Musical Acoustics

Homework 4: Design of a piano

Homework 4 Report

Students

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The aim of this homework is to design the plate of a piano in Comsol, to use the results to approximate the shape of a possible piano bridge and evaluate the results by simulating the decay behaviour of couples of strings attached to the bridge.

Soundboard

Characterization

The size of the soundboard is assumed to be of $1[m] \times 1.4[m]$, $1[cm]$ thick and clamped at the edges. The material is Sitka Spruce and the grain of the wood is assumed to be parallel to the shorter dimension of the plate. The simulation was done in Comsol. The model was created via a simple block Geometry and the clamped condition is achieved via a Fixed Constraint on the four lateral faces of the plate in the Solid Mechanics extension. The material parameters of the Sitka Spruce **[3]** that were used are reported as follows:

- Young's Modulus $[Pa]$: $E_l = 10.8e10$, $E_R = 8.42e8$, $E_l = 4.644$
- Poisson's Ratio: $\nu_{lr} = 0.372$, $\nu_{LT} = 0.467$, $\nu_{RT} = 0.435$
- Shear Modulus $[Pa]$: $G_{lr} = 6.58e8$, $G_{lt} = 3.24e7$, $G_{rt} = 3.24e7$

The longitudinal values are aligned along the x axis, the tangential along the y axis and the radial ones along the z axis according to the desired direction of the grain of the wood. By its own nature the model of the material is orthotropic. The last necessary parameter is the damping. This is modeled via a Rayleigh damping model under the Linear Elastic Material node of Solid Mechanics with constants $\alpha = 10[s^{-1}]$, $\beta = 2e-6[s]$**[2]**.

Simulation

The study for the plate has been designed by using a Frequency Domain Study with a Parametric Sweep. The Frequency Domain study allows us to iterate the simulation of the response of the plate to a harmonic force over a set of frequencies. In this case, given the requests of the following points, we chose the interval of $[300, 800][Hz]$ with a step of $2[Hz]$ and later made another simulation with the specific frequencies requested to gather more accurate results. The Parametric Sweep was used to iterate the study spatially for different components of the simulation. In particular the harmonic excitement of the plate was simulated using a Point Load in the desired position with a force of $1[N]$ perpendicular to the face of the plate. A point was then added to the geometry with two different purposes: the first is to give a reference to the Point Probe node that collects the velocity that results from the excitement, the second one is to force the mesh, set as a fine free triangular on the forced face of the plate and then swept

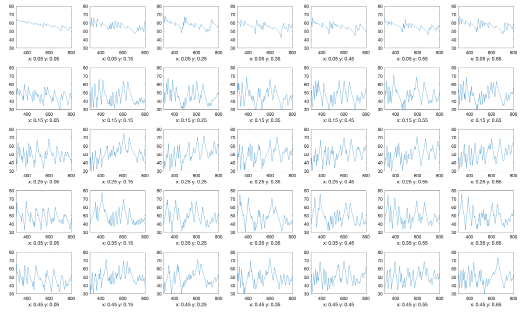

Figure 1: Frequency Response Matrix of the soundboard at the chosen points

f	Z
349.23	34.18 - 164.52j
440	19.36 - 149.98j
523.25	134.91 - 35.74j
659.25	53.50 - 275.11j
783.99	118.90 + 135.97j

Table 1: Chosen values for the impedance

onto the other, to take into consideration the driving point. Since the plate has symmetry along the two axis that cut the length and the width of the board in half it is possible to compute just the bottom left quarter of the board and then use the symmetry to obtain the driving impedance at points in all the four quarters of the plate. The resulting velocity data is then converted into an Impedance matrix for each point that was chosen. The resulting Impedance matrix is shown in Fig.1.

String Pairing

Bridge Shape Design

Five pairs of strings need to be taken into consideration now, with the following frequencies: $349.23[Hz]$, $440[Hz]$, $523.25[Hz]$, $659.25[Hz]$ and $783.99[Hz]$. As stated before a second simulation was done to get more accurate results at these exact frequencies with the same methodology described above. The method used to design the bridge shape was to plot the 10 points for each frequency where the measured absolute value of the impedance is the lowest, mirror them around the two axis and then choose the points that would give a shape which could be roughly similar to the one of a real piano. The strings are thought as going along the x axis along the grain of the wood. Placing the strings perpendicular to the grain would probably break the plate in a real scenario. The plot for the 10 best points is shown in Fig.2, the resulting bridge is shown in Fig.3 and the values of the impedance are shown in Tab.1

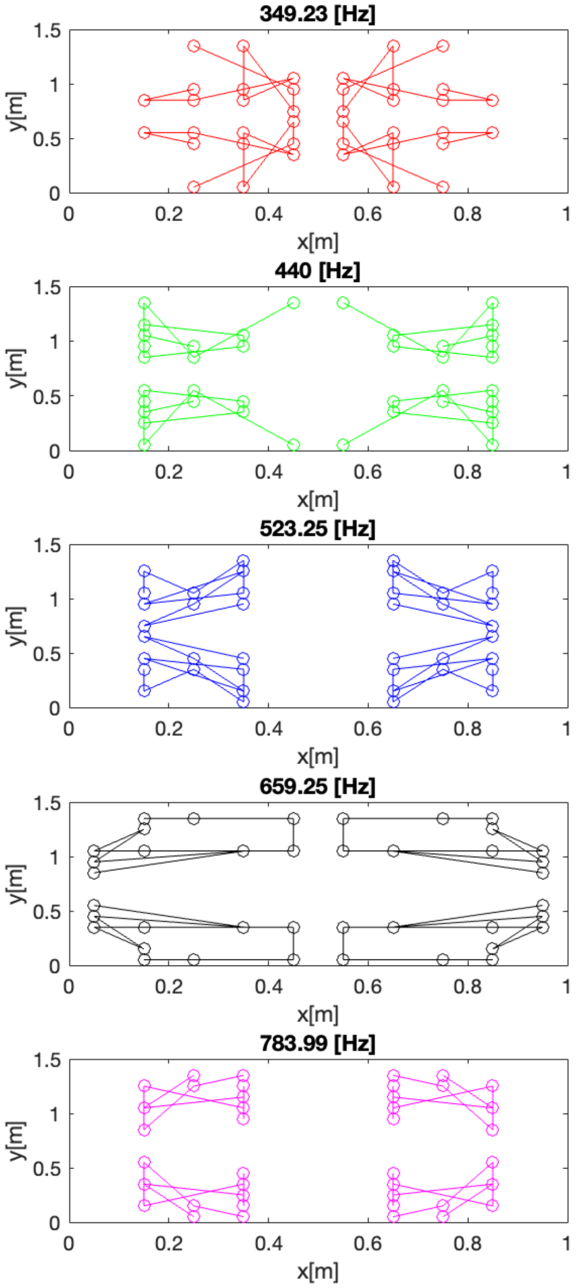

Figure 2: 10 best points in terms of absolute value of the impedance for each couple of strings

Eigenfrequencies

The next step is to compute how the detune between the two strings, modeled with a parameter ϵ, affects the eigenfrequencies of the string-bridge coupled system. First the linear density ρ for the strings is assumed to be $10.8[g/m]$. The next assumption is that all the strings start from a tension of around $800[N]$ which is

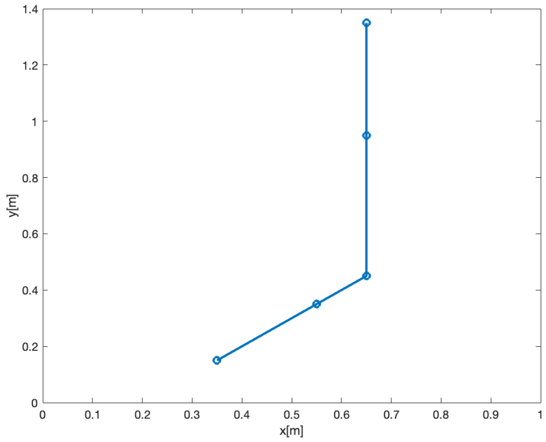

Figure 3: Designed Bridge Shape

in the middle of the usual tension range between 700 and $900[N]$. From this data we can start our analysis of the eigenmodes. The formula for the eigenfrquency shift[1] is shown in Eq.1

$$a = \Im(\chi) + \epsilon \pm \sqrt{\chi^2 + \epsilon^2} \tag{1}$$

where χ is obtained by Eq.2, which, when inverted gives us Eq.3

$$Y_B = \frac{\pi}{jZ_0\omega_0}\chi \tag{2}$$

$$\chi = \frac{jY_B Z_0 \omega_0}{\pi} \tag{3}$$

Y_B is the admittance of the bridge, χ has the dimension of $[1/s]$ and Z_0 is the characteristic impedance of the single string, computed as:

$$Z_0 = \sqrt{\rho T}$$

where ρ is the linear density of the string. In our computations ϵ is defined as a vector of 300 linearly spaced angular frequency elements that go from $-0.02\omega_0$ to $0.02\omega_0$. From this formula we can see how the eigenfrequency shift when we apply a detune of $\epsilon[Hz]$ to one of the strings. The results are shown in Fig.4.

Time Decay

The last analysis is the one for the time decay of the string couples. The bridge velocity can be considered [1] from Eq.4

$$V_B = V_0 e^{j\beta t} \tag{4}$$

where β is given by $a + \omega_0$. Since, generally, a is a complex value, we can write it as the sum of a real and an imaginary part $\gamma + j\delta$. If we subsbtitute β with the new expression of a in Eq.4 we can recognize a phase parameter and a damping parameter as shown in Eq.5

$$V_B = V_0 e^{j(\omega_0+\gamma)t} e^{-\delta t} \tag{5}$$

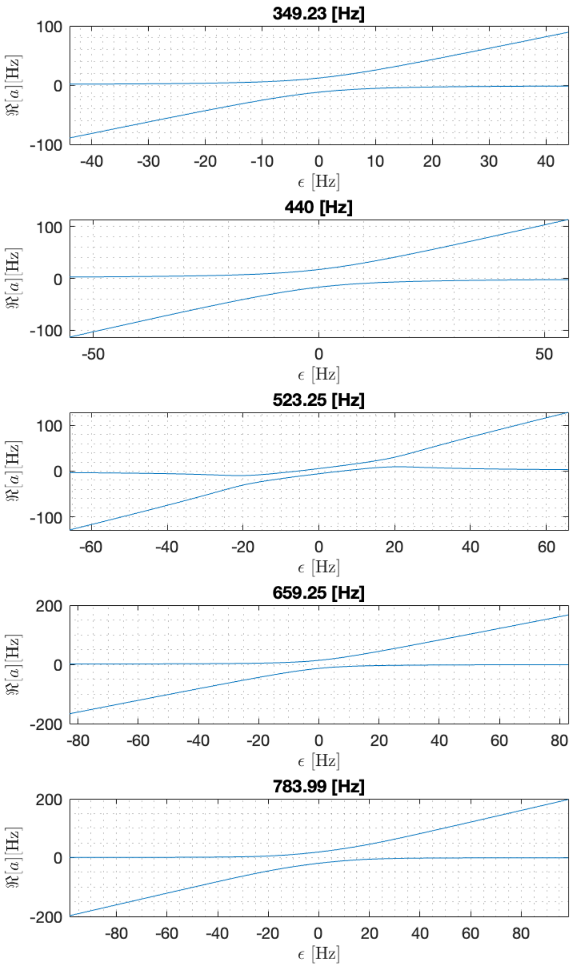

Figure 4: Eigenfrequency Shift as function of ϵ

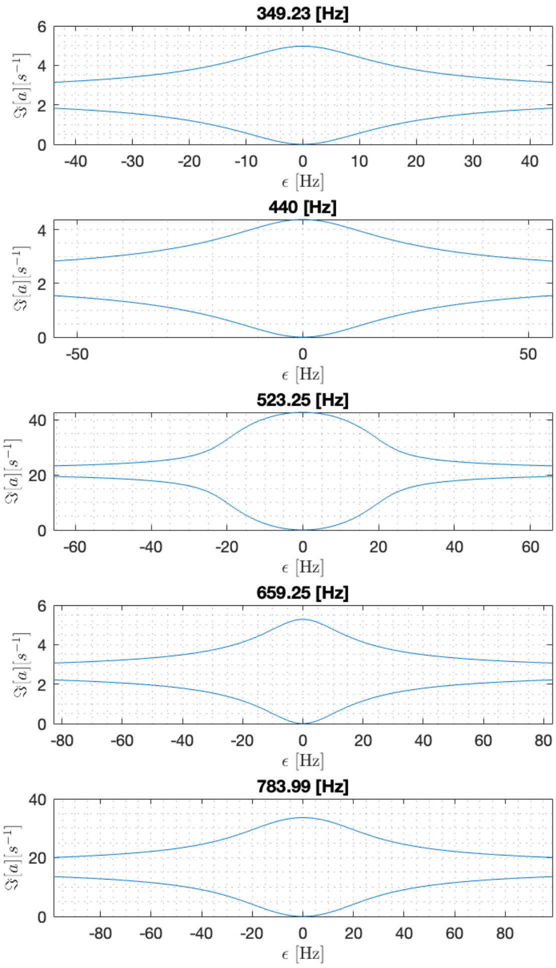

Figure 5: Damping rate as function of ϵ

From these considerations it is clear that the imaginary part of the eigenfrequency shift determines the decay time for the two coupled strings. The results are shown in Fig.5. We can clearly see that the two strings will have very different decay times. This gives as a result the typical double decay effect where one string will decay faster than the other.

By equating the decaying exponential with a general decay factor $e^{-t/\tau}$ we can obtain the time constants for the strings as a function of ϵ as in Eq.6. Results for the trend of τ are shown in Fig.6

$$\tau = \frac{1}{\delta} = \frac{1}{\Im(a)} \tag{6}$$

A good metric to evaluate the decay time is the T60, the time it takes a signal to decay by 60dB. To evaluate this we take into consideration the parameter $R(t)$[**1**], the theoretical relative level of energy transmission to the bridge of a two-string system as a function of time. This is evaluated as shown in Fig.7

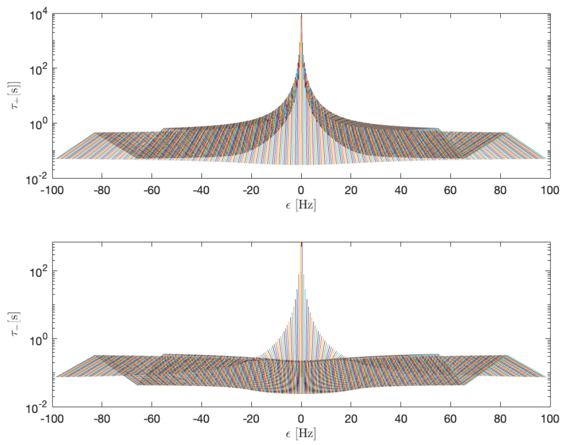

Figure 6: Trend of τ as function of ϵ, represented on a semi-logarithmic scale

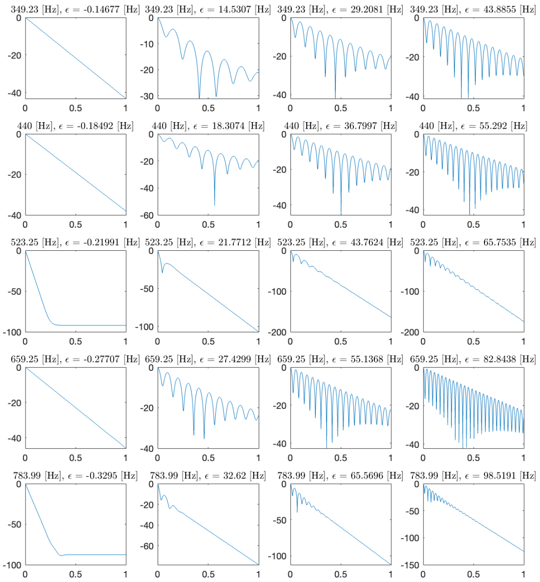

Figure 7: $R(t)$ for different ϵ

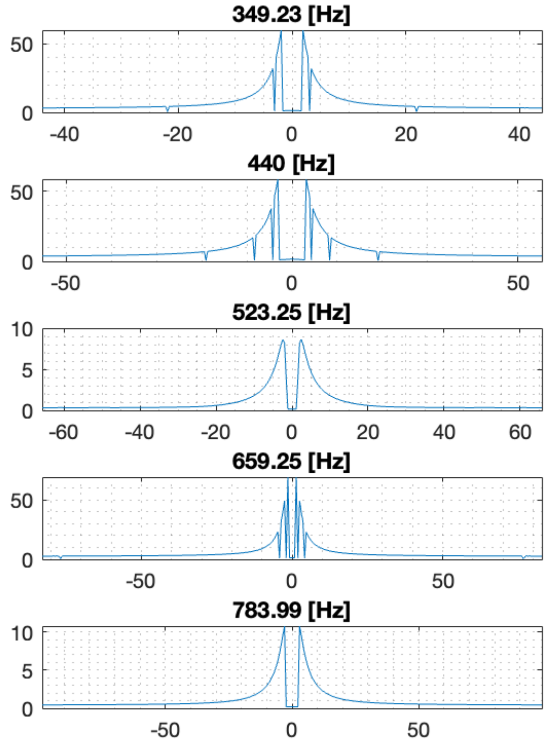

Figure 8: T60 for different ϵ

$$R(t) = \left| \frac{(\chi+\mu)e^{j(\chi+\mu)t} - (\chi-\mu)e^{j(\chi-\mu)t}}{2\mu} \right|^2 \qquad (7)$$

where

$$\mu = \sqrt{\epsilon^2 + \chi^2}$$

Examples for $R(t)$ at different values of ϵ are shown in Fig.7
Now by computing the dB value of $R(t)$ as

$$R(t)_{dB} = 10 \log \left(\frac{R(t)}{R(0)} \right)$$

we can look for when the value is -60dB. Results are shown in Fig.8
We can clearly see how some values of ϵ give longer decay times than others which was clear also from Fig.6. This estimation is a good representation only when the amplitude of the beating between the two coupled strings is small and only the double decay is visible. The results, however, are in agreement with the ones of τ: a small detune will give a longer decay time than a bigger one.

Comments

Overall the decay times seem reasonable with respect to the decay time of a real piano and the model should be reasonably close to a real piano soundboard. A better bridge design could be obtained by having more points to choose from but this will increase the computation time very rapidly. A better model for the damping could also be used as Rayleigh damping is, intrinsically, an approximation that enables more convenient computations since all the equations of motion become linearly independent when using this method. Fig.5 shows that also the way we choose points is a trade off between different characteristics that we might want to achieve. Although the simulation results were all ordered in terms of the absolute value of the impedance the time decay is very different: for some strings the damping rates have a smaller span while for others the rates have a span of $40[s^{-1}]$. This is given by the fact that the decay rate is a function of $\sqrt{2\Re[\chi]\Im[\chi]}$ which means that it is a function of both the real and imaginary part of the impedance and both can influence the results. A more resistive or a more reactive part of the bridge will give different results even if the absolute value of the impedance is similar.

Bibliography

[1] Weinreich, G. (1977) "Coupled piano strings," The Journal of the Acoustical Society of America, 62(6), p. 1474

[2] Malvermi, R., Albano, M., Gonzalez, S. et al. The impact of alkaline treatments on elasticity in spruce tonewood. Sci Rep 12, 13335 (2022

[3] Wood handbook: Wood as an engineering material (2011). Madison, WI: Forest Products Society, Table 5-1 and 5-2