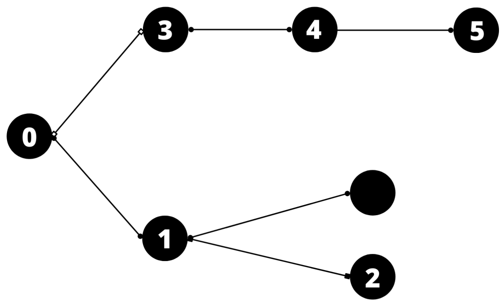


Figure 11: Generic Incomplete Helmholtz Resonator Tree geometry

Resonances

From the general complete tree case, and also from general circuit theory we know that resonances are a topographical property of a circuit which means that no matter where we do our measurement, we get resonances at the same frequencies. How many there are is, however a property of the shape of the tree. To find this number it is possible to follow a scheme: starting from the root we go down each individual branch leaf for leaf. For each individual leaf which is not part of an equivalent resonator made with leaves that only come after or are in parallel with the one taken into consideration(combination in series and in parallel of n different resonators) that we didn't already considered the number of resonances increases by one. Taking for example the geometry of Fig.11: starting from leaf 0 the only equivalent resonator which it is part of is the whole circuit. Going to leaf 1 it is part of the equivalent resonator given by the parallel between the two leaves after it and the sum with itself. Leaf 2 has a parallel with the blank one. The blank leaf is part of the same equivalent resonator as leaf 2 so we don't count it. Leaf 3 is part of the series of Leaf 3,4 and 5. Leaf 4 is part of the series of leaf 4 and 5. Leaf 5 is part only of a single resonator but since we never encountered this case it counts. This all amounts to 6 unique equivalent resonators that are present in the circuit, each one of them providing a resonance to the circuit. If, for example we added a leaf in parallel to leaf 5 (attached to leaf 4) we would still get 6 resonances because the new equivalent resonator made by the parallel of 5 and 6 and the sum of 4 has the same geometry of the one made by leaves 1,2 and blank but the first one is part of the bigger equivalent resonator made starting from leaf 3.

Anti-resonances

In contrast to resonances, anti-resonances are a local property of a circuit, which means that they can change according to where we are taking our measures along the considered circuit. This means that the number is both a function of the placement of the sensor and of the topography of the circuit. To find the number we can think again about equivalent resonators and the impedance that they can show. In particular we get an anti-resonance for each equivalent resonator that we can make, on the same branch as the one considered, that can show an infinite impedance and that wouldn't allow current to flow through it, or for each equivalent resonator in every other branch that can show zero impedance and drain the current away from the considered resonator. Taking now as example the geometry shown in Fig.7. Let's take leaf 3 as our measuring point. The only equivalent that can show infinite impedance is the parallel of 5 and blank. Both leaves 2 and 4 can show zero impedance and drain current away from 3. This accounts for 3 anti-resonances which, according to Fig.12 are all at the same frequency as we can clearly see from the 6π jump. The phase has been unwrapped to more clearly see these jumps. Taking now leaf 1 we have that the parallel of blank and 5, its sum with 3 and then also the parallel of all the above with 4 can show infinite impedance while only 2 can have zero impedance. This amounts to 4 anti-resonances which matches with the simulation. Important to note is that although the parallel of 5 and blank can show infinite impedance the current can still follow the path 0-1-4 but that combination still gives rise to an anti-resonance. Last interesting case is leaf 5 where we have a double anti-resonance given by 4 and 2 but not blank. This because since blank is in parallel to just 5 they are considered the same leaf and we are not considering the measurement leaf in the equivalent resonators so we don't count it. Hence why that leaf is not given a label. Both the method to find resonances and the one to find anti-resonances have gathered convincing results with different topographies but they are mostly theories that would need a more analytical derivation in order to be considered as valid across all possible topographies.

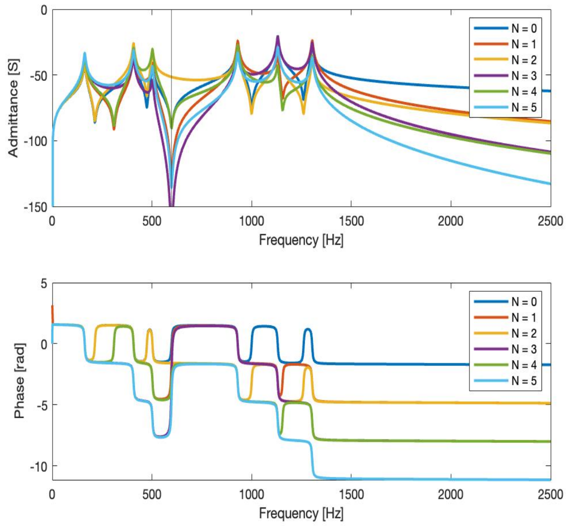


Figure 12: 3x2 incomplete Helmholtz Resonator Tree Impulse Response for each position

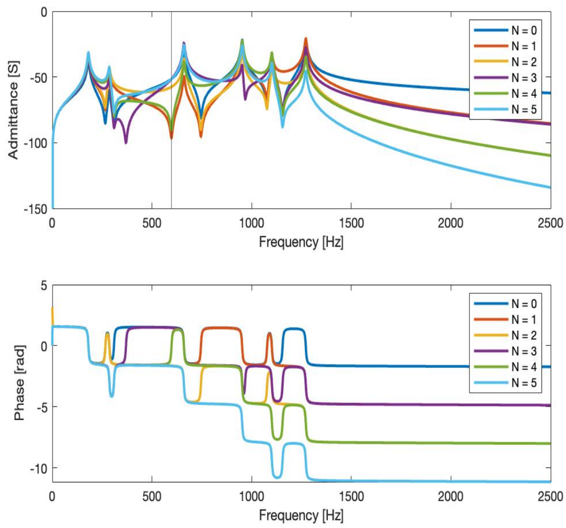


Figure 13: Generic Incomplete Helmholtz Resonator Tree Impulse Response for each position